# Table of Contents

## Acknowledgements

## Foreword

## Background

## Introduction

Purpose of the Document ................................................................. 2
Beliefs About Students and Mathematics Learning .......................... 2
Affective Domain ............................................................................ 3
Early Childhood ............................................................................ 3
Goal for Students .......................................................................... 4

## Conceptual Framework for K–9 Mathematics

Mathematical Processes ................................................................. 5
Nature of Mathematics .................................................................. 9
Strands ......................................................................................... 12
Outcomes and Achievement Indicators ......................................... 13
Summary ....................................................................................... 13

## Instructional Focus

Planning for Instruction ............................................................... 14
Resources .................................................................................... 14
Teaching Sequence ...................................................................... 15
Instruction Time per Unit ........................................................... 15

## General and Specific Outcomes

## General and Specific Outcomes by Strand Grades 5 – 7

Numeration .................................................................................. 31
Number Relationships .................................................................. 55
Patterns in Mathematics .............................................................. 85
Data Relationships ....................................................................... 115
Motion Geometry ......................................................................... 147
Ratio & Percent ........................................................................... 171
Fractions ..................................................................................... 199
Multiplication and Division of Decimals ...................................... 225
Measurement ............................................................................... 251
2D Geometry ............................................................................... 295
Probability .................................................................................. 313

## Appendix A: Outcomes with Achievement Indicators (Strand)

## References
Acknowledgements

The Department of Education would like to thank the Western and Northern Canadian Protocol (WNCP) for Collaboration in Education, *The Common Curriculum Framework for K-9 Mathematics* - May 2006 and *The Common Curriculum Framework for Grades 10-12* - January 2008, which has been reproduced and/or adapted by permission. All rights reserved.

We would also like to thank the provincial Grade 6 Mathematics curriculum committee, the Alberta Department of Education, the New Brunswick Department of Education, and the following people for their contribution:

- **Trudy Porter**, Program Development Specialist – Mathematics, Division of Program Development, Department of Education
- **Paulette Jayne**, Teacher – Sprucewood Academy, Grand Falls-Windsor
- **Millie Walsh**, Teacher – Baie Verte Academy, Baie Verte
- **Elaina Johnson**, Teacher – Bishop White School, Port Rexton
- **Megan Wamboldt**, Teacher – Queen of Peace Middle School, Labrador City
- **Larry Doyle**, Teacher – Numeracy Support Teacher, Nova Central School District
- **John Power**, Teacher – Numeracy Support Teacher, Eastern School District
- **Damien Lethbridge**, Teacher – St. John Bosco, St. John’s
- **Daryl Rideout**, Teacher – Mary Queen of Peace, St. John’s

Every effort has been made to acknowledge all sources that contributed to the development of this document. Any omissions or errors will be amended in future printings.
Foreword

The *WNCP Common Curriculum Frameworks for Mathematics K – 9 (WNCP, 2006)*, formed the basis for the development of this curriculum guide. While minor adjustments have been made, the outcomes and achievement indicators established through the WNCP Common Curriculum Framework are used and elaborated on for teachers in this document. Newfoundland and Labrador has used the WNCP curriculum framework to direct the development of this curriculum guide.

This curriculum guide is intended to provide teachers with the overview of the outcomes framework for mathematics education. It also includes suggestions to assist teachers in designing learning experiences and assessment tasks.
The province of Newfoundland and Labrador commissioned an independent review of mathematics curriculum in the summer of 2007. This review resulted in a number of significant recommendations. In March of 2008, it was announced that this province accepted all recommendations. The first four and perhaps most significant of the recommendations were as follows:

- That the WNCP Common Curriculum Frameworks for Mathematics K – 9 and Mathematics 10 – 12 (WNCP, 2006 and 2008) be adopted as the basis for the K – 12 mathematics curriculum in this province.

- That implementation commence with Grades K, 1, 4, 7 in September 2008, followed by in Grades 2, 5, 8 in 2009 and Grades 3, 6, 9 in 2010.

- That textbooks and other resources specifically designed to match the WNCP frameworks be adopted as an integral part of the proposed program change.

- That implementation be accompanied by an introductory professional development program designed to introduce the curriculum to all mathematics teachers at the appropriate grade levels prior to the first year of implementation.

As recommended, the implementation schedule for K - 6 mathematics is as follows:

<table>
<thead>
<tr>
<th>Implementation Year</th>
<th>Grade Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>K, 1 and 4</td>
</tr>
<tr>
<td>2009</td>
<td>2, 5</td>
</tr>
<tr>
<td>2010</td>
<td>3, 6</td>
</tr>
</tbody>
</table>
INTRODUCTION

Purpose of the Document

The Mathematics Curriculum Guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for K–9 Mathematics: Western and Northern Canadian Protocol, May 2006* (the Common Curriculum Framework). These guides incorporate the conceptual framework for Kindergarten to Grade 9 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Beliefs About Students and Mathematics Learning

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students’ experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must realize that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.
A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Early Childhood

Young children are naturally curious and develop a variety of mathematical ideas before they enter Kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, in preschools and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, beading, baking, storytelling and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in, and talking about, such activities as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs and building with blocks.

Positive early experiences in mathematics are as critical to child development as are early literacy experiences.
Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

<table>
<thead>
<tr>
<th>STRAND</th>
<th>GRADE</th>
<th>K1 2 3 4 5 6 7 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns and Relations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variables and Equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape and Space</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D Objects and 2-D Shapes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chance and Uncertainty</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematical Processes - Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

Nature of Mathematics

Change, Constancy, Number Sense, Patterns, Relationships, Spatial Sense, Uncertainty
Mathematical Processes

- **Communication [C]**
- **Connections [CN]**
- **Mental Mathematics and Estimation [ME]**
- **Problem Solving [PS]**
- **Reasoning [R]**
- **Technology [T]**
- **Visualization [V]**

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

**Communication [C]**

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.
Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding…. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001, p. 442).

Mental mathematics “provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers” (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.
Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type How would you? or How could you?, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.
Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

• explore and demonstrate mathematical relationships and patterns
• organize and display data
• extrapolate and interpolate
• assist with calculation procedures as part of solving problems
• decrease the time spent on computations when other mathematical learning is the focus
• reinforce the learning of basic facts
• develop personal procedures for mathematical operations
• create geometric patterns
• simulate situations
• develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).
Nature of Mathematics

- Change
- Constancy
- Number Sense
- Patterns
- Relationships
- Spatial Sense
- Uncertainty

Change

Change is an integral part of mathematics and the learning of mathematics.

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curriculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry. Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.
Number Sense

An intuition about number is the most important foundation of a numerate child.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections to their own experiences and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students’ interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and nonroutine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.
Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.
The learning outcomes in the mathematics program are organized into four strands across the grades K–9. Some strands are subdivided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

### Strands

- **Number**
- **Patterns and Relations**
- **Shape and Space**
- **Statistics and Probability**

### Number

**Number**

- Develop number sense.

### Patterns and Relations

**Patterns**

- Use patterns to describe the world and to solve problems.

**Variables and Equations**

- Represent algebraic expressions in multiple ways.

### Shape and Space

**Measurement**

- Use direct and indirect measurement to solve problems.

**3-D Objects and 2-D Shapes**

- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

**Transformations**

- Describe and analyze position and motion of objects and shapes.

### Statistics and Probability

**Data Analysis**

- Collect, display and analyze data to solve problems.

**Chance and Uncertainty**

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
Outcomes and Achievement Indicators

The mathematics program is stated in terms of general outcomes, specific outcomes and achievement indicators.

**General Outcomes**

*General outcomes* are overarching statements about what students are expected to learn in each strand/substrand. The general outcome for each strand/substrand is the same throughout the grades.

**Specific Outcomes**

*Specific outcomes* are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given grade.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

**Achievement Indicators**

*Achievement indicators* are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. Achievement indicators are context-free.

**Summary**

The conceptual framework for K–9 mathematics describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.
INSTRUCTIONAL FOCUS

Planning for Instruction
Consider the following when planning for instruction:

• Integration of the mathematical processes within each strand is expected.
• By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
• Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
• There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
• Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Resources
The resource selected by Newfoundland and Labrador for students and teachers is Math Focus 6 (Nelson). Schools and teachers have this as their primary resource offered by the Department of Education. Column four of the curriculum guide references Math Focus 6 for this reason.

Teachers may use any resource or combination of resources to meet the required specific outcomes listed in column one of the curriculum guide.
Teaching Sequence

The curriculum guide for Grade 6 is organized by units from Unit 1 to Unit 11. The purpose of this timeline is to assist in planning. The use of this timeline is not mandatory; however, it is mandatory that all outcomes are taught during the school year so a long term plan is advised. There are a number of combinations of sequences that would be appropriate for teaching this course. The arrow showing ‘estimated focus’ does not mean the outcomes are never addressed again. The teaching of the outcomes is ongoing and may be revisited as necessary.

Instruction Time Per Unit

The suggested number of weeks of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested weeks includes time for completing assessment activities, reviewing and evaluating.
GENERAL AND SPECIFIC OUTCOMES BY STRAND
(pages 17–28)
This section presents the general and specific outcomes for each strand, for Grade 5, 6 and 7.

Refer to Appendix A for the general and specific outcomes with corresponding achievement indicators organized by strand for Grade 3.

GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (beginning at page 31)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding to be used to determine whether or not students have achieved a given specific outcome. Teachers should use these indicators but other indicators may be added as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.
GENERAL AND SPECIFIC OUTCOMES BY STRAND

(Grades 5, 6 and 7)
## General and Specific Outcomes by Strand

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
</tr>
<tr>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
<tr>
<td>1. Represent and describe whole numbers to 1 000 000. [C, CN, V, T]</td>
<td>1. Demonstrate an understanding of place value, including numbers that are: • greater than one million • less than one thousandth. [C, CN, R, T]</td>
<td>1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. [C, R]</td>
</tr>
<tr>
<td>2. Use estimation strategies, including: • front-end rounding • compensation • compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]</td>
<td>2. Solve problems involving large whole numbers and decimal numbers. [ME, PS, T]</td>
<td>2. Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected). [ME, PS, T]</td>
</tr>
<tr>
<td>3. Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine, with fluency, answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]</td>
<td>3. Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors. [CN, PS, R, V]</td>
<td>3. Solve problems involving percents from 1% to 100%. [C, CN, PS, R, T]</td>
</tr>
<tr>
<td></td>
<td>4. Relate improper fractions to mixed numbers. [CN, ME, R, V]</td>
<td>4. Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions. [C, CN, R, T]</td>
</tr>
</tbody>
</table>
## Number

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Outcome</td>
<td>General Outcome</td>
<td>General Outcome</td>
</tr>
<tr>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
</tr>
<tr>
<td>Specific Outcomes</td>
<td>Specific Outcomes</td>
<td>Specific Outcomes</td>
</tr>
<tr>
<td>4. Apply mental mathematics strategies for multiplication, such as:</td>
<td>5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically.</td>
<td>6. Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.</td>
</tr>
<tr>
<td>• annexing then adding zero</td>
<td>[C, CN, PS, R, V]</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td>• halving and doubling</td>
<td>6. Demonstrate an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems.</td>
<td>7. Demonstrate an understanding of integers, concretely, pictorially and symbolically.</td>
</tr>
<tr>
<td>• using the distributive property.</td>
<td>[C, CN, ME, R, V]</td>
<td>[C, CN, R, V]</td>
</tr>
<tr>
<td>5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems.</td>
<td>5. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:</td>
<td>7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:</td>
</tr>
<tr>
<td>[C, CN, PS, V]</td>
<td>• create sets of equivalent fractions</td>
<td>• create sets of equivalent fractions</td>
</tr>
<tr>
<td>6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems.</td>
<td>• compare fractions with like and unlike denominators.</td>
<td>• compare fractions with like and unlike denominators.</td>
</tr>
<tr>
<td>[C, CN, ME, PS, R, V]</td>
<td>[C, CN, PS, R, V]</td>
<td>[C, CN, PS, R, V]</td>
</tr>
</tbody>
</table>
## Number

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Outcome</td>
<td>General Outcome</td>
<td>General Outcome</td>
</tr>
<tr>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
<td>Develop number sense.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
<tr>
<td>8. Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially and symbolically.</td>
<td>8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1 digit natural number divisors).</td>
<td>5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).</td>
</tr>
<tr>
<td>[C, CN, R, V]</td>
<td>[C, CN, ME, PS, R, V]</td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td>9. Relate decimals to fractions and fractions to decimals (to thousandths).</td>
<td>9. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).</td>
<td>6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.</td>
</tr>
<tr>
<td>[CN, R, V]</td>
<td>[C, CN, ME, PS, T]</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td>10. Compare and order decimals (to thousandths) by using:</td>
<td>11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).</td>
<td>7. Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:</td>
</tr>
<tr>
<td>• benchmarks</td>
<td>[C, CN, PS, R, V]</td>
<td>• benchmarks</td>
</tr>
<tr>
<td>• place value</td>
<td></td>
<td>• place value</td>
</tr>
<tr>
<td>• equivalent decimals.</td>
<td></td>
<td>• equivalent decimals.</td>
</tr>
<tr>
<td>[C, CN, R, V]</td>
<td></td>
<td>[CN, R, V]</td>
</tr>
<tr>
<td>11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Patterns and Relations
(Patterns)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
</tr>
<tr>
<td>Use patterns to describe the world and to solve problems.</td>
<td>Use patterns to describe the world and to solve problems.</td>
<td>Use patterns to describe the world and to solve problems.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
<tr>
<td>1. Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]</td>
<td>1. Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]</td>
<td>1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]</td>
</tr>
<tr>
<td>2. Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V]</td>
<td>2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
## Patterns and Relations
(Variables and Equations)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Outcome</td>
<td>Represent algebraic expressions in multiple ways.</td>
<td>General Outcome</td>
</tr>
<tr>
<td>Specific Outcomes</td>
<td>2. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]</td>
<td>Specific Outcomes</td>
</tr>
<tr>
<td></td>
<td>4. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially. [C, CN, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Shape and Space
*(Measurement)*

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
</tr>
<tr>
<td>Use direct or indirect measurement to solve problems.</td>
<td>Use direct or indirect measurement to solve problems.</td>
<td>Use direct or indirect measurement to solve problems.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
</tbody>
</table>
| 1. Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions. [C, CN, PS, R, V] | 1. Demonstrate an understanding of angles by:  
- identifying examples of angles in the environment  
- classifying angles according to their measure  
- estimating the measure of angles, using 45°, 90° and 180° as reference angles  
- determining angle measures in degrees  
- drawing and labelling angles when the measure is specified. [C, CN, ME, V] | 1. Demonstrate an understanding of circles by:  
- describing the relationships among radius, diameter and circumference  
- relating circumference to π  
- determining the sum of the central angles  
- constructing circles with a given radius or diameter  
- solving problems involving the radii, diameters and circumference of circles. [C, CN, PS, R, V] |
| 2. Demonstrate an understanding of measuring length (mm and km) by:  
- selecting and justifying referents for the unit mm  
- modelling and describing the relationship between mm and cm units, and between mm and m units.  
- selecting and justifying referents for the unit km.  
- modelling and describing the relationship between m and km units. [C, CN, ME, PS, R, V] | 2. Demonstrate that the sum of interior angles is:  
- 180° in a triangle  
- 360° in a quadrilateral. [C, R] | 2. Develop and apply a formula for determining the area of:  
- triangles  
- parallelograms  
- circles [CN, PS, R, V] |
## Shape and Space (Measurement)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td>Use direct or indirect measurement to solve problems.</td>
<td>Use direct or indirect measurement to solve problems.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 3. Demonstrate an understanding of volume by:  
  • selecting and justifying referents for cm³ or m³ units  
  • estimating volume, using referents for cm³ or m³  
  • measuring and recording volume (cm³ or m³)  
  • constructing right rectangular prisms for a given volume. | 3. Develop and apply a formula for determining the:  
  • perimeter of polygons  
  • area of rectangles  
  • volume of right rectangular prisms. | |
| [C, CN, ME, PS, R, V] | [C, CN, PS, R, V] | |
## Shape and Space
### (3-D Objects and 2-D Shapes)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong>&lt;br&gt;Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.</td>
<td><strong>General Outcome</strong>&lt;br&gt;Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.</td>
<td><strong>General Outcome</strong>&lt;br&gt;Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong>&lt;br&gt;5. Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:&lt;br&gt;• parallel&lt;br&gt;• intersecting&lt;br&gt;• perpendicular&lt;br&gt;• vertical&lt;br&gt;• horizontal.&lt;br&gt;[C, CN, R, T, V]&lt;br&gt;6. Identify and sort quadrilaterals, including:&lt;br&gt;• rectangles&lt;br&gt;• squares&lt;br&gt;• trapezoids&lt;br&gt;• parallelograms&lt;br&gt;• rhombuses (or rhombi) according to their attributes.&lt;br&gt;[C, R, V]</td>
<td><strong>Specific Outcomes</strong>&lt;br&gt;4. Construct and compare triangles, including:&lt;br&gt;• scalene&lt;br&gt;• isosceles&lt;br&gt;• equilateral&lt;br&gt;• right&lt;br&gt;• obtuse&lt;br&gt;• acute in different orientations.&lt;br&gt;[C, PS, R, V]</td>
<td><strong>Specific Outcomes</strong>&lt;br&gt;3. Perform geometric constructions, including:&lt;br&gt;• perpendicular line segments&lt;br&gt;• parallel line segments&lt;br&gt;• perpendicular bisectors&lt;br&gt;• angle bisectors.&lt;br&gt;[CN, R, V]&lt;br&gt;5. Describe and compare the sides and angles of regular and irregular polygons.&lt;br&gt;[C, PS, R, V]</td>
</tr>
</tbody>
</table>
### Shape and Space

**(Transformations)**

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
</tr>
<tr>
<td>Describe and analyze position and motion of objects and shapes.</td>
<td>Describe and analyze position and motion of objects and shapes.</td>
<td>Collect, display and analyze data to solve problems.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
<tr>
<td>7. Perform a single transformation (translation, rotation or reflection) of a 2-D shape, and draw the image. [C, CN, T, V]</td>
<td>6. Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]</td>
<td>4. Identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V]</td>
</tr>
<tr>
<td>8. Identify and describe a single transformation, including a translation, rotation and reflection of 2-D shapes. [C, T, V]</td>
<td>7. Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V]</td>
<td>5. Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]</td>
</tr>
<tr>
<td>8. Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs. [C, CN, V]</td>
<td>8. Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs. [C, CN, V]</td>
<td></td>
</tr>
<tr>
<td>9. Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Statistics and Probability
### (Data Analysis)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
<td><strong>General Outcome</strong></td>
</tr>
<tr>
<td>Collect, display and analyze data to solve problems.</td>
<td>Collect, display and analyze data to solve problems.</td>
<td>Collect, display and analyze data to solve problems.</td>
</tr>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Specific Outcomes</strong></td>
</tr>
<tr>
<td>1. Differentiate between first hand and second-hand data. [C, R, T, V]</td>
<td>1. Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V]</td>
<td>1. Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range • determining the most appropriate measures of central tendency to report findings. [C, PS, R, T]</td>
</tr>
<tr>
<td>2. Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]</td>
<td>2. Select, justify and use appropriate methods of collecting data, including: • questionnaires • experiments • databases • electronic media. [C, CN, PS, R, T]</td>
<td>2. Determine the effect on the mean, median and mode when an outlier is included in a data set. [C, CN, PS, R]</td>
</tr>
<tr>
<td></td>
<td>3. Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T]</td>
<td>3. Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]</td>
</tr>
</tbody>
</table>
Statistics and Probability
(Chance and Uncertainty)

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Outcome</td>
<td>General Outcome</td>
<td>General Outcome</td>
</tr>
<tr>
<td>Collect, display and analyze data to solve problems.</td>
<td>Collect, display and analyze data to solve problems.</td>
<td>Collect, display and analyze data to solve problems.</td>
</tr>
</tbody>
</table>

Specific Outcomes

<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcomes</td>
<td>Specific Outcomes</td>
<td>Specific Outcomes</td>
</tr>
<tr>
<td>4. Demonstrate an understanding of probability by:</td>
<td>4. Express probabilities as ratios, fractions and percents.</td>
<td>5. Identify the sample space (where the combined sample space has 36 or fewer elements) for probability experiment involving two independent events.</td>
</tr>
<tr>
<td>• identifying all possible outcomes of a probability experiment</td>
<td>[C, CN, R, T, V]</td>
<td>[C, ME, PS]</td>
</tr>
<tr>
<td>• differentiating between experimental and theoretical probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• determining the theoretical probability of outcomes in a probability experiment</td>
<td>6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.</td>
<td>[C, PS, R, T]</td>
</tr>
<tr>
<td>• determining the experimental probability of outcomes in a probability experiment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• comparing experimental results with the theoretical probability for an experiment.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Numeration

Suggested Time: 3 Weeks
Unit Overview

Focus and Context

Students have already had an opportunity to develop a good conceptual understanding of large numbers to one million and of small numbers to one thousandth. In Grade 6, students will continue to develop an understanding of the base-ten number system through continued work with larger numbers as well as with decimals.

Students will explore key concepts of number including:

- whole numbers greater than one million
- decimal numbers less than one thousandth
- solving problems involving whole and decimal numbers
- understanding the context when large or small numbers are used in real life
- using estimation to help determine the reasonableness of an answer
- determining the appropriateness of using technology to solve a given problem and then using it to solve the problem

Math Connects

Students will see large and small numbers in newspapers, on television, in stores and in texts. Understanding these large and small numbers is essential in helping students make sense of larger numbers as well as decimal numbers. Understanding these numbers enables students to make real world connections. This understanding will be beneficial in other subject areas, as well as other strands in the mathematics curriculum. In science, for example, students explore the distances and size of each planet in relation to the earth. In social studies, students look at populations of other countries as well as sizes of land masses. Olympic events, for example, can be used to help students appreciate the necessity of small numbers; a gold medal can be lost due to a difference of one thousandth of a second. As students become more comfortable with large and small numbers, their ability to use these numbers in computation will strengthen.
### Process Standards Key

<table>
<thead>
<tr>
<th>Key</th>
<th>Curriculum Outcomes</th>
<th>Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>[C]</td>
<td>Communication</td>
<td>[PS] Problem Solving</td>
</tr>
<tr>
<td>[CN]</td>
<td>Connections</td>
<td>[R] Reasoning</td>
</tr>
<tr>
<td></td>
<td>and Estimation</td>
<td>[V] Visualization</td>
</tr>
</tbody>
</table>

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
</table>
| Number | 6N1 Demonstrate an understanding of place value, including numbers that are:  
• greater than one million  
• less than one thousandth. | [C, CN, R, T] |
| Number | 6N2 Solve problems involving whole numbers and decimal numbers. | [ME, PS, T] |
Strand: Number

Outcomes

Students will be expected to

6N1 Demonstrate an understanding of place value, including numbers that are:
• greater than one million
• less than one thousandth.
[C, CN, R, T]

Achievement Indicator:

6N1.1 Explain how the pattern of the place value system, i.e., the repetition of ones, tens and hundreds within each period, makes it possible to read and write numerals for numbers of any magnitude.

Elaborations—Strategies for Learning and Teaching

The beginning of this unit focuses on building a conceptual understanding of numbers greater than one million. Work on numbers that are less than one thousandth will be done later in the unit. Students have already worked with numbers to one million in Grade 5. However, it may be necessary to review numbers to one million to strengthen students' conceptual understanding of large numbers.

It is important that students explore the place value chart to discover the pattern of ones, tens and hundreds to express a period of three digits in the place value system. An example such as 345 345 345 written in words or reading the word name may help students see the pattern more clearly. Students also need to see that, three million forty six thousand five hundred twenty one is written in standard form as 3 046 521, not 3 46 521. Students need to see that they need to have three digits in the thousands period where zero would indicate the absence of hundred thousand in the number. The importance of “0” as a place holder needs to be stressed.

Students should understand that the place value system follows a pattern such that:
• each position represents 10 times as much as the position to its right
• each position represents \( \frac{1}{10} \) as much as the position to the left
• place values are grouped in threes for purposes of reading them and the groups are called 'periods'.

Using base ten materials will help students understand that it will take 10 units to create one rod, 10 rods to create one flat and 10 flats to create one large cube.

As students begin working with a place value chart to represent large numbers, concentrate on their understanding of the number as a whole, as well as the value of each digit in the number. To simply place digits in the proper space in the place value chart does not indicate an understanding of the value of that digit. Students need to understand how to transfer the numbers from the place value chart to standard form with correct spacing, as well as in written form and expanded form.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Create cards with various number words on each as shown:

```
   six  hundred  billion  two  four  million
    thousand  eight  five
```

Ask students to rearrange the cards creating different number combinations. Next, students are instructed to record the numbers they create, in various ways such as in standard form, expanded form, in a place value chart, and in words. Students can then order these numbers using a number line and explain how they chose the endpoints and benchmarks, as well as how they placed their numbers on the line. Challenge students to create the greatest and the least number using all the cards. (6N1.1)

- Ask students to use the number 619 723 766 to answer the following questions.
  a) What is the value of the 9?
  b) What is the value of the 3?
  c) Choose a digit and show how it is ten times greater than the digit to its immediate right. (6N1.1)

**Student-Teacher Dialogue**

- Write a number, such as, 32 765 345 on the board. Ask students how many millions are in the number. How many thousands? Ten thousands? Ask them to justify their thinking. (6N1.1)

**Journal**

- Pose the following problem to students:
  Joe said 3 450 000 is greater than 27 450 000 because three is greater than two. Ask students to decide whether or not he is correct and explain using words, numbers and/or pictures. (6N1.1)

---

**Resources/Notes**

- **Math Focus 6**
  - **Lesson 1: Representing Numbers in the Millions**
  - **6N1**
  - TG pp. 13 – 17

- **Additional Reading (provided):**
Strand: Number

Outcomes

Students will be expected to

6N1 Continued

Achievement Indicators:

6N1.1 Continued

Elaborations—Strategies for Learning and Teaching

When students begin exploring the value of numbers and are presented with a number such as 7,324,169, they may be asked to indicate what the 7 represents. Students need to see the 7 means 7,000,000. Students should also understand, for example, in the number 345,461, five is in the thousands place, so it represents five thousand, but there are actually 345 thousands in this number.

In groups, give students 6-10 single digit number cards. It may be necessary to give different combinations of cards to different groups so each group will not come up with the same number. For example, group 1 may have the cards with digits 2, 3, 5, 6, 8, 1, 9, 0. Ask each group to come up with the greatest and the least possible number using their number cards. Ask the groups to display their number by writing it on the board and then by saying the number. After each group has presented their number, ask them to order all the numbers from greatest to least and place the numbers on a number line and/or a place value chart. Ask students to present their number lines and/or charts explaining how they were able to place each number. Ask students to choose one number and give an example where the number could be used in a real life situation.

As students begin to work with numbers greater than one million, it can become more challenging to provide meaningful examples to represent these numbers. Various texts, media and technology may provide students real-life examples of large numbers and it can give students a context in which they can understand what these numbers mean. The Guinness Book of World Records, and other subject text such as Social Studies and Science may also provide examples of large number usage.

An activity such as the following could be used to help students develop a good understanding of a million.

- Asking them to estimate how many jelly beans are in a jar.
- Record student responses, asking them to justify their thinking. Discuss any unreasonable responses.
- Ask students to think about how many jelly beans could fit inside their classroom. Is it reasonable to think at least one million could fit? A billion?
- Begin a discussion about these large numbers and ask students where they would possibly see these numbers in their life.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Paper and Pencil**

- Tell students that Lori read the number twenty-three million sixty-five thousand one. She recorded it as 23 651. Ask students if she is correct. Ask them to use words, pictures, and/or numbers to explain their answers.

  *(6N1.1)*

**Performance**

- Give students the following problem: We can use base-ten blocks to represent 1, 10, 100 and 1 000 using the unit, rod, flat and cube respectively. Ask: If we were to create new base-ten blocks to represent 10 000, 100 000, 1 000 000 and 10 000 000 what would they look like?

  *(6N1.1)*

- Number Curling - Using painters tape, create different sized squares inside the other on the floor. Using different colored beanbags, assign a value for each color. (E.g., red = 5, blue = 2 and green = 6). Ask students to throw/slide several beanbags into the squares. Using the value on each bean bag, ask them to record the number they created using the value in which the bean bags land. For example, a red and a green bean bag in the million square would equal 11 million, a red and blue in the thousand square would total 7 thousand and a green in the hundred square would equal 6 hundred. Students would then take each period and add the numbers together creating the number 11 007 600. They can continue play until they create several numbers. Ask students to compare them with their classmates to decide who had the greatest number.

  *(6N1.1)*

- Give students various resources such as newspapers, magazines, etc., and ask them to look for large numbers. They can cut out the article with the large numbers in it and display them in the classroom. Students can look at the articles and compare the numbers – to order them and talk about the contexts in which they are used.

  *(6N1.2)*

---

**Resources/Notes**

*Math Focus 6*

**Lesson 1 (Cont’d): Representing Numbers in the Millions**

*6N1*

TG pp. 13 – 17
Outcomes

Students will be expected to

6N1 Continued

Achievement Indicator:

6N1.2 Continued

6N2 Solve problems involving whole numbers and decimal numbers.

[ME, PS, T]

Elaborations—Strategies for Learning and Teaching

The book *How Much is a Million?* can initiate good discussion. For example, you could ask students if they agree with the suggestion that it would take 23 days to count to one million. If not, try it out. Begin by timing students counting to 100, 200, 300. Use this time to find how long it would take to count to 1,000, then 100,000, then 1,000,000!

In keeping with the theme of the book, ask students if we were to take 1,000,000 unsharpened pencils and lay them down from end to end, how far would they stretch? Would they reach all around Newfoundland? Canada? The world? Students could work in groups then present to the class, discussing their reasoning.

Students will be formally introduced to numbers greater than a million for the first time. Further work on the place value chart can be incorporated here to help students see the connection between a million and a billion. As students extend the place value chart using the patterns they have been exploring, they should see that one billion equals 1,000 millions. Providing students with problem solving experiences involving large numbers may help them gain a better understanding of the value of these numbers.

Problem solving situations should be embedded in a meaningful context as often as possible. Problem solving requires students to build on existing knowledge and develop their own strategies. Encourage students to demonstrate persistence as they work through a challenging problem. It is understandable for them to not know how to go about solving the problem immediately. Solving these challenging problems with minimal assistance will help empower students. Encourage students to communicate their thinking processes about the problem, write out the information they know and what they need to know. Then, using this information to make a plan of how to go about solving the problem will help them be successful with the problem solving process.

In previous years, students have worked with a variety of problem solving strategies. These strategies will prove to be useful to students...
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Student-Teacher Dialogue**

- Ask students: When might 1,000,000 of something be a big amount? When might 1,000,000 of something be a small amount? (6N1.2)

- Using several number lines, incorrectly place one or two numbers on each and ask students to work in small groups to explain why they are in the wrong places. They can share their responses with the group. Ask them to justify their thinking. (6N1.1, 6N12.1, 6N2.2, 6N2.3, 6N2.4, 6N2.5)

**Performance**

- The chart shows annual salaries for some famous sports stars:

<table>
<thead>
<tr>
<th>Sport</th>
<th>Annual Salary ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hockey</td>
<td>$6,500,000.00</td>
</tr>
<tr>
<td>Baseball</td>
<td>twelve million</td>
</tr>
<tr>
<td>Basketball</td>
<td>$18,000,000.00</td>
</tr>
<tr>
<td>Golf</td>
<td>one million two hundred thousand</td>
</tr>
<tr>
<td>Tennis</td>
<td>$750,000.00</td>
</tr>
</tbody>
</table>

Ask students to decide which benchmarks they wish to use and arrange the salaries in order from least to greatest by sport. Students should be able to explain how they decided on the order. (6N1.2)

- Ask students to research the population of the provinces in Canada. In a chart, list them in order from least to greatest and graph the results. Ask students to compare populations of the provinces and write a journal entry on what they have found. (6N1.2)

Resources/Notes

**Math Focus 6**

**Lesson 1 (Cont’d):** Representing Numbers in the Millions

6N1
TG pp. 13 – 17

**Children's Literature** (provided):

Schwartz, David. M. *How Much is a Million?*

**Math Focus 6**

**Lesson 2:** Exploring Billions

6N2
TG pp. 18 – 21

**Lesson 2 and Lesson 3 may be combined.**

**Lesson 3:** Solving Problems That Involve Large Numbers

6N2
TG pp. 24 – 27

**Curious Math:** Pandigital Numbers
TG pp. 28 - 29

**Math Game:** What’s in a Name?
TG pp. 22-23
Outcomes

Students will be expected to

6N2 Continued

Elaborations—Strategies for Learning and Teaching

in approaching a wide variety of problems. In order for students to successfully use problem solving strategies, they should be explicitly discussed with students, preferably after a student successfully uses it. There is a value in naming the strategy for students so they can discuss them and use them again. As students work through solving problems and begin to learn new strategies to solve these problems, you may consider posting these strategies in the classroom. A math wall or bulletin board designated to problem solving could be displayed for students. Students can select from the strategy list when they are facing a new problem and when they are not sure how to proceed.

Some possible problem strategies are:

act it out    use a model
draw a picture    guess and test
look for a pattern    use an open sentence
make a chart/table or graph    solve a similar problem
consider all possibilities    consider extreme cases
make an organized list    work backwards
use logical reasoning    change your point of view


Achievement Indicator:

6N2.1 Identify which operation is necessary to solve a given problem, and solve it.

Provide students with ample problem solving opportunities and situations to practice their already developed strategies to divide, multiply, add and subtract whole numbers. It is through this problem solving that students will become more efficient with these strategies and the concept of performing operations on whole numbers. This will also become a prerequisite to performing operations on decimals.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Paper and Pencil**

- Ask students what they would rather have for their allowance: one penny a day which doubles each day or $1000 a month? Explain their thinking and encourage them to use a table to show their solution.
  
  (6N2.1, 6N2.2, 6N2.3)

- The following chart shows the populations of five countries from least to greatest:

  Ask students the following:

  (i) The population of Russia is 141,862,011. If the population of Russia is added to the chart, between what two countries would it be listed? Explain how you know.

  (ii) If we combined the population of Mexico and Japan, would it be more or less than the population of the USA?

  (iii) Is there any one country that has about double the population as another? Explain.

  (iv) If all the populations of the listed countries were combined, would we have more than or less than 1 billion people? How do you know? Students show how they arrived at the answer.

  (6N2.1, 6N2.2, 6N2.3)

**Performance**

- Tell students Amy has 500 daytime minutes on her cell phone. She is using on average, 37 minutes a day. Ask students how many days she will have before her minutes are used? Ask them to suggest the number of minutes Amy should use each day to ensure she will not run out of daytime minutes each month.

  (6N2.1, 6N2.2)

- Tell students John's class sold 104 magazine subscriptions and Jane's class sold 108. The profit on each subscription was $11.00. One student estimated the total profit was $230.00. Ask students if this estimate is reasonable. Explain how you know.

  (6N2.2)
Outcomes

Students will be expected to

6N2 Continued

Achievement Indicators:

6N2.2 Determine the reasonableness of an answer.

Elaborations—Strategies for Learning and Teaching

Estimation must be encouraged in all problem solving situations, rather than always asking students to estimate and then calculate. Help students understand that we estimate to ensure we come to a reasonable solution to a problem. Students should always be asked to check the reasonableness of their answers by estimating and employing mental math strategies. ‘Front end estimation’ and ‘using compatible numbers’ are some ways they can do this.

When a student asks whether or not their answer to a question is correct, ask the student if the answer makes sense to them, having them justify their response. Having students determine the reasonableness of an answer and explaining their thinking is a powerful way to assess understanding and learning.

Students need to be given opportunities to see that there will be times when getting an exact answer is not possible. E.g., knowing exactly how many hot dogs to buy for the school population on sports day is difficult. Estimation would be used to come close to a reasonable number. Other real life examples could be explored to emphasize the importance of knowing how to estimate such as knowing how much money would be needed to buy a list of items at the store or how much lumber is needed to build a dog house.

Children’s Literature link – Read Betcha! Estimation by Stuart Murphy to help students understand the concept of estimation. Many times students confuse estimation with guessing. Often, students may blurt out answers or “an estimate” without thinking or giving any thought to the number. Students must realize that in order to estimate, students must do something with the numbers (rounding, comparing, using referents/benchmarks, using compatible numbers to do mental math computations, etc.) whereas guessing is a random response without using a strategy.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Provide sample problems such as the following and ask students to solve them. Ask them to share with their classmates how they found their answer:
  
  (i) Mr. Ron collects recyclables. He counted 32 full bags of bottles. Mr. Ron knows he has more than 400 recycled bottles. Ask students to come up with some possible amounts of bottles that could be in each bag if there were the same number of bottles in each bag.

  (ii) Mr. Ben’s greenhouse has rows of flowers. There are 72 flowers in each row with 1080 flowers in total. Ask students to find the number of rows in Mr. Ben’s greenhouse.

  
  \((6N2.1, 6N2.2, 6N2.3)\)

Portfolio

- Ask students to think about 1 million pennies and answer the following questions:

  (i) How many loonies would that be worth?

  (ii) How many $100 bills would it take to equal the 1 million pennies?

  (iii) How high would the stack of pennies be if you piled one on top of the other?

  (iv) What would the mass of 1 million pennies be?

  (v) Estimate how long it would take to count 1 million pennies.

  (vi) What could you buy with 1 million pennies?

  (6N2.1)

Journal

- Ask students to write about a situation in which estimated numbers were used.

  (6N2.3)

- Pose the following: Jane has $500 to buy 8 games. Each game costs $37. Jane wonders if she has enough money to buy them all. Invite students to help Jane out by explaining how they know if she really does have enough money. How can estimation help to determine the solution to the problem?

  (6N2.1)

Resources/Notes

\textit{Math Focus 6}

\textbf{Lesson 2 (Cont’d): Exploring Billions}

\textbf{6N2}

TG pp. 18 – 21

\textit{Lesson 2 and Lesson 3 may be combined.}

\textbf{Lesson 3 (Cont’d): Solving Problems That Involve Large Numbers}

\textbf{6N2}

TG pp. 24 – 27

\textbf{Children’s Literature} (provided):

Murphy, Stuart. \textit{Betcha! Estimation}
Students need to be aware of when to use technology. There are situations where using a calculator is more appropriate than using Paper and Pencil.

Although it is very important to ask students to master the basic facts and become comfortable using mental math strategies, calculator use can enhance student learning if used appropriately. Sometimes it is beneficial to involve students in solving problems that focus on the problem solving process and not the actual calculations within the problem. Using a calculator can free a student’s mind where they are focused on solving the problem and not hung up on the calculations of the numbers.

Solving problems involving large numbers may seem overwhelming to some students. Breaking the problem down into smaller steps, or using smaller numbers will help students understand how to go about solving the problem. It may also be important to emphasize the necessity to estimate the answer to the problem before or after the solution is found.

Calculators can assist students in solving problems involving large numbers. Ask students what kind of jobs would use calculators as part of the work environment? Example – bankers, accountants, stock brokers, etc.

Ask students to determine, using a calculator, how many Grade 6 students would be needed to reach from St. John’s to Port aux Basques if they were standing hand to hand.

(It is necessary for students to realize this is a question that requires estimation as each student will have different arm lengths.) Students can get an average arm length for their particular class. Breaking down the problem by determining how many students it would take to reach 10 m, 100 m and then 1 km could help determine the total number of students needed to reach the destination. Students could use the internet to determine many kilometres needed to reach Port aux Basques from St. John’s and their calculators can be used to calculate the total number of people needed. It may also then be determined if there are enough people in Newfoundland to reach from St. John’s to Port aux Basques and if not, how many more would be needed.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance
- Provide students with index cards folded in half with numbers written on each card. Ask students to work in small groups and distribute the number cards to each group. Give students time to discuss within their group the value of the numbers on the cards. Ask students to place their numbers on a number line made of string. You may also choose to give students blank index cards that could be used to create end points and benchmarks for their groups’ set of numbers. Give students time to reflect on their work looking at the placement of the numbers. When students are comfortable and confident of the placement, ask them to explain to the class their placement of the numbers.

  Model this task as a whole class activity before going into independent work. (6N1.1, 6N2.2, 6N2.3)

Paper and Pencil
- Ask students to solve the following problem. Mary’s class sold tickets for a fundraiser. There were 22 students in Mary’s class and each student sold 14 tickets. How many tickets were sold? (6N2.1)

Journal
- Ask the students to make a list of 3-4 situations where they would use a calculator to solve a problem. Ask them to explain why they would use a calculator rather than pencil/paper to get the answer. (6N2.4)

Resources/Notes

Math Focus 6
Lesson 2 (Cont’d): Exploring Billions
6N2
TG pp. 18 – 21

Lesson 2 and Lesson 3 may be combined.

Lesson 3 (Cont’d): Solving Problems That Involve Large Numbers
6N2
TG pp. 24 – 27
Strand: Number

Outcomes

Students will be expected to

6N1 Continued

As students continue work with large numbers, they sometimes need to rename numbers using decimals. They need to see that a number such as 3 450 000 is about 3.5 million. The focus should be on students’ reasoning and estimating of these numbers. When we are dealing with extremely large numbers it is very difficult to be accurate. If asked to find the population of Canada, students could use a search engine to see that in the 2008 census the population was 33 311 389 people. However, it is impossible to calculate the population of the nation at any given time since it is continuously changing. Therefore, giving this population count is only an estimate. When asked about the population of Canada, a reasonable estimate would be 33.3 million people.

Students are required to rename large numbers using decimals. They need to understand that the number to the left of the decimal names the whole number and the digits to the right of the decimal names the part of the number. For example, in the number 43 431 509 there are 43 whole millions, and 431 thousands. We would write this as 43.4 million.

A place value chart will reinforce students understanding of the values of numbers within periods.

Students could generate large numbers from texts and/or media that are meaningful to them. Using these numbers, ask students to place the numbers in the place value chart and rename the numbers using decimals. Ask them to round the numbers to the nearest tenth and hundredth and show how they decided to rename the numbers.

<table>
<thead>
<tr>
<th>Million</th>
<th>Thousand</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Rounded to Decimal Millions

33.3 million

For example, if asked to round 33 311 389 to the decimal million, students could write 33.3 million.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Portfolio/ Journal**
- Tell students that Jim and Tom each rounded numbers to 2.4 million. Ask them if this means they both started with the same exact number? Ask them to explain their thinking using pictures, numbers and words. (6N1.1, 6N2.2)

**Performance**
- Find news articles that include large numbers written in different forms. Students may use the Internet and search databases such as Statistics Canada.
  Ask students to rename the numbers in the article using decimal millions. (6N1.1, 6N1.2)
- Matching game – Prepare two sets of cards. One set of cards will have numbers written in standard form or in words and the other set will have the same number written in decimal form. Ask students to lay all cards face down. The first player turns over two cards. If they match, he/she keeps the cards. If they do not match, the two cards will be turned face down again. Play continues until all of the matches have been found. The player with the most cards at the end of the game wins. (6N1.1, 6N2.2, 6N2.3)
- Ask students to write five numbers between 5 000 000 and 10 000 000 and rename these numbers using decimal millions. They can also place the numbers on a number lines. (6N1.1, 6N2.2, 6N2.3)
- Ask students to research the distance of each planet from the sun. Record these distances in decimal millions. Ask students to create problems that could be solved using this information and get their partner to solve them. E.g., order the planets from least to greatest according to their distance from sun. Rename the numbers and record in a place-value chart. (6N1.1, 6N1.2)

Resources/Notes

*Math Focus 6*
Lesson 4: Renaming Numbers
6N1
TG pp. 30 – 33
Strand: Number

Outcomes

Students will be expected to

6N2 Solve problems involving whole numbers and decimal numbers.

[ME, PS, T]

Elaborations—Strategies for Learning and Teaching

Communicating mathematical understanding and thinking is important in learning new skills and concepts in math. When a student can explain how they know an answer is reasonable, this means a student understands the process and the problem in a way that makes sense to them. As students continue to solve problems, it is necessary to work on their communication skills – showing how they know. Using pictures, numbers and words can help show their understanding and can improve their communication skills.

Consider the following situation:

The Toronto Maple Leafs have 42 regular season games and sell out all their games each season. Their stadium has 18 800 seats.

If we ask students to calculate how many tickets would be sold in a year, they are simply computing two given numbers. A richer way to ask students to engage in problem solving using large numbers would be to ask the following:

Would the Toronto Maple Leafs sell 1 000 000 tickets in a year? If not, how many years would it take to sell 1 000 000 tickets?

Therefore it is necessary to engage students in appropriate questions and tasks that allow such a process. Open ended tasks can allow students an opportunity to effectively communicate their reasoning in multiple ways. These representations strengthen the connections students make among the various strands in the math curriculum.

Achievement Indicator:

6N2.5 Use technology when appropriate to solve a given problem

Working with large numbers may cause some difficulty with computation without the use of technology. It is suggested that students use calculators to work with these large numbers for computational purposes when the focus is not on mental math. Take the opportunity to observe students as they use calculators when computing large numbers. Assess students’ understanding as they communicate the reasonableness of the answer they found on the calculator.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Performance*

- Present the following situation:
  Betty sends at least 50 text messages a day to her friends. Each text message has about 140 characters. Ask students how many days it would take Betty to text one and a half million characters. Explain your thinking. (6N1.1, 6N2.1, 6N2.2, 6N2.3, 6N2.4, 6N2.5)

- Ask students to find the 5 most populated countries in the world and record the population. Ask them to estimate the total population of these countries and, using a calculator, determine if the estimate is reasonable. Explain why or why not. (6N2.3, 6N2.4, 6N2.5)

*Resources/Notes*

*Math Focus 6*

*Lesson 5: Communicating about Large Numbers*

*6N2*

TG pp. 34 - 37
Students will be expected to

6N1.1 Continued

elaborations—strategies for learning and teaching

Work at this level should concentrate on having students understand that the place value system extends to the left of the decimal as it does to the right. It is here that it should be emphasized that the majority of real life exposure to decimal numbers is unlikely to extend below the thousandths. However, it is important for students to know that the place value system extends beyond thousandths and that they can use the patterns of the place value chart to assist them in reading and writing these decimal numbers.

Students can often use the same strategies when reading and writing small decimal numbers as those used to read and write large whole numbers.

Show students how to write the numbers that are less than one thousandth in words to get them familiar with saying the words. For example, when talking about two ten thousandths (0.000 2), students could begin writing or saying this as two parts out of ten thousandths, or two in ten thousandths. They could then place this number on the place value chart to show that the 2 is in the ten thousandths place. Therefore we say that the number is 2 ten thousandths.

Some students may have trouble reading and writing small numbers. When writing numbers in the place value chart, demonstrate to students how the value of the last digit depends on which period it is placed in.

In example (a) we would read the number as thirteen ten thousandths. In example (b) the last digit is in the hundred thousandths place, therefore we would read or write the number as two and four thousand five hundred sixty seven ten thousandths.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to use the following numbers to answer the questions below:
  8.0254  2.086  0.83  24.918
  i) In which number does 8 represent a value of 8 hundredths?
  ii) In which number does 2 represent a value of 2 tens?
  iii) In which number does 0 represent the value of 0 ones?  (6N1.1)

- Ask students to write the number 23.0876 in words.  (6N1.1)

Performance

- Using a set of 24 decimal ten thousandths problem cards, have 12 cards with decimal ten thousandths written in words and 12 cards with the same numbers written in numbers.

  Students can play the game “Go Fish” where players pick 4 cards each. They take turns asking the other if they have the card that matches one in their hand. For example, do you have a card that matches 2 and three hundred thousandths? If the other student has the card with the number 2.0003 on it, they would give the card to the other player.

  The game continues until all cards are gone. The player with the most cards at the end of the game wins.  (6N1.1, 6N2.2, 6N2.3)

- Ask students to roll a die 5 times. Using the numbers rolled, create a decimal number with a value between 1.000 1 and 9.999 9  
  (6N1.1, 6N2.2, 6N2.3)

- Using number cards with digits 0-9 on them, ask students to create decimal ten thousandths numbers that are in a desired range. For example, using 5 different number cards create a number that can be found between 1.000 9 and 1.500 1. Ask students to write a journal entry explaining their thinking.  (6N1.1, 6N2.2, 6N2.3)

Resources/Notes

Math Focus 6
Lesson 6: Representing Millionths
6N1
TG pp. 42-45

Lesson 6 and Lesson 7 may be combined

Math Focus 6
Lesson 7: Exploring Decimals to Millionths
6N1
TG pp. 42-45

The focus of lesson 6 and 7 in the resource is on decimals to millionths; however, the outcome states that students will demonstrate an understanding of place value to numbers less than one thousandths. Therefore it is important not to spend a great amount of time on numbers less than ten thousandths.
Strand: Number

Outcomes

Students will be expected to

6N1 Continued

Achievement Indicators:

6N1.1 Continued

As students begin comparing small decimal numbers, connections to how they compare large numbers should be made. However, it must be noted that not all strategies to compare whole numbers will work for decimal numbers. For example, when comparing 3 456 with 345, it is evident to see that 3 456 is greater because there are more digits in the number. When comparing decimals, however, this strategy may not work. Ask students which number is greater - 0.234 or 0.228 7. Students may think that 0.228 7 is greater because it has more digits. Help students understand that 0.234 is greater because 0.23 is greater than 0.22. It is through this work that student understanding of the place value system will be seen. The place value chart, as well as number lines, can be used in the comparison of decimals. For most students, it will be easier to compare decimals with the same number of decimal places. For decimal numbers that do not have the same number of decimal places, students can be shown that they can place a desired number of zeros at the end of the number without changing the value of the number. It will be necessary here to discuss how one tenth, for example is the same as 10 hundredths, or 100 thousandths or 1 000 ten thousandths. Writing these numbers on a place value chart or under each other to show the relationship would be helpful. E.g.,

\[
\begin{align*}
0.1 \\
0.10 \\
0.100 \\
0.1000 \\
\end{align*}
\]

6N1.2 Continued

Students understanding of decimal numbers involving ten thousandths is an extension of their understanding of the place value system. It can be difficult to provide meaningful examples of numbers that extend beyond the thousandth. Using examples such as ppm (parts per million) in science to explain the amount of a chemical in a solution, may be one way to help students understand where these numbers could be used.

As students continue to work with decimal numbers beyond thousandths they can be shown how these very small numbers are related to larger numbers (millions) using the place value chart.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

<table>
<thead>
<tr>
<th>Performance</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provide students with a list of numbers written both in words and in numeral form. Ask them to compare these numbers by placing them in a place value chart.</td>
<td>Math Focus 6</td>
</tr>
<tr>
<td>26.004 3</td>
<td>Lesson 8: Using Decimals</td>
</tr>
<tr>
<td>0.001 3</td>
<td>6N1</td>
</tr>
<tr>
<td>Seventy ten thousandths</td>
<td>TG pp. 49-52</td>
</tr>
<tr>
<td>four and fourteen ten thousandths</td>
<td>Curious Math: Googols and Googolplexes</td>
</tr>
<tr>
<td>(6N1.1, 6N2.2, 6N2.3)</td>
<td>TG pp. 53-54</td>
</tr>
</tbody>
</table>
Number Relationships

Suggested Time: $4\frac{1}{2}$ Weeks
Unit Overview

Focus and Context

This unit focuses on finding factors and multiples of whole numbers to 100, classifying numbers as being prime or composite, exploring integers and using the order of operations. Previously, students have been working with multiplication, which will help them quickly find factors and multiples of numbers once this concept is explored. Students who have not mastered their basic multiplication facts may be able to use this work on factors and multiples to help strengthen and solidify the skills and understanding of multiplication. Working with positive and negative integers will further develop students place value knowledge by extending the number line to the left of zero. Students need to recognize that negative numbers are the mirror reflection of their positive counterpart on the number line. Grade 6 students are formally introduced to the rules of the Order of Operations. This is merely an introduction as further work will be completed in Grade 7. It should be noted that students need to apply these rules in a problem solving situation to discover why they exist.

Math Connects

Connecting student learning with life outside the classroom, providing meaningful contexts where math can be used in students’ lives and encouraging students to explore and begin to wonder about math should be the focus of any math program. Introducing factoring and prime numbers where students recognize that all numbers can be decomposed into prime numbers gives them a foundation of knowledge and skills which will be further developed in later grades. The introduction of integers allows students to extend their knowledge about the number system and begin to see that every situation that happens in life can be represented in some way with numbers. Keeping students curious about number and having them constantly question our number system will help keep students engaged and interested in mathematics. Practicing with the rules of the order of operations will only strengthen student understanding of how numbers are affected by operations and in turn, increase their number sense.
## Number Relationships

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
</table>
| Number | 6N3 Demonstrate an understanding of factors and multiples by:  
- determining multiples and factors of numbers less than 100  
- identifying prime and composite numbers  
- solving problems using multiples and factors. | [CN, PS, R, V] |
| Number | 6N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically. | [C, CN, R, V] |
| Number | 6N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). | [C, CN, ME, PS, T] |

---

### Process Standards Key

- **[C]** Communication
- **[CN]** Connections
- **[ME]** Mental Mathematics
- **[PS]** Problem Solving
- **[R]** Reasoning
- **[T]** Technology
- **[V]** Visualization
Strand: Number

Outcomes

Students will be expected to

6N3 Demonstrate an understanding of factors and multiples by:

- determining multiples and factors of numbers less than 100
- identifying prime and composite numbers
- solving problems using multiples and factors.

[CN, PS, R, V]

Achievement Indicator:

6N3.1 Determine all the whole number factors of a given number, using arrays.

Elaborations—Strategies for Learning and Teaching

Begin this unit by introducing students to factors. Factors are numbers that when multiplied, produce a product. Factors always come in pairs. For example, the factors of 12 are 1 and 12, 2 and 6, 3 and 4. A review of basic multiplication facts may be necessary for students experiencing difficulty, multiplication tables can be used to help identify factors of numbers. Students may also begin working with factors of numbers they are comfortable with.

The term ‘factor’ can be added to the class math word wall with students defining it in their own words, giving examples to illustrate their understanding.

When students are introduced to factors, it would be a good idea to use various manipulatives such as square tiles or snap cubes to form arrays or rectangles to identify the factors of a given number. To find the factors of 12, give each student 12 tiles or snap cubes and ask them to form a rectangle(s) or an array using only these 12 tiles. Ask students to find other ways to arrange the 12 tiles to make complete rectangles. Students should conclude that they can make a rectangle of $1 \times 12$, $2 \times 6$ and $3 \times 4$. These numbers then, would represent the factors of 12.

Students sometimes forget to list 1 and the number itself, as factors of a given number. Remind students they are to find all whole numbers factors.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Finding Factors – Present numbers that have several factors – E.g., 12, 18, 24, 30 or 36. With counters and snap cubes ask students to attempt to find a way to separate them into equal sets. With arrays, students build rectangles that have the given number of squares. For each arrangement, a multiplication equation should be written. These equations would represent the factors of that number.


  (6N3.1)

- Ask students to find a number that has exactly 4 factors and another one that has 5 factors.  
  (6N3.1)

- Ask students to draw one or more rectangles to show that 8 is a factor of 16 and 24.  
  (6N3.1)

- Factor Patterns - Tell students that they are going to look for multiplication expressions and the corresponding rectangular array for several numbers. Their task includes trying to find all the multiplication expressions and rectangular arrays for each number. Have square tiles available that students can use to explore possible arrays. Once they have created an array, they should draw it on grid paper. Ask students to write a multiplication sentence for each array. Students should group together all arrays with the same number of squares. After identifying the multiplication expression and the rectangular arrays, students are to look for patterns in the factors and rectangular arrays. E.g., which numbers have the least number of arrays and therefore, the least number of factors? Which numbers have arrays that form a square? Which numbers have a factor of 2? What can you say about the factors for even numbers? Do even numbers always have 2 as a factor? What do you notice about the factors of odd numbers? Encourage students to think about why different patterns occur.  
  (6N3.1)

**Resources/Notes**

*MATH FOCUS 6*

**Lesson 1: Identifying Factors**

**6N3**

TG pp. 13 – 17

**Additional Reading (provided):**

Strand: Number

Outcomes

Students will be expected to

6N3 Continued

Achievement Indicator:

6N3.2 Identify the factors and multiples for a given number, and explain the strategy used; e.g., concrete or visual representations, repeated division by prime numbers, factor trees.

Elaborations—Strategies for Learning and Teaching

Manipulative use is helpful for students as they learn to identify the factors of numbers. Allowing students to use manipulatives to break the numbers apart into arrays will give them a visual of the factors for a given number. This hands-on method then becomes a strategy. Using manipulatives will allow students to become comfortable with identifying factors and be ready to create visual representations of the number.

One or two centimetre grid paper can be used to create arrays or rectangular shapes to represent a given number and its factors. When finding the factors of a number, students may color squares on the grid paper to form different rectangles. The width and the length of these rectangles will represent the factors of that number.

Factorize - In this game students are able to create rectangles of a given area on a grid that represent the number they are finding factors for. The dimensions of the rectangle can then be entered as factors for that number.

Students can also create an organized list of the factors of a given number where students identify the number and then list the factors in a row. Suggest that students, when listing the factors of a number, start on the left and list them in ascending order to ensure they do not omit any.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Give students the following journal prompt:
  - Find a number which has 4, 7, 28 and 12 as factors. Ask them if there is a smaller number which will meet the conditions and to explain why or why not. 

Resources/Notes

*Math Focus 6*

Lesson 1 (Cont’d): Identifying Factors

6N3

TG pp. 13 – 17
Outcomes

Students will be expected to

6N3 Continued

Achievement Indicators:

Factor Rainbows - Another visual representation that students can use to help factor numbers. It is recommended that students have prior experience factoring numbers before moving on to using factor rainbows. In a factor rainbow, students begin listing the factors of the number, horizontally, starting at 1. When the factor pairs are connected, the coloured, curved lines create a rainbow shape.

A factor rainbow for 18

The factors of 18 are: 1, 2, 3, 6, 9, 18

Posing real life problems that involve factors will allow students opportunities to see how finding factors of numbers are used outside of the math classroom. These real world connections help put a value on student work and understanding. Helping students understand the concept of factors can be done through problem solving where a problem can be posed and once it is solved the term “factor” can be introduced.

Consider the following example: Farmer Joe is trying to figure out how to plant a new potato garden. He has a plot of land that can cover a maximum of 100 m² and a minimum of 10 m². He needs to decide how big to have his garden, but wants some choice on the dimensions of the area. He needs to know which area would give him the most choice of dimensions, but still have enough room to plant his stock of potatoes. What could you suggest?

Here, students are asked to find areas for Farmer Joe’s potato garden that would yield different dimensions, or factors of that area. They should see that numbers like 13, 37 and 59 only have one way to plant the area. Other areas, such as 36, 48 and 54 have many. They should also see that the larger the area, the more potatoes that can be planted.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Tell students: Jill has an ice cream sandwich which measures 10cm by 10cm. She wants to cut the ice cream into squares. What possible sizes could the squares be? How many squares of each size would be cut? (6N3.1, 6N3.2, 6N3.3)

- Ask students to solve the following problem by using words, pictures and numbers:

  Harry's dad had 36 Halloween treats that he wanted to share evenly among treat bags. What are the different possibilities of the number of bags that he could fill? (6N3.2, 6N3.3)

Resources/Notes

Math Focus 6
Lesson 1 (Cont'd): Identifying Factors
6N3
TG pp. 13 – 17
Strand: Number

Outcomes

Students will be expected to

6N3 Continued

Achievement Indicator:

6N3.4 Identify multiples for a given number, and explain the strategy used to identify them.

Elaborations—Strategies for Learning and Teaching

Students have worked previously with factors and will now be introduced to multiples.

Discuss with students that when we multiply two factors, the product is a multiple of those two factors. E.g., \(2 \times 5 = 10\). 2 and 5 are the factors of 10, whereas 10 is a multiple of 2 and 5. Students sometimes do not recognize 0 as a multiple of any number. Small (2008) states that there are two ways to approach this. One is to observe that, for example, \(0 = 0 \times 3\), so 0 is a multiple of 3. The other is to use patterns. The multiples of 4 are 4 apart. So going down from 4 you get to 0. E.g., 24, 20, 16, 12, 8, 4, 0. (p.155)

Multiples of a number can be identified using a hundred chart. Students can start at 0 and then skip count the specified number. For example, when asked to find the multiples of 8, students can start at 0, and shade in every 8 numbers. The shaded numbers are the multiples of 8. To emphasize 0 as a multiple of every number, it is suggested that a hundred chart that includes 0 be used.

Students can also use a number line to skip count by the specified number.

Students can also use an organized chart to identify multiples of a number. Here they can use multiplication of the number by various factors to determine multiples of the number.

<table>
<thead>
<tr>
<th>Multiply by 8</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>0</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

The multiples of 8 are: 0, 8, 16, 24, 32...

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance/Paper and Pencil

- Present the following situations to students and ask them to find the answer:

  Joe bought some $10 computer games. Damian bought some $15 computer games. They each spent less than $200, but they both spent the same amount. How much could they have spent?

    (6N3.4, 6N3.3)

- Tell students you are thinking about a number that is a multiple of 2 and 6. Ask them to identify some possible numbers that you could be thinking about.

    (6N3.4, 6N3.3)

Journal

- Tell students you were trying to figure out the multiples of 8 and here is the list you came up with: 0, 8, 16, 23, 32 and 40. Ask if they agree and have them explain their thinking.

    (6N3.4, 6N3.3)

- Ask students to choose a number from 2 – 10 and ask them to list at least 5 or 6 multiples of that number. Ask them to explore any patterns they see in the multiples and discuss why this may happen.

    (6N3.4, 6N3.3)

Resources/Notes

Math Focus 6
Lesson 2: Identifying Multiples
6N3
TG pp. 18 – 21

Curious Math: String Art
6N3
TG pp. 22-23
Strand: Number

Outcomes

Students will be expected to

6N3 Continued

Achievement Indicators:

6N3.4 Continued

Elaborations—Strategies for Learning and Teaching

Students may use various manipulatives such as snap cubes, counters, buttons, etc, to create equal groups of the specified number to find its multiples. The total amount of items would be the multiple of that number. E.g., if a student was asked to find the multiples of 5, they could create one group of 5 and list 5 as a multiple, two groups of five would be 10 and 10 can be listed as a multiple of 5. Five groups of five would be 25 and 25 would be multiple. Remind students that 0 is a multiple of every number and it should be included as a multiple here.

6N3.3 Continued

Encourage students to develop their own strategies in determining multiples of numbers. They can use these strategies to help solve various problems involving multiples.

Ask students to figure out how many packs of wieners with 12 wieners in a pack and how many packs of hot dog buns with 8 buns in a pack would be needed to fill an order of 72 hot dogs.

Spiders have 8 legs and ants have 6 legs. There is a container on the table that contains both spiders and ants. The number of spider legs equal the number of ant legs. What are some possible numbers of spiders and ants that would produce this result?
### General Outcome: Develop Number Sense

#### Suggested Assessment Strategies

**Performance/Paper and Pencil**
- Present the following situations to students and ask them to find the answer:

  The cafeteria is having a promotion. Every second student receives free milk and every sixth student receives a slice of pizza. If 60 students are served at the cafeteria that day, which students received free milk? A slice of pizza? Both?

  (6N3.4, 6N3.3)

**Journal**
- Tell students that Olivia was in a class of 24 people. Her teacher told the class to line up and that every second student would receive a pencil and every sixth student would receive a smelly sticker. If Olivia wanted both a pencil and a smelly sticker, ask students which position in the line Olivia should be in.

  (6N3.4, 6N3.3)

#### Resources/Notes

*Math Focus 6*

*Lesson 2 (Cont’d): Identifying Multiples*  
6N3  
TG pp. 18 – 21
Outcomes

Students will be expected to

6N3 Continued

Achievement Indicators:

6N3.5 Provide an example of a prime number, and explain why it is a prime number.

6N3.6 Provide an example of a composite number, and explain why it is a composite number.

6N3.7 Sort a given set of numbers as prime and composite.

Elaborations—Strategies for Learning and Teaching

The work that students have completed on factors will help them as they work on prime and composite numbers. Students see how factors help determine prime and composite numbers. Arrays and rectangular arrangements provide students with a visual and concrete representation of how a number can be broken down. Numbers that can only be arranged in one array are prime numbers. Numbers that can be arranged in more than one array are composite. Give students several numbers to explore. E.g., 3, 6, 9, 13 and 16. Ask them to find the factors of the numbers. Ask students if they notice any similarities or differences with the factors of the numbers. Discuss with students the fact that some numbers only have 2 factors, which are prime and others have more than 2 factors, which are considered composite.

Students continue to work with prime and composite numbers identifying whether a number is prime or composite, factoring a number into its prime factors and continuing to develop personal strategies to solve problems. Students need opportunities to apply these skills and ideas in problem solving situations where they are thinking about and reasoning through their work. Students, through working with factoring numbers, are now asked to demonstrate an understanding that composite numbers can be created by multiplying prime numbers together.

An interesting way to help students determine whether a number is prime or composite is a technique called “The Sieve of Eratosthenes”.

Use a hundred chart to find the primes from 1-100:

Step 1: Color the number 1 blue.
Step 2: Color every multiple of 2 red but not 2 itself.
Step 3: Color every uncoloured multiple of 3 yellow but not 3 itself.
Step 4: Color every uncoloured multiples of 5 green but not 5 itself.
Step 5: Color every uncoloured multiple of 7 orange but not 7 itself.

The remaining uncoloured numbers are prime numbers!

After completing “The Sieve of Eratosthenes” activity, ask students to explore the result of multiplying two prime numbers. Students should realize that when they multiply any two prime numbers the resulting product will always be a composite number. Ask students to explain why this happens. If students work with smaller prime numbers as factors, they will be able to identify on the hundred chart that the resulting product is a composite number. (Small, 2008)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Student-Teacher Dialogue**

- Ask students: Is it possible for an even number, other than 2, to be prime? Explain. (6N3.5, 6N3.6)

**Performance**

- Factor Game - Using a number chart, ask students to work in groups. Students would need one game board and different colored markers/crayons. The object of the game is to circle the factors of a number. Player One chooses a number on the number board and scores that many points. Player Two crosses out all the factors of that chosen number that are not already circled. Player Twos’ score is the sum of the circled factors. E.g., Player One chooses the number 16. They score a total of 16 points. Player Two would then circle the available factors of 16, which are 1, 2, 4, 8. (The number 16 would already be circled by Player One). Player Two would add the factors together to get a total score of 15. Player Two would then choose a number and score that many points, for example, 21. Player One would circle all available factors of 21 which are 3 and 7 and score a total of 10. As play continues, it may be possible for a player to select a number where all possible factors are circled, therefore they would get that many points with the opponent not scoring any. Play continues until all numbers are circled where students would then total their points. The player with the most points, wins.

<table>
<thead>
<tr>
<th>Player One</th>
<th>Player Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>15 (1+2+4+8)</td>
</tr>
<tr>
<td>10 (7+3)</td>
<td>21</td>
</tr>
</tbody>
</table>

(6N3.5, 6N3.6, 6N3.7, 6N3.3)

Resources/Notes

**Math Focus 6**

**Lesson 3: Prime and Composite Numbers**

6N3

TG pp. 24 – 28

**Math Game: Colouring Factors**

6N3

TG pp. 29 - 30

**Additional Reading** (provided):

Strand: Number

Outcomes

Students will be expected to

6N3 Continued

Achievement Indicators:

6N3.3 Continued

Students can employ personal strategies to solve problems involving factors and prime and composite numbers. Solving problems becomes a vehicle which allows students to apply the knowledge and skills they are learning in a meaningful way.

Ask students to think about the following situation: If you had 17 tissue boxes, how many different ways could you design a carton to package your tissue boxes with no empty space in the carton? How would this change if there were 34 tissue boxes? (students may want to model this problem using snap cubes to create a rectangular prism)

6N3.8 Explain why 0 and 1 are neither prime nor composite.

It may be necessary to point out that the number 1 is neither prime nor composite, since it does not fit the definition of a prime or composite number. To be prime, the number must have only two factors, one and itself. The number 1, only has one factor. Therefore, it cannot be composite (it does not have more than 2 factors).

0 is another special number. 0 cannot be a prime number because every number is a factor of zero. 0 x 1 does equal 0, but 0 multiplied by anything equals 0.

0 is not a composite number because it cannot be written as a product of 2 factors, neither of which is itself.

6N3.2 Continued

Students have had opportunities to work on factoring numbers using various strategies that involved using concrete and visual representations. Students should begin to understand that our whole number system can be built up by prime numbers and that any one number can be broken down into a series of prime numbers. This is called prime factorization.

E.g., 24 = 2 x 12 = 2 x 2 x 6 = 2 x 2 x 2 x 3

Students may also see that if they use repeated division of prime numbers, they will end up with prime factors.

E.g., 36 ÷ 3 = 12
    12 ÷ 3 = 4
    4 ÷ 2 = 2

so the prime factors would be 3, 3, 2, 2.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance
- Ask students to use colored tiles or grid paper to determine whether the numbers 7, 10, and 18 are prime or composite numbers. Ask them to explain their thinking. (6N3.5, 6N3.6)

Student-Teacher Dialogue
- Ask the student how he/she can determine, without factoring, that certain large numbers such as 17932 and 19875 are not prime. (6N3.7)

Journal
- Give students 6 – 8 numbers and ask them to sort them as either prime or composite numbers. Ask students to justify their reasoning. (6N3.7)

Resources/Notes

Math Focus 6
Lesson 3 (Cont'd): Prime and Composite Numbers
6N3
TG pp. 24 – 28

Math Focus 6
Lesson 4: Identifying Factors by Dividing
6N3
TG pp. 31 - 34
Ask students to work in groups to factor the number 36. Give students time to employ different strategies to find the prime factors. Have each group present their strategies to the class, looking at similarities and differences in each groups approach and their starting point to factor the number. It is important that students discover that regardless of where they start the factorization process, they will always end with the same prime factors.

![Factor Trees]

In Grade 6, students should realize they can use division to find the factors of a given number. For example, when asked to find all the factors of 12, students can look for different numbers that can divide evenly into 12. These are the factors of 12. $12 \div 1 = 12$, so 1 and 12 are factors. $12 \div 2 = 6$, so 2 and 6 are factors and $12 \div 3 = 4$; 3 and 4 are factors.

Review with students that they can find the factors of numbers in different ways:
- forming arrays
- factor rainbows
- repeated division
- factor trees (factorization)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Paper and Pencil**
- Tell students that a number has exactly 4 prime factors. Ask them to think about what could this number be. (6N3.2, 6N3.3)

- Ask students to draw two different factor trees for 56 and for 32. Ask them why it is possible to draw two different factor trees for each number. (6N3.2, 6N3.3)

- Ask students if they can name a composite number for which you can draw only one factor tree. How many factor trees can they draw for the number 13? Explain. (6N3.2, 6N3.3)

**Performance**
- There are 84 students in four grades and they are arranged into teams with the same number on each team. How many teams are there and how many students might there be on each team? How many possible solutions can you find to this problem? (6N3.4, 6N3.2, 6N3.3)

### Resources/Notes

**Math Focus 6**

**Lesson 4 (Cont’d):** Identifying Factors by Dividing

6N3

TG pp. 31 - 34

---

(Math Focus 6)

**Lesson 5:** Creating Composite Numbers

6N3

TG pp. 35 – 38

*This lesson does not require a lot of time.*
# Strand: Number

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Solving Strategy:</strong></td>
<td>As students continue to develop their own problem solving strategies, they may wish to include creating an organized list. Continue to build on various problem solving strategies and when students use one of the strategies, ask them to present how they used the strategy and how it helped them solve the problem.</td>
</tr>
<tr>
<td><strong>Using an Organized List</strong></td>
<td>An organized list helps students organize their thinking about a problem. Students may wish to use an organized list to begin a problem, or to organize given information in a problem. Students can record their thinking in an organized list which makes it easy for them to review what has been done, the stage of the problem solving process they are in and what is left to be done. Using an organized list provides a good way to record computations and any other important information needed to solve the problem.</td>
</tr>
</tbody>
</table>

Ask students to create a problem whereby they could use an organized list to help them solve the problem. Ask them to write this problem on a large sheet of paper and then show how they would solve it using the list. Students could then add this strategy to their math wall.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students the following stories and ask them to find the answers:

  (i) Michael and his sister, Rebecca, both play soccer. They are on different teams and both have a tournament this weekend. Michael and Rebecca’s parents are trying to figure out if there is a time when they are playing against each other. Michael plays every three games and Rebecca is playing every two games. If there are 12 games in the tournament, is it possible for Michael and Rebecca to play against each other? If so, how many times will they play against each other? (6N3.4, 6N3.2, 6N3.3)

  (ii) Both Craig and John bought packages of granola bars. Craig had 24 bars in total and John had 30 bars in total. Find all the possible sizes of packages for each student.

  (iii) If both Craig and John bought packages with the same number of bars in them, what would be the number of bars in each package? (6N3.4, 6N3.1, 6N3.2, 6N3.3)

**Resources/Notes**

*Math Focus 6*

**Lesson 6: Solving Problems Using an Organized List**

TG pp. 43 - 46
Strand: Number

Outcomes

Students will be expected to

6N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically.
[C, CN, R, V]

Elaborations—Strategies for Learning and Teaching

Students are familiar with numbers 0 and above. This unit introduces them to integers which includes numbers above and below 0. The main ideas for students to understand are that:

- each negative integer is the mirror image of a positive integer with respect to the 0 mark
- 0 is neither positive nor negative
- negative integers are all less than any positive integer

Using the word ‘negative’ and not ‘minus’ is important when describing negative integers. This will avoid confusing the sign of the integer with the operation of subtraction.

Negative integers are the opposites of natural numbers. Each integer is the reflection of its opposite across the number line. A number line is a useful tool in helping students see the relationship between positive and negative integers. Tape a number line on the floor, use cash register tape or string, whereby the middle, or center of the number line is marked with a 0. Ask students to think about what numbers would go to the left, or below the 0. Ask students to explore these numbers asking them to think about the placement of the numbers that are ‘less’ than 0. Show students that numbers such as +1 and -1 are the same distance away from 0.

Students, almost daily, have experiences that can be modelled with negative numbers. For example, slowing down a car is negative acceleration. Students may also gain an understanding of negative integers by putting them into meaningful context such as driving in reverse, describing floors below the main floor in a building, talking about below sea level or golf scores that are below par. It could also be described in terms of owing money. Discuss some of these situations where it can be represented by negative numbers. E.g., losing $15 or when trying to walk up an icy hill and you fall backward a few steps.

Using a number line (painter tape works well), ask students to stand on a negative integer and tell where they would see this number in real life, or describe a situation that could be represented by this number.

Use play money to represent various situations where students can see a gain or loss, emphasizing a loss would represent a negative situation.

It may be necessary to help students understand that a situation can be represented by a negative number without the result being a negative number. For example, if you have $5.00 in your pocket and you gave $3.00 to your friend for his birthday, you would still have $2.00 left, but it would be a loss of $3.00.

Achievement Indicators:

6N7.1 Extend a given number line by adding numbers less than zero, and explain the pattern on each side of zero.

6N7.2 Describe contexts in which integers are used; e.g., on a thermometer.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Student-Teacher Dialogue

- Students may illustrate / discuss a situation in which they have encountered something that could be represented with a negative number. (6N7.2)
- Joe and John are standing on a number line. Joe is 6 spaces away from John. Joe is standing on a negative number and John is on a positive number. Ask students to determine some possible numbers Joe and John may be standing on. Ask the students to explain the strategies used to solve this problem. (6N7.1, 6N7.3, 6N7.5)

Performance

- Provide students with a blank number line. Give them positive and negative integers to place on the line with them choosing their end points and benchmarks. (6N7.1, 6N7.3, 6N7.4, 6N7.5)

- Ask students to design a game for which positive and negative points may be awarded. Ask students to play and keep track of their score. (6N7.1, 6N7.3, 6N7.5)

- A number is 12 jumps away from its opposite on a number line. Ask students what the number could be and how do they know. (6N7.1, 6N7.3, 6N7.5)

- Ask students to choose two negative integers. Ask them to compare these numbers by describing a context in which they could be used (temperature) and use this context to compare using the less than/greater than symbols. (6N7.2, 6N7.4)

Paper and Pencil

- Ask students how many negative integers are greater than -7. Ask them to explain how they know. (6N7.1, 6N7.3, 6N7.5)

Journal

- Ask students to explain, in writing why -4 and +4 are closer to each other than -6 and +6.
- Ask students to explain, in writing, why an integer is never an odd number of jumps away from its opposite on a number line. (6N7.1, 6N7.3, 6N7.5)

Resources/Notes

Math Focus 6
Lesson 7: Representing Integers
6N7
TG pp. 47 – 50

Curious Math: Countdown Clock
6N7
TG pp. 51 - 52

Children’s Literature (not provided):
Murphy, Stuart, Less than Zero
ISBN 9780060001261
Outcomes

Students will be expected to

6N7 Continued

Achievement Indicator:

6N7.3 Place given integers on a number line, and explain how integers are ordered.

Elaborations—Strategies for Learning and Teaching

When comparing and ordering numbers, students should refer to a number line and look at the position of the number on the line in relation to 0 to determine its value. A common misconception students have is the greater the digit, the greater the value will be. They may see a number like -8 and think it is greater than -1 or +5 just because the digit 8 is greater than 1 or 5. Ask students to refer to the position of the number in relation to 0. Remind students that any number to the right is always greater than any number to the left. This applies to both positive and negative integers.

When placing integers on a number line, remind students to first look at the sign to see whether it is a positive or negative number. Then ask them to look at the digit to determine how far away from 0 the number should go. Ensure students understand that all negative numbers are less than 0 and placed to the left of the 0 on the number line.

Students can stand on number lines to help order integers. To illustrate the idea of whether a negative integer is greater than or less than another, ask students to stand on the number in question and ask them to hop to 0 counting how many times they needed to hop to get to 0. Ask them to do the same for the other number and then compare the number of hops it takes to get to 0.

E.g., When comparing -5 and -8, have a student stand on -5 and jump to 0. They would see it takes 5 jumps. Next, ask them to stand on -8 and jump to 0 where they will jump 8 times. They should see that because it took more jumps to get to 0 from -8, or -8 is farther away from 0, then -8 would be less than -5.

If students are struggling with this concept, ask them to refer to a thermometer or use a vertical number line to represent the thermometer.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students if any positive number is greater than any negative number. Ask them to use a number line to explain their thinking.  
  (6N7.1, 6N7.3, 6N7.4)

- Provide students with incomplete number lines where they would fill in any missing numbers.  
  (6N7.1, 6N7.3, 6N7.4, 6N7.5)

- Ask students to research cities in North America that are below sea level, at about sea level and some that are above sea level. Using a chart, ask them to list these cities from lowest elevation to highest.  
  (6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)

- Students may wish to research pro golf players databases where they compile data on scores. Explain that golf scores are reported in positive and negative numbers, where positive numbers show how many shots above par was needed to sink the ball. A negative number would be how many shots under par was needed to sink the ball. For example, on a hole that was par 5, it is suggested that it would take 5 shots to sink the ball. If a player took 3 shots, he/she would score -2 for that hole. If it took him 6 shots to sink the ball, his score would be +1. Ask students to rank players according to their scores.  
  (6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)

- There are 10 numbers between a set of integers. What are some possible numbers this set could be?  
  (6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)

- Create a number line on the board and incorrectly place a negative number on the positive side of the number line. Ask students to decide if this number line is correct and ask them to justify their thinking.  
  (6N7.1, 6N7.3, 6N7.4, 6N7.5)

Resources/Notes

Math Focus 6
Lesson 8: Comparing Integers
6N7
TG pp. 53 - 57
Strand: Number

Outcomes

Students will be expected to

6N7 Continued

Achievement Indicators:

6N7.4 Compare two integers; represent their relationship using the symbols <, > and =; and verify the relationship, using a number line.

6N7.5 Order given integers in ascending or descending order.

6N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).

[C, CN, ME, PS, T]

Achievement Indicator:

6N9.1 Explain, using examples, why there is a need to have a standardized order of operations.

Elaborations—Strategies for Learning and Teaching

There are many different strategies to help students remember the symbols of less than and greater than. One way to help students remember the symbols would be to get students to look at the shape where they would see a sideways “V”. Tell students there are two sides to the symbol, a big side and a small side.

A number line is a useful tool to help students order integers.

The order of operations is a set of rules that tells you which order to compute numbers so you will get the same answer as everyone else. It is just merely a convention of mathematics and something that students will practice.

When applying these rules, multiplication and division will always come first unless there are brackets in the equation. Here, you can explain that sometimes you may not want or need to follow the rules and need to highlight a specific computation. It is here you need to use brackets.

For example, consider the following question: Each of 6 friends got a full box of bars that contained 10 bars in each box and then each of them got an extra 4 bars. Write an equation to show how many bars all 6 friends got.

Students may write the following:

6 × 10 + 4

The order of operations would tell us to multiply 6 × 10 and then add 4. But that would not make sense as 6 × 10 would show there are 6 friends with 10 bars each totalling 60 bars and then if you added 4 there would only be a total of 64. We know from the problem there are 4

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Create number cards where there are both positive and negative numbers. Play Integer War where students would pick a card and then compare each. The player with the greatest value would keep the cards. Play continues until all cards are used. The player with the most cards wins. (6N7.1, 6N7.3, 6N7.4, 6N7.5)

- Mrs. Smith bought 2 boxes of bars for her three children. Each box of bars has 6 bars in it. Ask students to determine how many bars each child receives. Ask students to write an expression to show the order of operations you would use to solve the problem. (6N9.1, 6N9.2)

- Ask students to place a set of brackets into this equation to explore how many different solutions are possible.
  \[10 + 2 \times 8 - 6 \div 2 = ?\]

  (6N9.1, 6N9.2)

- Ask students to find some skill testing questions that can be found on contests. Ask students to answer the question and compare the answers when following the order of operations and the answers when you do not follow them. Discuss the importance, in terms of the contest, of following these rules. (6N9.1, 6N9.2)

**Resources/Notes**

*Math Focus 6*

**Lesson 8 (Cont’d): Comparing Integers**

6N7

TG pp. 53 - 57

*Math Focus 6*

**Lesson 9: Order of Operations**

6N9

TG pp. 58 – 61

**Math Game:**

Four in a Row

6N9

TG pp. 62 - 63
Strand: Number

Outcomes

*Students will be expected to*

6N9 Continued

**Achievement Indicators:**

6N9.1 Continued

Elaborations—Strategies for Learning and Teaching

Extra bars for each of the 6 friends. Therefore you would have to add the 10 and 4 to show that all 6 friends would have a total of 14 bars each. Then you would multiply $14 \times 6$ to get the total number of bars. So the expression then would look like:

$$6 \times (10 + 4)$$

From here, students would see that you would compute the numbers inside the brackets first, then continue with multiplication/division and end with addition or subtraction.

To introduce students to the idea of the order of operations, provide the class with a question similar to the following: $4 + 8 \times 2 - 7$ and ask them to find the solution. Ask students to share their solutions and discuss if there are other possible solutions for this problem.

Discuss why people may have different answers for this question. Some may add 4 and 8 and then multiply by 2 and subtract 7 to get 14, whereas some others may add 4 to $8 \times 2$ and then subtract 7 for an answer of 13. Explain that we need to have rules to make sure everyone is getting the same answer. Many times in real life people are in a situation where there are various operations to calculate in order to solve the problem. To ensure the correct solution is found, these rules must be followed.

To illustrate the necessity of the rule, pose this problem to students: Mac bought 6 pairs of socks for $7.00 each and a scarf for $4.00. How much money did Mac spend?

To find the amount of money spent the equation could be written as: $6 \times 7 + 4$ or $4 + 6 \times 7$. Explain to students in order to find the correct answer we would have to multiply $6 \times 7$ and then add 4 in order for the amount to make sense. If we have 6 pairs of socks and spent $7.00 on each pair we would have to multiply these two numbers. It would not make sense to add 4 and 6 and then multiply this number by 7.

Students must be aware that most calculators will not use the order of operations to calculate equations automatically. Therefore they cannot rely on their calculator to solve problems involving multiple operations. Students will need practice entering the digits on the calculator in the order that the operation should be performed.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Give students an equation with the operations missing. Ask them to fill in the missing mathematical symbols to make the statement correct. This could also be done in a riddle type activity.

  (6N9.1, 6N9.2)

- Molly was doing her math homework when all of a sudden her pet mouse came along and began chewing her paper. When she looked, she noticed all the operation symbols were missing. Ask students to help Molly put these symbols and numbers back to make the statements true. Brackets should be included where necessary.

  \[
  12 \ ? \ 8 \ ? \ 3 \ ? \ 2 = 26 \\
  8 \ ? \ 6 \ ? \ 4 \ ? \ 2 = 10 
  \]

  (6N9.1, 6N9.2)

**Resources/Notes**

*Math Focus 6*

**Lesson 9 (Cont’d): Order of Operations**

**6N9**

TG pp. 58 – 61
Patterns in Mathematics

Suggested Time:  3 Weeks
Unit Overview

Focus and Context

A solid foundation in analyzing and understanding patterns is fundamental to student success and progress throughout intermediate and high school algebra. The use of tables, such as graphic organizers, allow students to see the relationship between input and output values. Students need to recognize patterns within columns of a given table of values as well as the relationship between rows. This relationship is described through the use of algebraic expressions. The focus is to generate a table of values from an algebraic expression and to derive the expression from a given table of values. Through their understanding of representing patterns concretely, extending patterns, finding missing values and creating algebraic expressions, students are encouraged to use their knowledge of patterns to solve problems.

In Grade 6, the focus is on the preservation of equality within algebraic equations. The solving of these equations will be addressed in Grade 7. Students will use concrete models, primarily a balance scale to demonstrate preservation of equality, with emphasis on the two sides of the equation balancing. This can be done by changing each side of an equation in the same manner.

Math Connects

Patterns are an effective way to demonstrate the relationship between variables. Providing students with opportunities to analyze, model and extend patterns will help prepare them for algebra in higher grades. The concept of modelling equivalent equations, using a balance scale is also important. The skills acquired in Grade 6 contribute to the development of a deeper understanding of mathematics and will continue to be useful in real world situations.
<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patterns and Relations (Variables and Equations)</td>
<td>6PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.</td>
<td>[C, CN, PS, R]</td>
</tr>
<tr>
<td></td>
<td>6PR3 Represent generalizations arising from number relationships, using equations with letter variables.</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td>6PR4 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.</td>
<td>[C, CN, PS, R, V]</td>
</tr>
</tbody>
</table>

### Process Standards Key

- **[C]** Communication
- **[CN]** Connections
- **[ME]** Mental Mathematics and Estimation
- **[PS]** Problem Solving
- **[R]** Reasoning
- **[T]** Technology
- **[V]** Visualization
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.

[C, CN, PS, R]

Achievement Indicators:

6PR1.1 Create a concrete or pictorial representation of the relationship shown in a table of values.

6PR1.2 Identify errors in a given table of values.

Elaborations—Strategies for Learning and Teaching

Give students a table of values and encourage them to visualize the change in each variable, as well as model it pictorially (drawing pictures) or concretely (using manipulatives). Patterns can be modelled using any available manipulatives. E.g., students could model the table of values below using toothpicks.

<table>
<thead>
<tr>
<th>Number of Squares</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

Students should first identify that the number of squares starts with 1 and increases by 1. They will then determine that the number of toothpicks starts with 4 and increases by 3. This means that each new square in the pattern must be constructed using only 3 additional toothpicks. Based on this identified relationship students must construct each new square by starting with one side of a pre-existing square.

It is important that students are able to identify errors in a given table of values so that they do not extend the pattern incorrectly. They should be able to support their answer. E.g., Sam has a weekly paper route. He gets paid $30 a week. The following table of values shows his earnings over a 5 week period. Identify the error in this table.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Earnings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>130</td>
</tr>
</tbody>
</table>
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Provide the following table of values. Ask students to use the green triangles from the set of pattern blocks to represent what the worm would look like on day 3, 4 and 5.

<table>
<thead>
<tr>
<th>Age/Days</th>
<th>Number of Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

E.g., This is a one-day-old worm:

This is a two-day old worm:

A train has 8 wheels and pulls cars each having 4 wheels. The table of values below shows the number of wheels on a train with different numbers of cars being pulled. Draw or make a model of the train with each number of cars being pulled.

<table>
<thead>
<tr>
<th>Number of cars</th>
<th>Number of wheels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

Paper and Pencil

- Create tables containing an error in the right column and ask students to try to identify the value that does not fit the pattern. Ask students to explain why the value in question is incorrect. Ask them to justify their choice.

Resources/Notes

Math Focus 6
Lesson 1: Identifying Number Patterns
6PR1
TG pp. 13 - 17
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.3 Describe the pattern within each column of a given table of values.

6PR1.4 Create a table of values to record and reveal a pattern to solve a given problem.

Elaborations—Strategies for Learning and Teaching

When describing patterns in a given column of a table of values, students sometimes overlook stating the starting value of the pattern. The table below shows changes in the height of a plant over time.

<table>
<thead>
<tr>
<th>Week</th>
<th>Plant Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
</tbody>
</table>

Most students would describe the change in weeks as “going up by 1” and the change in plant height as “going up by 4”. Both of these descriptions do not acknowledge the starting value. A more appropriate description of each would be, “Weeks start at 1 and increase by 1 each time. Height starts at 6 and increases by 4 each time”.

Students have previously extended patterns concretely and pictorially. Creating a table of values to record the pattern is new to Grade 6. At this point, students are simply extending the pattern in a given column to solve a problem rather than using a pattern rule that relates one column to the other.

Students are expected to create their own table of values. They may have difficulty determining how each column should be labelled. Remind students that the values in the second column result from (or is dependent on) the change in the first column. For instance, in the previous example, plant growth is determined by the number of weeks that have passed.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

**Journal**

- Ask students to identify the pattern within each column of the given table:

<table>
<thead>
<tr>
<th>Hour</th>
<th>Snowfall (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
</tr>
</tbody>
</table>

(6PR1.3)

**Performance**

- Tell students that a statue is shaped like a tower and is made of a single column of cubes (see diagram below). A painter has been hired to paint all the cube faces that are visible. This would include the side and top faces as well. The bottom face of each cube is not visible. Students may build the model using multilink cubes or blocks. Ask students to create a table of values to record the number of faces that need to be painted on towers 1, 2, 3, 4 and 5 blocks high. Ask students to find the number of faces that would have to be painted for a tower containing 10 blocks. 20 blocks.

(6PR1.4)

Resources/Notes

**Math Focus 6**

**Lesson 1 (Cont'd):** Identifying Number Patterns

6PR1

TG pp. 13 - 17
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

The focus is to identify patterns that increase or decrease between values within and between the columns of a table of values chart and write the pattern rule. It is important that students first identify the value at which the pattern begins and then indicate the amount they increase or decrease from that given value.

In previous grades students completed tables of values given simple expressions involving one operation, addition or subtraction. In Grade 6 they will be exposed to expressions that involve two operations, most commonly multiplication followed by addition or subtraction. E.g., Judy pays $10 for her cell phone. Each time she uses the Internet it costs $2. This relationship can be represented by the pattern rule $2n + 10$. Use this pattern rule to complete the table shown here.

<table>
<thead>
<tr>
<th>Number of Internet Visits</th>
<th>Cost per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

In describing the relationship between or within the columns of a given table of values, students should be encouraged to use appropriate mathematical language. Invite students to share how they arrived at their conclusions. To create a comfortable environment for students to justify their reasoning, model appropriate language. Brainstorm, with students, other words or phrases that could be used and display these mathematically, appropriate words.

As students explore relationships in and between numbers in columns of a table of values, they will predict what missing values might be and figure out values for numbers not covered in the table.

Previously, students found relationships (pattern rules) within a column and then used that rule to predict terms not in the table of values. This strategy works well when the numbers in the column are in sequence, and when they are asked to find larger terms in the sequence in which extending the pattern is not practical (E.g., find the 50th term in this sequence). Problems such as these will require developing a pattern rule which can be used to determine the value in the second column based on the corresponding value in the first column.

The objective is that students are able to derive a pattern rule that relates one column of a table of values to the other column.

Students will initially see the pattern in each column of the table but may have difficulty identifying the relationship between the input and output columns. This concept can be introduced using guess and check to derive a pattern rule.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

• Give students the following situation: You are going to play paintball with your friends. It costs $20 for admission and an additional $5 for every round of balls. This relationship can be represented by the expression $5b + 20$. Use this pattern rule to complete the table of values below.

<table>
<thead>
<tr>
<th>Number of Rounds</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(6PR1.5)

• Ask students to create a table of values to represent the pattern below.

i) Write a pattern rule to describe the change within each column.
ii) Predict the number of straws for diagram 10.

(6PR1.6, 6PR1.7)

Journal

• Ask students to find the missing values in the table below, based on the patterns observed. Ask:
  How many sandwiches would each person get? Following this pattern, predict how many sandwiches would be needed if 60 people attended the picnic. How many could attend if 90 sandwiches were provided?

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Number of sandwiches</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>15</td>
<td>36</td>
</tr>
</tbody>
</table>

(6PR1.7)

Resources/Notes

Math Focus 6
Lesson 2: Describing Relationships in Tables
6PR1
TG pp. 18 – 22
Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicator:

6PR1.8 Continued

“What's My Rule” box - Prepare sentence strips with numbers on both ends as shown below. Insert the sentence strip so students can see input and output numbers. Show students and challenge them to come up with several different operations or combinations of operations that could be used to get the output from the initial input.

E.g., 4 is inserted into the box and 12 comes out.

Ask “What rule could have been used to get an output of 12 from an input of 4”?

Students may reply “We multiplied by 3”, “We added 8”, “We doubled the input and added 4”, etc.

Record all the students' suggestions and then present another input/output situation based on the same pattern rule. For example, 5 is put in and 15 comes out.

Ask students, “Which rule from the first input/output situation could also be used to describe an input of 5 and an output of 15”?

Students will likely reply “Adding 8 will not work, because 5 + 8 = 13. However, multiplying by 3 will work. Therefore the rule for this pattern must be multiply the input by 3”.

Verify that this is the correct rule by presenting another input/output scenario from the same pattern.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Use the “What’s my Rule?” box to model the input and output for each row in the tables below. Ask students to come up with a pattern rule that can be used to describe the relationship between all the input/output combinations in the table.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Remind students that the pattern rule has to apply to all the input/output combinations in the table, not just the first one. (6PR1.8)

- Ask two student volunteers to model how a “What’s my Rule?” box (as described on the previous page) works. Give one student a card with an operation written on it (E.g., +2, × 3, -1 etc.). No one else should know what operation is on this student’s card. Have the other volunteer put in an input number between 1 and 10. The student with the operation card will perform the operation mentally on this input value and put the resulting output card through the output slot and say it aloud to the class. Ask the class to identify what operation was performed on the input. Repeat the process using the same operation but a new input to verify the class’s answer. Once students become proficient at identifying a single operation ask another volunteer to come forward and give them a second operation card. The first student will say another input aloud. The student with the first operation card will mentally perform his/her operation and say the output aloud. The student with the second operation card will perform their operation on that output mentally and say the resulting number aloud. Have the class deduce the two operations performed. Verify using another input number. Once students gain proficiency at identifying two operations have the student with the first operation card whisper their output to the student with the second operation card so that the class is only hearing the final output of both operations and not the output of each. This will require more verification using different inputs to deduce the pattern rule. (6PR1.8)

Resources/Notes

Math Focus 6
Lesson 2 (Cont’d): Describing Relationships in Tables
6PR1
TG pp. 18 – 22
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicator:

6PR1.8 Continued

Elaborations—Strategies for Learning and Teaching

From this point, concrete examples can be used. For example, four students could be seated around a square table, with one on each side. Tables could be added with another chair on either side in the pattern shown below. This could be concretely modelled using actual tables and chairs if available. Pattern blocks or interactive whiteboard graphics could also be used to create the model.

This could be represented in the following table of values.

\[
\begin{array}{c|c}
\text{Number of Tables} & \text{Number of Chairs} \\
1 & 4 \\
2 & 6 \\
3 & 8 \\
4 & 10 \\
5 & 12 \\
\end{array}
\]

Once this table has been generated, attention should be drawn to how the increase in the second column (constant change) relates to the operation(s) performed on the values in the first column to find the corresponding value in the second.

With 1 table there will be 4 chairs.
With 2 tables there will be \(4 + 2 = 6\) chairs
With 3 tables there will be \(4 + 2 + 2 = 8\) chairs
With 4 tables there will be \(4 + 2 + 2 + 2 = 10\) chairs

Notice that the number of chairs is increasing by 2. This would be logical because for every table added, two new chairs are added (one on each side of the table). However, there will always be two extra chairs (one on each end of the line of tables) that are constant.

Thus, the pattern rule for this scenario would be \(2n + 2\), where \(n\) = the number of tables.
General Outcome: Use Patterns to Describe the World and to Solve Problems

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math Focus 6</td>
</tr>
<tr>
<td></td>
<td>Lesson 2 (Cont'd): Describing Relationships in Tables</td>
</tr>
<tr>
<td></td>
<td>6PR1</td>
</tr>
<tr>
<td></td>
<td>TG pp. 18 – 22</td>
</tr>
</tbody>
</table>
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicator:

6PR1.8 Continued

Elaborations—Strategies for Learning and Teaching

By extension, I can use this rule to predict the number of chairs needed for 9 tables. Double the number of tables (9 × 2 = 18), and add the two constant chairs at the ends (18 + 2 = 20). Therefore, we would need 20 chairs for 9 tables.

Students should recognize that the amount of constant change in the output is the number by which the input is being multiplied. If the product of this amount of change and a given input is not equal to its corresponding output, we must then determine the amount of increase or decrease required to arrive at that output. For example, in the table below students should first determine the amount of constant change in the output values (second column).

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>

Draw attention to the output values. Ask “How much do the output values increase by each time”? Students will observe a constant increase of 3 in the outputs. This means that each input must have been initially multiplied by 3. Thus, the first part of our pattern rule is 3n, where n represents any given input value.

Starting with the first input, multiply by 3.

1 × 3 = 3

Now ask, “What did we add to or subtract from this product to get the output of 5?” Students should recognize that 2 was added. So, this means our pattern rule is 3n + 2, or in words, “multiply the input by 3 and add 2.” Confirm that this is correct by evaluating the expression using the other inputs in the table of values.

3(2) + 2 = 8 correct
3(3) + 2 = 11 correct
3(4) + 2 = 14 correct
3(5) + 2 = 17 correct, 3n + 2 must be the pattern rule.

It is important to note that using the difference in the outputs to determine a pattern rule in this manner will only work if the inputs are consecutive numbers. (e.g. 1, 2, 3, 4, 5…). Therefore, when assessing this outcome be sure to use consecutive input numbers.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Journal

- Present the following scenario to the class:
  Ted is having a pot-luck dinner. He has prepared 4 dishes of food for the dinner and told all his invited guests to bring two dishes each. The number of dishes at the party depends on the number of guests that come.

  i) Write a pattern rule that could be used to determine the number of dishes that will be at the dinner for any number of guests that might attend.

  ii) Use this pattern rule to complete the table of values below.

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Number of Dishes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

(6PR1.5, 6PR1.6, 6PR1.8)

Resources/Notes

Math Focus 6
Lesson 2 (Cont’d): Describing Relationships in Tables
6PR1
TG pp. 18 – 22
Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.9 Identify missing elements in a given table of values.

Elaborations—Strategies for Learning and Teaching

Students are familiar with using a given pattern rule to complete the right column of a table of values. However, they will now be expected to find missing values in either column of an incomplete table using a given pattern rule. Inverse operations may be addressed here as a possible strategy.

E.g.,

<table>
<thead>
<tr>
<th>n</th>
<th>3n</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Students will complete the first two missing values by simply applying the pattern rule, $3n$. To find the third and fourth missing values they will have to work backwards (use the inverse operation). For example, to determine the third missing value the student should think “Three groups of an unknown number gave me 18. If I share 18 into three equal groups, how many will be in each group?” (18 divided by 3).

Students will now be expected to develop an expression and table to solve a given problem. Consider the following example:

Gloria is going to a community celebration. Admission is $5 and each activity costs $2.

i) Use words to describe how to find the total amount of money Gloria will spend for any number of activities that she may participate in. (Multiply the number of activities by 2 and add 5.)

ii) Write an expression to represent the above situation.

iii) Use your expression to create a table of values showing how much Gloria will spend if she takes part in 0 to 5 activities.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

• Present the following situation to students:
  (i) Jill works in a store for a wage of $9 per hour. Help Jill complete the following table to show her total earnings after each hour worked in a day. Some values were omitted on each side of the table. Find the missing values:

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>72</td>
</tr>
</tbody>
</table>

Ask students to explain how they derived each omitted value using the pattern rule.

(ii) Jill wants to buy two pairs of jeans that cost $46.00 each. How many hours does she need to work in order to buy the jeans? (6PR1.9)

• Present the following situation:
  Sheila works in a computer repair shop. She gets paid $75 a day plus $5 for every computer she fixes.
  i) Create a table to display the total amount of money Sheila could make in a day for any number of computers she might fix.
  ii) Write a pattern rule that you could use to find the total amount of money Sheila could make in a day for any number of computers she might fix.
  iii) Use your rule to determine how much money Sheila would make if she fixed 12 computers in one day. (6PR1.4)

Resources/Notes

Math Focus 6
Lesson 3: Using Expressions to Create Tables
6PR1
6PR3
TG pp. 23 – 27
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR3 Represent generalizations arising from number relationships, using equations with letter variables.

[C, CN, PS, R, V]

Achievement Indicators:

6PR3.1 Describe the relationship in a given table, using a mathematical expression.

6PR3.2 Represent a pattern rule, using a simple mathematical expression such as \(4d\) or \(2n + 1\).

Elaborations—Strategies for Learning and Teaching

Students have worked on writing a word rule to describe the relationship in a given table. Students will now extend this skill and write the pattern that is found in the table using a mathematical expression, or numbers and variables.

Students will take a word rule and use it to generate a mathematical expression using variables. E.g., the cost to join minor hockey is $120.00 per player. Each player must pay an additional fee of $5.00 for each practice. To represent the total cost for any given player, the word rule would be to multiply the number of practices by $5.00 and add $120.00. This can be now written as a mathematical expression, \(5p + 120\), where ‘\(p\)’ represents any number of practices. Students could then use this expression to generate a table of values showing total costs for various possible numbers of practices.

<table>
<thead>
<tr>
<th>Number of Practices</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>145</td>
</tr>
<tr>
<td>10</td>
<td>170</td>
</tr>
<tr>
<td>15</td>
<td>195</td>
</tr>
<tr>
<td>20</td>
<td>220</td>
</tr>
<tr>
<td>25</td>
<td>245</td>
</tr>
</tbody>
</table>
General Outcome: Use Patterns to Describe the World and to Solve Problems

**Suggested Assessment Strategies**

**Journal**

- Ask students to create a situation for each expression:
  
  (i) \( p - 3 \)
  
  (ii) \( 3 - p \)

Explain how the position of the variable changes the meaning of the expression in each case.

Would this also apply to the following expressions?

- (i) \( p + 3 \)
- (ii) \( 3 + p \)  

**Performance**

- Ask students to write an algebraic expression to represent each pattern rule below:
  
  i) Double a number
  
  ii) Five more than a number
  
  iii) Three less than a number
  
  iv) A number less than ten
  
  v) Six more than twice a number
  
  vi) One less than triple a number

- Ask students to match each of the following situations with the correct expression:
  
  - Harry is twice as old as Noel \( 4n + 2 \)
  - Susan has a bag of candy and gave away four \( 2n \)
  - Margot has four packages of hockey cards and two individual cards \( n + 2 \)
  - Harry is two years older than Noel \( 4 - n \)
  - Susan has 4 dolls and gives some away \( n - 4 \)

**Resources/Notes**

- *Math Focus 6*
- Lesson 3 (Cont’d): Using Expressions to Create Tables
- 6PR1
- 6PR3
- TG pp. 23 – 27
Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.8 Continued

6PR1.3 Continued

6PR3 Continued

Achievement Indicator:

6PR3.2 Continued

Elaborations—Strategies for Learning and Teaching

The focus is to compare patterns created by different expressions. It should become apparent to students that expressions that share the same numbers and variables will not necessarily produce the same pattern. For example, $2n + 3$ will not produce the same pattern as $n + 3$. Neither will $4n + 2$ create the same pattern as $4n + 3$. Although each expression has a common feature, each generates a different table of values.

This concept should be introduced using manipulatives to generate two different patterns. From these patterns two separate tables of values would be generated and two different mathematical expressions (pattern rules) derived.

Give students counters. Students will start with 3 counters placed in a row. Each student will then create rows where each is increased by any desired constant amount. Encourage students to keep their amount of increase small so as not to run out of counters. Students will continue their pattern up to five rows.

Ask students to record the number of counters used to make each row in a table for values as shown:

<table>
<thead>
<tr>
<th>Row</th>
<th># of counters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

Ask students to analyze their table of values and create a word rule and an expression that could be used to find the number of counters for any row number.
General Outcome: Use Patterns to Describe the World and to Solve Problems

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 4: Comparing Expressions</strong></td>
</tr>
<tr>
<td></td>
<td>6PR1</td>
</tr>
<tr>
<td></td>
<td>6PR3</td>
</tr>
<tr>
<td></td>
<td>TG pp. 28 – 31</td>
</tr>
<tr>
<td><strong>Curious Math Activity:</strong></td>
<td></td>
</tr>
<tr>
<td>Clock Number Patterns</td>
<td></td>
</tr>
<tr>
<td>6PR1</td>
<td></td>
</tr>
<tr>
<td>TG pp. 32-33</td>
<td></td>
</tr>
<tr>
<td><strong>Additional Reading:</strong></td>
<td></td>
</tr>
<tr>
<td>Small, Marion (2008), <em>Making</em></td>
<td></td>
</tr>
<tr>
<td>Math Meaningful to Canadian</td>
<td></td>
</tr>
<tr>
<td>Students K-8, pp. 582 - 588</td>
<td></td>
</tr>
</tbody>
</table>
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.8 Continued

6PR1.3 Continued

6PR3 Continued

Achievement Indicator:

6PR3.2 Continued

Elaborations—Strategies for Learning and Teaching

After analysis of the expressions students developed, they should come to the conclusion that even though everyone started with the same number, they did not all get the same expression because their patterns increased by different amounts.

Next, ask the class to start their rows with any desired number of counters (advise students to keep their numbers small). They will then create rows where each increases by 2.

Ask each student to generate a table of values to show the change in the number of counters in each row.

E.g.,

Ask students to analyze their table of values and create a word rule and an expression that could be used to find the number of counters for any row number.

After analysis of the expressions students developed, they should come to the conclusion that even though everyone increased their rows by the same amount, they did not all get the same expression because their patterns began with different amounts.
General Outcome: Use Patterns to Describe the World and to Solve Problems

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td></td>
<td><em>Lesson 4: Comparing Expressions</em></td>
</tr>
<tr>
<td>6PR1</td>
<td><em>6PR3</em></td>
</tr>
<tr>
<td></td>
<td><em>TG pp. 28 – 31</em></td>
</tr>
</tbody>
</table>
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR4 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.

[C, CN, PS, R, V]

Elaborations—Strategies for Learning and Teaching

In order for equations to be equivalent, the same operations have to be performed on each side where the value of the variable does not change. E.g., \(3n + 1 = 7\) and \(3n = 6\) are equivalent equations because 1 is subtracted from each side in the first equation to make the second equation. This is called ‘preservation of equality’.

Students will now be introduced to equations involving multiplication as well as equations involving two operations, pictorially and concretely. Students will learn the difference between an equation and an expression in Grade 7 but it is important for them to hear the correct terminology modelled. An equation is a complete number sentence stating that two amounts are the same. Equations must contain an equal sign, e.g., \(2 + 3 = 5\). A number sentence with a variable is an algebraic equation, e.g., \(p + 2 = 3\) reads ‘Two more than a number is equal to 3’. This is an algebraic equation because it is stating that an unknown amount plus 2 is the same as 3. An algebraic expression on the other hand is simply a statement without the notion of equivalency. For example, \(p + 2\) reads ‘A number plus 2’. In this case, the variable \(p\) could be any value. The emphasis here is on modelling these equations and showing that if you add or subtract the same amount to/from each side, the equality is preserved. At this point students are not expected to solve equations although some students may be inclined to do so.

Use pan balance scales and number lines when introducing this concept.

When modelling equations, using the pan balance, bags can be used to represent variables (unknown amounts) and multi-link cubes or blocks used to represent numbers.

Model a simple equation on a pan balance such as \(3n = 9\).

Students can now add a constant amount to each side. No matter how much they add, the scale will remain balanced as long as they add the same amount on each side. This will help the student observe how the equality of the two pans (sides of the equation) is preserved.
General Outcome: Use Patterns to Describe the World and to Solve Problems

**Suggested Assessment Strategies**

**Performance**
- Ask students to draw or model (using a two pan-balance or number line) each set of equations below. Determine if each pair of equations are equivalent or not. Explain how you know.
  - \( n + 2 = 6 \) and \( n + 3 = 7 \)
  - \( 2m + 1 = 9 \) and \( 2m + 2 = 8 \)
  - \( 5p + 3 = 18 \) and \( 4p + 3 = 18 \)
  - \( 4y = 20 \) and \( 8y = 40 \)
  - \( 3k = 12 \) and \( 9k = 24 \) (6PR4.1)

**Journal**
- Ask students if \( 2n + 2 = 6 \) and \( 2n + 4 = 6 \) are equivalent equations. Explain why or why not? Use a model to represent each equation. (6PR4.1)

**Student - Teacher Dialogue**
- Tell students that Dave says that the following number lines show that \( 8 = 2n \) and \( 16 = 4n \) are equivalent equations. Is he correct? Ask them to justify their answer.

- Present the following: Victoria says that the following number lines show that \( 3p = 9 \) and \( 6p = 12 \) are equivalent equations. Is she correct? Explain how you know. (6PR4.1)

- Provide cards with the following equations:
  
  \[
  \begin{align*}
  15 - s &= 9 \\
  3s &= 12 \\
  2 + s &= 7 \\
  7 + s &= 15
  \\
  11 &= 22 - s \\
  4s + 2 &= 26 \\
  14 - s &= 7 \\
  15 &= 13 + s \\
  17 - s &= 11 \\
  3s - 4 &= 8 \\
  2 + 2s &= 12 \\
  9 + s &= 17 \\
  4s &= 24 \\
  21 - s &= 14 \\
  18 &= 16 + s \\
  0 &= 11 - s
  \end{align*}
  \]

Lay out the equation cards (face up) so students can see what is on the cards. Match the equation card with its corresponding equivalent equation. When all equation cards are matched, choose one equation and create a word problem. Ask students to switch word problems and solve one belonging to a classmate. (6PR4.5)
Strand: Patterns and Relations (Patterns)

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will be expected to</td>
<td>Therefore, $3n = 9$ and $3n + 2 = 11$ are equivalent equations.</td>
</tr>
<tr>
<td>6PR4 Continued</td>
<td>Once students have had experience creating equivalent equations using the pan balance, additional activities may be modelled using a digital balance from the National Library of Virtual Manipulatives or another on-line applet but this should not be a substitute for students using the actual pan balance as a hands on activity.</td>
</tr>
<tr>
<td>Achievement Indicators:</td>
<td>Equivalent equations can also be modelled on a number line. E.g.,</td>
</tr>
<tr>
<td>6PR4.1 Continued</td>
<td>Is $3p = 15$ equivalent to $6p = 30$?</td>
</tr>
<tr>
<td>6PR4.2 Continued</td>
<td>Show students a model of $3p=15$ and $6p=30$.</td>
</tr>
<tr>
<td>6PR4.3 Continued</td>
<td>These equations are equivalent because the “jumps” are all the same size.</td>
</tr>
<tr>
<td>6PR4.4 Continued</td>
<td>To ascertain preservation of equality for multiplication it must be</td>
</tr>
<tr>
<td>6PR4.5 Write equivalent forms of a given equation by applying the</td>
<td>determined whether each side of the equation was multiplied by the</td>
</tr>
<tr>
<td>preservation of equality and verify using concrete materials, e.g., $3b$</td>
<td>same amount. E.g., $3n + 2 = 8$ and $6n + 4 = 16$ would be equivalent equations because all terms on both sides were doubled ($\times 2$).</td>
</tr>
<tr>
<td>$= 12$ is same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$.</td>
<td>$2n + 3 = 7$ and $6n + 9 = 14$ would not be equivalent because the terms on the left side were tripled ($n \times 3$) but the terms on the right were doubled ($n \times 2$), therefore equality is not preserved. Use a balance scale to verify.</td>
</tr>
</tbody>
</table>
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Ask students to draw or model another equation (pan balance) that is equivalent to the first equation shown below. Explain why the model you created is equivalent to the original.

E.g.,

Answer:

Are there other ways of showing equivalent equations using other operations? Explain. (6PR4.1, 6PR4.2, 6PR4.3, 6PR4.4)

- Ask students to write an equation to represent the following situations:

  (i) Bethany is 3 years older than Toby. Toby is 21 years old. Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.

  (ii) There are 11 muffins on a tray. There were 24 at the start. Some have been eaten. How many muffins are missing from the tray? Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality. (6PR4.1, 6PR4.2, 6PR4.3, 6PR4.4)

- Draw or model another equation (two pan balance) that is equivalent to each equation shown below using multiplication. Explain why the model you drew/made is equivalent to the original.

  i) \( n + 4 = 6 \)

  ii) \( 2p + 2 = 4 \) (6PR4.3)

Resources/Notes

Math Focus 6
Lesson 5 (Cont’d): Equivalent Equations
6PR4
TG pp. 34 - 41

Other Resources:
National Library of Virtual Manipulatives
http://nlvm.usu.edu
Patterns can be used to solve problems. Previously, students have extended a pattern within a column of a table of values and developed a pattern rule to represent the relationship between/within columns in a given table of values. At this point, students should be encouraged to try different methods to solve a single problem. In some instances it may be more practical simply to extend the pattern in a column. In other cases students may find it easier to use a pattern rule (expression) to solve a problem. Students will now have to determine when it would be more appropriate to take each approach.

Present the following situation. Allow students time to explore solving this problem using counters before attempting to create a table. Using a table to record data will allow students to see patterns that can then be used to solve problems. Tell students that when Mary visits her grandparents she likes to go to the beach to collect beach rocks. She collected 12 on the first visit. Students can use the table below to help them figure out how many beach rocks Mary would have collected after 4 visits. Encourage students to explore, without continuing the table, how many rocks Mary would have collected after 8 visits to her grandparents.

<table>
<thead>
<tr>
<th>Visits</th>
<th>Number of Rocks Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

• Tell students that the average person’s hair grows 3 cm in one month. Ask students to create a table that would help them determine how much Dan’s hair would have grown after 6 months.

Journal

• Give students the following prompt:

   The distance from Irishtown to Pouch Cove is 720 km. A bus left Pouch Cove at 8:00 am travelling west at an average speed of 90 km/hr. A car leaves Irishtown at 9:00 am travelling east at an average speed of 120 km/hr. At what time will the bus and the car meet each other?

Sample solution: In order to solve this problem, students should create a table of values for each vehicle and compare these to solve the problem:

   The bus and car will meet at 12:00 noon.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 AM</td>
<td>0</td>
</tr>
<tr>
<td>9:00 AM</td>
<td>90</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>180</td>
</tr>
<tr>
<td>11:00 AM</td>
<td>270</td>
</tr>
<tr>
<td>12 noon</td>
<td>360</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hour</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 AM</td>
<td>0</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>120</td>
</tr>
<tr>
<td>11:00 AM</td>
<td>240</td>
</tr>
<tr>
<td>12 noon</td>
<td>360</td>
</tr>
<tr>
<td>1:00 PM</td>
<td>480</td>
</tr>
</tbody>
</table>

Resources/Notes

Math Focus 6

Lesson 6: Solving Problems Using Patterns

6PR1

TG pp. 42-45

Omit question 1 on p.26 of text.

Students have also had opportunities in lesson 3 to use a table of values to help them reveal a pattern and solve a given problem. You may wish to revisit some of the practice throughout this unit.

Math Game:

Rolling Equations

TG pp. 46 – 47
Data Relationships

Suggested Time: 3 Weeks
Unit Overview

Focus and Context
As students begin to understand ways of representing data, they will be ready to compare two or more data sets. Books, newspapers, the World Wide Web, and the media are full of displays of data, and by Grade 6, students need to learn to analyze these displays. Students should be able to compare the effectiveness of various types of displays (graphs) in organizing the data for further analysis or in presenting the data clearly to an audience.

Students should explore effective ways to gather their data and explore the most effective ways to display what they have found. This involves questioning techniques, and knowing the different types of graphs to display their data and various ways to organize their findings.

Once the data is organized, students should be encouraged to analyze the data, make inferences, comparisons and predictions.

Math Connects
Investigations involving data analysis offers a natural way for students to connect mathematics with other school subjects (Social Studies, Science, Physical Education, Health, etc) and with experiences in their daily lives.

Students need to know about data analysis in order to reason statistically and to develop skills necessary to become informed and intelligent citizens.

Through the study of data, students can learn that solutions to some problems depend on assumptions and they need to learn how to make inferences based on data.
### Process Standards Key

| [C] Communication | [PS] Problem Solving |
| [CN] Connections   | [R] Reasoning         |
| [V] Visualization |

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics and Probability (Data Analysis)</td>
<td>6SP1 Create, label and interpret line graphs to draw conclusions.</td>
<td>[C, CN, PS, R, V]</td>
</tr>
</tbody>
</table>
| Statistics and Probability (Data Analysis) | 6SP2 Select, justify and use appropriate methods of collecting data, including:  
• questionnaires  
• experiments  
• databases  
• electronic media. | [C, CN, PS, R, T] |
| Statistics and Probability (Data Analysis) | 6SP3 Graph collected data, and analyze the graph to solve problems.         | [C, CN, PS, R, T] |
| Shape and Space (Transformations) | 6SS8 Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs. | [C, CN, V] |
| Patterns and Relations (Patterns) | 6PR2 Represent and describe patterns and relationships, using graphs and tables. | [C, CN, ME, PS, R, V] |
Outcomes

Students will be expected to

6SP2 Select, justify and use appropriate methods of collecting data, including:

• questionnaires
• experiments
• databases
• electronic media.

[C, CN, PS, R, T]

Achievement Indicator:

6SP2.1 Select a method for collecting data to answer a given question, and justify the choice.

Elaborations—Strategies for Learning and Teaching

Give students opportunities to experiment with organizing and displaying data in a wide variety of ways. This can lead to discussions about which methods of data organization and display are the most effective and the easiest to understand.

In previous grades students were exposed to collecting information through first hand data and second hand data using questionnaires. Remind students that questionnaires are one way to gather information and brainstorm other methods for gathering information. They will come up with several examples from previous years of working with data. Some ideas include observation, surveys, interviews, polls, past records, searching the Internet and simulations.

Have a discussion about whether a questionnaire has multiple choice responses or Yes/No responses. Another way to gather information is through an interview, where the person doing the interview can probe for more information. Students may also suggest that they could also do an experiment or use databases and/or electronic media.

Write various questions on chart paper and have a discussion about whether or not these are good questions and justify.

Encourage students to think about what they already know about posing questions. Your questions could be related to Social Studies with reference to community – numbers of police officers, restaurants, recycling centres, etc.

Use teacher-created graphs or graphs from different sources (newspapers, Stats Canada, Social Studies textbook, etc) and ask students to generate questions based on that data. These questions will help students to think about the information on the graph and guide them in analyzing that information.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

**Journal**

- Ask students to write about:
  1. what they have learned about the different methods of data collection.
  2. which method for gathering information they find the easiest and explain why.
  3. what sample / data source they would use to determine the amount of milk an average grade 6 student would drink.
  4. where they might go to find out the number of school-aged children in their province. (6SP2.1)

- Ask students to write in their journals about some topics that interest them and explain why they are important. Students will refer to this information later. (6SP2.1)

- Ask students to write one question for which a questionnaire would be the best choice to find the answers to the question. Write one question for which some type of experiment would be conducted to find the answers to the question. Ask them which question they would prefer to solve and explain why. (6SP2.1)

**Resources/Notes**

*Math Focus 6*

**Lesson 1: Creating a Questionnaire**

- 6SP2
- 6SP3

TG pp. 13 – 17

*Additional Reading:*

Small, Marion (2008), *Making Math Meaningful to Canadian Students K-8*, pp. 472 - 513
## Strand: Statistics and Probability (Data Analysis)

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
</table>
| **6SP2 Continued**  
**Achievement Indicator:** 6SP2.2 Design and administer a questionnaire for collecting data to answer a given question, and record the results.  
**Brainstorm different topics for which they would like to create a survey or questionnaire. Ask students to select one topic on which they will gather information. They will create the survey questions or questionnaire, carry out the survey, and record the results. Where possible, students should be encouraged to carry out school-wide surveys or even community surveys. These surveys should be shared with the class. Discuss the various questionnaires and how whether their method of gathering information was effective.**  
**Remind students the importance of formulating a good question.**  
**Review the different types of graphing that are used to display data. Students have been previously exposed to bar and double bar graphs in Grade 5. In Grade 6 the focus will be 6SP1 - the line graph.**  
**Ask students to search magazines and newspapers or various websites (weather sites, Stats Canada) to find different examples of graphs (or you can have several examples of various types of graphs already prepared to show the students). Guide students in a discussion as to why the various types of graphs are used for particular data.**  
**“When students interpret graphs by others, they learn to appreciate the features that can help them make sense of a visual display of data. A good graph should communicate some overall impressions of the data to the reader at a glance. This goal is facilitated by the choice of the graph type that suits the data, clear labelling and titling, and accuracy in representing the data.” Small, Making Math Meaningful to Canadian Students K-8 (2008) p. 503.**  
**Remind students that different types of graphs are used to display different types of information. Some choices that they may offer are pictographs, Venn Diagrams, and bar graphs.**  
**Brainstorm several types of data that could be collected within the class, the school or even to the community. Then ask students to justify which type of graph that they would use to display that particular data.** |
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Performance

• Present the following situation to students:
  Carmen designed and handed out 100 questionnaires to students in her school. She asked this question:
  ‘What do you want to be?’
  - Doctor/Dentist - Teacher - Lawyer
  - Sports Manager - Coach
  50 questionnaires were returned. Here are the results:

<table>
<thead>
<tr>
<th></th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctor/Dentist</td>
<td>###</td>
<td>###</td>
</tr>
<tr>
<td>Teacher</td>
<td></td>
<td>###</td>
</tr>
<tr>
<td>Lawyer</td>
<td>###</td>
<td>###</td>
</tr>
<tr>
<td>Sports Manager</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carmen reached the conclusion that most students will become doctors or dentists. Do you agree with her conclusions? Explain. Describe what she might have done to improve:

• the wording of her question;
• the method of gathering data; and,
• the sample she chose to survey. (6SP3.1)

• Ask students to create a graph that compares two sets of data such as the number of pizza slices ordered by grade 5 and grade 6 students OR the number of books that students in Grade 5 and Grade 6 read over a 4 week period. (6SP3.1)

• Ask students to create a question that they would like to use for a survey. Sample Questions: What is your favourite genre of music? Favourite types of snacks / foods? After-school activities? How much time do you spend watching TV? How do you spend your leisure time? What is the most important issue that our community is currently facing? Etc.
  • Ask them to use the question to gather the information and record the results. (6SP2.2)

Resources/Notes

Math Focus 6
Lesson 1 (Cont’d): Creating a Questionnaire
6SP2
6SP3
TG pp. 13 – 17
Strand: Statistics and Probability (Data Analysis)

Outcomes

Students will be expected to

6SP3 Continued

Achievement Indicator:

6SP3.2 Solve a given problem by graphing data and interpreting the resulting graph.

Elaborations—Strategies for Learning and Teaching

Students need to be aware that graphs give us all types of information. It is one way of sharing information other than using the written language. Students have to analyze graphs to get the information they are looking for. Students, in pairs, could create a problem, collect the data and graph the results. Ask them to write 3 questions based on the graph for other groups to answer.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Performance

- Provide an untitled and unlabeled graph and ask students to come up with different sets of data that might realistically be represented by the graph.

E.g.,
- It might represent the number of students with each hair colour – red, brown, black and blond. (Using a scale of 2)
- The number of books read by 4 family members over the summer. (6SP3.1)

Journal

- Ask students to write about how they decide which type of graph best displays a set of data. Ask them to use examples in their explanations. (6SP3.1)

Resources/Notes

Math Focus 6
Lesson 1 (Cont’d): Creating a Questionnaire
6SP2
6SP3
TG pp. 13 – 17
Strand: Statistics and Probability (Data Analysis)

Outcomes

Students will be expected to

6SP2 Select, justify and use appropriate methods of collecting data, including:
- questionnaires
- experiments
- databases
- electronic media.

[C, CN, PS, R, T]

Achievement Indicators:

6SP2.3 Explain when it is appropriate to use a database as a source of data.

Elaborations—Strategies for Learning and Teaching

Students may have experience looking for information in an online database. Databases are used to store large amounts of information on various topics, from populations, birth and death rates to gathering information on lifestyles and careers.

Have a discussion about databases and what kinds of information would be contained within a particular database. Students need to know that a database is used for large amounts of data and that the type of database they choose to explore will depend on the type of question they want answered. (E.g., NHL, music data, Stats Canada) You could use a database to look for information from the past, or to look for information that covers a particular period of time.

Brainstorm topics where students might need to search a database to gather information. Allow students to visit particular databases so they can see how the information is stored, and how it is organized.

Ask students where they might search to find data about the number of school-aged children in their province.

6SP2.4 Gather data for a given question by using electronic media, including selecting data from databases.

Present students with a variety or topics for which they can research information. E.g., use the Weather Network website to investigate how the high and low temperatures in a given area have changed over the last 10 years.

Encourage the use of the Stats Canada website to find information on a given topic. A sample topic could be the number of immigrants who came to Canada in each of the last 5 years.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

**Presentation**

- Students can visit different databases such as Stats Canada, Guinness Book of Records, and the NHL Database. Provide opportunities for them to explore the sites and to discover the layout, to see how things are organized and to get used to navigating the database. Ask students to choose one database of interest, to collect some data that they are interested in and to share it with the class. (6SP2)

- Ask students to choose one of the following questions to answer by using an appropriate database.
  - How much has the moose population in Newfoundland and Labrador increased over the past 25 years?
  - What are some of the genres of Newfoundland musicians?
  
  Search online.

  Ask them to share their results with the class. (6SP2.4)

**Performance**

- Ask students to create a graph that shows the growth of the Canadian population over a period of 20 years. Discuss the various databases that could be accessed to find the information. (6SP2.4)

- Ask students where they would look if they wished to find out information about the population growth of their town or city, or the nearest centre to them, or a city they would like to visit. (6SP2.3)
Strand: Statistics and Probability (Data Analysis)

Outcomes

Students will be expected to

6SP2 Continued

Achievement Indicator:

6SP2.5 Answer a given question by performing an experiment, recording the results and drawing a conclusion.

Elaborations—Strategies for Learning and Teaching

The focus is on using experiments to collect data.

Have a discussion about experiments and when and why we would carry out experiments. Most students would think of Science experiments, which explain particular scientific processes.

Make the association that experiments can be conducted to investigate concepts such as which brands of particular products are best (e.g., which paper towel absorbs the most water).

We can use experiments to gather information. We can then analyze this information to make choices or to determine if one factor affects another (the age of a hockey player to the number of goals scored in a season). Students need to look at graphed data and be able to make inferences and draw conclusions about the information. For this performance indicator, you could show the students several graphs that show the results of experiments and ask them to answer specific questions about what they see.

Sample questions:

• What do you notice about the results? Is there another method you could use to answer the same question?
• Why was this experiment a good method to find out the answer to your question?
• Why would you not use another method of collecting your data?
General Outcome: Collect, Display and Analyze Data to Solve Problems

### Suggested Assessment Strategies

#### Performance

- Working in pairs, ask each student to roll a die 25 times. Record how many times each number occurs and then combine the individual’s results with those of her/his partner. You can inform students that this is an example of an experiment. You can ask students to draw conclusions from this experiment. E.g. the number 4 was rolled the most often.  
  
  \[ (6SP2.5) \]

- Give students a choice of questions that could be answered by performing experiments. Ask them to conduct the experiments to answer the questions. Some possibilities are:
  - If you rolled a pair of dice 10 times, how many times would a double number show up?
  - If you used a spinner that is colour coded and spun it 20 times, how many times would you land on a particular colour?
  - If you flipped a coin 50 times, how many times would TAILS show up?  
  
  \[ (6SP2.5) \]

#### Portfolio

- Ask students to design an experiment to answer a question that is important or of interest to them. Discuss the student-created questions in class and ask them to determine if the experiment answers the question(s) investigated and whether changes may need to be made. Refer to journal activity for topics of interest that students created earlier.  
  
  \[ (6SP2.5) \]
Strand: Statistics and Probability (Data Analysis)

Outcomes

Students will be expected to

6SP3 Continued

Achievement Indicator:

6SP3.1 Continued

Elaborations—Strategies for Learning and Teaching

Students often have little difficulty with creating graphs; however, they may be challenged to analyze the information depicted in the graphs. Continue to provide opportunities for students to practice making graphs from previously collected data. Present them with some data that you create and ask them to graph it. Students should now be able to justify the reasons why they create the graph as they do. Students will need to be reminded to choose an appropriate scale and an appropriate graph.

Students can work in pairs to analyze the graphs – asking questions about what the graph shows and what kinds of predictions they can make from the data.

Consider writing some questions on the board to guide their thinking. Some possible choices for discussion are:

- State the facts that the graph shows.
- Which element on the graph is the greatest?
- Which element on the graph is the least?
- What trend does the graph show?
- Can you make a prediction based on the information provided in the graph?
- What could influence the trend in the data?
- Pose WHAT IF types of questions.

Students should be able to justify why they select one type of graph over another. There are many reasons as to why one type of graph is better suited.

Have a discussion with the class on the benefits of using one type of graph over another. Ask them which type of graph they prefer and ask them to explain why. Ask students to explain which type of graphs they see the most in magazines and newspapers and explain why they think that is the case.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Performance

- Ask students to answer the following question by carrying out the experiment:

  What is the relationship between the 'drop height' and the 'bounce height' of a ball? Make a prediction first before you do the experiment. Conduct trials to gather the data and to answer the question. Create a graph of the results and answer the questions: What can you say about the relationship between drop height and bounce? Was your prediction correct? Explain.

  (Make sure that the ball is simply released and let the ball bounce on a hard surface. Try heights other than the ones that are in your experiment to confirm the results).  

- Ask students to work in groups. On separate sheets of chart paper, write the names of types of graphs: bar graph, double bar graph, line graph, pictograph and Venn Diagram. Ask them to create two columns on each sheet: Column A - Advantages and Column B - Disadvantages. After they complete the chart, you may wish to ask them to share the results with the class.

  Extend this activity by adding the kinds of questions that could be used for each type of graph. E.g. a bar graph is best used to graph comparisons, while a double bar graph can be used to compare the same information, which contains two distinct parts (e.g., male and female, cats and dogs, etc.) A Venn Diagram could be used to show how things are the same, as well as how they are different, etc.

Resources/Notes

Math Focus 6
Lesson 3 (Cont’d): Performing an Experiment
6SP2
6SP3
TG pp. 22 – 27

Children's Literature (provided):
Scieszka, Jon and Smith, Lane.
Math Curse

Refer to pages 8-9 which shows a bar graph and makes reference to children's birthday by month.
### Strand: Shape and Space (Transformations)

**Outcomes**

*Students will be expected to*

- **6SS8** Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs.

  [C, CN, V]

**Elaborations—Strategies for Learning and Teaching**

Students need to develop an understanding of ordered pairs as this will be a prerequisite for creating line graphs a little later. Have a discussion about where in everyday life and in what professions we use grids. (GPS Systems, shipping lanes, mapping, etc). This will give students some frame of reference for using grids.

Use the game of Battleship to teach coordinates. Without explaining what coordinates are, let the students play the game. Afterwards, talk about how they found the points that were given. They will discover that if they go over a number of places and up a number of places, they will find the coordinates. The students need to be exposed to the idea that these coordinates are called an ordered pair.

They should know that the first number in an ordered pair tells the horizontal distance from the origin and the second number in the ordered pair tells the vertical distance from the origin.

When describing an ordered pair, the pair is represented in alphabetical order. E.g., (x, y).

It is important that you draw attention to the Cartesian Plane. Students will be working with one quadrant of four on the Cartesian Plane, making the link to the notion that a Cartesian Plane is created when two lines, one horizontal and one vertical, meet at a point called the origin.

Lead a discussion on the concept of a coordinate grid, focusing on the visual aspect of the grid. Explain to the students that a coordinate grid is another name for the quadrants of a Cartesian Plane. It is used to locate points on the plane.

Model for students that in order to construct a picture of a coordinate grid, they need to draw a horizontal number line, called the x-axis and a vertical number line called the y-axis. These two lines intersect at a point called the Origin (0, 0). Use a coordinate grid on the board while you discuss these concepts with the class. Be sure to label the axes accurately. A common error students make when labelling the axes is putting the number in the middle of the blocks. This causes problems when plotting points.

Refer the students back to the game of Battleship and how students used the coordinates to locate the ships of their opponents. Also, you may discuss that the playing board of the Battleship game is the same as the first quadrant of the Cartesian Plane.

---

**Achievement Indicators:**

- **6SS8.1** Label the axes of the first quadrant of a Cartesian plane, and identify the origin.

- **6SS8.2** Plot a point in the first quadrant of a Cartesian plane, given its ordered pair.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

• Battleship - Ask students to change the scale on the horizontal (x-axis) and the vertical (y-axis) axes to other numbers and letters. Give them a strip of paper to create their scale and tape to the side of the game. Use the “new” coordinates to locate the ships. (6SS8.1)

• Twister - Using masking tape, create an x- and y-axis and number it 10 x 10 (use your floor tiles as a guide to your grid). Prepare two sets of number cards, 0 through 10. Place one set in a bag labelled X and the other Y. You will pull out a number from each bag to form a coordinate point (Replace it when you are done for the next round). Using a spinner, divide four equal sections and name each one as either right hand, left hand, right leg or left leg. Spin the spinner to find out which hand or foot must remain on that point. You could divide the class into two teams to help with the large scale of the grid and allow each team to work together to touch each of their coordinate points. If a student “falls” or moves from his coordinate point, the other team wins a point. Any student not wanting to participate could plot each team’s coordinates in alternating colors on a Cartesian Plane. (6SS8.2)

Paper and Pencil

• Ask students to practice locating points along the x-axis and the y-axis. One of the coordinates of the ordered pair will be zero. They need to make the connection that if one coordinate is 0, the point will need to be plotted on one of the axes. Present students with a grid showing points labelled on the x- and y-axis and ask the students to provide the coordinates for the labelled points. (6SS8.2)

Resources/Notes

Math Focus 6
Lesson 4: Plotting Points on a Grid
6SS8
TG pp. 28 - 32
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS8 Continued

Achievement Indicators:

6SS8.3 Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.

6SS8.4 Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5 or 10 on its axes, given whole number ordered pairs.

6SS8.5 Draw shapes or designs, given ordered pairs, in the first quadrant of a Cartesian plane.

Elaborations—Strategies for Learning and Teaching

Explain to students that sometimes an ordered pair on a quadrant is given a letter name to identify the point in a quadrant. That is, a letter name is used in place of the coordinate pair. You may want to ask students to plot points on a grid and then give them letter names. This can be completed as a whole group using the overhead projector or interactive white board.

The students are now familiar with finding and plotting points on a Cartesian Plane. Students will learn to draw designs, shapes or block letters on a grid. Encourage students to be creative in their design. They will plot the points then connect them to complete the image. As an extension, they can give other students the coordinates for the points on their image and have others create the same image on their own grid.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

*Performance*
- Give students a blank grid on which the axes have been labelled. Ask the students to randomly place 10 points anywhere on the grid. Call out points at random, and if the students have that point on their grid, they mark an X through that point. The first student to have all of their points marked with the X is the winner! (6SS8.2)

- Give students a grid that has points already labelled with letters as shown below. Ask students to find the letter on the grid represented by each ordered pair. Record the letters, in order, to figure out the message. (6SS8.3)

(1, 4) (9, 9) (6, 2) (0, 5) (8, 1) (3, 3) (10, 5) (2, 8) (7,10)

*Journal*
- Ask students to explain, using words, numbers, and/or pictures, how to use ordered pairs to describe and locate points on a grid. (6SS8.3)

*Presentation*
- Give students a coordinate grid to plot the points listed below and join to them in order. The last point should be joined to the first point. Ask students to describe the figure they have drawn to the class.

A(2,2) B(5,3) C(8,2) D(7,5) E(9,8) F(6,7) G(5,10) H(4,7) I(1,8) J(3,5)

(6SS8.5)
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS8 Continued

Achievement Indicators:

6SS8.6 Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.

6SS8.7 Draw shapes or designs in the first quadrant of a Cartesian plane, and identify the points used to produce them.

Elaborations—Strategies for Learning and Teaching

Students need to focus on the distance between points along each of the horizontal and vertical lines. They often make the mistake when counting spaces to include the points of each square, instead of the number of squares between the points. Some practice may be needed.

Relate the movement along the axis as being similar to making jumps along a number line.

Give grids to the students with several points along horizontal and vertical lines. Ask them to count the distance between these points. They can work in pairs. This may not take a lot of practice but it is important that they understand how to measure the distance.

Using the interactive white board or an overhead projector, plot points on a grid and join the points together to create a closed figure. Ask students to label the points and to name the figure. You could also recite coordinates and ask students to plot the points on their own grid paper.

When you join points together on the graph, you are creating a line segment. Draw the students’ attention to this fact. We can use line segments to measure the distance between two points on a Cartesian Plane.

This is a good way to encourage interaction and to motivate the class to focus on points on the grid and finding coordinates easily.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

• Give students a coordinate grid with axes labelled from 0 to 10. Ask students to plot each pair of points on the grid, join the points with a line segment and find the length of each line segment.
  (i) (4,2) and (7,2)
  (ii) (5,7) and (10,7)

Students can then work with a partner to share pairs of points and find the distance between them. (6SS8.6)

• Give students a grid with the x- and y-axes labelled and with various shapes drawn on it. Ask students to name the coordinates of each shape. Extension: Draw partial closed figures and ask students to complete the figure and label its coordinates. (6SS8.7)

Resources/Notes

Math Focus 6
Lesson 4 (Cont’d): Plotting Points on a Grid
6SS8
TG pp. 28 - 32
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR2 Represent and describe patterns and relationships, using graphs and tables.

[C, CN, ME, PS, R, V]

Elaborations—Strategies for Learning and Teaching

Students are already familiar with working with tables of values. In this unit students will represent a pattern using a line graph. Line graphs are new for Grade 6 students. A line graph is used when there is a numeric value associated with equally spaced points along a continuous number scale. Points are plotted and a line is drawn to connect the points. Every point on the line should have a value.

Achievement Indicators:

6PR2.1 Create a table of values from a given pattern or a given graph.

6PR2.2 Describe, using everyday language, orally or in writing, the relationship shown on a graph.

As well as creating graphs from a table of values, students need to be able to create a table of values from a pattern or graph.

Students should be presented with graphs and expected to create tables of values from the graphs. They will need to look for the patterns within the table of values and make the connection between that and the graph. Students should not only be able to create graphs, but they need to be able to explain what the graph shows. They need to be able to explain what information they get from the graph and what questions can be answered by looking at the graph.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

**Performance**

- Present students with tables and graphs and ask them to look for patterns. Keep the graphs and tables simple so that the pattern can be readily seen. *(6PR2)*

- Tell students that a relationship can often be found between two different body measurements, such as the measurement around a person's head to his or her height. Brainstorm, with students, some key questions that might guide the investigation of body relationships? Can they predict what conclusion might be expected for each? Ask them to write and carry out a plan to answer one of the questions. Include information on the following:
  - sources of your data;
  - sample size and type of people studied;
  - method of data collection. *(6PR2.2)*

- Give students multi link cubes and ask them to create an odd number pattern – starting with one cube, then add 2 each time with one to the bottom right and one on the top (L shape). The L shape will increase each time. Ask students to create a table of values and graph it. *(6PR2.1)*

**Journal**

- Ask students to write about as many ways in everyday life, as they think of, where patterns can be seen. Encourage them to use examples in nature as well as those they are able to identify such as in buildings, clothing, games, decorating, etc. *(6PR2)*

**Presentation**

- Students can be involved in collecting data from the class on the number of hours each person in their family sleeps each night for a week. At the end of the week, ask students to present their findings to the class in a graph. Ask students to create three questions for classmates to answer by looking at the graph. Students may want to share their results with other classes in order to talk about the importance of sleep (cross curricular – also meets Health outcomes). Extension: Students may survey other classes to see if most students get the recommended amount of sleep. *(6PR2.2)*

Resources/Notes

* Math Focus 6
  - Lesson 5: Interpreting Line Graphs
  - 6PR2
  - 6SP1
  - TG pp. 36 - 40

* Additional Reading:

* Curious Math: A Graph is Worth a Thousand Words
  - 6SS8
  - TG pp. 41 - 42

* Math Game: Wei-Chi
  - 6SS8
  - TG pp. 43 - 44
Strand: Statistics and Probability (Data Analysis)

Outcomes

Students will be expected to

6SP1 Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V]

Achievement Indicator:

6SP1.1 Determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs.

Elaborations—Strategies for Learning and Teaching

Students often see things differently and they may not use the same scales and/or titles for the same graphs. The information may be the same but the way in which they represent it may be different.

Discuss the term ‘attribute’ with the class and identify the attributes they may need to consider when working with graphs (title, axis, and intervals). Discuss with students that they may have to adjust the attributes they are using to fit the data they are analyzing. Ask them if the attributes that they have chosen are appropriate and ask them to justify their choices.

The scale will determine the size of the bars on a bar graph and likewise, the lines on a line graph will be different depending on the scale they have used.

Make sure that you discuss the importance of scale to the students. A scale has to represent the data accurately. Remind students that the scale is related to number lines. Look at the smallest value and the largest value and then determine the scale.

E.g.

![Annual Science Fair Participation graph](image)

Ask students to determine why each graph is different and which one has the best scale to show the trend.

Tell them that an inappropriate scale can skew (distort or depict unfairly) the data and be misleading. Provide other examples to reinforce the importance of scale. Stress to the students that although different graphs can show the same data, one graph may be a better choice to answer a particular question.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

**Student-Teacher Dialogue**

- Give students data on a topic of your choice and ask them to graph it. The scale, the title and the intervals may be different among students. Ask students to explain and justify why they used the attributes they did (the scale, the intervals and the title). (6SP1.1)

<table>
<thead>
<tr>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td><strong>Lesson 5 (Cont’d): Interpreting Line Graphs</strong></td>
</tr>
<tr>
<td>6PR2</td>
</tr>
<tr>
<td>6SP1</td>
</tr>
<tr>
<td>TG pp. 36 - 40</td>
</tr>
</tbody>
</table>
**Strand: Statistics and Probability (Data Analysis)**

**Outcomes**

*Students will be expected to*

6SP1 Continued

<table>
<thead>
<tr>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to now, students have been working mainly with bar graphs and double bar graphs. Expose students to the fact that the horizontal axis is called the x-axis and the vertical axis is called the y-axis as this is the terminology that is used in Grade 7.</td>
</tr>
<tr>
<td>Line graphs are used to show trends in data, usually over time. The points are plotted to show relationships between two variables (one is usually time) and the points are joined with a line to make it easier to see trends. Like bar graphs, line graphs have a title, they are labelled and they use a clear scale.</td>
</tr>
<tr>
<td>Provide opportunities for students to learn the difference between continuous data and discrete data. If the data are continuous, the points on the graph are joined. The points on a line are connected when all the values between the points are permitted. Discrete data is a series of points that are not joined. When data are discrete, there are numbers between those given that are not meaningful in the context of a problem. For example, consider the points (1.3) and (2.6) plotted on a graph. These points can be joined if they represent distance against time, since distance could include values between 3 and 6 and time could include values between 1 and 2, such as 1.5. However, if the graph represents costs against the number of DVDs rented, the points should not be joined since it is not possible to rent 1.5 DVDs.</td>
</tr>
<tr>
<td>• Examples of discrete data – the temperature of a given day at each hour of the day. You can predict what the temperature would be at 9 PM based on the temperature trend over the day.</td>
</tr>
<tr>
<td>• Examples of continuous data – the temperatures of a particular place over a month. You cannot predict the temperature for tomorrow based on the temperature for the previous days because it changes.</td>
</tr>
<tr>
<td>After you have exposed students to various graphs, ask them to create their own.</td>
</tr>
<tr>
<td>Students should look at and analyze various line graphs that display data on different topics. They should distinguish between continuous and discrete graphs.</td>
</tr>
<tr>
<td>Have a whole group discussion about the differences between continuous and discrete data and when to use each type of graph.</td>
</tr>
<tr>
<td>Remind students that graphs can be used to see trends, to find out facts, or to determine information on many different topics. Provide students with several line graphs that are titled and ask them to interpret the data. The graphs should be pre-made and posted in the classroom for reference.</td>
</tr>
</tbody>
</table>

**Achievement Indicators:**

6SP1.2 Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data), and explain why.

6SP1.3 Create a line graph from a given table of values or a given set of data.

6SP1.4 Interpret a given line graph to draw conclusions.
General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Performance

- Give students several examples of data. They have to determine whether they would graph the information using continuous or discrete data. Examples of data:
  
  (i) The amount of recycling that each class does over a month.
  
  (ii) Population trends of our province over the last 20 years.
  
  (iii) The number of students who are absent in school for the month.
  
  (iv) How much homework you have throughout the month.

  \( \text{(6SP1.2)} \)

- Physical Education connection - Ask students to record how long it takes them to run 5m, 10m, 15m, 20m, 25m and 30m in gym class and bring that data to math class.

  Model how to plot that information on a line graph. Ask the students what they see in the graph – what conclusions they can make. Ask the class if this is discrete or continuous data.

  Ask students to create their own line graph using this information.

  \( \text{(6SP1.3)} \)

- Students may enjoy interpreting line graphs that tell a story. Ask students to describe the graph below that shows Marc’s hike.

  \( \text{(Small, 2008)} \) \( \text{(6SP1.4)} \)

Presentation

- Use the graph to show that we can predict the number of students who will participate in the next Science Fair.

  \( \text{(6SP1.4)} \)

Resources/Notes

\textit{Math Focus 6}

\textit{Lesson 5 (Cont’d): Interpreting Line Graphs}

\textit{6PR2}

\textit{6SP1}

TG pp. 36 - 40

Additional Reading:


\textit{Math Focus 6}

\textit{Lesson 6: Constructing Line Graphs}

\textit{6PR2}

\textit{6SP1}

TG pp. 45 - 49
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR2 Continued

Achievement Indicator:

6PR2.3 Translate a pattern to a table of values, and graph the table of values (limited to linear graphs with discrete elements).

Elaborations—Strategies for Learning and Teaching

Students need to make a connection between the information on a graph and the table of values. They need practice in transferring the information on a graph to a table of values and vice versa. Focus on students’ ability to explain how to collect, record and display the data appropriately.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

**Performance**

- Give students a table of values and ask them to create a graph with the same information. They will need to label each axes in the same way as the table is labelled. Then do the reverse – give them a graph and ask them to create a table of values. Some simple data that could be used are (a) time and distance information and (b) growing patterns with multi-link cubes.  
  \[(6PR2.3)\]

- Ask students to create a graph to display the relationship between the number of tricycles and the number of wheels. They will discover that as the tricycles increase in number, the wheels will increase by 3. They should represent this data in a table of values as well.  \[(6PR2.3)\]

- Present a graph drawn on the board and have a discussion about what it shows. Label the graph and give it a title. Guide the discussion with questions about what this graph tells us. Drawing conclusions from the graph may be a challenge, but encourage your students to look beyond what the graph shows and make an inference. Use the following graph to guide discussion.

\[\text{Graph: Cost of Rides at an Amusement Park}\]  
\[(6PR2.3)\]

Resources/Notes

*Math Focus 6*

*Lesson 6 (Cont’d): Constructing Line Graphs*

*6PR2*

*6SP1*

*TG pp. 45 - 49*
Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR2 Continued

Achievement Indicator:

6PR2.2 Describe, using everyday language, orally or in writing, the relationship shown on a graph.

Elaborations—Strategies for Learning and Teaching

The focus is on expanding the students’ ability to explain what they see in a graph, rather than creating the graph. If they are able to explain relationships involving data shown on graphs, they are better able to make connections. Encourage each student to explain everything about the data and the relationships that are shown. They need to draw on previous knowledge to make those connections clear.
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

• After looking at graphs, encourage class discussions about what the graph tells them. Ask students to look at more graphs individually. You can use the following questions as prompts to help guide the students’ thinking.

Some guiding questions or prompts to help students with their explanations are:

• Why do you think that?
• Would it also be true if….?
• Could there be a different answer?
• How did you figure that out?
• What strategies did you use to…?  

Resources/Notes

Math Focus 6
Lesson 7: Communicating about Data
6PR2
6SP2
6SP3
TG pp. 50 - 54
Motion Geometry

Suggested Time: 2 Weeks
Unit Overview

Focus and Context

As students work through activities involving motion geometry, they will have opportunities to use their understanding of transformations to identify, perform, and describe these transformations and use these to solve problems. The focus for students is on the ability to visualize, perform and describe the positional change of the vertices for single and successive transformations in the first quadrant of a Cartesian plane. This unit builds on students’ knowledge of plotting ordered pairs, as was introduced in previous work on Data Relationships. Transformational geometry was introduced in Grade 5, where students worked with single transformations. In Grade 6, students will be introduced to combining transformations in the first quadrant of the Cartesian plane. It is important to make use of manipulatives such as pattern blocks, Miras, overhead projectors, and grid paper. If available, use of the interactive white board and interactive websites can be very effective in helping students visualize the various transformations.

Math Connects

Geometry is an important aspect of the mathematics curriculum as it helps students relate to their physical surroundings. The activities and concepts in this unit will help students further develop their spatial sense, including the ability to mentally visualize objects and spatial relationships. Being able to visualize the orientation and movement of shapes is important as we use it in everyday life when driving, rearranging furniture, and map reading. Developing geometric thinking students will improve knowledge of geometric forms and help develop an appreciation for art, nature and architecture. Skills in describing and predicting location will help develop spatial sense which is also a necessary component in many career paths such as engineering, design, and carpentry.
### Process Standards Key

<table>
<thead>
<tr>
<th>[C]</th>
<th>Communication</th>
<th>[PS]</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>[CN]</td>
<td>Connections</td>
<td>[R]</td>
<td>Reasoning</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[V]</td>
<td>Visualization</td>
</tr>
</tbody>
</table>

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape and Space (Transformations)</td>
<td>6SS6 Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image.</td>
<td>[C, CN, PS, T, V]</td>
</tr>
<tr>
<td>Shape and Space (Transformations)</td>
<td>6SS7 Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.</td>
<td>[C, CN, T, V]</td>
</tr>
<tr>
<td>Shape and Space (Transformations)</td>
<td>6SS9 Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).</td>
<td>[C, CN, PS, T, V]</td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS9 Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).

[C, CN, PS, T, V]

Elaborations—Strategies for Learning and Teaching

In Grade 5, students described and performed single translations, reflections and rotations. In Grade 6, students will continue this work but identify the coordinates of the vertices of a 2-D shape in the first quadrant of a Cartesian plane and describe the positional change in the vertices as a result of a transformation. Work on plotting points in the first quadrant of the Cartesian plane was done in the Data Relationships unit. This knowledge base will be important as students learn about transformations. If this topic has not been introduced previously, it would be necessary to cover the material before starting work on this particular outcome.

Students will have greater success if they are able to visualize the transformations before they perform them. To encourage this, ask students to predict the location of the transformed image before they perform the transformation.

Generate a discussion about orientation and how it is affected by each of the three transformations. (E.g., In a translation, the shape moves without rotating or resizing. Every point of the shape moves the same distance and in the same direction. It’s orientation does not change). When focusing on translations, discuss key words such as horizontal, vertical, diagonal, etc. Note that the terms up, down and across are equally acceptable.

Students may find it helpful when performing transformations to use tracing paper, a small piece of overhead plastic and dry-erase marker or a traced and cut out image of the shape they are transforming to help perform each transformation.

As students begin working on various transformations, they will benefit from working with ‘hands on’ materials such as pattern blocks or attribute blocks to physically manipulate each block as indicated by the transformation. When using symmetrical shapes to apply transformations, it may be a good idea to highlight or mark one of the vertices so students can indicate the orientation of the image. Students should also be given ample opportunities to work with less symmetrical shapes where it is easier to identify the effect of the transformations.

Students will begin their study of transformations by learning about translations. A translation is a slide where the shape being translated does not change its direction or orientation. Students should already be familiar with key terms such as coordinate plane, ordered pairs, origin, x-axis (horizontal axis), y-axis (vertical axis), x-coordinates and y-coordinates from previous work on data relationships. Continued use of this terminology is very important.

Achievement Indicator:

6SS9.1 Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).

(continued)
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td></td>
</tr>
<tr>
<td>• Provide a 2-D shape on grid paper and ask students to translate the shape according to specific instructions. E.g., 4 units left and 1 unit down. Ask students to identify the vertices of the image. (6SS9.1, 6SS9.2)</td>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 1: Translating Shapes</strong></td>
</tr>
<tr>
<td></td>
<td><strong>6SS9</strong></td>
</tr>
<tr>
<td></td>
<td><strong>TG pp. 13 – 17</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Additional Reading:</strong></td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS9 Continued

Achievement Indicators:

6SS9.1 Continued

Elaborations—Strategies for Learning and Teaching

Model plotting and identifying the coordinate points corresponding to the vertices of a given 2-D shape. Students are expected to identify the coordinates of the vertices of shapes drawn on a coordinate plane. Identify the coordinates of vertex A. Vertex A is named by (2,3).

A common error when identifying and plotting points is to reverse the order of the x-coordinate and the y-coordinate. Encourage students to always label the x- and y- axes of a Cartesian plane to avoid making this mistake. Remind students that ordered pairs are communicated in alphabetical order (x, y).

Ask students to play a Hide and Seek game where they draw a shape on their Cartesian plane and hide it from their partner. In turn, each student will say a coordinate trying to find the vertices of the shape. The player to find all vertices, wins.

6SS9.2 Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image (limited to the first quadrant).

When identifying a translation remind students that:
- the 2-D shape and its image are congruent
- the 2-D shape and its image have the same orientation (the vertices of the translated image will be in the same relative position as the original image.

Remind students to label the vertices of the shape (e.g. A, B, C, D) and the corresponding vertices of the reflected image (A’, B’, C’, D’). A’ is read as A prime.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

- Provide students with a Cartesian plane. Ask them to draw a three-sided figure on the plane. Tell them to choose another coordinate in the plane that the image is not already on. Tell them this new point is now a vertex of the translated image. Ask them to show and explain how they would know what the other coordinates of their translated image would be just by knowing one of the vertices. Ask students to describe the positional change of the vertices after the translation.

(6SS9.2, 6SS9.3)

Resources/Notes

Math Focus 6
Lesson 1 (Cont’d): Translating Shapes
6SS9
TG pp. 13 – 17
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS9 Continued

Achievement Indicators:

6SS9.3 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to the first quadrant).

Elaborations—Strategies for Learning and Teaching

When describing the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a translation, students should keep in mind the following:

- the shape and its image will have the same orientation
- all vertices move together
- each vertex moves the same way
- if the translation is:
  - to the left, the x-coordinate will decrease
  - to the right, the x-coordinate will increase
  - downward, the y-coordinate will decrease
  - upward, the y-coordinate will increase

Based on the transformation shown here, students should be able to answer questions such as the following:

“Describe the translation.”
- Left 4, up 3

“Describe the change in the x-coordinates of the vertices.”
- They decreased. They are 4 less.

“Describe the change in the y-coordinates of the vertices.”
- They increased by 3.

Use masking tape to create the x- and y-axis of the first quadrant coordinate grid on the classroom floor (use floor tiles as the grid if applicable). Label the x- and y-coordinates for each axis on the masking tape. Ask students to create a shape by standing on the vertices of the shape. To create the sides of the figure have the first student hold the end of a ball of yarn and toss it to the next student and so on, until the shape is complete. Give students a transformation rule and ask students to predict their new position on the grid. Finally, ask students to who are not part of the shape direct the students to where their new position should be. The students can now move or another group of students can move onto the grid to form the image.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Journal</strong></td>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td>• Ask students to describe how the translation rule can help them</td>
<td><strong>Lesson 1 (Cont’d): Translating</strong></td>
</tr>
<tr>
<td>identify the positional change of the vertices.</td>
<td>Shapes</td>
</tr>
<tr>
<td></td>
<td><strong>6SS9</strong></td>
</tr>
<tr>
<td></td>
<td><strong>TG pp. 13 – 17</strong></td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS9 Continued

Students will continue their study of transformations by learning about reflections. Students have had prior experience reflecting images in Grade 5. At this point, reflections should now be performed in the first quadrant of the Cartesian plane. Ask students to reflect a shape over four different lines (up, down, left and right) and ask them to compare and contrast the four different images created. Students should see that no matter where the shape is in relation to the line of reflection, the image is always the same distance from the line as the original shape. Some students may think that a reflection can only be horizontal or vertical. They may not recognize that a reflection can also be diagonal. It is important to use illustrations of other reflections as the topic is being discussed.

Achievement Indicators:

6SS9.2 Continued

Providing students with 'hands on' activities where they actually manipulate objects to reflect them will help students visualize the effect of this type of transformation on the object. Using pattern blocks, model the act of reflecting the object by 'flipping it' on the line of reflection. Students can trace these objects and the resulting image. The use of Miras can also be helpful to students when performing reflections, especially when working with diagonal reflections. Review Mira use with students as they should already be familiar with them, modelling how a reflected image can be produced. As students reflect images, they must label each of the vertices of the image and be able to name the ordered pair for each.

6SS9.3 Continued
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

**Performance**

- Ask students to reflect ABC using the given line of reflection. Ask them to describe the position and orientation of the reflected image and justify why it is correct.

![Diagram of triangle ABC with line of reflection](image)

- Tell students that \( \triangle ABC \) with coordinates A(1,5), B(0,2) and C(4,0) is reflected. The resulting image has vertices (6, 2), (4, 6) and (1, 5). Ask students to determine where the line of reflection is located and to write the coordinates of 2 different points on the line of reflection.

*(6SS9.2, 6SS9.3)*

**Resources/Notes**

*Math Focus 6*

Lesson 2: Reflecting Shapes

6SS9

TG pp. 18 – 21
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS9 Continued

Achievement Indicators:

6SS9.2 Continued

6SS9.3 Continued

Elaborations—Strategies for Learning and Teaching

When describing the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a reflection, remind students that:

- the shape and its image are of opposite orientation
- a 2-D shape and its image are congruent
- there is an equal distance from the mirror line to both the 2-D shape and its reflected image
- when reflecting a 2-D shape across a horizontal line of reflection, the x-coordinates of the vertices do not change, but the y-coordinates do.

- when reflecting a shape across a vertical line of reflection, the y-coordinates of the vertices do not change, but the x coordinates do.

- when reflecting a 2-D shape across a diagonal line of reflection, both the x- and y-coordinates of the vertices change.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

**Pencil and Paper**

- Parallelogram Reflection - Students can work in pairs. Provide them with a similar copy of a coordinate grid and a table for recording vertices.

  ![Coordinate Grid]

  Ask students to do the following:
  
  (i) Reflect Parallelogram ABCD using each reflection line and draw the image each time.
  
  (ii) Label the vertices of each image and record them in the table.
  
  (iii) Describe the distance of the image from the line of reflection.
  
  (iv) Describe the orientation.
  
  (v) Examine the table and describe any changes in the vertices when comparing the original shape to each image.

  \[(6SS.9.1, 6SS.9.2, 6SS.9.3)\]

Resources/Notes

**Math Focus 6**

**Lesson 2 (Cont’d): Reflecting Shapes**

6SS9

TG pp. 18 – 21
Strand: Shape and Space (Transformations)

6SS9 Continued

Outcomes

Students will be expected to

Elaborations—Strategies for Learning and Teaching

The next transformation students will be studying is rotation. When students describe a rotation, their description should include the amount of rotation, the direction of turn and the center of rotation. Rotations can be described in terms of degrees (E.g., 90° turn and 180° turn) or fractions (E.g., $\frac{1}{4}$ turn and $\frac{1}{2}$ turn). When describing the direction of rotation students should be using the terms clockwise and counter clockwise.

To help students solidify their understanding of rotations, ask them to use their own bodies to explore turns. A half turn (180º) means turning from front to back. A quarter turn (90º) means turning right or left. Placing a large wooden protractor on the floor may also help students rotate accordingly.

Achievement Indicators:

6SS9.2 Continued

In Grade 5, students rotated shapes about a vertex of that shape. In Grade 6, students will rotate images about a centre of rotation on a vertex, outside the shape and within the shape.

Students need a lot of ‘hands on’ experiences performing rotations to become comfortable with this particular transformation as they will later use these skills to combine various transformations on a given shape.

Using tracing paper or overhead transparencies, ask students to trace the shape on the paper and place a dot on the point of rotation. Place the tip of a pencil on the point of rotation and turn tracing paper the indicated direction and amount for that particular rotation to see the position of the rotated image. Students can then transfer the traced shape on the grid.

6SS9.3 Continued

When describing the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a rotation, students should keep in mind the following:

- all vertices move together $\frac{1}{4}$ (90º), $\frac{1}{2}$ (180º), or $\frac{3}{4}$ (270º) of a turn in the same direction, either clockwise or counter clockwise
- the shape and its resulting image are congruent
- the orientation of the shape and its image are different
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

**Paper and Pencil**

- Provide students with coordinate grid paper and colored pencils.
  On the grid paper ask students to draw trapezoid ABCD with vertices A(2, 4), B(9, 4), C(9, 8) and D(4, 8).
  Ask students to draw the image of the trapezoid each time and color it as indicated.
    - Rotate trapezoid ABCD $\frac{1}{4}$ ($90^\circ$) turn clockwise about C(9, 8) and color the image blue.
    - Rotate trapezoid ABCD $\frac{1}{2}$ ($180^\circ$) turn about a point at (11, 9) and color the image red.
    - Rotate trapezoid ABCD $\frac{3}{4}$ ($270^\circ$) turn counter clockwise about a point at (10, 4) and color the image yellow.
  Ask students to write the coordinates of the vertices of each rotated image. Ask them to describe if the centre of rotation has an effect on the location of the image. (6SS9.1, 6SS9.2, 6SS9.3)

**Student-Teacher Dialogue**

- Provide students with a variety of completed transformations. Ask them to identify the type of transformation and explain how they know. Ask them to describe a transformation required to produce the image by discussing the positional change of the vertices. (6SS9.3)

**Resources/Notes**

- Math Focus 6
- **Lesson 3: Rotating Shapes**
- 6SS9
- TG pp. 22 – 26
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS6 Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image.

[C, CN, PS, T, V]

Elaborations—Strategies for Learning and Teaching

Combining transformations is a new concept for Grade 6 students. After working with each of the three transformations, combining transformations of the same type should become a natural progression for students.

When students are given a combination of transformations, they should focus on one transformation at a time and recognize that each transformation in succession would be applied to the image resulting from the previous transformation.

Students should investigate combining transformations each time, trying to visualize the result in order to make a prediction before actually carrying out the transformations.

At this point students will combine only like transformations.

Achievement Indicators:

6SS6.1 Model a given set of successive translations, successive rotations or successive reflections of a 2-D shape.

Place three geoboards as shown here.

Students work in groups of threes. Ask one student to make a scalene triangle on the first geoboard. Ask another student to construct on the second geoboard the image of this triangle if the right side of the first geoboard is used as a mirror line. Ask another student to construct on the third geoboard the image of the triangle on the second geoboard if the top of the second geoboard is used as the mirror line. Repeat this activity using other shapes and/or other transformations.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td>• Ask students to create a triangle, then locate the image of ΔABC after a reflection in line 1 followed by a reflection in line 2 and indicate the coordinates of the final image. Ask them what single transformation of ΔABC would have the same result. (6SS6.1, 6SS6.5)</td>
<td>Lesson 4: Combining Transformations of the Same Kind 6SS6 TG pp. 31 – 35</td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS6 Continued

Achievement Indicator:

6SS6.2 Describe the transformations performed on a 2-D shape to produce a given image.

Elaborations—Strategies for Learning and Teaching

Students have been describing successive like transformations. Now they will describe successive transformations of all three types. When describing the transformations performed on a shape, students should be encouraged to use appropriate math language. Review with students the appropriate description for each type of transformation.

Some students may be able to describe combinations of differing transformations and can quickly move on to identifying these combinations.

Students but may describe any number of combinations of transformations. Students may present differing ways to identify how an image was obtained; a specific number of transformations used cannot always be determined or, in fact, if a combination was used.

Investigate questions such as:

- If a shape undergoes 2 translations, does it matter in which order they take place?
- Could this image have been obtained by a single transformation?

Students have worked on single transformations and performing a combination of like transformations. Now, focus is on performing a combination of different transformations on a shape. Combinations should include:

- a reflection followed by a translation
- two translations
- two reflections
- a translation followed by a rotation
- two rotations

Remind students when performing a combination of transformations to focus on only one transformation at a time where each new transformation is applied to the previous transformed image. Each new image should be labelled with an additional prime symbol at each vertex.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

- Motion Commotion- Provide students with a sheet of \(8 \frac{1}{2}\) by 14 inch paper to fold and cut according to the diagram below. Students must cut along the dotted edge so that the top half of the strip has flaps that can be folded over to cover the images. Place one figure in the first (lower left-hand) box of the strip. Students must perform a transformation and write on the flap a description of the movement performed. Students should continue performing successive transformations until all blocks have been filled. This activity shows students the orientation of each image, not the position in reference to the shape undergoing the transformation. Students can exchange strips and predict the shape they will see before they raise each flap. It provides a great opportunity for students to make predictions.

\(6SS6.4, 6SS6.5, 6SS6.2, 6SS6.6, 6SS6.7\)

Source: Navigating Through Geometry (Grades 3 – 5)

Paper and Pencil

- Provide students with a coordinate grid and figure as shown:

Ask students to translate the figure 1 square left and 5 squares down. Rotate the translated image \(\frac{1}{4}\) turn counter clockwise about (4, 1). Write the coordinates of the final image. What do you notice about this final image?

\(S66.5, 6SS6.2, 6SS6.6, 6SS6.7\)

Resources/Notes

Math Focus 6

Lesson 5: Combining Transformations of Different Kinds

6SS6
TG pp. 36 - 40

Lesson 5 and 6 may be combined.

Lesson 6: Communicating About Transformations

6SS6

6SS7
TG pp. 45 – 48

Curious Math: Single Transformations or Multiple Transformations

6SS6
TG pp. 41 - 42

Math Game:

Wei-Chi

6SS8
TG pp. 43 - 44

Capturing Squares
TG pp. 43 - 44

Additional Reading:


“Motion Man” activity – p. 233
Tessellations – p. 237-239
Outcomes

Students will be expected to

6SS6 Continued

Achievement Indicators:

6SS6.3 Demonstrate that a 2-D shape and its transformation image are congruent.

6SS6.4 Model a given combination of two different types of transformations of a 2-D shape.

6SS6.5 Draw and describe a 2-D shape and its image, given a combination of transformations.

Elaborations—Strategies for Learning and Teaching

Demonstrate congruency to students, using pattern blocks. Choose two pattern blocks, such as the triangle and the trapezoid, and place them side by side. Have the students choose a combination of 2 transformations. Perform these combinations to create a pattern with the blocks. Ask students if the image in the pattern is congruent.

Ask students to check an image's congruency by tracing it and overlaying it on the initial shape. Students will see that the size and shape is maintained. If not, this is an indication that there is an error in their creation of the image or they have performed the transformation incorrectly. Students will discover that the images are congruent because the transformations do not change the size or form of the shape.

During modelling, place an emphasis on which image is being transformed. Students must be aware that when performing a combination of transformations the second transformation is performed on the first image, not on the original shape.

As students begin working on modelling and performing given combinations of different types of transformations, it is suggested that they limit their work to combining two different transformations only. Generally students will require practice in this area in order to successfully model a combination of transformations on their own.

Using an overhead projector or interactive white board, ask a student to give a single transformation direction where another student would perform this transformation on the overhead/interactive white board. Invite other students to provide different directions to transform the resulting image. Once three or four combinations of different transformations have been used, invite students to brainstorm other transformations that could be used to get the original shape to its final image.

Repeat this activity using two other transformations.

Students have had practice drawing and describing single and combined like transformations and now will apply these skills and strategies to drawing and describing a 2-D shape when given a combination of different transformations. When students are drawing combined transformations encourage them to appropriately label their images. For example, when transforming \( \triangle ABC \), its image after the first transformation should be labelled \( \triangle A'B'C' \). The second image should be labelled \( \triangle A''B''C'' \), and so on.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Journal

- Present students with two congruent shapes on grid paper (the first and the third shapes after two transformations were performed.) Ask students to write in their journals:
  
  (i) What two transformations do you predict were performed? Explain your reasoning.
  (ii) Draw the second image
  (iii) Could this have been done more than one way?
  (iv) Could this have been done by a single transformation?

Performance

- Provide each student with grid paper marked with a coordinate grid and three pattern blocks of the same type. Ask students to place one block on the grid so that one of its vertices is at (4, 3). Ask them to place a second block so that it would be the image of the first block under a vertical translation of 10 units up. Then ask them to place the third block so that it is the image of the second block under a reflection in the vertical line through the point (11, 10). Ask them to compare the first and third blocks.

Resources/Notes

Math Focus 6

Lesson 5 (Cont’d): Combining Transformations of Different Kinds

6SS6
TG pp. 36 - 40
Lesson 5 and 6 may be combined.

Lesson 6 (Cont’d):
Communicating About Transformations

6SS6
6SS7
TG pp. 45 – 48
Strand: Shape and Space (Transformations)

Outcomes

Students will be expected to

6SS6 Continued

Achievement Indicators:

6SS6.6 Model a given set of successive transformations (translations, rotations and/or reflections) of a 2-D shape.

6SS6.7 Perform and record one or more transformations of a 2-D shape that will result in a given image.

6SS7 Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.

C, CN, T, V

Elaborations—Strategies for Learning and Teaching

Rotations can be challenging. It may be a good practice to leave a rotation in a set of combined transformations as the final transformation in the set. This would ensure that in assessment students could model their understanding of combining transformations without making a rotational error at the beginning that would continue that error throughout.

Provide each student with a coordinate grid and pattern blocks. Ask each student to carry out two transformations of their choice on the grid and leave only the first and third blocks in place. Ask them to exchange grids with a partner and predict the two transformations that took place. Share their predictions and actual transformations.

Demonstrate a design that could be created using a combination of transformations. Discuss the various transformations that could have been applied to create this design. Initiate a class discussion about how transformations can be used to create various designs, such as company logos and symbols.

Model some examples of creative tessellations. M.C. Escher (graphic artist) is well known for his tessellations which often take the shape of various objects. Explain that these designs are created by using transformations or by combining compatible polygons. Invite students to create their own tessellation designs. This particular activity can help students see that math can be used in other areas such as art.

Create a design using pattern blocks (preferably on the overhead or board) for all students to see. Ask students to analyze the pattern to identify and describe the transformations used to create the design.

Ask students to draw their initials in a block letter symbol.

Ask them to perform a combination of two different transformations on their symbol. Repeat these transformations to create a design with their symbol. Ask students to exchange symbols with a partner and have the partners describe which combinations of transformations got them to form the design.
General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

• Ask students to create a design using 3-5 pattern blocks (there must be at least 3 different shapes). Using a sheet of grid paper they will move their design from the top-left corner of the page to the bottom-right corner. The design should be in its original orientation when finally in the bottom right corner. Each move must be a reflection, a rotation or a translation. Encourage students to use more rotations and reflections than translations. Students should note how many moves were needed and draw each move on a piece of paper (or use additional blocks) to show what their design looks like. Extension: Students can complete the task again in fewer moves and describe the strategies used. (6SS7.1, 6SS7.2)

• Ask students to create a tessellation design using a combination of one type of transformation. (6SS7.2)

Presentation

• Ask students to choose from preselected company logos or symbols that illustrate different combinations of transformations. E.g., the recycling symbol, Pepsi symbol, etc. Ask them to present the logo/symbol to the class explaining the transformations that have taken place. (6SS7.1)

Resources/Notes

Math Focus 6

Lesson 5 (Cont’d): Combining Transformations of Different Kinds

6SS6
TG pp. 36 - 40

Lesson 5 and 6 may be combined.

Lesson 6 (Cont’d):
Communicating About Transformations

6SS6
6SS7
TG pp. 45 – 48
MOTION GEOMETRY
Ratio and Percent

Suggested Time: $2 \frac{1}{2}$ Weeks
Unit Overview

Focus and Context

“Students can develop a deep understanding of numbers through experiences with a variety of models, such as fraction strips, number lines, 10×10 grids, area models and objects. These models offer students concrete representations of abstract ideas and support students’ meaningful use of representations and their flexible movement among them to solve problems.” Principles and Standards for School Mathematics. (2000)

Math Connects

Students will encounter fractions, decimals, percentages and ratios in everyday situations. Being able to make sense of these concepts is necessary to be informed citizens and consumers to work in today’s technological society. As students make the necessary connections between decimals, fractions, ratios and percentages, it enhances their knowledge of and flexibility in thinking about number. The opportunities for students to work with these numbers and see these numbers in everyday situations is limitless. Going to the store and seeing 20% off, sharing pizza with their friends, understanding various sporting statistics such as batting averages, and making sense of what this information means to the sport are just some examples of when students would use these skills.
### Process Standards Key

<table>
<thead>
<tr>
<th>Key</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[C]</td>
<td>Communication</td>
</tr>
<tr>
<td>[CN]</td>
<td>Connections</td>
</tr>
<tr>
<td>[ME]</td>
<td>Mental Mathematics and Estimation</td>
</tr>
<tr>
<td>[PS]</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>[R]</td>
<td>Reasoning</td>
</tr>
<tr>
<td>[T]</td>
<td>Technology</td>
</tr>
<tr>
<td>[V]</td>
<td>Visualization</td>
</tr>
</tbody>
</table>

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>6N5 Demonstrate an understanding of ratio, concretely, pictorially and symbolically.</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td>Number</td>
<td>6N6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.</td>
<td>[C, CN, PS, R, V]</td>
</tr>
</tbody>
</table>
Strand: Number

Outcomes

Students will be expected to

6N5 Demonstrate an understanding of ratio, concretely, pictorially and symbolically.

[C, CN, PS, R, V]

Elaborations—Strategies for Learning and Teaching

Work involving ratios and percents will be new to Grade 6 students. As they begin to work on ratios and percents, it is necessary to show students how ratios and percents can also be represented by decimals and fractions. Connecting these four concepts is essential in the development of student’s number sense. Throughout the year, there will be many different opportunities where these concepts can be discussed.

In Grade 5, students explored the connections between fractions and decimals. This work will help them to now connect this with ratios and percents. Students should be able to fluently move between naming a number as a fraction, ratio, percent and decimal. For example, when given a number such as 0.50, students should see this as 50%, \( \frac{5}{10} \), 5:10 but also see this as one half.

Students may need review with fraction concepts, which would be beneficial in beginning work on ratios and percentages. Ask students to represent examples of different fractions in a variety of ways.

A ratio is a comparison of any two quantities. When investigating the concept of ratios, provide students with various concrete materials to represent these ratios. Using things like snap or linking cubes, pattern blocks, buttons or candy can help students see the part-to-part and part-to-whole relationships.

Models used to represent fractions can also be used when working with ratios.

Give students a ratio, for example, 2:5. Ask them to create a design to represent this ratio using pattern blocks and explain how they know their design represents this ratio.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Ask students to select 20 tiles of four different colours so that pairs of colours show the following ratios: 4 to 3, 2:1.  

- Provide students with a problem such as:
  
  The Easter Bunny left 6 Hershey Kisses and some Jujubes. The ratio of kisses to jujubes was 3:2. Altogether, how many Hershey Kisses and Jujubes did the Easter Bunny leave? Explain your answer pictorially, symbolically and concretely.

**Journal**

- Tell students that John’s family has a mother, father, 2 daughters and John. The part-to-part ratio of male to female is 2:3. The part-to-whole ratio (males:whole family) is 2:5. Ask students to represent these ratios using counters.

**Resources/Notes**

*Math Focus 6*

**Lesson 1: Ratios**

6N5

TG pp. 13 – 17
Strand: Number

Outcomes

Students will be expected to

6N5 Continued

Achievement Indicators:

6N5.2 Write a ratio from a given concrete or pictorial representation.

6N5.3 Express a given ratio in multiple forms, such as 3:5, or 3 to 5.

Elaborations—Strategies for Learning and Teaching

Use the students themselves, counters, or other simple models to illustrate the concept of ratio as a comparison between two numbers (or among three or more numbers).

Encourage the use of appropriate language. (e.g., Students should read the ratio “3:2” as “3 to 2” or “3 ___ for every 2 ___”)

Through exploration and making meaningful connections, ratios can be related to everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or “3 to 1”) or in relation to other topics in mathematics (e.g., students can explore the ratio of the length of one side of a rectangle to the perimeter).

Ask students to create a poster on ratios found in the classroom. They could include such ratios as:

- boys:girls
- teacher:pupils
- desks:students
- tables:students
- pencils:students

As students continue to work with ratios, provide them with many opportunities to show and understand that ratios can be written in many forms. It is beneficial for students to be able to move easily among different forms when expressing a number.

Throughout this unit it is suggested that students get daily practice using the different forms of ratios. Ask students to create a book where each page represents a different ratio. Ask them to draw pictures to show the ratio and then demonstrate the related fraction, percentage and decimal.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Read the book *Math Curse* by Jon Cieszka. Ask students to draw the shirts and then create ratios to represent the shirts in the main character’s closet.

- Invite your students to a mathematical yogurt party. Everyone should bring a single-serving container of a favourite flavour of yogurt. Look on the carton to find out how many calories each yogurt has and how many of those are from fat. Write the ratio of fat calories to total calories as a fraction, then convert the fraction to a percent. Compare the percent of fat in your yogurt with the values obtained by others. Does the information on the containers explain the differences? What might account for caloric differences? Discuss why you think the ratios differ.

- Ask students to find body ratios such as wrist size: ankle size, wrist size: neck size, hand width: hand length, arm span: body height. Ask students to compare their results with others and express the ratios in multiple forms.

Paper and Pencil

- Ask students to find and record the ratio of odd numbers to even numbers in their home phone number.

Student-Teacher Dialogue

- Give students a handful of two or three different coloured snap cubes. Ask them to describe all possible ratios that exist using these cubes.

- Ask: Why might you describe the data set below as 4:1? As 1:4? Are there other ratios that you can use to describe the boys and girls?

  B B B B G
  B= boy G=girl

Resources/Notes

*Math Focus 6*

Lesson 1 (Cont’d): Ratios

6N5

TG pp. 13 – 17

Children’s Literature (provided):

Cieszka, Jon. *Math Curse*. 
Outcomes

Students will be expected to

6N5 Continued

A scale on a map provides a real life example of ratios. Discuss with students the need for this scale, or ratio (it is impossible to show the actual size and/or distances on a map). Another example of a ratio in the real world is in mixing gas and oil for chain saws and snowmobiles. The gas:oil ratio might be 50:1. This means that for every 50 L of gas there would be 1 L of oil needed.

Achievement Indicators:

6N5.4 Identify and describe ratios from real-life contexts, and record them symbolically.

To illustrate the difference between part-to-part and part-to-whole ratios of the set, provide small groups of students with a bag containing two different colored counters. Ask students to compare the counters in as many ways as they can. Invite groups of students to share their findings. They can describe their comparisons by explaining if their ratio is a part-part or part-whole ratio.

6N5.5 Explain the part/whole and part/part ratios of a set; e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8 and 5:8.

Discuss part-to-part and part-to-whole ratios with students identifying examples of each type. Once the ratio is given, students could be told to represent it as a part-to-part ratio or it could be left open where students choose which ratio type to use. After creating their representations, encourage students to explain why they chose to represent the ratio the way they did.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Paper and Pencil**
- Give students the following information and ask them to write/read ratio comparisons (including part-to-part and part-to-whole ratios) and to identify those that can be expressed as fractions.
  
  4 cats, 3 goldfish, 2 hamsters

- Ask students whether or not he/she believes that the ratio of the population of any city in Canada to the total population of Canada could be 1:2. Students should explain their responses.

- Ask students to model two situations which could each be described by the ratio 3:4. Specify that the situations must involve a different total number of items.

**Performance**
- Ask students to explore the ratios of the different colors in a box of Smarties®, bag of Skittles®, or a set of pattern blocks.
- Ask students to write their full name indicating the ratio of vowels to consonants, vowels to all letters, consonants to vowels and consonants to all letters.

**Performance**
- Ask students to use snap cubes to show the ratio 5:6. Then ask them to use more cubes to create an equivalent ratio.
- Ask students to use counters to display a ratio of 3:5. Ask them to show an equivalent ratio and justify their answers.
- Provide students with a problem such as:
  
  Donald’s punch recipe calls for 3 L of ginger ale, 1 L of strawberry juice and 2 L of orange juice. Suppose Donald uses 9 L of ginger ale, how much strawberry juice and orange juice should he use? Justify your answer.

---

### Resources/Notes

**Math Focus 6**

**Lesson 1 (Cont’d): Ratios**

6N5

TG pp. 13 – 17
Strand: Number

Outcomes

Students will be expected to

6N5 Continued

Achievement Indicators:

6N5.6 Demonstrate an understanding of equivalent ratios.

Elaborations—Strategies for Learning and Teaching

Many students will recognize the similarity between equivalent ratios and equivalent fractions.

Using pattern blocks, ask students to explore equivalent ratios by seeing that when the yellow hexagon is one whole, one blue rhombus represents 1:3 or $\frac{1}{3}$ of the hexagon. To create an equivalent ratio, students could use the green triangles to match the same area as one blue rhombus. They will see that it takes 2 green triangles to create a blue rhombus, therefore the ratio of triangles to the whole is 2:6 and this clearly illustrates that 1:3 is equivalent to 2:6. Ask students to explore other equivalent ratios using the pattern blocks.

Students have worked with equivalent fractions in Grade 5 and should be able to use this concept to help them understand equivalent ratios. E.g., in the diagram below, $\frac{2}{5}$ of the counters in the top row are white, which also illustrates the ratio 2:5. In total $\frac{4}{10}$ of the counters are white or 4:10, so $\frac{2}{5} = \frac{4}{10}$. Therefore, the ratios 2:5 and 4:10 are also equivalent. If 2 of every 5 counters are white, then 4 of every 10 would also be white.

To help students visualize the concept of equivalent ratios, ask them to create a given ratio using two different colored snap cubes. E.g., when given the ratio of 3:5, students can build a model using 3 black cubes and 2 white cubes (part-to-whole). When looking at the ratio of black to the whole, they would see 3 black to 5 in all or 3:5. Demonstrate for students an equivalent ratio for 3:5 by replicating the original model. We now have an equivalent ratio for 3:5, which is 6:10. By continuing to replicate their original model, they can create additional equivalent ratios.

Students can use equivalent ratios to make predictions. E.g., in a large bag of marbles, the ratio of blue marbles to the total number of marbles is 4:10 (i.e., 4 out of every 10 marbles are blue). Use this to predict the number of blue marbles you would expect in 100 selections.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Presentation

- Ask students to draw a picture of them and their families at the park. Ask students to write a part-to-part ratio and a part-to-whole ratio to describe their picture (e.g., Number of arms to legs and number of children to people) and allow time for them to share their pictures and ratios with the class. Next, ask students to switch pictures. Give each student a strip of paper and ask them to write a word problem involving equivalent ratios to go with their classmate’s picture. Display the pictures and word problems around the classroom. Allow students time to solve all the problems by having students do a gallery walk in pairs. (6N5.2, 6N5.6)

Paper and Pencil

- Present the following diagram to students:
  
  \[
  \begin{array}{c}
  x x x o \\
  x x x o \\
  x x x o \\
  \end{array}
  \]

  Ask students to write equivalent ratios demonstrated through this diagram and to explain their thinking. (6N5.2, 6N5.6)

Performance

- For each of the following ratios, ask students to find an equivalent ratio in which one of the terms is 20.
  
  \[
  4:6 \hspace{1cm} 10:30 \hspace{1cm} 3:5 \hspace{1cm} 4:5
  \]
  
  (6N5.6)

- Ask students to list:
  
  \[
  \begin{align*}
  & five ratios that are equivalent to 1:2 \\
  & three ratios that are equivalent to 8:6.
  \end{align*}
  \]
  
  (6N5.6)

Student-Teacher Dialogue

- Ask students to explain how a multiplication chart can be used to generate equivalent ratios. (6N5.6)

- Ask students: Why do you get an equivalent ratio by multiplying both terms of a ratio by 3? (6N5.6)

Resources/Notes

Math Focus 6
Lesson 1 (Cont’d): Ratios
6N5
TG pp. 13 – 17

Math Focus 6
Lesson 2: Equivalent Ratios
6N5
TG pp. 18 – 22
Strand: Number

Outcomes

Students will be expected to

6N5 Continued

Achievement Indicators:

Elaborations—Strategies for Learning and Teaching

Multiplication tables can be used to look for equivalent ratios. Ratios equivalent to 2:4 (4:8, 6:12, etc.) are found by looking at numbers in the same column of the table.

While it is not a requirement to ask students to represent ratios in the simplest form, it is suggested that students explore the idea of writing ratios in the simplest form.

Ask students to work in pairs or small groups to discuss all possible ratios, including equivalent ratios, that could be represented by the following situation:

During a student council election, Sue received 36 votes and Sam received 9 votes. Students should recognize the following ratios:

- 36:9 or 4:1 (Sue received 4 votes for every 1 vote Sam received.)
- 9:36 or 1:4 (Sam received 1 vote for every 4 Sue received.)
- 36:45 or 4:5 (Sue received 4 votes for every 5 votes cast.)
- 9:45 or 1:5 (Sam received 1 vote for every 5 cast.)

Allow students time to write their own scenarios that would demonstrate equivalent ratios.

Ask students to use fraction strips to represent the ratio 1:2. For example, the ‘1 whole’ strip could be used with the $\frac{1}{2}$ strip where it could be seen that it takes two $\frac{1}{2}$ to make up 1 whole.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Journal**

- Tell students that in a class of 30 students, there are 20 girls. Ask them to explain why the ratio of boys to girls is 1:2. (6N5.6)

- Ask students to create a picture representing various groups of items and write two equivalent ratios that can be found in the picture. Ask them to explain their thinking. (6N5.2, 6N5.6)

**Resources/Notes**

*Math Focus 6*

**Lesson 2 (Cont’d): Equivalent Ratios**

6N5

TG pp. 18 – 22
Strand: Number

Outcomes

*Students will be expected to*

6N6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.

[C, CN, PS, R, V]

Elaborations—Strategies for Learning and Teaching

This is the first time Grade 6 students will work with percents.

Percent is a ratio and, therefore, another name for a fraction. Percent should be viewed as a part-to-whole ratio that compares a number to a whole divided into 100 equal parts. Students may note the connection to the word “cent” where a cent is \( \frac{1}{100} \) of a dollar. Students should not be computing percentages where they are procedurally finding the percentage of fractions or ratios at this time and need not work with percentages greater than 100, but should recognize:

- situations in which percent is commonly used
- diagrams that represent various percentages
- the relationship between percents, decimals and fractions (e.g., 
  \( 48\% = 0.48 = \frac{48}{100} \))
- that percent is a ratio or a comparison of the percent value to 100 and can be written as \( \frac{\_\text{___}}{100} \).
- that finding a percentage is the same as finding an equivalent ratio out of 100

Demonstrate to students how to use a hundredths grid to represent percents by shading in the desired portion of the grid. E.g., to represent and model 25%, students could use a hundredths grid to shade 25 blocks out of 100. This will help students to understand and see the connection among fractions, decimals, percents and ratios as 25 blocks shaded out of 100 could be seen as \( \frac{25}{100} \), or \( \frac{1}{4} \), 25:100 or 1:4, 0.25 or 25%.

Discuss with students that you evaluate their progress in many different ways. One form of assessment that they may remember is a test and getting the results in the form of a percentage. The greatest score you can get is 100%; therefore, to give a percent, it must always be ‘out of 100’. E.g., if you get 87% on a test, this means you got 87 marks out of a possible 100 marks \( \frac{87}{100} \). As connections are made to fractions, 100% can be seen as a whole where anything less than that whole is a part or percent. To connect decimals to percents, ask students to use a calculator to calculate \( \frac{87}{100} \) where they would see an answer of 0.87. Ask students to explore and discover how 0.87 means the same as 87% or \( \frac{87}{100} \).

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students Emma is making a quilt. She has 60 patches. Help Emma create a quilt with the following colors:
  - 25% red
  - 0.10 green
  - 3:10 yellow
  - The rest is blue

  Ask students to draw a picture to show your thinking and explain how you were able to find out how many of each color Emma will need to complete her quilt. (6N6.1)

- Tell students they have been hired by a graphic design company to design a new logo for the company. They tell you the logo can be any shape and have the following criteria:
  - Less than one third of the logo is blue
  - About 60% red
  - The rest yellow.

  Ask students to design the logo and write a description telling the company how you were able to come up with the percentages of each color. You may need to represent each portion (color) of the logo in decimal and fractional form, just in case they may want to make sure you have met the criteria. (6N6.1)

- Ask students to explain which of the following numbers represents the least and which represents the most: \( \frac{1}{20}, 0.25, 0.020 \). (6N6.1)

- Ask students to choose three percentages. Write these numbers in fractions, decimals and ratios. Order the numbers from least to greatest by placing them on a number line. (6N6.1)

- Ask students if 68% of a hundredths grid is shaded, what ratio of the grid is not shaded? (6N6.1)

- Ask students if a red and blue quilt has 50 squares and 20% are blue, what ratio of the quilt squares is red? (6N6.1)

Resources/Notes

- *Math Focus 6*
  - **Lesson 3: Percents**
  - **6N6**
  - TG pp. 23 - 27
Strand: Number

Outcomes

Students will be expected to

6N6 Continued

Achievement Indicators:

6N6.1 Continued

6N6.2 Explain that percent is a ratio out of 100.

6N6.3 Use concrete materials and pictorial representations to illustrate a given percent.

6N6.4 Record the percent displayed in a given concrete or pictorial representation.

6N6.5 Identify and describe percents from real-life contexts, and record them symbolically.

Elaborations—Strategies for Learning and Teaching

Focus on mathematical language, using 87 hundredths, or 87 out of 100 to help students see these connections.

Many problems involving percents will also require students to use their knowledge of equivalent fractions and equivalent ratios. Provide students with an example such as, there were 10 cartons of milk ordered for recess; 7 were chocolate. Therefore \( \frac{7}{10} \) milks were chocolate. This also means \( \frac{70}{100} \) or 70%, of the milk was chocolate.

Provide students with a blank hundredths grid and ask them to use four different colors to shade in the grid. Ask them to, for example, shade 30 blocks red, 20 blocks blue, 45 black and 5 yellow. Ask students to describe each color using a fraction, decimal, percent and a part to whole ratio. This activity will help students connect these four ways of representing a number.

Ask students to work with various concrete materials to represent percents (e.g., cut sheets of paper and/or lengths of string to show 50%, 10%, 25%, etc.).

Hundredths grids are excellent resources to use with students to help develop their understanding of percents. Encourage them to also use other concrete representations to facilitate their understanding. Share with students the book *Piece = Part = Portion: Fractions = Decimals = Percents* by Scott Gifford. The images provided demonstrate concrete examples of percents of objects, as well as equivalences of fractions, decimals and percents.

Ask students to predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of

- red counters when fifty 2-coloured counters are shaken and spilled
- each colour of Bingo chips, if a total of 100 blue, red, and green chips are shown on an overhead for 10 seconds
- a hundredths grid that is shaded to make a picture

Ask students to bring in a flyer from a local store. Ask them to go through each flyer identifying different percents that are used throughout. Discuss what these percents mean (e.g., Walmart, Zellers, or Canadian Tire, Riffs may have 33.3% off sale).
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask the student to shade hundredths grids to show particular percentages such as 20%, 60%, etc. (6N6.3)

- Ask students to place the following on a number line. Then choose one number and justify their thinking.
  
  0.40  76%  2/10  95%  

(6N6.3)

- Ask students to use the Internet or print resources to find out such things as:
  - What percent of the Earth is water?
  - What percent of the rainforests are in danger?
  - What percent of animals are endangered? (6N6.5)

- Ask students to create a collage showing how percents are used in daily life. (6N6.5)

- Ask students to draw a design in a hundredths grid (or partially cover a flat) and describe the percentage of the grid covered.
  Ask further questions such as: How many more squares would you have to cover to fill in the grid? (6N6.4)

- Ask students to use the Internet, a geography book, or other print resource to locate the flags of various countries. You will notice that many flags are created with a number of colors or combinations of those colors. Have the students choose 3 different countries to reflect on the design of their flags. What percentage of a flag is a particular color? What fraction? What would this look like as a ratio to the whole flag? Sort and graph flags that represent halves, thirds, and fourths. (6N6.1)

Journal

- Ask students to draw a picture to show why a decimal can be represented as a percent. (6N6.1, 6N6.3)

- Ask students to choose a fraction and a percent that are not equivalent. Ask them to use pictures, numbers and words to explain which is greater. (6N6.1, 6N6.3)

Resources/Notes

Math Focus 6
Lesson 3 (Cont'd): Percents
6N6
TG pp. 23 - 27

Children's Literature (provided):
Strand: Number

Outcomes

Students will be expected to

6N6 Continued

Achievement Indicator:

6N6.6 Express a given percent as a fraction and a decimal.

Elaborations—Strategies for Learning and Teaching

Throughout the unit students have been making connections using fraction, decimals, percents and ratios to represent any given number. Working with a hundredths grid is essential to this type of work so students can have a visual representation of the work they are doing. Provide students with a hundredths grid, asking them to shade a percentage of the grid. Once the grid is shaded, ask them to create a corresponding card that illustrates the percent shaded on the grid using a decimal, fraction, ratio and in words.

(These grids and cards will be similar to the materials used in the Decimal Square kit.)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Student-Teacher Dialogue*

- Ask students to compare 20% and 0.02 on a hundredth grid. Which is greater? Explain your answer. (6N6.3)

- Ask students: What percent of a metre stick is 37 cm? How do you know? (6N6.4)

- Ask the students to name percents that indicate:
  - almost all of something
  - very little of something
  - a little less than half of something (ask students to explain their thinking) (6N6.4)

- Ask students to estimate the percentage of red that is shown on the Canadian flag. Justify your thinking. (6N6.4)

*Portfolio*

- Ask students to create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages, ratios or fractions of each colour within the patch and then estimate the percent of the total quilt that is each colour. (6N6.2, 6N6.4)

- Tell students to use a hundredths grid and shade in 25% of the grid. Ask what percent is left unshaded? What are other ways of representing the unshaded part? (6N6.3)

*Resources/Notes*

*Math Focus 6*

**Lesson 4: Percents as Fractions or Decimals**

6N6

TG pp. 32 - 35

*Math Game: Ratio Match*

6N5

TG pp. 36 - 37
Outcomes

Students will be expected to

6N5 Demonstrate an understanding of ratio, concretely, pictorially and symbolically.

[C, CN, PS, R, V]

Achievement Indicator:

6N5.7 Solve a given problem involving ratio.

Elaborations—Strategies for Learning and Teaching

There are many different applications to the real world where students can use percents and ratios to solve problems. One such example is through the use of scale diagrams; however, students’ work and discussion should not be limited to this particular situation.

In using scale diagrams as a means of having students solve problems involving ratios and percents, students can look at maps investigating the particular scale used to represent distances and sizes of countries. For example, based on this scale, they could calculate distances between identified places.

Students may be familiar with model toys, and can readily identify that a model car or motorcycle has a scale of 1:30 to an actual car. Ask them to explore the dimensions of the actual size of the car or motorcycle where they come to realize that the model dimensions are in the numerator and the actual car dimensions are in the denominator. If the model car has a door that is 4 cm in height, students can use their understanding of scales and ratios to determine the height of the actual car door.

It is very important to keep numbers simple when representing or comparing various ratios.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to use 20 tiles of four different colors to show pairs of colors that show the following ratios:
  - 4 to 3
  - 2:1
  - \( \frac{1}{3} \)

- Ask students to create their own scale diagram. Ask them to let every one block of one centimetre-grid paper represent two meters of real, outdoor playground space. To visualize this scale, use string or chalk to mark off a two-meters-by-two-meters space on the floor. Use small cubes or blocks to build a scale model of a playground structure on your paper. Trace and draw on the paper a top-view plan of the structure. How could you represent yourself on your playground? (6N5.6, 6N5.7, 6N6.7)

- Provide students with a problem such as: 758 people were surveyed to determine their favourite laundry detergent. 248 individuals responded that they used Brighto detergent. Working in pairs, ask the students to estimate what ratio would best describe the number of people who use Brighto. Ask students to explain their reasoning. Ask them to make up similar situations for their classmates to solve. (6N5.7)

Resources/Notes

* Math Focus 6
  Lesson 5: Exploring Scale Diagrams
  6N5
  TG pp. 38 - 41

Scale diagrams is ONE example of solving a given problem involving percents and ratios. You may wish to explore other ways to ask students to solve problems involving these concepts.
RATIO AND PERCENT

Strand: Number

Outcomes

Students will be expected to

6N6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.

[C, CN, PS, R, V]

Achievement Indicator:

6N6.7 Solve a given problem involving percents.

Elaborations—Strategies for Learning and Teaching

Students have been working on making connections between percents, decimals, ratios and fractions. Through this work, students understanding of percent will now be extended to calculate and estimate percents to solve problems. They will now be expected to understand how to estimate and find a given percent of a number. E.g., they may be asked to estimate what 50% of 80 is.

Number lines are helpful tools when working with percents. Students can see that when they are asked to get a percentage of a given number, the given number is the whole and is represented at the end of the number line. E.g., Shawn wanted to save 60 dollars for his sister’s birthday gift. He thought about it and decided he should have 50% saved by June. How much money would Shawn have saved by June?

Students would then use their knowledge of benchmarks, using \( \frac{1}{2} \) as 50%, \( \frac{1}{4} \) as 25% and \( \frac{3}{4} \) as 75% to help them estimate and calculate the given percentage of the number. E.g.,

Present the following problem:

The school has raised $800.00 to buy new sports equipment. 50% of the money will be spent on volleyball equipment. 30% will be spent on basketball equipment. The remaining money will be used to purchase new scooters. How much money was spent on each item?

Students need to realize that if they want to use a number line to help represent a problem, they need to see that while the end points of their number line begin at 0 and go to 800, for example, that this number line represents 100% of the total amount of money raised. Therefore, when they establish the benchmarks of one half or 50% it represents $400.00. Through this exploration it is hoped that students will discover that 100% of the money is used and the total for all equipment purchased is $800.00.

Begin instruction with establishing the benchmark of 50% or the half way point. Practice getting 50% of numbers where students can explore how this is really finding the midpoint on their number line between 0 and the number in question. When students are comfortable with this idea, lead them to getting 25% and 75% of given numbers. These two percentages would represent the \( \frac{1}{4} \) and \( \frac{3}{4} \) mark on their number line.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- In a set of tangrams, a large triangle is 25% of the whole set. Ask students what percentage of the set is the square? Parallelogram? Small triangle? Medium triangle? (6N6.7)

- Tell students Sandra bought a pizza for her slumber party. Jaime and Maria ate 25% of it. Louisa and Abby ate one-third or 33% of what was left. Chantel and Sammie ate 50% of what was left. Manuela ate two slices. Sandra was left with two slices. How many slices were in the pizza? How many slices did each of the girls eat? Draw a picture and show how you went about solving this problem. (6N6.7)

- Tell students that approximately 50% of all people in Canada over 18 years old vote when it is time to elect a new prime minister. If 50% of your class voted, how many people would that equal? How about in your grade? How about in your school? How about in your community? Was this percentage easy or difficult to work with and why? What would happen if the percentage was 75%? Would you use the same strategy or a different strategy to find your answer? (6N6.7)

- Ask students to assign a percentage value to each letter in the word **HEART**. Assign the values so that the sum of the letters equals one hundred percent. All the letters can have the same value or each letter can have a different value. Show 4 different ways you can do this. (6N6.7)

- Changing to newer, more energy-efficient light bulbs can save up to 70 percent on your electric bill. If a person's electric bill was $30 before changing his bulbs, what would the bill be with the newer bulbs? Talk to your family about your electric bill. How much could you save? Or how much are you already saving? Make a list of additional ways your family could both conserve energy and save money. (6N6.7)

Resources/Notes

**Math Focus 6**

**Lesson 6: Solving Percent Problems**

**6N6**

TG pp. 42 – 45

**Curious Math: Interesting Percents**

**6N6**

TG pp. 46 - 47

**Children's Literature (provided):**

Merrill, Jean. *The Toothpaste Millionaire*

*Some problems presented in this literature use the Imperial system so you will need to modify to reflect the Metric System.*
Students will be expected to

6N6 Continued

Students can then use these benchmarks to help them estimate and calculate any other given percentages.

For some students, it may be difficult to estimate and calculate various percentages of numbers. It may be possible to give these students numbers for which the percent is easier to calculate or think about, such as 10%, 20%, 25%, 50% and 75%.

Using the book “The Toothpaste Millionaire”, by Jean Merrill, ask students to review page 27 where Mr. Conti caught Rufus passing a note to Kate which read “If there are $\frac{\frac{1}{2}}{\frac{1}{2}}$ billion tubes of toothpaste...”

Achievement Indicator:

6N6.3 Continued

Students can use base-ten blocks, counters or a number line to find given percentages. When asked to find 60% of 30, for example, students could count out 30 blocks, or use 3 rods or 30 units. They should see that 60% is the same as $\frac{60}{100}$ or $\frac{6}{10}$. They can use this relationship to discover if they broke the group of 30 into 10 groups, they would have made tenths. Since 60% means 6 tenths, they could show that 60% of 30 would be 6 groups out of 10, where each group consists of 3 counters, so 6 groups of 3 counters would be 18. Therefore, 60% of 30 is 18:

Students should be asked to justify their thinking and provide reasons for answering the question the way they did. This allows students the opportunity to reflect on their answers and decide if it is reasonable.

For the above example, students who got an answer of 18 could have thought about the fact that one half of 30 is 15 and 60% is one tenth more than 50%. Therefore 18 would be reasonable, as it is a little more than 15, or 50%.

continued
### General Outcome: Develop Number Sense

#### Suggested Assessment Strategies

*Paper and Pencil*

- If the Montreal Canadiens won 75% of their hockey games this season, and their season consists of 60 games, what is the ratio of their games won? Lost? Justify your answers. **(6N6.7)**

- There are 50 students in a choir with 32% boys. How many girls are in the choir? Is it possible to use a hundredths grid to solve the problem? **(6N6.7)**

<table>
<thead>
<tr>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td><strong>Lesson 6 (Cont’d): Solving Percent Problems</strong></td>
</tr>
<tr>
<td><strong>6N6</strong></td>
</tr>
<tr>
<td>TG pp. 42 – 45</td>
</tr>
</tbody>
</table>

**Children's Literature** (provided):

Merrill, Jean, *The Toothpaste Millionaire*

*Please note that this book is written using Imperial system. Please adjust to the metric system.*
Strand: Number

Outcomes

Students will be expected to

6N5 Continued

Achievement Indicator:

6N5.7 Continued

6N6 Continued

Achievement Indicator:

6N6.7 Continued

Elaborations—Strategies for Learning and Teaching

Students have been working on understanding the concepts of ratio and percent. They have been solving problems involving these concepts and representing this understanding. The models they have been using, such as hundredths grids, pictures, and pattern blocks, are very useful in helping students communicate their understanding of the problems that are given. Students need to focus on how they communicate their understanding as this is an important skill they need to practice. Students sometimes find it helpful talking out their understanding of a topic as this helps solidify and organize their thinking. Also, analyzing students’ communication about a topic becomes a critical assessment tool for teachers.

Students are expected to demonstrate their understanding using pictures, numbers and words, using appropriate mathematical language. Students should be encouraged to use many forms of mathematical communication, including oral, written, physical and symbolic.

Ensuring the use of modelling appropriate responses to various types of questions will allow students to see what is expected and how to use pictures, numbers and words effectively to show their understanding of a concept.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Performance*

- Ask students to explain how they know that the ratio of 1:5 represents 20%.

- Think of and explain two situations which could be described by the ratio 3:4.

- The ratio of boys to girls in Sarah’s class is 7:13. Sarah says there are at least 50% girls in her class. Is she correct? Explain.

*Resources/Notes*

*Math Focus 6*

**Lesson 7:** Communicating about Ratios and Percents

6N5

6N6

TG pp. 48 – 51
Fractions

Suggested Time: 3 Weeks
Unit Overview

Focus and Context

In Grade 6, students will deepen their understanding of number by continuing to work on fractions. They will develop an understanding of fractions that are greater than one and how these improper fractions can be expressed as mixed numbers. Students will have many opportunities to develop these concepts as they model, draw, name and write about mixed numbers and improper fractions while solving meaningful problems. These ‘hands on’ experiences will enable students to visualize the connections between improper fractions and mixed numbers. The study of improper fractions and mixed numbers should build on students’ prior knowledge of whole number and proportional concepts and skills and their encounters with decimals, proper fractions, ratios and percents in previous work and in their everyday life. Students will be engaged in problem solving situations where they will have to compare improper fractions and mixed numbers. As they work through various situations, students will continue to develop their problem solving strategies as they learn how to effectively use models, pictures and logical reasoning to solve problems.

Math Connects

Fractions are a part of everyday life; cooking, measuring, building and understanding quantities are just a few examples of where people encounter fractions. Learning about fractions that have a value greater than one becomes merely an extension of what students have already learned previously. This not only helps in the development of a strong sense of number, but also gives students a better understanding of how these numbers are used in everyday life. Thus, developing this number sense, understanding what a number means, what it represents and how it can be used becomes a reason to understand and become familiar with the concepts of improper fractions and mixed numbers. As students learn about these concepts through solving problems in context, they can consider the advantages and disadvantages of various representations of these quantities. For example, they could consider the reasonableness of showing how much flour is needed to make pizza dough as a mixed number rather than an improper fraction. Their use of logical reasoning to help solve problems such as this becomes stronger as they add more strategies to helping them solve everyday, real world problems.
### Process Standards Key

<table>
<thead>
<tr>
<th>Communication</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connections</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Mental Mathematics and Estimation</td>
<td>Technology</td>
</tr>
<tr>
<td></td>
<td>Visualization</td>
</tr>
</tbody>
</table>

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>6N4 Relate improper fractions to mixed numbers.</td>
<td>[CN, ME, R, V]</td>
</tr>
</tbody>
</table>
Strand: Number

Outcomes

Students will be expected to

6N4. Relate improper fractions to mixed numbers.

[CN, ME, R, V]

Elaborations—Strategies for Learning and Teaching

Improper fractions and mixed numbers are new to Grade 6 students. This work provides students with opportunities to build on their prior understanding of equivalent fractions and their ability to compare fractions with like and unlike denominators where the numerator is less than the denominator. In Grade 6, students now work on fractions that are greater than 1 and relate this to mixed numbers. This is completed through the use of models and pictures such as pattern blocks, fraction pieces and number lines.

Students will have an understanding of fractional parts, or equal shares which they refer to as thirds, fourths, fifths, tenths, etc. They will recognize that these fractional parts can be counted in the same way as any other set of objects. Fractions greater than one whole can be understood this way. For example, six fourths is just six parts called fourths. By counting fractional parts we can help students develop a completely generalized system for naming fractions before they learn about fraction symbolism. Counting fractional parts lays the groundwork for several important ideas. The idea that fifths are smaller than halves, for example, can be a difficult concept for students to understand since 5 is usually 'more' than 2. Counting the different size parts and seeing how many parts it takes to make one whole is an opportunity to begin thinking on this idea.

To illustrate this idea, show students manipulatives representing five or six fourths. Ask students to tell how many fourths. Ask if the collection is more or less than one whole, two wholes? While doing this, prompt students to make informal comparisons among different counts. For example, ask them to explore the reasons they would get almost two wholes out of eight fifths when they would not get one whole out of five eighths. Ask them questions such as “What is another way we could say seven fourths? To introduce this topic, use fractional parts that students are most comfortable with, such as thirds, fourths and halves. Challenge students to come up with ways to represent what the fractional pieces show. For example, if they have $\frac{5}{4}$, ask students to explore other ways of showing how they are thinking about this fraction using models, pictures and numbers.

Models can help students clarify ideas that are often confused in a purely symbolic mode. Sometimes it is useful to do the same activity with two different models. From the view point of the student, the activity may be quite different. Using two different models, for example pattern blocks and fraction tiles, students visually see how the same fraction can be modelled in two different ways. This may help students understand the concept of improper fractions holding a value that is greater than 1.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Tell students Billy went to the store. While he was there he noticed a sign saying bars were on sale. “4 for $1.00. Billy thought this was a great deal and bought 4 bars. Billy wanted to share the bars among his two friends. Show how Billy can share this equally among him and his two friends. (6N4.1, 6N4.2)

Student-Teacher Dialogue

- Ask students to explain why \( \frac{5}{3} \) must be greater than one whole. (6N4.1, 6N4.2)

- Ask students to model \( \frac{7}{4} \) to show that it is greater than one whole.
  Ask students to model an improper fraction. Ask them to explain how they know it is an improper fraction. (6N4.1)

- Tell students you modelled the improper fraction \( \frac{9}{5} \) using pattern blocks. Ask them how they know you used 9 blocks. Ask them to determine which pattern block was used to model \( \frac{9}{5} \) and explain how they know. (6N4.1)

- Tell students that the improper fraction \( \frac{10}{3} \) can be modelled using pattern blocks. How many different ways can you model this improper fraction using these pattern blocks? (6N4.1)

- Ask students to represent the following improper fractions using rectangles:
  (i) \( \frac{5}{4} \)
  (ii) \( \frac{3}{2} \)
  Ask students to explain their thinking. (6N4.1)

Resources/Notes

Math Focus 6
Lesson 1: Modelling Fractions
6N4
TG pp. 13 – 17
Strand: Number

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will be expected to</td>
<td></td>
</tr>
<tr>
<td>6N4 Continued</td>
<td></td>
</tr>
<tr>
<td>Achievement Indicator: 6N4.1 Continued</td>
<td></td>
</tr>
</tbody>
</table>

Making meaningful connections to the world can also help students understand the concepts they are learning. While talking about improper fractions, and even mixed numbers, money can be used to help students understand these concepts. A dime, for example, can be seen as \( \frac{1}{10} \) of a dollar where 10 dimes make one whole dollar. Use this context to ask students to explore ways to make more than one dollar using these coins. For example, if they use 5 quarters, which can be represented as \( \frac{5}{4} \), they will have $1.25 or \( \frac{5}{4} \) of a dollar. Since students are really familiar with money, it may be a good place to start when talking about these concepts. This can also help students see the connections between improper fractions and mixed numbers.

It is important that students have a strong conceptual understanding of improper fractions. Students need to be able to understand and explain that an improper fraction represents more than one whole and that its numerator is greater than its denominator. To create this conceptual understanding it is suggested that students engage in many hands on activities that require them to solving problems using improper fractions. Using this approach and encouraging the use of models and pictures, students will begin to get a good idea about what improper fractions and mixed numbers really mean.

Two central concepts students need to develop about improper fractions is that of the purpose of the denominator and the numerator and the fact that the number of parts in a whole may change depending on the situation. Remind students that the denominator names the number of parts in a whole and the numerator names how many parts in all. Similarly, students also need to recognize that the number of parts in a whole can change. Doing activities such as using pattern blocks with the yellow hexagon as being one whole and then changing the whole to be two yellow hexagon, can help students understand this concept.

When using one hexagon as a whole, the red trapezoid is \( \frac{1}{2} \), the blue rhombus is \( \frac{1}{3} \), and the green triangle is \( \frac{1}{6} \).

When two yellow hexagons becomes one whole, the red trapezoid, then becomes \( \frac{1}{4} \), the blue rhombus becomes \( \frac{1}{6} \), and the green triangle becomes \( \frac{1}{12} \).

It may be a good idea to do several examples of this using these blocks to help students recognize this difference.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

• Write the following on the board “All improper fractions must be greater than one whole”. Ask students to explore this statement using models and pictures to agree or disagree with the statement. Ask them to justify or prove their thinking about the statement.

(6N4.1, 6N4.2)

Performance

• Tell students there are 12 eggs in a carton. Ask them if they had 17 eggs, would $\frac{11}{2}$ cartons be enough to hold the eggs? (6N4.1, 6N4.2)

• Ask students to use any type of manipulative available to model an improper fraction. Ask students to pass this model to their shoulder partner where their partner will determine if the model represents an improper fraction. Ask each partner to explain how they know it is or is not an improper fraction and use a picture and the symbolic form to explain.

(6N4.1, 6N4.2)

• Using snap cubes, show students a model of a whole. (E.g., 5 same coloured snap cubes would equal one whole) Ask students to explore different ways to create an improper fraction using this whole that would come between 1 and 2.

(6N4.1)

• Using the numbers 2, 5, 7, 8, ask students to create as many improper fractions as they can. Ask them to choose one improper fraction and represent it using a model, picture and in symbolic form.

(6N4.1, 6N4.2)

Resources/Notes

Math Focus 6
Lesson 2: Fractions Greater Than 1
6N4
TG pp. 18 - 22
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.2 Translate a given improper fraction between concrete, pictorial and symbolic forms.

Elaborations—Strategies for Learning and Teaching

To help students understand that an improper fraction represents a number greater than one, use a hundreds grid. Presenting the idea that 100 blocks in the grid equals one whole (grid), students can explore ways they can represent more than one whole. In the example below, students are asked to shade in 245 blocks and use their picture to name the improper fraction. Using this type of activity will also help students strengthen their understanding of the purpose of the denominator and numerator.

Once students have had ample opportunity building, creating, modeling, drawing, and naming improper fractions, they will be ready to use symbolic forms to represent the fractions they are working with. Students should be able to easily translate a given improper fraction between various representations such as models, pictures and then in numbers. Ask students to model an improper fraction and then draw this representation. Ask students to then use numbers to name this fraction.

To help students see the relevance of translating their models of improper fractions to pictures and then to a symbolic form ask students to prove that $\frac{8}{6}$ is less than $\frac{5}{3}$. To do this ask students to use pattern blocks to build $\frac{8}{6}$ and then draw this on paper as a part of their written response to the question. Students could then go on to show how their picture of the pattern blocks show that $\frac{8}{6}$ is less than $\frac{5}{3}$. 

Students should also see this as $\frac{45}{66}$.

\[ \begin{array}{ccc}
\frac{100}{100} & \frac{100}{100} & \frac{45}{100} \\
\end{array} \]
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to explore the different possibilities of creating an improper fraction with two hexagons being one whole. Challenge them by limiting the number of blocks they can use. For example, if two hexagons equals one whole, ask students to find various ways to represent an improper fraction with 7 blocks. Ask them to record their answers in a chart. Make this a class challenge where students can find as many representations as possible, receiving one point for each representation found and 5 points for each representation that is unique among the students. The winner would be the student with the most points. (6N4.2)

- Provide index cards and ask students to create cards that name an improper fraction and that represent the improper fractions. E.g., students can draw five rhombuses to represent $\frac{5}{3}$ when one hexagon is a whole and then create the corresponding card with the symbolic form of $\frac{5}{3}$. Students can then combine their completed cards and play a matching game whereby they have to match the picture with the number. (6N4.2)

Resources/Notes

Math Focus 6

Lesson 2 (Cont’d): Fractions

Greater Than 1

6N4

TG pp. 18 - 22
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.3 Express improper fractions as mixed numbers.

Elaborations—Strategies for Learning and Teaching

After working with improper fractions, students are formally introduced to mixed numbers. Students need to see the connection between improper fractions and mixed numbers as both represent numbers greater than one whole. They also need to recognize that every improper fraction can be converted to a mixed number and every mixed number can be converted to an improper fraction.

Pattern blocks can be used to help students see these connections. Ask students to model \( \frac{7}{2} \) when one yellow hexagon equals one whole. As they build this model they will see that they have created 3 whole hexagons with one piece or one half of another hexagon left over. Ask students to then think about how they could report what they see. Ask them to think of another way to represent this improper fraction.

Using effective math language while modeling these concepts may become important with regards to student understanding. E.g., if a student was asked to model seven over two, or \( \frac{7}{2} \), he/she may not have a good understanding of what this really means. Compare this to asking a student to model seven halves. As long as a student has a good sense of what a half is, it may be easier to think about seven halves rather than seven over two.

Through working with these numbers, students may discover the relationship in multiplying the denominator with the whole number and adding the numerator to get the improper fraction, but it is not the recommended way to introduce or teach the topic.

“There is absolutely no reason ever to provide a rule about multiplying the whole number by the bottom number and adding the top number. Nor should students need a rule about dividing the bottom number into the top to convert fractions to mixed numbers.” Van de Walle (2006) p. 141.

Providing students with ample opportunities to explore these concepts through the use of hands on activities, using models and pictures will help students develop an understanding but in their own words and in their own way.

(continued)
**General Outcome: Develop Number Sense**

### Suggested Assessment Strategies

**Performance**

- Tell students: Susan wrote the improper fraction \(\frac{7}{4}\). Ask students to think about what this improper fraction could represent.
  
  (6N4.1, 6N4.3, 6N4.2)

- Ask students to think of a mixed number that is a little less than \(\frac{9}{4}\). Ask them to show how they know.
  
  (6N4.1, 6N4.3, 6N4.6)

- Ask students to make a model to represent \(\frac{15}{6}\). Ask them to explain how this number can be expressed as a mixed number using models, picture and numbers.
  
  (6N4.3, 6N4.4)

- Ask students the following: If Daniel ate 9 half donuts, how do you know he ate between 4 and 5 full donuts? Use models, pictures and numbers to show your thinking.
  
  (6N4.3, 6N4.2 6N4.4)

### Resources/Notes

**Math Focus 6**

**Lesson 3: Representing Improper Fractions as Mixed Numbers**

6N4

TG pp. 23 - 27

**Additional Reading** (provided):

Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

6N4.3 Continued

Give students a mixed number such as $3 \frac{2}{5}$. Their task is to find a single fraction that names the same amount. They may use any familiar materials or make drawings, but they must be able to give an explanation for their result. Similarly, ask students to start with a fraction greater than 1, such as $17 \frac{7}{4}$, and ask them to determine the mixed number and provide a justification for their result. This activity can be repeated using other numbers. Similarly, students may find it easier if they are able to choose the numbers they work with for this task. (Van de Walle, 2006).

Give students 5 green triangles and using the yellow hexagon as a whole. Ask students to write the mixed number that represents the triangles. (The purpose of this activity is to help students realize that in order to write a mixed number, the numerator must be greater than the denominator, or there has to be at least one full whole in the number.)

Provide students with opportunities to use concrete, pictorial and symbolic forms to represent numbers. This helps students become exposed to the number in more than one way, where they are physically working with the number using materials. Drawing a picture to represent the number they are working with helps students solidify the concrete image of the number. The next step in this progression of learning is naming the number using symbols.

Ask students to model a given mixed number, for example $3 \frac{2}{5}$ using manipulatives. You may wish to give them a choice of several different mixed numbers depending upon their understanding of mixed numbers. Allow students time to discuss their choice of manipulatives and how their model represents their chosen mixed number. Ask students to then draw a picture to represent this number (it could be a picture of the model they already used, or the number in a different context). Again, ask them to explain how their drawings represent the given mixed number. Ask them to then represent this mixed number in symbolic form (using numbers).

6N4.4 Translate a given mixed number between concrete, pictorial and symbolic forms.

Elaborations—Strategies for Learning and Teaching

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students that a cookie recipe calls for $\frac{4}{3}$ cups of flour. Mr. Bob is not sure what this means. Ask students to help Mr. Bob by explaining to him what $\frac{4}{3}$ means and tell him how much flour he needs using a mixed number.  
  \((6N4.3)\)

- Ask students to model and then draw a picture to show that $\frac{5}{2} = 2\frac{1}{2}$.  
  \((6N4.3, 6N4.5, 6N4.2, 6N4.4)\)

- Tell students to explain a situation when it would be a good idea to express an improper fraction as a mixed number.  
  \((6N4.3)\)

**Resources/Notes**

*Math Focus 6*

*Lesson 3 (Cont'd): Representing Improper Fractions as Mixed Numbers*

*6N4*

TG pp. 23 - 27
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Elaborations—Strategies for Learning and Teaching

As students continue to work with improper fractions and mixed numbers, they may begin to use and refine personal strategies they have created to make sense of this concept. Encourage the use of these strategies and provide students with a lot of opportunities to talk about the strategies they are using and ask them to demonstrate these to other students.

Many types of manipulatives can be used to explore improper fractions and mixed numbers. For example, coloured snap cubes can be used to create different lengths where students can compare the lengths of the joined cubes. Students could create a length of 4 blue cubes and another with 8 yellow cubes. Students could then talk about how the lengths of these two compare. So, blue is \( \frac{1}{2} \) the length of yellow. If they then create a length of 6 red cubes, they would say that red is \( 1 \frac{2}{4} \) the length of blue.

Another suggested activity in using the snap cubes would be to provide students with 20 same colored snap cubes. Ask students to represent an improper fraction such as \( \frac{17}{5} \). They should understand from previous work that \( \frac{17}{5} \) means there are 5 in a whole with 17 parts in all. They should then go on to create towers of 5 snap cubes where they will see that they can create 3 complete towers with 2 cubes left over. This can be used then to help them see that \( \frac{17}{5} \) is the same as \( 3 \frac{2}{5} \).
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Invite students to make a plan to teach their parents about improper fractions and mixed numbers. Ask students to use models, pictures, numbers and words to show their parents how to express a mixed number as an improper fraction.  

  (6N4.3, 6N4.5)

- Improper Fraction War - Using a deck of cards containing numerals 1-9, ask students to work in small groups. Shuffle the cards and deal 4 cards to each player. Students can use any two of the 4 cards in their hand to create the greatest possible improper fraction. In turn, each player will reveal their improper fraction determining who has the greatest number. Students may have to convert these improper fractions to mixed numbers to help them compare the numbers. The player with the greatest improper fraction will score one point. The first player with 5 points wins. 

  (6N4.3)

- Ask students to create their own riddles about improper fractions and mixed numbers. Ask their classmates to solve the riddles. E.g.,

  I am a unit of time.
  My denominator is 2.
  My numerator is the number of days in one week.
  How many days did Mary work?  

  \( \frac{7}{2} = 3 \frac{1}{2} \text{ days} \)  

  (6N4.3, 6N4.5)

Resources/Notes

**Math Focus 6**

**Lesson 4: Exploring Improper Fractions and Mixed Numbers**

**6N4**

TG pp. 28 – 31

The lesson activity calls for the use of colored rods. If these are not available, colored snap cubes can be used instead.

**Children’s Literature (provided):**

Merrill, Jean *The Toothpaste Millionaire*

Please note: The book is written using Imperial units. Please modify to reflect the metric system.
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.5 Express mixed numbers as improper fractions.

Elaborations—Strategies for Learning and Teaching

Ask students to use any manipulative to model or represent $\frac{5}{3}$. Discuss with students their choice of models and lead the discussion to help them see or link the $\frac{5}{3}$ to $1 \frac{2}{3}$. Invite students to explain some of their personal strategies that help them to understand that $\frac{5}{3}$ is the same as $1$ and $\frac{2}{3}$.

Using pattern blocks, have the yellow hexagon represent 1 whole. Give students 7 blue rhombuses and ask them how many complete hexagons they could cover. Ask them to explore and decide what fraction of the hexagon they could cover with the left over blue rhombus. Then ask students to think about how to write the mixed number and the improper fraction that the 7 blue rhombuses represent.

Students have been modeling, creating, describing, drawing and naming improper fractions and mixed numbers. They have been exploring with naming improper fractions as mixed numbers and mixed numbers as improper fractions. Now, students will strengthen their understanding that every improper fraction can be written as a mixed number and every mixed number can be written as an improper fraction. They will learn that when an improper fraction can be expressed as a mixed number these numbers then are equivalent. Equivalent fractions were taught in Grade 5.

The figure below illustrates this idea by showing that the improper fraction $\frac{7}{4}$ and the mixed number $1 \frac{3}{4}$ refer to the same fraction of a region, or area, therefore they are equivalent.

![Diagram of fractions](image)

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students that it takes 4 – 250 mL cups to fill a 1 L bottle. If they had 10 – 250 mL cups, how many 1 L bottles would they fill? Represent this answer as a mixed number and an improper fraction. 
  
  \[
  \text{(6N4.3, 6N4.5)}
  \]

- Ask students to choose one mixed number and one improper fraction. Ask them to write these numbers on separate pieces of paper. Next ask students to create a model for each number and draw a picture to represent each. Next, students to switch papers and models with a partner and asks their partner to express the improper fraction as a mixed number and express the mixed number as an improper fraction. Finally, partners model these new numbers using a different type of manipulative and draw their representation of the numbers and show the number in symbolic form. 
  
  \[
  \text{(6N4.2, 6N4.4)}
  \]

\[\begin{align*}
\text{a)} & \quad \begin{array}{c}
\text{\includegraphics{image1.png}} \\
\text{5/4} \\
\text{- student A chooses 5/4}
\end{array} \\
\text{b)} & \quad \begin{array}{c}
\text{\includegraphics{image2.png}} \\
\text{1\frac{1}{4}} \\
\text{- student B expresses 5/4 as 1\frac{1}{4}}
\end{array}
\end{align*}\]

- Ask students to think of possible values for \(13/4\) when \(13/4\) is an improper fraction that is between 2 and 3. Ask students to determine if there is more than one answer and explain how they know. 
  
  \[
  \text{(6N4.3, 6N4.5, 6N4.6)}
  \]

- Tell students it takes \(\frac{1}{3}\) of an hour to bake one batch of cookies. If they had 5 batches of cookies to bake, how long would it take. Ask them to represent their answer using a mixed number and an improper fraction. 
  
  \[
  \text{(6N4.3, 6N4.5)}
  \]

**Resources/Notes**

*Math Focus 6*

**Lesson 5: Representing Mixed Numbers as Improper Fractions**

6N4

TG pp. 36 - 40
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.5 Continued

The figure below illustrates another way to show that an improper fraction can be expressed as a mixed number, where both name the same quantity. In this example, it can be seen that there is one full egg carton, or \( \frac{12}{12} \) eggs and a part of another carton, namely \( \frac{5}{12} \). The resulting mixed number then, would be \( 1 \frac{5}{12} \) or the equivalent improper fraction that would name the amount of the set would be \( \frac{17}{12} \).

Using the book *Funny and Fabulous Fraction Stories* ask students to read the section on Improper Fractions and Mixed Numbers found on page 22. Set up this activity as a reader’s theatre. Students take on the roles of the various characters but extend the scenes, whereby, the callers of the radio station need further clarification about the nature of converting mixed numbers to improper fractions. Students can rewrite the script to do this. (It may not be necessary to do the second scene with Joe and Al as this section deals with simplifying the fractions which is not a requirement for Grade 6). Once Joe and the caller is established, give the remaining students the role of the Math Experts in which Joe calls upon to help out the callers “Betty” and “Pauline”. In the scenes Betty and Pauline call Joe Trella, the Fraction Fella, to get advice on a problem they have involving converting mixed number and improper fractions. The Math Experts have to come up with a plan on how to show Betty and Pauline how to convert these numbers. Encourage students to use different ways of teaching Betty and Pauline as they may need to see how it can be done in different ways. These scenes should be done separately with Joe and the callers being different people allowing all students to be involved in the lesson. You may also decide that Joe could have a role where he is showing Betty a way to convert and then have the others demonstrate their thinking to coincide with Joe’s lesson. Alternatively or as a follow up, other types of situations can be created where students role play the scenarios demonstrating their understanding of a concept.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Mixed Number War - Using a deck of cards containing numerals 1-9, ask students to work in small groups. Shuffle the cards and deal 3 cards to each player. Students use the cards to create the greatest possible mixed number. In turn, each player will reveal their mixed number. The player with the greatest mixed number will score one point. The first player with 5 points wins. (6N4.5)

- Place a mixed number/improper fraction on students’ back. Instruct the class to question the student about the number to determine what it is. Limit the number of questions the student can ask. This could also be done as a class activity where all students have improper fractions/mixed numbers on their back where they have to go around the class asking questions to get clues about what their number is. (6N4.3, 6N4.5)

- Tell students that you have 18 quarters in a coin collection. How many dollars would this be? Can you model the amount as an improper fraction? Mixed number? Which one would be easier to understand how much money you have altogether? (6N4.3, 6N4.5)

- Mario and Sydney are great hockey players. To prepare for the season, Mario practiced $3 \frac{3}{4}$ weeks. Sydney practiced $\frac{25}{7}$ weeks. Who practiced more? Explain your thinking using pictures, numbers and words. (6N4.3, 6N4.5)

Resources/Notes

- Math Focus 6
- Lesson 5 (Cont’d): Representing Mixed Numbers as Improper Fractions
- 6N4
- TG pp. 36 - 40

Children’s Literature (provided):
- Greenberg, Dan. *Funny and Fabulous Fraction Stories*
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

Give students a mixed number such as $2 \frac{1}{3}$. Using pattern blocks, ask students to use the blue rhombi to create a design on triangular dot paper that represents this mixed number. Ask them to use their design to explain how this mixed number also represents the equivalent improper fraction, where they would write the improper fraction in symbolic form.

This is just one example of how the blue rhombi can be used to model $2 \frac{1}{3}$. Encourage students to be creative in their designs.

The picture below shows another example of how to use pattern blocks, namely the green triangles, to represent an improper fraction on triangular dot paper. Ask students to represent $\frac{13}{6}$ and then explain how this also shows $2 \frac{1}{6}$. 
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to choose a mixed number and tell them to keep it a secret from their classmates. Set up centers around the room where students could model their number in one center, draw it in another and then represent it using an improper fraction in yet another center. After everyone has had an opportunity to represent their number in the three centers, bring the class together. Ask them to match all the models to the corresponding pictures and numbers.

  (6N4.2)

- Give students an equivalent improper fraction and a mixed number (e.g., \( \frac{12}{5} \) and \( 2 \frac{2}{5} \)). Ask them to represent these two numbers concretely, pictorially and symbolically to show they are equivalent.

  (6N4.4, 6N4.5)

Resources/Notes

Math Focus 6

Lesson 5 (Cont’d): Representing Mixed Numbers as Improper Fractions

6N4

TG pp. 36 - 40
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

6N4.6 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.

Elaborations—Strategies for Learning and Teaching

Students have been working with mixed numbers and improper fractions by representing, modelling, naming and expressing them in different forms. As students continue working with these numbers they can begin thinking about strategies that would enable them to compare improper fractions and mixed numbers. In Grade 5, students worked with comparing proper fractions with like and unlike denominators which will now help them compare mixed numbers and allow them to extend their personal strategies to compare improper fractions.

When comparing improper fractions, encourage students to recognize that it may be easier to express the improper fraction as a mixed number where they would compare the whole number first and then look at the proper fraction if needed. For example, when comparing $\frac{6}{4}$ and $\frac{9}{5}$, students could express both as a mixed number namely, $1\frac{2}{4}$ and $1\frac{4}{5}$. Here they could easily see that they are both 1 whole with the first having an extra $\frac{2}{4}$ or $\frac{1}{2}$ and the second having an extra $\frac{4}{5}$. Students should be able to see that $\frac{4}{5}$ is greater than $\frac{2}{1}$ because $\frac{4}{5}$ is greater than $\frac{1}{1}$.

Using a number line when solving problems is another strategy that students can use to help show their understanding. As students encounter problems that require them to compare mixed numbers and improper fractions, a logical representation of the understanding would be to place the given numbers on a number line.

Fractional Clothesline - Stretch string across the classroom with various points marked for 0, 1, 2, 3, and 4. You may want to ask students to do this. The clothesline will be used to show students that all proper fractions are between 0 and 1 and all mixed numbers and improper fractions are greater than 1. Students will clip index cards with various proper fractions, improper fractions and mixed numbers on the clothesline. You may wish to ask students to place various benchmarks and ask them to choose the numbers they want to place. For example, you may wish to ask them to write any mixed number or improper fraction that would come between 1 and 2, or 3 and 5 that could go on the number line. After each student has had a chance to place some numbers on the line, have a class discussion to decide if all the placed numbers are in relative positions. You may even want to give the class a chance to change some already placed numbers before the discussion takes place.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Ask students: Bill said he ate \( \frac{1}{3} \) bags of popcorn. Ed said he ate \( \frac{4}{3} \) bags of popcorn. Assuming each bag of popcorn is the same size, is it possible for Bill to have eaten more popcorn than Ed? Explain your thinking with pictures, numbers and words.

  \((6N4.3, 6N4.5, 6N4.6)\)

- Ask students to determine which number is greater – \( \frac{26}{5} \) or \( 4 \frac{3}{4} \). Ask students to explain using pictures, numbers and words how they know.

  \((6N4.6)\)

- Tell students: 4 friends were at a party. Joe stated he ate \( \frac{5}{3} \) of pizza while Amy stated she ate \( \frac{5}{4} \) of pizza. Larry said that Amy ate more pizza than Joe. Is Larry correct? Explain your thinking.

  \((6N4.1, 6N4.6)\)

- Ask students why someone would think that \( \frac{5}{4} \) is greater than \( \frac{3}{2} \)? Use pictures, numbers and words to explain.

  \((6N4.1, 6N4.3, 6N4.5, 6N4.6)\)

- On the number line below, place the following: \( \frac{3}{2}, \frac{1}{2}, \frac{5}{4}, \frac{15}{4}, 1 \frac{2}{5} \).

  \( \quad 2 \quad \frac{1}{2} \quad \frac{3}{2} \quad 2 \quad \frac{1}{2} \quad \frac{5}{4} \quad \frac{15}{4} \quad 1 \frac{2}{5} \)

  Choose one to explain its placement.

  \((6N4.6)\)

- Ask students how they would immediately know that \( 2 \frac{2}{3} \) is greater than \( 1 \frac{7}{8} \)? Explain.

  \((6N4.6)\)

- Ask students to choose two improper fractions or two mixed numbers, order them and compare them. Ask them to explain to a friend how they know they have ordered their numbers correctly.

  \((6N4.1, 6N4.3, 6N4.5, 6N4.6)\)

- Tell students two athletes were competing in a biathlon. Athlete one finished the race in \( \frac{7}{6} \) hours and the other athlete finished in \( \frac{12}{10} \) hours. Ask students to determine who won.

  \((6N4.6)\)
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

- 6N4.3 Continued
- 6N4.5 Continued
- 6N4.6 Continued

Elaborations—Strategies for Learning and Teaching

As students continue working with improper fractions and mixed numbers, they can use this understanding to help them solve meaningful problems using logical thinking. Using logical reasoning is yet one strategy for solving problems students have developed. Engaging students in problems that allow them to synthesize and analyze the information presented, having them plan out their solutions and then represent their thinking using pictures, numbers and words will help strengthen their problem solving process.

Another strategy that may help students solve problems is that of solving a simpler problem that is related to a given problem. For example, if a problem deals with both improper fractions and mixed numbers, ask students to solve the problem in which only improper fractions are used. Similarly, students could also change the numbers in the problem (if that is what is causing difficulty) to numbers they are more comfortable with and note how they would go about solving the problem using simpler numbers and how this could be transferred to solving the problem with the given numbers.

Encourage students to pick out the most important parts of the problem, draw or use a model to help them visualize the given information and identify exactly what the problem is asking. Students should be given many opportunities to work together while solving problems and talk out their thinking with their partner, as it is through this collaboration that students learn best.

Students may find it interesting to explore how the fraction name changes if the whole changes. If the blue rhombus is the whole, using pattern blocks, for example, the green triangle would be $\frac{1}{2}$ and the red trapezoid would be $1 \frac{1}{2}$ or $\frac{3}{2}$. Playing with these different scenarios will help students better understand the relationship between the improper fractions and mixed numbers.
### General Outcome: Develop Number Sense

#### Suggested Assessment Strategies

**Performance**

- Tell students there are 24 cans of soft drink in one case. 60 cans of drink are needed for the volleyball tournament. How could knowing about mixed numbers and improper fractions help you determine the number of cases of drinks that are needed for the tournament? (Remind students that a part of a case of drink can be bought)  
  \((6N4.1, 6N4.3)\)

- Ask students to write two improper fractions and two mixed numbers that are between 4 and 5. Ask them to explain their thinking.  
  \((6N4.6)\)

- Pose the following to students: You just baked one dozen brownies. Seven of your friends just arrived and you want to share the brownies equally among all of you. Show how you would do this and write the amount you all get using a mixed number and an improper fraction.  
  \((6N4.3, 6N4.5)\)

- Tell students that the answer to a problem is \(2 \frac{1}{5}\). Ask them what the question might be?  
  \((6N4.6)\)

**Student-Teacher Dialogue**

- Ask students: Which of the following improper fractions and mixed numbers are between 2 and 3? Explain.  
  \(\frac{7}{5}, \ 2 \frac{1}{5}, \ 3 \frac{1}{2}, \ \frac{7}{4}\)  
  \((6N4.6)\)

### Resources/Notes

**Math Focus 6**

**Lesson 7: Solving Problems Using Logical Reasoning**

**6N4**  
TG pp. 48 – 50

**Curious Math: Growing Shapes**

**6N4**  
TG pp. 51 – 52
FRACTIONS
Multiplication and Division of Decimals

Suggested Time: $3 \frac{1}{2}$ Weeks
Unit Overview

Focus and Context

This unit will draw upon students’ previous knowledge of multiplication and division of whole numbers. The strategies that were previously taught and developed when working with whole numbers should be brought forward for use in this unit. Mental Math strategies are very important in this unit as well.

Estimation strategies are very important as students work with computing decimals. Students should be encouraged to estimate the product or the quotient before calculating the solution. In doing so, they will be able to determine if their calculated answer is reasonable. Estimation is also an essential tool for students to correctly place the decimal point in the product or quotient. As students work through this unit, they will be encouraged to use pencil and paper as well as technology and manipulatives to solve problems involving multiplying and dividing decimals.

Students will be working on multiplication and division of decimals to the thousandths involving single digit multipliers and divisors. Students should be encouraged to use concrete materials, such as base-ten blocks or decimal squares, which will promote a stronger conceptual understanding of the content rather than simply following a set procedure. These concrete and visual representations of student thinking will encourage them to develop, analyze and explain methods for solving problems involving multiplication and division of decimals.

Math Connects

It is very important to make real life connections involving decimals. The most obvious connection for students involves money. Students are familiar with using money and this connection needs to be stressed, especially at the start of this unit. This will help develop a sound understanding of working with decimals. Previous skills involving multiplication and division will be used to further develop the understanding of multiplication and division of decimals.
Process Standards Key

| [C] Communication | [PS] Problem Solving |
| [CN] Connections   | [R] Reasoning        |
|                  | [V] Visualization |

Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>6N8 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).</td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
</tbody>
</table>
Outcomes

Students will be expected to

6N8 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

[C, CN, ME, PS, R, V]

Elaborations—Strategies for Learning and Teaching

The focus is on multiplication of decimals with a one digit whole number multiplier. Division of decimals will be addressed later.

Estimation should play a significant role in students’ development of personal strategies for multiplication of decimals. As a beginning point, consider this problem:

The farmer fills each jug with 3.7 litres of cider. If you buy 4 jugs, about how many litres of cider is that?

Students should see that 3.7 is about 4 and $4 \times 4 = 16$.

The students will then use any method to determine if their answer is correct. (Repeated addition, doing the actual multiplication, or using a concrete representation).

Students have had prior experience with estimating decimals. At this time, it may be necessary to review these concept with students. The strategies for estimation of decimals are the same as estimation of whole numbers. Students may benefit from practice in estimation of whole numbers prior to focusing on decimals.

Achievement Indicator:

6N8.1 Predict products and quotients of decimals, using estimation strategies.

Students will begin work on predicting products of decimals using estimation strategies. Quotients will be addressed later. They may need to be reminded of the various methods of estimating:

- **front-end estimation** – a method by keeping the first digit of the number and changing all the other digits to zero.
- **compatible numbers** – a method of using friendly or nice numbers that can be easily calculated mentally.
- **rounding** – a method by which a number is rounded up or down to the nearest whole, tenth, etc.
- **compensation** - a method of adjusting a computational estimate to make it closer to the calculated answer.

E.g., When estimating the product of 2.629 and 4:

* front-end estimation - $2.000 \times 4 = 8$
* compatible numbers - $2.5 \times 4 = 10$ or $3 \times 4 = 12$
* rounding - $3 \times 4 = 12$ or $2.6 \times 4 = 10.4$
* compensation - $2.629 \times 4$. Think $2 \times 4 = 8$ (front end estimation)

0.629 is about $0.5 \times 4 = 2$, thus $8 + 2 = 10$.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Journal**

- Give students a choice of several multiplication expressions involving decimals multiplied by a single digit whole number. Ask them to choose one or two sentences and estimate the solution. Next, ask students to explain how they estimated the product(s) and to justify their thinking.

  \[(6N8.1)\]

**Performance**

- Ask students to use the information given to determine the better buy:
  
  A. Apple juice – 2 L for $1.99 or 4 L for $3.89
  B. Oranges – 4 for $0.99 or 6 for $1.59
  C. Bananas – 3 kg for $1.89 or 5 kg for $3.19

  Ask students to explain how they know. \[(6N8.1, 6N8.2)\]

- Tell students a decimal number was rounded to 3. Ask them what that number could be. \[(6N8.1)\]

**Resources/Notes**

*Math Focus 6*

**Lesson 1: Estimating Products**

*6N8*

TG pp. 12 – 15

The text does not state dollar values with a decimal point. E.g., $1200 should be written as $1200.00. The decimal point should be put in for even dollar amounts.
Strand: Number

### Outcomes

*Students will be expected to*

6N8 Continued

**Achievement Indicators:**

**6N8.1 Continued**

<table>
<thead>
<tr>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give students many opportunities to practice their estimating skills. As a class warm up, ask students to work in pairs. Each partner gives the other a decimal number and they take turns estimating the given number. As students begin working with estimating products of decimal and whole numbers, this warm up activity could be extended and partners take turns giving each other a number sentence and estimating the product sharing their strategies.</td>
</tr>
</tbody>
</table>

| To make real life connections, create problems involving money using catalogues, and flyers, and also sport related data (e.g., speed times for downhill skiing). |

| As students continue to solve problems involving multiplication of decimals and whole numbers, they are only expected at this point to use estimation to solve the problem and not calculate the actual solution. |

| It is suggested that students estimate decimal numbers to the nearest whole, at first to make computation of estimated products easier. It is not necessary, at this point, to expect students to mentally calculate products of decimal numbers, as the focus is on estimation. |

| Base-ten materials, decimal squares, and number lines can be used to represent decimal numbers to help students recognize, for example, that when 3.7 is estimated students can see that it is closer to 4 than 3. |

| Giving students many opportunities to represent decimals with manipulatives is essential as they progress through their work in the multiplication and division of decimals. **It is important that students need to recognize when representing decimals, the one whole can be represented using different base-ten blocks.** |

| The book *Piece=Part=Portion: Fraction=Decimal=Percent* can be used to review decimals as it gives a good visual representation of each decimal indicated. Have a discussion about the various pictures and how they represent the decimal. |

*(continued)*
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- As a mental math activity, ask students to find the product of 5.6 × 2, using an estimation strategy of their choice. Then expand that to decimals in the hundredths, then the thousandths place. Ask students to explain how they know their estimate is reasonable.

  (6N8.1, 6N8.2)

- Give students a grocery store flyer and ask them to select any item for purchase. Ask them if they were going to purchase 6 of the same item, about how much would it cost. They will need to be able to explain the strategy for estimation. Extend this to a specific dollar amount (e.g., $100.00) and ask students to select the number of this item that they could purchase with this amount of money without going over.

  (6N8.1, 6N8.2)

Resources/Notes

Math Focus 6
Lesson 1 (Cont’d): Estimating Products
6N8
TG pp. 12 – 15

Related Literature (provided):
Gifford, Scott. Piece=Part=Portion: Fraction=Decimal=Percent
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicators:

6N8.1 Continued

Elaborations—Strategies for Learning and Teaching

Students may want to use these pictures to help them understand the various ways to estimate the given decimal. This book can also be used to model ways to create problem solving situations, where students can practice using their estimation skills to estimate products of decimals and whole numbers. Ask students, for example, to look at the page where there is 1 stick of gum shown out of 5 in the package. Review how this fraction can be written as a decimal, 0.20 where this would represent the remaining sticks of gum in the package. Use this context to engage students in finding out how many packs of gum they would have. The visual representation will help students make the connection that when multiplying a whole number by a decimal, you can get a decimal number. They will be trading parts for a whole and counting what is left to get the answer.

Students should have a solid understanding of multiplying a single-digit whole number by a decimal number in the tenths place before they move on to multiplying a whole number by a decimal in the hundredths and thousandths place. The use of base-ten grids or decimal squares will help students visualize of the concept.

In previous work, students were only expected to solve problems using estimation. Now, students will continue to use their estimation strategies to solve problems but will also be expected to find the exact answer. Their estimate answer will now verify the reasonableness of their exact answer.

Students should be given opportunities to experience ‘real life’ estimation situations and realize that the most important estimate is not always the closest to the actual answer. E.g., If you needed 2.2 metres of string to wrap a parcel and you have 3 parcels to mail, how much string would you need? Most students would say 2 × 3 = 6 metres. While this is a close estimate, it would be inappropriate since you would not have enough string to wrap the parcel.

Most students should be able to easily relate money to decimals. This will be an easy concept to work with as flyers and catalogues can be used to show real life examples of decimals. Students can also talk about saving their allowances to buy something special, or solve problems where they are given a certain amount of money and can go on a shopping spree.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Ask the students to draw or build a model to illustrate \( 4 \times 1.36 \) and show the answer. Ask students to create a story problem based on the multiplication sentence and share it with a partner to solve.
  
  \( (6N8.2) \)

- Give students a grocery flyer and ask them to choose 3 of their favourite items. Ask them to determine how many of each item they could buy with $90.00.
  
  \( (6N8.1, 6N8.2) \)

**Paper and Pencil**

- Ask students to determine how much more five cans of juice cost at $1.29 each than 6 cans at $0.99 each.
  
  \( (6N8.2) \)

- Present the following: Milk at school costs $0.55. If there are 8 students in your class who order one milk per day, how much would it cost each day for milk in your class? How much would it cost for one week?
  
  \( (6N8.1, 6N8.2) \)

- Pose the following to students: If one school basketball jersey costs $18.49, about how much would it cost to buy 9 jerseys. Show how you got your answer pictorially and explain your thinking.
  
  \( (6N8.1, 6N8.2) \)

Resources/Notes

*Math Focus 6*

**Lesson 1 (Cont’d): Estimating Products**

\( 6N8 \)

TG pp. 12 – 15

*Math Focus 6*

**Lesson 2: Multiplying Money Amounts by One-Digit Numbers**

\( 6N8 \)

TG pp. 16 - 19
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicator:

6N8.2 Continued

Elaborations—Strategies for Learning and Teaching

It is important that students draw or build models to show the multiplication sentence involving decimals. E.g., \(2 \times 0.7\)

Show the students (or ask them to find) 2 decimal squares for 0.7. Overlap them to illustrate the total shaded amount so students can see how one whole is created and how many parts of another whole there are.

Another way to illustrate this example is to use base-ten blocks. Students can represent 0.7 using seven rods, where one flat is equal to one whole. They could make 2 groups of 7 rods and see that there is a total of 14 rods. Knowing there are 10 rods in a flat, students should understand that 14 rods equals one whole and 4 tenths. Students can also use a number line to represent \(2 \times 0.7\). After placing 0.7 on the line, students could move another 0.7 along the line to arrive at 1.4.

Students need to know about the placement of the decimal points in the product of a multiplication problem. To do this, they could use front-end estimation.

E.g., There are 3 CDs that cost $12.69 each. How much money would you need to buy these 3 CDs?

Which is the best choice for the correct answer?

A) 380.70
B) 3.807
C) 38.07
D) 3807.00

Think: Round 12.69 to 12 and \(12 \times 3 = 36\). The answer from our choices has to be close to 36.

Patterns can be used to help students understand the placement of the decimal in the product of two decimal amounts.

E.g., \(420 \times 4 = 1680\) \(42 \times 4 = 168\) \(4.2 \times 4 = 16.8\)

Decimal placement can also be explored using a calculator but it is important that students practice applying mental math strategies.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Tell students to look at the following number sentence where the decimal point has been left out. Ask them to estimate the product to determine the placement of the decimal. Ask them to explain how estimation helped them correctly place the decimal point in the product.

\[ 3 \times 16.17 = 4851 \]

(6N8.3, 6N8.1)

Portfolio

- Each product is missing a decimal point. Place the decimal in each product. Explain one answer.

(i) \[ 15.97 \times 3 = 4791 \]

(ii) \[ 4.326 \times 7 = 30282 \]

(6N8.4)

Resources/Notes

Math Focus 6
Lesson 2 (Cont’d): Multiplying Money Amounts by One-Digit Numbers

6N8
TG pp. 16 - 19
Outcomes

Students will be expected to

6N8 Continued

Achievement Indicators:

6N8.2 Continued

Elaborations—Strategies for Learning and Teaching

Estimation is essential when determining the correct placement of the decimal in the product when multiplying a whole number by a decimal. Applying estimation strategies will help students correctly place the decimal point in the product.

E.g., You go to the store to buy 9 cases of water. Each case costs $6.69. The cashier tells you the total is $602.10. You know immediately that this is wrong. Explain what you think the cashier did to get this total.

Having students solve problems involving money will help them make a connection between using decimals in school and real life experiences. Putting decimals in the context of money rather than letting the decimal number stand alone will make learning more meaningful.

Using the book, Math Curse students can explore the various ways the character in the book finds decimals in his daily life. Ask students to discuss where they see and use decimals in their lives. Using the same context of the book, ask students to record all the ways they use decimals in one day, especially when they use multiplication of decimals to figure out a problem. For example, they may go to the cafeteria and buy three cookies for $0.35 each. They would then need to know if $1.00 is going to be enough money to buy all 3.

Give students ‘play’ money, (e.g., $100.00) and various catalogues or flyers. Ask them to spend their money buying items of interest. The tricky part is that they have to buy the same item for their 2 friends. Ask them to make a list of things to buy and keep a record of how much they spend. Ask them to present to the class, what they would buy and how they were able to come up with the total cost. Students could also be encouraged to talk about how estimation of prices helped them to determine if the item(s) were affordable.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

• Ask students to explain why the product of 0.6 and 3 will have a digit in the tenths place. Use words, pictures and numbers to explain your thinking.

• Ask students to explain whether or not John is right when he says that the answer to $4 \times 4.5$ is 0.18.

Presentation

• Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.

  E.g., $4.35 \times 6 = 2.615$
  $6.487 \times 2 = 129.74$

Resources/Notes

Math Focus 6
Lesson 2 (Cont’d): Multiplying Money Amounts by One-Digit Numbers
6N8
TG pp. 16 - 19

Children's Literature (provided):
Scieszka, Jon and Smith, Lane. Math Curse
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicators:

6N8.2 Continued

Elaborations—Strategies for Learning and Teaching

It is important for students to use manipulatives such as base-ten materials or decimal squares, as it may help them visualize the concept of multiplication more easily. Also, previous work on multiplying decimals in a money context may enable students to think about what the decimal means in a more meaningful manner. Although Grade 6 students have had prior experience modelling decimals where they would have used different blocks to represent one whole, it may be necessary to review this concept again as some students may struggle with the idea that one whole can be represented in different ways.

Using decimal squares, ask students to shade a blank hundreds grid (decimal square) to represent 0.27. Ask them how they could now use this square to represent or show $5 \times 0.27$. Students should come to recognize that if they shaded 5 groups of 0.27 on the grid that they could represent the product of these numbers. Students can then be asked to explain other ways they could obtain the product.

The squares shown here indicate that the shading was done by repeatedly shading 27 parts (x’s and o’s) to fill up the squares where 135 blocks in total are shaded or 1 whole and 35 hundredths.

Another strategy students could use to help them solve multiplication problems involving decimals is the area model. This model may have already been introduced to students in Grade 5 in multiplying whole numbers and now can be used to multiply whole numbers and decimals.

Demonstrate using a rectangle and base-ten materials students, as shown, to multiply $4.6 \times 2.2$. The dotted lines divide the rectangle into sections.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Portfolio**
- Tell students that a person’s hair grows an average of 0.83 cm a month. Ask students to predict how long a child’s hair would grow in 9 months if they never had a haircut. Ask them to explain their estimate.

Resources/Notes

*Math Focus 6*

**Lesson 3:** Multiplying Decimals by One-Digit Numbers

6N8

TG pp. 20 - 24

Additional Reading (provided):
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicator:

6N8.3 Continued

As students continue working with multiplication of decimals and whole number multipliers, they should be encouraged to continue to use estimation to verify that the answers to problems they solve are reasonable. One of the suggested estimation strategies is front–end estimation, where students simply look at the first digit and round all other digits to zero. Students should be cautioned that although this is a sound strategy for estimation, especially in correctly placing decimal points, it may be misleading. To illustrate this point, ask students to consider the following example:

Jillian is building a wood box. She measures the length to be 2.98 m, the width, 1.87 m and the height 1.6 m. To determine the amount of board she needs, Jillian uses front end estimation to get an idea of how much material to buy. What might happen if Jillian uses this strategy without finding the exact amount of board needed?

6N8.4 Continued

Students need to practice where to place the decimal point in a given product. Teach students to place the decimal point in products by methods other than simply counting decimal places in the factors as this does not promote an understanding of place value or number sense. The important concept students need to understand is the place value of the digit in the product will change according to the placement of the decimal. In an example such as 1.255 × 2 = 2.51 students should see that counting the decimal places will not help them verify if this answer is correct.

Help students to see the decimal number in terms of its value. For example, students should see that 1.62 is a little more than 1 and one half. When solving a problem that involves multiplying, for example 1.62 by 5, students should see the product would be a little greater than 5, but less than 10 as 1.62 is less than 2. They should also see that the exact product will be a little more than 7.5, as 5 groups of 1.5 is 7.5.

When making predications about the product of a decimal number and its whole number multiplier, the product has to be reasonable based on previous knowledge of decimals and place value. When students look at problems involving multiplication, they should be able to come up with a reasonable answer and be able to justify how they came up with that answer. This will not be automatic for some students and practice with manipulatives will be beneficial.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students Jack wanted to pay each of his 3 friends $10.15 each for helping him paint his shed. Ask students to estimate the total amount of money that Jack will have to pay to his friends.  
  \((6N8.3, 6N8.2)\)

- Ask students to explain why the decimal point in each of the following problems, is in the wrong place.
  (i) Fred calculated that \(315.2 \times 2 = 63.04\)
  How do you know his answer is incorrect? What is the correct answer? Show your workings.
  (ii) The postman gave 9 students stamps that were valued at $10.45 each. “The total cost of the stamps is $940.50” exclaimed one student. Was she correct in saying that? Explain.  
  \((6N8.3, 6N8.4, 6N8.1)\)

- Tell students Mr. Brown took his family of 8 to a local fast food restaurant. A meal for each person cost $9.59. Ask students to estimate what Mr. Brown's bill will be before taxes and to calculate the total cost. Show your workings and strategies.  
  \((6N8.1, 6N8.2)\)

- Allie bought three bags of bird seed. Each bag weighed 0.398 kg. What is the total mass of all three bags of bird seed? Demonstrate your understanding in two different ways.  
  \((6N8.2)\)

**Journal**

- Ask students if they were to predict the answer for \$21.57 \times 5\, , would it be greater or less than one hundred? Ask them how do they know.  
  \((6N8.1)\)

**Paper and Pencil**

- Ask students to use base-ten blocks or decimal squares to solve the following:
  (i) \(4.8 \times 2\)  
  (ii) \(7.3 \times 8\)  
  (iii) \(3.1 \times 7\)  
  \((6N8.3, 6N8.2)\)

- Ask students to find the numbers that, when multiplied, give the products shown:

\[
\begin{array}{c}
\square \cdot \square \\
\times \square \\
\square \square \square
\end{array}
\]

\[
\begin{array}{c}
\square \cdot \square \\
\times \square \\
\square \square \square
\end{array}
\]

\((6N8.3, 6N8.1, 6N8.2)\)
Focus is now on division of decimals involving 1 digit natural number divisors. Many of the personal strategies students use to compute division using whole numbers can also be applied to division of decimals. Using base-ten materials hand in hand with these strategies will help students to become more efficient and accurate when dividing decimals.

The concept of division involving decimals is new to students in Grade 6. Therefore, it will be necessary to begin with a review of division of whole numbers (2-digit and 3-digit by 1-digit divisors) before beginning work on dividing decimal numbers.

As with multiplication of decimals, estimation should precede all calculations when beginning work on division of decimals. Students will only be expected to estimate quotients up to this point and will be required to calculate exact quotients later in the unit. These estimations will be a basis for students to test the reasonableness of their answers in later work. It is important that students use proper terminology when working with division. The use of key words such as quotient, divisor and dividend, should be encouraged.

Achievement Indicators:

Students have used front-end estimation as they multiplied decimal numbers by 1-digit whole numbers. Now they will use this estimation strategy to work with division of decimals and natural numbers in appropriately placing the decimal point in the quotient. Show students that by rounding the decimal number to the nearest whole number (that is a multiple of the divisor), they can then more easily identify a reasonable solution to the problem.

As in predicting products using estimation strategies, students can now use these same strategies to predict quotients of decimals. Students could also use the ‘think multiplication’ strategy as they predict quotients of decimals.

Also, students should be encouraged and provided with opportunities to use mental math to complete estimation problems. This can be achieved by having students practice estimating numbers on a regular basis.

Solving problems involving division of decimals may be a little more challenging for some students than multiplication of decimals. Using real life situations when presenting problems so that the numbers and operations have some sort of context for the students may make it easier for them to grasp. It is imperative to ask students to use manipulatives to model given problems where they can physically break apart the decimal numbers. This will help reinforce the concept of division.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

Present the following stories to students:

- Paula is building a bird house and she needs to have 24.6 m of lumber to complete the project. Each piece of lumber is 3 m long. Ask students about how many pieces of lumber will she need to build the bird house? (6N8.1, 6N8.5)

- Philip is going to the store. He has $15.00 in his pocket and he wants to buy as many trays of strawberries as he can. One tray of strawberries cost $3.69. Ask students how can estimation help Philip determine how many trays he can buy with his $15.00? (6N8.5, 6N8.1, 6N8.2)

- Ask students to think of a situation where it is more practical to estimate the quotient involving a decimal dividend rather than finding the actual answer. Explain. (6N8.1, 6N8.2)

- Ask students to think of a situation where front-end estimation would not be the best estimation strategy to use when solving a division problem involving decimals. (6N8.1, 6N8.2)

- Engage students in Mental Math activities encouraging the use of front – end estimation. Ask them to estimate the following quotients.
  
  (i) \( 36.317 \div 2 \)  
  (ii) \( 45.036 \div 3 \)  
  (iii) \( 16.02 \div 4 \)  
  (iv) \( 80.987 \div 9 \)  
  (6N8.5)

Portfolio

- Ask students to create their own math assignment, accompanied by the answer key, that would require them to use different estimation strategies to solve division problems. Encourage students to share their assignments with the class. (6N8.5, 6N8.1, 6N8.2)

Resources/Notes

Math Focus 6
Lesson 4: Estimating Quotients
6N8
TG pp. 29-32
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicator:

6N8.2 Continued

Elaborations—Strategies for Learning and Teaching

Division can be approached in a manner exactly parallel to multiplication. In fact, the best approach to a division estimate generally comes from thinking about multiplication rather than division.

Students will need a lot of practice with division of decimals. Use story problems and real life situations to create problems to be solved.

It is very important that students have a lot of opportunities to practice division of decimals using manipulatives and visual representations. Students can not be expected to solve division problems involving decimals using symbolic notation before completing work with this concept using ‘hands on’ materials. A strategy to introduce division of decimals by whole numbers would be to show students how to break apart a decimal into the desired number of groups. In using base-ten blocks, ask students to model or represent the decimal number to be divided. Ask them to indicate how many equal groups they need to break the decimal number into. Following the idea of equal sharing, ask students to share each of the base-ten blocks into the desired number of groups where they would share out all the flats, then the rods and so on. In doing this, some students may not recognize they can not share, for example 4 rods into 5 groups. They may continue their sharing by placing the 4 rods into the four available groups with the fifth group not having a rod. Students will need to be reminded in this case, the rods can be broken into units where equal shares are then possible. It is important that students continue to use manipulatives to model division of a decimal by a whole number when solving problems. Present examples, such as the following:

\[3.42 \div 3 – \text{show your answer pictorially.}\]

In the thinking multiplication strategy, students can look at the number of equal groups and think about what possible decimal number would be needed in each group to total the dividend.

E.g., \[12.33 \div 3\]

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Paper and Pencil*

- Ask students to write a story problem using the following division sentence.
  
  \[ \frac{96.6}{7} \]

- Ask students to find the quotient of \( \frac{2.4}{4} \). Ask them to show how they would use base-ten blocks to help.  

*Resources/Notes*

*Math Focus 6*

*Lesson 5: Exploring Division of Decimals*

*6N8*

TG pp. 33 - 36
Strand: Number

Outcomes

Students will be expected to

6N8 Continued

Achievement Indicator:

6N8.2 Continued

Elaborations—Strategies for Learning and Teaching

Students would determine how much would be in 3 equal groups that would total 12.33. They could think that there has to be at least 4 wholes in each group as $4 \times 3 = 12$ with 0.33 left over. They could then see that there would be 0.1 in each group with 0.03 left over. From there, they could see that 0.01 would fit into each group. In total, there would be 4.11 in each group with no remainder. Students beginning this strategy would be strongly encouraged to use base-ten to model their thinking.

Students’ attention must be drawn to the fact that when there is a remainder in a division equation, it does not mean a whole number— it means part of a whole number (a decimal).

In addition to having used estimation and creating models to divide decimals by one digit whole numbers, students need to formalize their understanding and connect the concrete and pictorial models to symbolic representation (using the algorithms).

Students have been working with solving problems using concrete materials and visual representations. This work should have allowed them to gain a good conceptual understanding of division. Now, students will formalize their understanding and connect the concrete and visual models to symbolic representations of division. Students cannot be expected to successfully accomplish this concept without first physically manipulating materials to divide decimal numbers.

When solving problems involving division of decimals, students need to be encouraged to choose their own strategies and be able to communicate their understanding of how their strategy works. Students will need time to develop critical thinking skills as they discover and begin to use more efficient strategies.

It is suggested that students focus on creating their own strategies to divide decimal numbers rather than following the traditional procedure. Students who can follow the steps to divide a decimal number by a whole number using the traditional algorithm does not necessarily show that they have an understanding of division. They do show they understand, or remember how to do the algorithm.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Give students the following problems and ask them to solve them.
- Susie had 25.55 metres of string. She needed to hang 5 balloons from the gym ceiling. Ask students how much string did she use for each balloon if she hung each one equally? (6N8.1, 6N8.2)

- A group of 7 students ordered pizza and the total cost was $51.45. Ask students how much would each student have to pay if they shared the cost equally? (6N8.1, 6N8.2)

- Tell students five friends found 4 loonies on the road. They are now trying to figure out how to equally share this amount. Ask students to help the friends by showing them how this could be done. (6N8.2)

Resources/Notes

Math Focus 6
Lesson 5 (Cont’d): Exploring Division of Decimals
6N8
TG pp. 33 - 36

Math Focus 6
Lesson 6: Dividing Decimals by One-Digit Numbers
6N8
TG pp. 37 – 41

Math Game: Low as You Go
6N8
TG pp. 42 – 43

Curious Math: Magic Squares
6N8
TG pp. 44 - 45
Outcomes

Students will be expected to

6N8 Continued

Achievement Indicator:

6N8.2 Continued

Provide students with many examples of problems that could be solved using the working backward strategy. Model with the class how the strategy could be used. Explain that as in every strategy, the way it is used is determined by the context of the problem. To illustrate a problem that could be solved using the “working backward” strategy ask students to consider the following:

Dylan found a piece of lumber in his shed. He cut 12.34 cm off each end where the piece was broken. Then he cut the remaining piece of lumber into 3 equal pieces. Each piece is now 21.57 cm long. How long was the piece of lumber when Dylan found it?

6N8.5 Continued

Before students begin multiplying decimals, it is useful to look at patterns with respect to the effect multiplying whole numbers by 0.1 and 0.01 and decimals by 10, 100 and 1000.

Multiplying or dividing by powers of 10 does not change the digits of a number, only the POSITION of each digit within the number. E.g., if you begin with 3.4, dividing by 10 or multiplying by 0.1 decreases the value of each part of the number by a factor of 10, and so the digits in the product change value and move over one place to the right, e.g., 3 ones are now 3 tenths, 4 tenths are now 4 hundredths. It is actually the digits that move, not the decimal point. (Small 2008. Making Math Meaningful to Canadian Students K-8)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Katie said that $3.45 \times 4$ must be 1.380 because there is only one digit before the decimal place in 3.45, so there must be one digit before the decimal place in the product. Respond to what Katie said. (6N8.3, 6N8.5, 6N8.4, 6N8.2)

Performance

- Ask students to answer the following questions:
  (i) Keith cut 0.5 m from a length of rope. Then he cut what was left into four equal pieces. If each of the four pieces was 1.25 m long, what was the length of the rope before Keith cut it? (6N8.2)

  (ii) Evan downloaded four songs on MP3 files from the Internet. Two file sizes were 2.7 MB, three were 4.6 MB, and the others were 2.7 MB, and 8.1 MB (MB - megabyte). After the downloads, the disk where he stored the files held 35.5 MB of data. How much data was on the disk before the downloads? (6N8.2)

Resources/Notes

Math Focus 6
Lesson 6 (Cont’d): Dividing Decimals by One-Digit Numbers
6N8
TG pp. 37 – 41

Math Focus 6
Lesson 7: Solving Problems by Working Backwards
6N8
TG pp. 46 - 48
Measurement

Suggested Time: 3 Weeks
Unit Overview

Focus and Context

The focus of this unit is to develop an understanding of measurement within the tangible realm of linear distance, two-dimensional area, three-dimensional volume and degree of rotation. Students have likely attained a degree of familiarity with measurement through real life experience. This background experience and knowledge is a good foundation on which to develop and further concepts in measurement. The nature of the material also provides ample opportunity for group based, hands on learning activities which can increase student interest and motivation and thus greatly enhance learning.

In Grade 6, students are expected to:

• extend previous experience with rotational benchmarks such as quarter, half, three quarter and full turns to angle classification and measurement in degrees.
• identify, classify and measure angles in their environment.
• estimate angle measures in degrees using benchmarks as reference angles.
• measure angles using a 180° and 360° protractor.
• draw angles and angle approximations with and without the use of a protractor.
• explore relationships within the interior angles of triangles and quadrilaterals.
• derive formulae for the perimeter of polygons, area of rectangles and volume of right rectangular prisms through exploratory activities.
• use derived formulae to solve problems involving perimeter of polygons, area of rectangles and volume of right rectangular prisms.
• solve complex measurement problems through a process of solving simpler problems.

Math Connects

Measurement involves the use of quantitative (numerical) values to describe a specific attribute. These measurable attributes may be tangible, or intangible (e.g., time, temperature etc.). In order for measurement to be meaningful a holistic understanding of units and their relation and conversion must be developed. A comprehensive understanding of measurement is fundamental to the development of concepts in other branches of mathematics such as Euclidian geometry, transformational geometry, algebra and statistics. It is also important to ensure that students have a sound understanding of the metric system and appropriate usage of units.

Measurement is an essential link between mathematics, sciences, the arts and other disciplines and has an endless variety of applications in real life situations. Encourage students to research how measurement is involved in careers and hobbies in which they are interested. Providing students with opportunities to engage in measurement based activities with real world application will enhance student learning, making it meaningful and relevant.
### Process Standards

**Key**

| [C] Communication | [PS] Problem Solving |
| [CN] Connections  | [R] Reasoning         |
| and Estimation    | [V] Visualization     |

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shape and Space</strong></td>
<td>6SS1 Demonstrate an understanding of angles by:</td>
<td>[C, CN, ME, V]</td>
</tr>
<tr>
<td><em>(Measurement)</em></td>
<td>• identifying examples of angles in the environment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• classifying angles according to their measure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• estimating the measure of angles, using 45°, 90° and 180° as reference</td>
<td></td>
</tr>
<tr>
<td></td>
<td>angles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• determining angle measures in degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• drawing and labelling angles when the measure is specified.</td>
<td></td>
</tr>
<tr>
<td><strong>Shape and Space</strong></td>
<td>6SS2 Demonstrate that the sum of interior angles is:</td>
<td>[C, R]</td>
</tr>
<tr>
<td><em>(Measurement)</em></td>
<td>• 180° in a triangle</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• 360° in a quadrilateral.</td>
<td></td>
</tr>
<tr>
<td><strong>Shape and Space</strong></td>
<td>6SS3 Develop and apply a formula for determining the:</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td><em>(Measurement)</em></td>
<td>• perimeter of polygons</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• area of rectangles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• volume of right rectangular prisms.</td>
<td></td>
</tr>
<tr>
<td><strong>Patterns and Relations</strong></td>
<td>6PR3 Represent generalizations arising from number relationships, using</td>
<td>[C, CN, PS, R, V]</td>
</tr>
<tr>
<td><em>(Variables and Equations)</em></td>
<td>equations with letter variables.</td>
<td></td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Demonstrate an understanding of angles by:
• identifying examples of angles in the environment
• classifying angles according to their measure
• estimating the measure of angles, using 45°, 90° and 180° as reference angles
• determining angle measures in degrees
• drawing and labelling angles when the measure is specified.

Elaborations—Strategies for Learning and Teaching

In Grade 5 students had experience with angles using fractions of a circle as benchmark angles. Up to this point they have been exposed to quarter, half, three-quarter and full turns within a circle. The measuring of angles in degrees and the subsequent use of a protractor will be new concepts in Grade 6. The concept of a circle being comprised of 360° will serve as a convenient starting point for measuring angles in degrees.

It may be necessary to review the concept of an angle as being the amount of rotation between two rays (arms) joined at a shared point (called the vertex). The amount of rotation required to get from one arm of the angle to the other is the angle's measure. The length of the arms does not affect the angles measure.

E.g., if students were asked which of the angles below has the greatest measure, some might respond that angle ABC is larger than DEF. In actual fact, both angles have the same measure (the amount of rotation between the arms is the same).

Achievement Indicator:

6SS1.1 Provide examples of angles found in the environment.

Initial exploration of angles will simply involve students identifying two line segments connected at a shared point and thus forming an angle in their surrounding environment. Examples in the classroom may include door and window frames (right angles), adjacent floor tiles (right or straight angles) or hands of a clock (any angle depending upon the given time).

Description of angles at this point will be based on the quarter turn (right angle), half turn (straight angle) and three-quarter turn benchmarks. For example, if a student identifies an obtuse angle they will describe it as being between a quarter turn and a half turn. Terms such as acute, obtuse and reflex may be unfamiliar to students at this point.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

**Performance**

- Ask students to identify angles in their surrounding environment. They may do this over the course of a class or a whole day. Keep a log of where and on what object the angle was identified. Create a sketch of the object, highlighting the identified angle in a different colour. Include a brief description of the angle measure relative to the quarter-turn (right angle), half-turn (straight angle) and three-quarter turn benchmarks. (6SS1.1)

- Using the log of angles that they previously identified from their surroundings ask students to label each of the angle they recorded as acute, right, obtuse, straight or reflex. (6SS1.2)

**Resources/Notes**

*Math Focus 6*

**Lesson 1: Identifying Angles**

6SS1

TG pp. 13 – 17

**Children’s Literature**

(not provided):

Alder, David A. *Shape Up: Fun With Triangles and other Polygons*. ISBN: 0-8234-1638-0
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.2 Classify a given set of angles according to their measure; e.g., acute, right, obtuse, straight, reflex.

Elaborations—Strategies for Learning and Teaching

As students are familiar with the quarter, half and three-quarter turn benchmarks it will now be necessary to introduce names of other angles that fall between these benchmarks.

- Right Angle (Quarter-Turn) – Two rays that connect to form a square corner.

- Straight Angle (Half Turn) – Two rays that connect to form a straight line.

- Acute Angle – An angle with a measure less than a right angle (half turn)

- Obtuse Angle – An angle with a measure greater than a right angle but less than a straight angle.

- Reflex Angle – An angle with a measure greater than a straight angle.

Some students will likely make the extension that an acute or obtuse angle also has a reflex angle on the outside of its arms. It may be appropriate to address this point as a class. Any given angle has a second angle outside of its arms (the remainder of the circular rotation).
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

**Journal**

- Ask students to identify the type of angle formed by the hands of a clock at various times of the day. Identify the type of angle created by each time in the clocks shown below.

![Clocks showing different times](image)

At what time will each type of angle be formed again?

Note: Reflex angles may also be possible answers for 8:30, 11:25 and 3:30. (6SS1.1, 6SS1.2)

**Performance**

- Ask students to draw a picture of a familiar object using only one type of angle, acute or obtuse. Check the angles you drew with a square corner (page, ruler etc.) to compare them to a right angle and ensure that they are all the type of angle you chose. (6SS1.2)

- Provide students with printed paper copies of flags from various provinces, states and countries that have linear designs or are composed of various shapes. The Newfoundland and Labrador flag would be a good example. Ask students to identify and record all of the angles they can find on the flag and classify them as either acute, right, obtuse, straight or reflex. (6SS1.2)

Resources/Notes

**Math Focus 6**

*Lesson 1 (Cont’d): Identifying Angles*

*6SS1*

TG pp. 13 – 17

**Additional Reading (provided):**

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

**Achievement Indicator:**

| 6SS1.3 Estimate the measure of an angle, using 45°, 90° and 180° as reference angles. |

Elaborations—Strategies for Learning and Teaching

Introduce the concept of the degree. Students will be familiar with the use of metric units such as cm, m, and mm for measuring linear distance. A degree is the unit used to measure rotation within a circle and subsequently can be used to measure angles.

Some students will likely attempt to make an association between degrees of a circle and degrees Celsius or Fahrenheit used for measuring temperature. It may be necessary to differentiate between the two. Although the word ‘degree’ is used in both cases, they are units of measurement for two entirely unrelated quantities.

Establish the fact that the measure of rotation for a full turn within a circle will always be 360°. This is true regardless of the size (diameter) of the circle. Students will likely have heard “360” as a stunt performed by snow boarders, skateboarders, ice skaters, etc., where the athlete makes a complete spin and ends up facing forward in his/her original position. This may be a good starting point for the introduction of this concept. Students will also likely be familiar with the stunt called a “180”, where the athlete performs a half spin and ends up facing opposite the original position. Hence, the term “180” is used because the athlete rotated his/her body in a half circle, and 180° is half of 360°.

Once it has been established that a full turn has a measure of 360° and a half turn measures 180°, the benchmarks of 90° and 45° can be easily introduced. Since a half turn has a measure of 180°, a quarter turn (half of 180°) would thus have a measure of 90° and an eighth of a turn (half of 90°) would measure 45°.

These benchmarks may also be established through student construction of a protractor.

Once the 45°, 90° and 180° benchmarks have been established students can use these to estimate the measure of other angles.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Ask students to construct their own 360° protractor using a circular piece of paper. Through this process 45°, 90° and 180° benchmarks may be established.
  - Students must first be aware that a circle (full turn) has a measure of 360°.

- Fold the circle in half. If a full circle measures 360°, a half of a circle must measure 180°.

- Fold the half circle in half again. This represents a quarter of the whole circle. If half of the circle measures 180°, a quarter must measure 90°.

- Fold the quarter circle in half again. This represents an eighth of the whole circle. If a quarter of the circle measures 90°, an eighth must measure 45°.

- Once the circle is unfolded it will be evident that there are eight sets of 45° in the whole. Choose a fold line as 0°; label each fold line in the counterclockwise direction as a consecutive multiple of 45°, as shown below.

Students now have a guide by which they can estimate the measure of any angle. (6SS1.3)

Resources/Notes

Math Focus 6
Lesson 2: Constructing a Protractor (optional)
6SS1
TG pp. 18 - 21

Students will need to learn to measure given angles using a protractor. However, construction of a protractor is optional.
Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.4 Sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them.

Elaborations—Strategies for Learning and Teaching

In addition to estimating the measure of an angle using the 45°, 90° and 180° benchmarks, students will be sketching these angles without the use of a protractor and describing relationships between them.

Having estimated angle measures previously students should be able to visualize each of the 45°, 90° and 180° benchmarks individually. They can now use combinations of these benchmarks to sketch angles without the use of a protractor.

It will be necessary here to introduce the correct method for labelling angles. As previously discussed, any angle can be viewed as two different angles, the smaller angle between the arms and a larger angle outside as shown below.

To avoid confusion as to which angle is being referred to in a particular drawing, an arc (hatch mark) is drawn between the two arms. Note that a special mark, a small square, is used to indicate a right angle. E.g.

The arc between the arms of the angle above indicates that this drawing is showing a 270° angle and not a 90° angle.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Angle “I Spy” Game – Ask students to identify an angle in the classroom. Write down the type of angle (acute, right, obtuse, reflex or straight), an estimate of its measure and draw a sketch of its orientation. Trade angles information with a partner and attempt to find a representation of each other’s angle. (E.g. Floor, wall, corner, etc.)

(6SS1.1, 6SS1.2, 6SS1.3, 6SS1.4)

Resources/Notes

* Math Focus 6
  * Lesson 3: Estimating Angle Measures
  * 6SS1
  * TG pp. 22 - 25
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.3 Continued

Elaborations—Strategies for Learning and Teaching

This would also be an appropriate time to introduce the correct method of naming angles. When an angle is presented with three labelled points, the angle would be named using the angle symbol $\angle$ and the three points written with the vertex in the middle. E.g.

This angle could be correctly named $\angle$LMN or $\angle$NML, because in each case the vertex point is in the middle of the name. Emphasize that writing the points in alphabetical order is not a requirement and will not necessarily be correct in all cases, depending on the letter name of the vertex.

This angle could be named $\angle$BAC or $\angle$CAB. Writing the points in alphabetical order ($\angle$ABC) in this case would be incorrect, as the vertex would not be in the middle.

Visualizations of the 45°, 90° and 180° benchmarks and their combinations can now be used to estimate the measures of other angles. E.g. Estimate the measure of the angle below.

Students may visualize or sketch benchmark angles on a separate sheet of paper or directly on the given angle, as shown. The angle in question is between 135° and 180°. Its measure might be estimated at somewhere between 150° to 160°.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

**Performance**

- Ask students to sketch each of the following angles without using a protractor. Label each angle correctly and use an arc to indicate the direction or rotation.
  
  (i) \( \angle ABC \) is 135°
  
  (ii) \( \angle DOG \) is 275°
  
  (iii) \( \angle LMN \) is 88°
  
  (iv) \( \angle ZYX \) is 190°
  
  (v) \( \angle PRQ \) is 100°
  
  (vi) \( \angle GEF \) is 290°
  
  (vii) \( \angle CAT \) is 376°

  Ask students to explain which benchmarks they found helpful in sketching each angle and how they used these to produce their sketch. Focus on the reasoning students used to arrive at the sketches they produced. (6SS1.4)

- Using the log of angles previously created from their surroundings, ask students to estimate the measure of the angle in each of their drawings and write the estimated measure between the arms of each angle they sketched. (6SS1.3)

- Provide students with various flags of provinces, states and countries that have linear designs or include various shapes. The Newfoundland and Labrador Flag would be a good example. Ask them to identify as many different angles as they can find. Ask students to estimate the measure of each angle they identified using benchmarks and explain how they arrived at each estimation. (6SS1.3)

**Resources/Notes**

*Math Focus 6*

*Lesson 3 (Cont’d): Estimating Angle Measures*

*6SS1*

*TG pp. 22 - 25*
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Elaborations—Strategies for Learning and Teaching

Building upon the benchmarks and estimation skills developed previously, students will now use an actual protractor to measure angles. The initial use of a circular (360°) protractor rather than a semi-circular (180°) protractor is recommended. This will reinforce students’ understanding of an angle as a rotation within a full circle.

Achievement Indicator:

6SS1.5 Measure, using a protractor, given angles in various orientations.

Students will now use a protractor showing all 360 degrees of a circle (full turn) to measure angles to the nearest degree. As this is the first time students have used a protractor, it will be necessary to explain that the degrees on the protractor are only labelled in multiples of ten, however each unlabeled tick represents 1°, thus allowing them to measure angles between the labelled values.

Not all angles will be oriented with one horizontal arm. This may present a challenge for some students. In any case, the centre point of the protractor must be placed on the vertex of the angle and the zero degree mark of the protractors baseline must be lined up with one of the angle's arms. If this orientation of the angle presents a problem, the page or book may be turned so that one arm becomes horizontal from the students point of view.

(continued)
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Provide students with illustrations from books. Ask them to identify and estimate the measure of angles found in each picture. Students can create a list of the angles and their approximated measures. Another possibility would be to provide photocopies of the illustrations so that students can highlight or colour the identified angles and write their estimated measures on the picture. (6SS1.5)

Journal

- Engage students in a journal entry activity using the following prompt:
  
  Pretend you work for a company that makes and sells protractors. You have been given the job of writing instructions for the protractor package. Write a detailed step-by-step set of instructions (with diagrams if needed) to explain how to measure any angle using the protractor. Assume the reader has never seen a protractor before.

  Trade instructions with a partner and follow them to measure a given angle.

  What changes would you make to your partner’s instructions?

  (6SS1.5)

Resources/Notes

Math Focus 6
Lesson 4: Measuring Angles
6SS1
TG pp. 26 - 30
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.5 Continued

Elaborations—Strategies for Learning and Teaching

When measuring angles using a protractor it is important that the reading always begins at zero. Incorrect measures often result from students beginning at the 180° mark. Emphasize to students that they must always line up one arm of the angle with the zero degree and then begin counting up from zero until they reach the next arm. This will avoid incorrect scale readings.

It is also important to recognize that the direction of rotation between angle arms will determine which scale is used on the protractor. Again, lining up the zero degree with the first arm of the angle and counting up to the next arm will avoid any confusion with regard to which scale should be used. Remind students that the arc (hatch mark) between the arms of the angle indicates which angle is being measured.

Estimating angle measures before using the protractor is important in helping students avoid mistaken scale readings. E.g., A student might estimate angle \( \angle ABC \) to have a measure of a little more than 90°.

If they measure the angle and mistakenly read the protractor scale as 88° as opposed to the correct measurement of 92°, they should realize that this measurement does not agree with their estimation and therefore is probably incorrect.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

• Create a Flag - Give students a list of angle measures that must be incorporated in an original flag design. Students can create their own flag design that incorporates the specified angle measures. A colour or line type key can be used to indicate the specified angle measures. E.g. “Create a flag design that contains at least one 45°, 120° and 155° angle.”

This activity can be extended by having students measure any additional angles that may have been formed in their flag creation.

Math Focus 6
Lesson 4 (Cont’d): Measuring Angles
6SS1
TG pp. 26 - 30
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.6 Draw and label a specified angle in various orientations, using a protractor.

Elaborations—Strategies for Learning and Teaching

Previously students have sketched angle approximations using benchmarks. At this point they will construct angles with a specified angle measure using a protractor and straightedge.

After students have become proficient measuring angles with a protractor ask them to reverse the process to construct angles with a specified measure using a ruler and protractor. When completed, students should measure their angle using the protractor to ensure that they completed the construction correctly. Discuss the process that was used to construct the angles with the class.

Students may find it easier to use a semi-circular (180°) protractor to construct angles to an accurate degree. It is important to note that as with some circular protractors, there is an inner and outer scale, and that to avoid confusion between the two, students must always start measuring from the 0° mark. Ask students to sketch an approximation of the angle using benchmarks before attempting the actual construction. A mental visualization of this approximation may be sufficient for some students at this point.

When constructing angles, students should always use a straight edge to construct the first arm. This arm may be horizontal unless otherwise specified. The centre mark of the protractor would then be placed on the endpoint of the first arm and the baseline rotated so that the 0° mark is in line with that arm. Keep in mind that the direction of rotation will determine whether the inner or outer scale of the protractor will be used and indicate from which 0° mark (left or right) measurement will begin. Students should then count up from zero to the desired measure, mark that degree and connect it to the vertex using a straight edge. The angle must be indicated by drawing an arc between the two arms and any given points should be labelled correctly remembering that the vertex is always found in the middle of the angle name. Students need lots of opportunities to practice drawing angles of different measures and orientations to attain proficiency.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Use a straightedge and protractor to draw the following angles:
  (i) 54°
  (ii) 135°
  (iii) 75°
  (iv) 156°

  Label each of the constructed angles and the specified measure.

  (6SS1.6)

Performance

- Angle Art – Ask students to create pieces of art that incorporate a number of specified angle measures. E.g., “Draw a scene that contains a 46°, 125°, 270° and a 285° angle.” Use a protractor to construct these angles. Label each of the constructed angles using an arc and the specified measure. If students do not want to write the angle measures on their artwork they may wish to use a colour key. This would involve colouring the arms of each specified angle a different colour and using a key/legend to indicate its measure.

  (6SS1.6)

- Ask students to make a stick person out of pipe cleaners or wire and pose it so that the assigned angles are displayed somewhere on its body.

  (6SS1.6)

Resources/Notes

Math Focus 6

Lesson 5: Drawing Angles

6SS1

TG pp. 31 – 34

Curious Math: Strange Buildings

6SS1

TG pp. 37 - 38

Math Game: Buried Treasure

6SS1

TG pp. 39-40
Outcomes

Students will be expected to

6SS2 Demonstrate that the sum of interior angles is:

- 180° in a triangle
- 360° in a quadrilateral.

[C, R]

Achievement Indicator:

6SS2.1 Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

Elaborations—Strategies for Learning and Teaching

To determine the sum of the interior angles of triangles and quadrilaterals it is recommended that an exploratory approach be employed. It may also be necessary to review the concept of an interior angle compared to an exterior angle.

To illustrate that the sum of the interior angles of a triangle is the same for all triangles in a visual/tactile manner, create several large triangles of varying shapes and sizes on construction paper or card stock. Use a protractor to trace an arc between the arms of all three angles of each triangle. Ask students to cut the corners off each triangle along the curved arc. Arrange the three corners so that they are all joined at the vertex forming a straight angle. Students will observe that when the three corners are arranged at a common vertex the three angles form a half circle (180°).

By using triangles of various sizes and shapes it becomes evident that the sum of the three interior angles will always be 180° for all triangles.

Cutting the corners of the triangles in an arc and labelling the vertices with letters or symbols will avoid confusion as to which vertices need to be adjacent when aligning the angles. The curved edges also provide a clearer visual for students that the three interior angles of any given triangle form a half circle.

It may be worthwhile to ask students to measure the interior angles of the triangles they used in the above activity using a protractor and then find the sum. They may notice that in some cases their sum does not equal 180° exactly but is very close. This would be a good opportunity to discuss sources of human error in measurement.

To illustrate that the sum of the interior angles of a triangle is the same for all triangles in a visual/tactile manner, create several large triangles of varying shapes and sizes on construction paper or card stock. Use a protractor to trace an arc between the arms of all three angles of each triangle. Ask students to cut the corners off each triangle along the curved arc. Arrange the three corners so that they are all joined at the vertex forming a straight angle. Students will observe that when the three corners are arranged at a common vertex the three angles form a half circle (180°).

By using triangles of various sizes and shapes it becomes evident that the sum of the three interior angles will always be 180° for all triangles.

Cutting the corners of the triangles in an arc and labelling the vertices with letters or symbols will avoid confusion as to which vertices need to be adjacent when aligning the angles. The curved edges also provide a clearer visual for students that the three interior angles of any given triangle form a half circle.

It may be worthwhile to ask students to measure the interior angles of the triangles they used in the above activity using a protractor and then find the sum. They may notice that in some cases their sum does not equal 180° exactly but is very close. This would be a good opportunity to discuss sources of human error in measurement.

(continued)
General Outcome: Use Direct and Indirect Measurement to Solve Problems

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math Focus 6</td>
</tr>
<tr>
<td></td>
<td>Lesson 5 (Cont’d): Drawing Angles</td>
</tr>
<tr>
<td></td>
<td>6SS1</td>
</tr>
<tr>
<td></td>
<td>TG pp. 31 – 34</td>
</tr>
<tr>
<td></td>
<td>Math Focus 6</td>
</tr>
<tr>
<td></td>
<td>Lesson 6: Angle Relationships in Triangles</td>
</tr>
<tr>
<td></td>
<td>6SS2</td>
</tr>
<tr>
<td></td>
<td>TG pp. 41 - 45</td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS2 Continued

Achievement Indicator:

6SS2.1 Continued

Elaborations—Strategies for Learning and Teaching

This concept can also be reinforced using FX Draw and Interactive Whiteboard technology computer software. Use the triangle shape tool \( \triangle \) to create a triangle of any size or shape. Using the angle measure tool \( \angle \) label the measure of each interior angle. The program will automatically place the correct measurement within the angle. Ask students to drag each vertex of the triangle to a different position on the Interactive Whiteboard to create a triangle of a new shape and size. The angle measure tool will automatically change the measure of the angle as the student drags the vertex. Students will observe that no matter how they change the shape and size of the triangle, the sum of the interior angles will always be 180°.

Extend this concept to find a missing angle in a given triangle when two other angle measures are known. By subtracting the sum of the two known angles from 180°, the unknown angle measure can be determined.

After establishing that the sum of interior angles for any triangle is 180°, students could use this information to explore the sum of the interior angles of any quadrilateral. They must first recognize that any given quadrilateral is composed of two triangles. This can be reinforced using tangram pieces. By combining two tangram triangles to form a quadrilateral students should deduce that since the sum of the interior angles of a triangle is 180°, then the sum of the interior angles of the quadrilateral would be 360° (180° + 180° = 360°).

Perhaps the most straightforward manner in which to demonstrate this concept is to ask students to draw a diagonal connecting opposite vertices of various quadrilaterals. Students can be provided with these quadrilaterals on construction paper or card stock or they may create their own. Students can then cut the quadrilateral along the diagonal they drew producing two triangles. Having already learned that the sum of the interior angles of a triangle is 180°, they can make the deduction that because any quadrilateral can be deconstructed into two triangles, the sum of the interior angles of any quadrilateral must be 180° + 180° = 360°.

(continued)
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

**Journal**

- Give students a measure of one angle in a triangle. Ask them to derive three other pairs of possible angle measures for the remaining two angles. Using a protractor draw these three triangles. E.g., “A triangle has a 45° angle. What are three possible sets of measures for the remaining two unknown angles? Draw these triangles using a protractor and a straight edge.”

  Three possible angle sets are:

  - $45^\circ$ (given) + $45^\circ$ + $90^\circ$ = $180^\circ$
  - $45^\circ$ (given) + $20^\circ$ + $115^\circ$ = $180^\circ$
  - $45^\circ$ (given) + $30^\circ$ + $105^\circ$ = $180^\circ$

  Etc…  \hspace{1cm} **(6SS2.1, 6SS1.6)**

**Resources/Notes**

**Math Focus 6**

**Lesson 6 (Cont’d): Angle Relationships in Triangles**

**6SS2**

TG pp. 41 - 45

**Math Focus 6**

Lesson 7: Angle Relationships in Quadrilaterals **6SS2**

TG pp. 46-50
Strand: Shape and Space (Measurement)

Outcomes

*Students will be expected to*

6SS2 Continued

**Achievement Indicator:**

6SS2.2 Continued

Elaborations—Strategies for Learning and Teaching

This concept may be approached in a similar fashion as triangles were previously. Ask students to cut out quadrilaterals of various shapes and sizes. Use a protractor to draw arcs between the arms of each interior angle. Cut out each angle along the arc and arrange them so that their vertices are adjacent. Students will observe that the four angles of any triangle form a full circle (360°).

![Diagram of quadrilateral angles]

FX Draw may also be used to illustrate this concept along with Interactive Whiteboard technology. Use the quadrilateral shape tool to draw a quadrilateral of any shape and size on the interactive whiteboard. Label the measure of the interior angles using the angle measure tool. Ask students to physically change the position of the vertices to create quadrilaterals of different shapes and sizes. The angle measure tool will automatically show the change in each angle, and students will observe that the sum of the four angles of any quadrilateral will always be 360°.

Extend this concept to find unknown angles within quadrilaterals given the measure of any three angles. By subtracting the sum of the three known angles from 360°, the unknown angle measure can be determined.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Organize stations within the classroom. Each station will have a quadrilateral or triangle cut-out. All interior angles would be labelled on the cut out except for one angle which has been torn off each shape. Assign students in small groups or pairs to a station. Ask them to find the missing angle measure for their given quadrilateral or triangle. Groups will rotate from station to station until they have found all the missing angle measures.  

(6SS2.2)

- Another way to approach this activity would be to have each group of students construct their own triangle or quadrilateral using a protractor, cut it out and tear off one angle and label the remaining angles. This could be the first station. Groups will then rotate, finding everyone else’s missing angle measures. When they arrive back at their own shape the activity is over.  

(6SS2.2, 6SS1.6)

Resources/Notes

Math Focus 6
Lesson 7 (Cont’d): Angle Relationships in Quadrilaterals
6SS2
TG pp. 46 - 50
Strand: Shape and Space (Measurement)

Outcomes

*Students will be expected to*

6SS3 Develop and apply a formula for determining the:
- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms.

[C, CN, PS, R, V]

Elaborations—Strategies for Learning and Teaching

It is important to approach this outcome as an investigation rather than providing a rote formula. Students should be given the opportunity to discover how to find the area of a rectangle independently.

This can be done using 1 cm$^2$ grid paper. Students must first understand that each square unit on the grid paper has a 2-D area of 1 cm$^2$ meaning it has linear dimensions of 1 cm in height and 1 cm in width.

Ask students to draw rectangles with various dimensions on 1 cm$^2$ grid paper. They will initially find the area by counting the number of 1 cm$^2$ units enclosed within the rectangle. Ask students to find the length and width of each rectangle. Prompt students to view these dimensions as rows of a given number of square centimetres. From this reasoning, students should deduce that by viewing the dimensions of a rectangle as groups of square units (width x length) the area of the rectangle can be found. E.g.,

Students should realize that a change in the unit of measurement (mm$^2$, cm$^2$, m$^2$) simply means that the uniform size of each unit within the rectangle has changed but the area can still be determined in the same way. This point can be illustrated by finding the area of rectangles on grid paper with units of varying dimensions.

This can be addressed using grid paper as previously mentioned. Students may also use fraction strips to model rectangles and squares of different areas. The strip format of the units will help students visualize area in terms of groups of units. It will first be important to establish the dimensions and area of each unit within the strip. E.g.,

A student uses a single strip of 4 units each with an area of 1 cm$^2$, to model a rectangle with dimensions of $1 \times 4$.

This rectangle consists of one row of four 1 cm$^2$ units, thus the area of the rectangle is 4 cm$^2$. The student might add another strip of 4 units to create the following rectangle.

This rectangle consists of two rows of four 1 cm$^2$ units, thus the area of the rectangle is 8 cm$^2$ ($2 \times 4 = 8$). Other rectangles can be modelled and the area found through this method.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Math Focus 6</em></td>
<td></td>
</tr>
<tr>
<td><em>Lesson 8: Area of a Rectangle</em></td>
<td></td>
</tr>
<tr>
<td>6PR3</td>
<td></td>
</tr>
<tr>
<td>6SS3</td>
<td></td>
</tr>
<tr>
<td>TG pp. 51 - 54</td>
<td></td>
</tr>
</tbody>
</table>
Strand: Shape and Space (Measurement)

**Outcomes**

*Students will be expected to*

6SS3 Continued

**Achievement Indicator:**

6SS3.2 Generalize a rule (formula) for determining the area of rectangles.

**Elaborations—Strategies for Learning and Teaching**

Students have not had exposure to the concept of a formula. A formula is a rule (process) that can be used to find a desired quantity or measurement. From the activity completed previously, students derived the general rule that the area of any rectangle can be found by multiplying its length by its width. Students should be able to express this rule in words and as a formula written using variables to represent the changing quantities.

In words: The area of a rectangle can be found by multiplying its length by its width.

As a formula: \( A = l \times w \), where \( A = \text{area} \), \( l = \text{length} \) and \( w = \text{width} \)
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

**Performance**

- Designing a Dream House - Provide students with 1 cm$^2$ grid paper. Ask them to design a floor plan for their dream house. Rooms can be rectangles of various dimensions. Students may even design two or three floors on separate sheets of grid paper. Once the floor plans are complete, ask students to use the formula they derived for area of a rectangle to find the area for each room in their floor plan. Once the floor area of each room is found, students can then find the combined floor area of the entire house. (6SS3.2)

- This activity could be extended to incorporate outcomes from other units. For instance students could be asked to develop a scale for their floor plan. What would each square centimetre represent in a real house? Students could also be given prices for various square areas of flooring and asked to calculate the cost of flooring their dream house. This could be part of a cumulative project, incorporating outcomes from multiple unit. (6SS3.2)

- Ask students to draw / construct rectangles of the same dimensions using cm$^2$ and pattern block grid paper. Using the formula developed previously ask them to find the area of each rectangle. Ask: Did the size of the unit affect the numerical value of the area in each case?

  Which area is actually larger? Explain how this is possible even though the numerical value of each area is the same. (6SS3.2)

<table>
<thead>
<tr>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td>Lesson 8 (Cont’d): Area of a Rectangle</td>
</tr>
<tr>
<td>6PR3</td>
</tr>
<tr>
<td>6SS3</td>
</tr>
<tr>
<td>TG pp. 51 - 54</td>
</tr>
</tbody>
</table>

| Math Focus 6 Masters Booklet p.40 and p.47. |
Outcomes

Students will be expected to

6PR3 Represent generalizations arising from number relationships, using equations with letter variables.

[C, CN, PS, R, V]

Achievement Indicators:

6PR3.3 Write and explain the formula for finding the area of any given rectangle.

Elaborations—Strategies for Learning and Teaching

The focus here is on deriving expressions to represent perimeter and area of various polygons as opposed to actually calculating these values given known measurements.

At this point it may be necessary to review area as the amount of flat (2-dimensional) surface within an enclosed shape. Students may find it easier to think of area in terms of what they would be colouring in if asked to colour a shape. This may be illustrated using computer drawing programs. Ask students to construct any enclosed shape and then use the “fill” or “paint” feature to colour in the shape.

It may be necessary to review the use of square units when measuring area. Students should realize that square units are used to measure 2-dimensional shapes. They will not be expected to understand \( n \times n = n^2 \), as they will not be dealing with exponents until junior high. However, at this level students may view the exponent of 2 on any squared unit as meaning 2-dimensional. Once students have derived an expression for the area of a rectangle, they will be able to use this formula to find the area of any given rectangle.
General Outcome: Represent Algebraic Expressions in Multiple Ways

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 8 (Cont’d): Area of a Rectangle</strong></td>
</tr>
<tr>
<td></td>
<td><strong>6PR3</strong></td>
</tr>
<tr>
<td></td>
<td><strong>6SS3</strong></td>
</tr>
<tr>
<td></td>
<td>TG pp. 51 - 54</td>
</tr>
</tbody>
</table>
### Strand: Shape and Space (Measurement)

**Outcomes**

Students will be expected to

**6SS3 Continued**

<table>
<thead>
<tr>
<th>Achievement Indicator:</th>
</tr>
</thead>
<tbody>
<tr>
<td>6SS3.3 Explain, using models, how the perimeter of any polygon can be determined.</td>
</tr>
</tbody>
</table>

**Elaborations—Strategies for Learning and Teaching**

Students have been introduced to perimeter in previous grades as the distance around a shape. Perimeter can be illustrated by having a student walk around the edge (perimeter) of the classroom, or trace the edges of their desk or textbook cover with their finger.

It may be beneficial to review the concept of a polygon as being enclosed shapes with three or more sides (polygon meaning “many sided figure”).

The use of variable \( l \) (length) and \( w \) (width) will be familiar to students at this point. However, in the case of squares, hexagons or other regular polygons having equal side lengths there will be no actual length and width dimensions as all side measures are equal. In this case the variable \( s \) (side) may be used.

Students have had experience finding perimeter by measuring the sides of a polygon and finding the sum. The focus here however is on deriving a formula that can be used to generalize the process by which the perimeter of any given polygon can be found.

Perimeter can be initially modelled and determined by using drawings of various polygons on 1 cm\(^2\) grid paper and using the number of units along the combined sides to find the perimeter. However, modelling of this nature would be restricted to squares, rectangles or combinations of these.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Farmer’s Fence - Provide students with a variety of polygons (squares, rectangle, triangles, parallelograms etc.) cut out from card stock or construction paper and having common side lengths labelled with a variable. Each polygon represents a different shaped field on a farm. Each different variable indicates the length of each side.

In groups students can arrange the polygons to create various combinations of different shaped farms. E.g.

You (The Farmer) want to build a fence around your farm. Write a formula for the perimeter of your farm. E.g. The formula for the above farm would be

\[ P = 3b + 2a + c + e \]

Once students have found the formula for the perimeter of their farm assign each variable a number value and ask students to use their formula to find the perimeter. For example: if \( a = 5 \) metres, \( b = 2 \) metres, \( c = 3 \) metres, \( e = 4 \) metres. The perimeter of the farm above would be

\[ P = 3(2) + 2(5) + 3 + 4 \]
\[ P = 6 + 10 + 3 + 4 = 23 \text{ metres} \]

(6SS3.3, 6SS3.4, 6SS3.7)

- This activity could also be extended to incorporate other outcomes involving multiplication with decimals by giving students a set price per metre of fencing and asking them to calculate the total cost of fencing their farm. (6SS3.7)

Resources/Notes

Math Focus 6
Lesson 9: Perimeter of a Polygon
6PR3
6SS3
TG pp. 55 - 58
Outcomes

Students will be expected to

6SS3 Continued

Achievement Indicator:

6SS3.4 Generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares.

Elaborations—Strategies for Learning and Teaching

Provide students with a variety of polygons on paper or cardstock. Ask them to identify which dimensions of that particular polygon are congruent (the same). Illustrate that the sum of sides having equal measures can be found through repeated addition and multiplication.

For example, in the square below the perimeter may be written as 3 cm + 3 cm + 3 cm + 3 cm or, through multiplication, 4 × 3 cm (Four groups of 3 cm) = 12 cm.

![Square with dimensions](image)

In the case where the measure of a side is not specified, students should be able substitute a variable for the unknown amount and use this to derive their formula. E.g. In the regular hexagon below the side length is unknown so we use the variable s to represent it.

![Regular Hexagon](image)

Thus, the expression for its perimeter may be written in repeated addition as \(s + s + s + s + s + s\) or more conveniently as 6s.

For polygons with two or more different side measures addition of the combined congruent dimensions will be required. For example;

![Polygon with different side lengths](image)

The expression for perimeter would be:

\[P = 2a + 2b + 2c.\]

Problems of this nature will not always necessarily involve regular polygons. However, as long as students can recognize congruent sides, a formula should be easily derived.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Provide students with pattern blocks and other manipulatives of varying shape and size. Ask them to identify which dimensions (sides) on the pattern blocks are of equal length. Trace these patterns on paper. Assign the equal sides a common variable or colour to represent the unknown length that these sides share. Write an expression to represent the perimeter of each pattern block.

E.g.,

```
\begin{align*}
\text{E.g.,} & \\
\text{\begin{tikzpicture}[scale=0.5]}
\draw (0,0) -- (2,0) -- (2,2) -- (0,2) -- cycle;
\draw (2,0) -- (4,2) -- (0,2) -- cycle;
\draw (4,2) -- (6,2) -- (4,0) -- cycle;
\end{tikzpicture}}
\end{align*}
```

\begin{align*}
P &= s + s + s + s \\
or\quad P &= 4s \\
\text{or}\quad P &= 6s
\end{align*}

\begin{align*}
P &= s + s + s + s + s + s \\
or\quad P &= 6s \\
or\quad P &= 6s \\
or\quad P &= 2s + s + s \\
or\quad P &= 5s \\
or\quad P &= 3s + 2s
\end{align*}

\textbf{Resources/Notes}

\textit{Math Focus 6}

\textbf{Lesson 9 (Cont'd): Perimeter of a Polygon}

6PR3

6SS3

TG pp. 55 - 58
Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

6PR3 Continued

Achievement Indicator:

6PR3.4 Write and explain the formula for finding the perimeter of any given rectangle.

Elaborations—Strategies for Learning and Teaching

Remind students that perimeter is a measure of linear distance and is thus written in one dimensional units (e.g., mm, cm, m, km etc…).

The focus here is not on actually finding the perimeter of a given rectangle but on deriving a rule that can be used to find the perimeter of any given rectangle. Present students with a variety of rectangles. Students must recognize that sides with same measure are indicated using the same hatch mark or variable.

Ask them to identify which sides are congruent (equal). The sum of these congruent sides can be found through repeated addition and therefore, by multiplication. For example, in the rectangle below students may write the combined width as \(w + w\) or \(2 \times w\) (Two sets of the width).

The combined width and lengths can be expressed as \(2w\) and \(2l\) respectively. Thus, the perimeter would be expressed as Perimeter = \(2w + 2l\). Students should recognize that this formula is the same as Perimeter = \(w + w + l + l\).

It will also be necessary to discuss the convention of writing multiplication involving a variable and a number without using the multiplication symbol. For example, \(2 \times w\) would be written as \(2w\). This is the form most commonly used in Grades 7, 8 and 9 when writing expressions.

The commutative property simply shows that the order in which terms are added or multiplied does not affect the final outcome.

This can be illustrated to students using real number examples such as:

\[2 + 4 = 6\text{ and }4 + 2 = 6\]
\[5 \times 2 = 10\text{ and }2 \times 5 = 10\]

In terms of developing formulae to find perimeter or area, the same would apply. E.g.,

\[P = 2w + 2l\] would yield the same results as \(P = 2l + 2w\)

\[A = w \times l\] would yield the same results as \(A = l \times w\)

6PR3.5 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication; e.g., \(a + b = b + a\) or \(a \times b = b \times a\).
General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Performance

• Ask students to measure the length and width of a rectangular room in the school using a measuring tape. If the floor has square tiles on it, students could find the length of one tile and find the length of each wall by determining the number of tiles that run along it. Partial tiles can be rounded off to the nearest whole. Once the length and width of the room has been determined ask students to use the formula they derived to calculate the room’s perimeter.

Extension:

“Approximately how many times would you have to walk around this room to have walked 1 km?”

“What is the area of this room?” (6SS3.1, 6PR3.4, 6SS3.7)

Resources/Notes

Math Focus 6
Lesson 9 (Cont’d): Perimeter of a Polygon
6PR3
6SS3
TG pp. 55 - 58
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Develop and apply a formula for determining the:
- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms.

[C, CN, PS, R, V]

Achievement Indicator:

6SS3.5 Explain, using models, how the volume of any right rectangular prism can be determined.

Elaborations—Strategies for Learning and Teaching

Students have been introduced to volume as the amount of 3-dimensional space occupied by an object. Students should be reminded that volume is measured in cubed units such as (mm$^3$, cm$^3$, m$^3$). Again, because students have not yet been introduced to exponents they may think of the exponent 3 as meaning 3-dimensional.

Use cubic centimetre blocks to model right rectangular prisms. As each cube has a volume of 1 cm$^3$, the total volume can initially be found by counting the number of cubes in the prism. As this will be very time-consuming for larger prisms it will be necessary to derive a formula to find its volume.

Begin by establishing the 3-dimensions of the prism. This is done simply by counting the number of cubes along the length, width and height. Keep in mind that these dimensions are linear distances and are not measured in cubed units. The height of the prism will indicate how many layers of cubes are in the prism. Students will be familiar with the concept of layers in a three dimensional solid from working with base-ten flats. First find the volume of one layer. For example in the prism below students would determine that the top layer is composed of four rows of five 1 cm$^3$ blocks. Therefore the volume of the top layer would be 20 cm$^3$ ($4\times5=20$).

Students should then draw the conclusion that because there are 4 layers and each has a volume of 20 cm$^3$, the total volume of the prism must be 80 cm$^3$ ($4\times20=80$ cm$^3$).
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Arrange students into small groups or pairs. Provide each group with a different number of cubic unit manipulatives. These could be cubic centimetre blocks, multi-link cubes or both depending on what is available. Have each group construct the largest right rectangular prism that they can, using the blocks they were given. Have each group determine the volume of their own prism. Students can then leave their prism at their desk as a station. Groups can then rotate around the room finding the volume of the prisms constructed by other groups. Once the activity is complete ask students to share their findings with those of other groups to compare their methods and verify their results. (6SS3.5)

Resources/Notes

Math Focus 6
Lesson 10: Volume of a Rectangular Prism
6PR3
6SS3
TG pp. 59 - 62
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to
6SS3 Continued

Achievement Indicator:

6SS3.6 Generalize a rule (formula) for determining the volume of right rectangular prisms.

Elaborations—Strategies for Learning and Teaching

From work with finding volume from the models used previously, students should have come to the conclusion that the volume of a given right rectangular prism can be found by finding the number of cubes in one layer (length x width), and then multiplying the volume of one layer by the number of layers (height).

Thus, the general formula \( V = l \times w \times h \) can be derived, where \( V \) = volume, \( l \) = length, \( w \) = width and \( h \) = height.

Students should also be able to interpret this formula in words as “The volume of a rectangular prism can be found by multiplying the length by the width by the height.”

Remind students of the commutative property and that the formula may be written with the three dimensions being multiplied in any order.

For example,

\[
\begin{align*}
V &= l \times w \times h \\
V &= w \times h \times l \\
V &= h \times l \times w
\end{align*}
\]

Etc…
**General Outcome:** Use Direct and Indirect Measurement to Solve Problems

**Suggested Assessment Strategies**

**Performance**
- Provide students with various base-ten blocks. Ask them to determine the volume of each different type of block (E.g. A unit is 1cm³, a rod is 10cm³, a flat is 100 cm³, and the cube block is 1000cm³.).

(6SS3.6)

**Resources/Notes**

*Math Focus 6*

**Lesson 10 (Cont'd): Volume of a Rectangular Prism**

**6PR3**

**6SS3**

TG pp. 59 - 62
Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued

Achievement Indicator:

6SS3.7 Solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.

Elaborations—Strategies for Learning and Teaching

This will involve the application of concepts previously developed to solve problems.

It is important for students to realize that in many cases solving a complex problem involves the process of solving several simpler problems and that it may take several steps to solve the whole problem.

Flag Colour Areas - Provide students with a variety of flags from different countries that have rectangular patterns, E.g. Newfoundland Independent Flag, Ireland, Latvia, Costa Rica, France, Germany, Austria, Yemen, Poland, Spain, Ivory Coast, etc… A collection of flags could be acquired as a cross-curricular activity in Social Studies or students could create replications of the flags in art class. Make sure all flags are the same size. The flags may also be printed or drawn and coloured on 1 cm$^2$ grid paper to make the length and width of each rectangle in the flag easier to identify.

Distribute the various flags amongst the students. Ask them to identify the length and width of each rectangle in their flag’s design. This could be done by measuring with a ruler or, if the flag is printed on grid paper, by counting the units along each edge. Using these dimensions students can determine the total area of each colour on the flag. E.g., If students are given the Newfoundland Independent flag they would find what area of the flag is pink, white and green. This activity could also be organized as a station activity with a different flag at each station.
General Outcome: Use Direct and Indirect Measurement to Solve Problems

**Suggested Assessment Strategies**

**Paper and Pencil**
- Ask students to design a patio/deck floor plan for their Dream House on 1 cm² grid paper. Set the condition that the deck design must be composed of two or more rectangles. Ask students to answer the following questions:
  - What is the total area of their deck that would need to be stained? (This would be done by calculating the area of each individual rectangle and then finding the sum.)
  - If you want to put patio lanterns around the perimeter of your deck. How much wire would you need?

(6SS3.3, 6SS3.4, 6SS3.1, 6SS3.2, 6SS3.7)

**Performance**
- 3-D Building - Ask students to construct right rectangular prisms of various dimensions using multi-link cubes or Lego/Duplo blocks. Combine these prisms to create a large building. Ask students to calculate the total volume of the building. This would be done by calculating the volume of each prism in the structure and finding the sum.

(6SS3.5, 6SS3.6, 6SS3.7)

**Resources/Notes**

*Math Focus 6*

*Lesson 11: Solving a Problem by Solving a Simpler Problem*

6SS3

TG pp. 63 - 66
2-D Geometry

Suggested Time: 3 Weeks
Unit Overview

Focus and Context

Students will use their knowledge of 2-D geometry and measurement to sort triangles and other polygons. They will further develop their knowledge of congruency by using transformational geometry and both formal and informal measurement strategies to establish congruent angles and line segments of polygons.

The use of appropriate mathematical language to enhance communication should be emphasized. Teachers should be aware of the importance of modelling for students the mathematical language necessary to demonstrate their understanding. Students should recognize that using this language will help them effectively communicate their understanding.

Math Connects

Students develop their spatial sense by making connections to their everyday life and environment. Recognizing 2-D shapes in the world around them will provide students the opportunity to develop the concepts they will learn in the classroom and apply them to these shapes. Spatial sense will encourage students to visualize objects, to understand the effects of changes made to objects, and to help with problem solving situations.
### Process Standards Key

<table>
<thead>
<tr>
<th>Communication</th>
<th>Problem Solving</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>[C]</td>
<td>[PS]</td>
<td>[R] Reasoning</td>
</tr>
<tr>
<td>Connections</td>
<td></td>
<td>[T] Technology</td>
</tr>
<tr>
<td>[CN]</td>
<td></td>
<td>[V] Visualization</td>
</tr>
<tr>
<td>Mental Mathematics and Estimation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ME]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[C, PS, R, V]</td>
<td>[C, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td>[C, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>

### Curriculum Outcomes

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape and Space</td>
<td>6SS4 Construct and compare triangles, including:</td>
<td>[C, PS, R, V]</td>
</tr>
<tr>
<td>(3-D Objects and 2-D Shapes)</td>
<td>• scalene</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• isosceles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• equilateral</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• right</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• obtuse</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• acute</td>
<td></td>
</tr>
<tr>
<td></td>
<td>in different orientations.</td>
<td></td>
</tr>
<tr>
<td>Shape and Space</td>
<td>6SS5 Describe and compare the sides and angles of regular and irregular polygons.</td>
<td>[C, PS, R, V]</td>
</tr>
<tr>
<td>(3-D Objects and 2-D Shapes)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Outcomes

Students will be expected to

6SS4 Construct and compare triangles, including:
- scalene
- isosceles
- equilateral
- right
- obtuse
- acute

in different orientations.

[C, PS, R, V]

Achievement Indicator:

6SS4.1 Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.

Elaborations—Strategies for Learning and Teaching

In Grade 5, students categorized quadrilaterals according to side length and pairs of parallel lines. Students will expand their knowledge of properties to categorize triangles based on side length and interior angles. Begin by exploring side lengths of triangles and naming the triangles as scalene, isosceles and equilateral.

To begin classifying triangles, focus on side length. There are three classifications when naming triangles according to their side lengths:
- Scalene – no equal sides
- Isosceles – two equal sides
- Equilateral – three equal sides

The use of concrete models will help students visualize side length and classify triangles. It will also help students realize that orientation does not affect a triangle’s classification. Provide students with manipulatives (toothpicks, string, geoboards and elastic bands, etc.) and ask them to make several triangles. Ask students to measure each side of the triangles they have constructed, and discuss what types of triangles they have made. When using geoboards, students commonly have the misconception that the diagonal distance between points is equal to the sum of the vertical and horizontal distances. It is important to ask students to measure the diagonal distances to see that this is not true.

Display an equilateral triangle. Ask students to replicate that triangle with any choice of manipulative and identify the type of triangle that is presented. Review the concept of perimeter and ask students to calculate the perimeter of their triangle. As a class, use the manipulatives and brainstorm to come up with other triangles with the same perimeter. Ask students to identify the type of triangle.

Students should come to realize that not just any combination of three lengths can become the side lengths of a given triangle. The sum of the two shorter sides of a triangle must be greater than the length of the longest side; otherwise the three sides could never connect. If students struggle with remembering the names for each triangle, a strategy such as the following may help. Place the three triangle names in alphabetical order. Then apply the 3, 2, 1 rule: the first triangle (equilateral) has 3 equal sides, the second (isosceles) has 2 equal sides and the third (scalene) has 1, or no equal sides.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

**Performance**
- Provide students with a geoboard, elastic bands and square dot paper. Ask them to create 3 different scalene triangles on the geoboard and then reproduce the triangles on the dot paper. Ask students to explain how they know they have created scalene triangles. Repeat the activity for isosceles and equilateral triangles. (6SS4.1)

- Provide students with a copy of a number of triangles. Be sure to include at least one of each of the three types. Ask students to sort the triangles. Rulers may be used. (6SS4.1, 6SS4.2)

- Ask students to create their own ‘What am I?’ cards (e.g., I am a triangle with one angle measuring 120˚ and two equal sides. What am I? Answer: obtuse isosceles triangle) and use them to play with a friend. (6SS4.1)

**Journal**
- Ask students to answer the following questions:
  (i) Can you create a triangle with more than one obtuse angle? Explain using words, pictures and/or numbers. (6SS4.1)
  (ii) Can an obtuse triangle be an equilateral triangle? Explain using words, pictures and/or numbers. (6SS4.1)
  (iii) Can a right triangle be an isosceles triangle? Explain using words, pictures, and/or numbers. (6SS4.1)

**Resources/Notes**

**Math Focus 6**

**Lesson 1: Classifying Triangles by Side Length**

*6SS4*

TG pp. 13 – 16

*Lessons 1 and 2 may be combined*

**Lesson 2: Exploring Triangles**

*6SS4*

TG pp. 17 - 20

**Additional Reading** (provided):


Outcomes

*Students will be expected to*

6SS1 Demonstrate and understanding of angles by:
- identifying examples of angles in the environment
- classifying angles according to their measure
- estimating the measure of angles, using 45°, 90° and 180° as reference angles
- determining angle measures in degrees
- drawing and labelling angles when the measure is specified.

Achievement Indicator:

6SS1.3 Estimate the measure of an angle, using 45°, 90° and 180° as reference angles.

6SS4 Continued

Achievement Indicator:

6SS4.2 Sort a given set of triangles and explain the sorting rule.

Elaborations—Strategies for Learning and Teaching

Students will focus on identifying the interior angles of a triangle as right, obtuse or acute, using 90° as a benchmark. Ask students to identify objects in the classroom that could be used as a 90° reference point (e.g., the corner of a sheet of paper, a textbook, the corner of a ruler, etc.). Students can use the object to identify the angles of a triangle as acute (smaller than the 90° angle), obtuse (larger than the 90° angle) or right (equal to the 90° angle).

At this point, introduce another three classifications of triangles, based on their interior angles:
- A right triangle has one 90° angle.
- An acute triangle has all angles less than 90°.
- An obtuse triangle has one angle greater than 90°.

Students can investigate the remaining angles in a right triangle and an obtuse triangle to determine what types of angles they must be.

Exploration should lead to the discovery of the angle relationships in equilateral triangles and isosceles triangles. The definition of equilateral triangles, isosceles triangles and scalene triangles should then be expanded to include interior angles.
- Equilateral triangle – has all sides equal and all angles equal
- Isosceles triangle – has two equal sides and two equal angles
- Scalene triangle – has no equal sides or equal angles

Give students the opportunity to sort a set of triangles and complete a table similar to the one below:

<table>
<thead>
<tr>
<th>Type</th>
<th>Equilateral Triangle</th>
<th>Isosceles Triangle</th>
<th>Scalene Triangle</th>
<th>Right Triangle</th>
<th>Acute Triangle</th>
<th>Obtuse Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

During this activity, students should discover that the triangles will have two classifications. They can be classified according to their side lengths and angles. For example, a right triangle can also be an isosceles triangle but an equilateral triangle can never be right or obtuse. Students should be allowed time to explore which combinations of right, acute and obtuse triangles and scalene, isosceles and equilateral triangles are possible. Those that exist are right isosceles, right scalene, acute isosceles, acute equilateral, acute scalene, obtuse isosceles and obtuse scalene.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a Venn diagram template and ask them to sort given triangles according to chosen properties, such as isosceles triangles vs. right triangles. (6SS4.2)

Journal

- Ask students to discuss the statement, “An equilateral triangle is a special type of isosceles triangle.” (6SS4.2)

Performance

- Ask students to create any triangle with construction paper. Put students in groups and ask them to sort their triangles into two groups using a sorting rule of their choice. (6SS4.2)

- Provide students with a sheet of paper. Using a ruler, ask them to tear off each of the four corners to form four different triangles. Ask them to use a ruler and protractor to name the triangles according to both their side lengths and angle measures. (6SS4.1, 6SS4.2)

Resources/Notes

Math Focus 6

Lesson 3: Classifying Triangles by Interior Angles

6SS1

6SS4

TG pp. 21 – 24

Children’s Literature (not provided):

Adler, David A. Shape up! Fun with Triangles and Other Polygons. ISBN 0-8234-1638-0
Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicators:

6SS1.5 Measure, using a protractor, given angles in various orientations.

6SS1.6 Draw and label a specified angle in various orientations, using a protractor.

6SS4 Continued

Achievement Indicator:

6SS4.3 Draw a specified triangle, e.g., scalene.

Elaborations—Strategies for Learning and Teaching

Review the use of a protractor. Students have been exposed to using a protractor to measure angles in the Measurement unit. While reviewing, ask questions such as: “What steps would you take to draw a 45° angle?” and “What steps would you take to draw a 120° angle?”

Ask students to draw specified angles as practice. Discuss with them that in this unit they will be drawing angles in order to create triangles.

Students need step by step instruction when they are first learning to draw triangles. It may be beneficial to ask students to begin by drawing a line segment and adding a specified angle to one of its ends. Then progress to asking students to draw a triangle with one angle specified. As students become more comfortable with this, add more specifications to the instructions. For example, give them two line segments and one angle, or two angles and one line segment, to include in the triangle.

When drawing triangles, students should be able to identify the unspecified angles and line segments. They should recognize that being given two angles and one side length results in a unique triangle. For example, if two students are asked to draw a triangle with a side length of 3 cm and angles measuring 40° and 70°, their resulting triangles will be congruent acute isosceles triangles. The orientation may be different, and students may need to be reminded that a change in orientation does not make it a different triangle. Unique triangles are also produced when two sides and the contained angle are specified, or when three sides are specified. This idea will be further developed later in the unit.

It is important that students be able to draw triangles of any specified type (e.g., acute scalene).

(continued)
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil
• Ask students to draw 3 different obtuse isosceles triangles.
  (6SS1.5, 6SS1.6, 6SS4.1, 6SS4.3)

• Ask students to construct a triangle with one angle measuring 65° and one angle measuring 40°. Ask questions such as:
  (i) What is the measure of the third angle?
  (ii) What kind of triangle did you make?
  (iii) How do you know?
  (6SS1.6, 6SS4.1, 6SS4.3)

Performance
• Provide students with a geoboard and geobands.
  Ask students to construct a triangle with two 45° angles. Ask students to make three additional triangles with the same attributes.
  (We are expecting the triangles to vary in size.) Ask students how their triangles are the same and how they are different. Ask them what kind of triangle they made.
  (6SS4.3)

Resources/Notes

Math Focus 6
Lesson 4: Drawing Triangles
6SS1
6SS4
TG pp. 25 – 28
Outcomes

Students will be expected to
6SS4 Continued

Achievement Indicator:

6SS4.3 Continued

Elaborations—Strategies for Learning and Teaching

Having an understanding of the properties of triangles is beneficial when drawing triangles. These properties include:

- The greatest angle is opposite the longest side, and the smallest angle is opposite the shortest side.

- The sum of the two shorter sides must be greater than the longest side.

- When two angles of a triangle are congruent, then the sides opposite them are congruent (and vice versa).

Students should be encouraged to use appropriate hatch marks on triangles to indicate when sides and angles are equal.

- The sum of the interior angles of any triangle is 180°.

- A triangle can never have more than one obtuse angle or one right angle.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td><em>Lesson 4 (Cont’d): Drawing</em></td>
<td></td>
</tr>
<tr>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td><em>6SS1</em></td>
<td></td>
</tr>
<tr>
<td><em>6SS4</em></td>
<td></td>
</tr>
<tr>
<td>TG pp. 25 – 28</td>
<td></td>
</tr>
</tbody>
</table>
Outcomes

Students will be expected to

6SS5 Describe and compare the sides and angles of regular and irregular polygons.
[C, PS, R, V]

Achievement Indicators:

6SS5.1 Sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.

Provide students with a template similar to the one below and a number of polygons and non-polygons that they can place into their template. Ask students to cut out polygons and glue them into the appropriate section of the template.

When defining polygons with your class, highlight examples of non-polygons as well.

6SS5.2 Demonstrate that the sides of a given regular polygon are of the same length and that the angles of a regular polygon are of the same measure.

It may be necessary to revisit what a polygon is - a closed 2-D figure bound by straight line segments that intersect at the vertices. Naming polygons may also require some review.

The definition of a regular polygon should be developed through a discovery approach. Display several regular polygons and ask students to measure the angles and side lengths, and discuss what they notice. This should highlight that, in regular polygons, all angles are equal and all side lengths are equal. Also include symmetry in the definition of a regular polygon. Students were exposed to symmetry in Grade 4.

Discussion should lead students to conclude that if all angles of a polygon are equal then all sides are also equal, and vice versa.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil
- Provide students with a template for the Frayer Model. Ask them to complete the sections individually to demonstrate their understanding of the geometric concept “polygon”. (6SS5.1)

<table>
<thead>
<tr>
<th>Definition</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| A polygon is a closed 2-D figure bound by straight line segments that intersect at the vertices. | - a closed 2-D figure with sides  
- the number of vertices is equal to the number of sides  
- does not have curves |

Journal
- Ask students to draw a polygon and a non-polygon, and explain why one is a polygon and the other is not. (6SS5.1)

- Ask students to agree or disagree with the statement below and to explain their thinking:
  Because all the angles of a rectangle measure 90˚, the angles are congruent. That means that rectangles are regular polygons. (6SS5.2)

Performance
- Ask students to sort these shapes into two groups; polygons and non-polygons.

Resources/Notes

Math Focus 6
Lesson 5: Sorting Polygons
6SS1
6SS5
TG pp. 34 - 37
Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS5 Continued

Achievement Indicators:

- 6SS5.3 Sort a given set of polygons as regular or irregular, and justify the sorting.
- 6SS5.4 Identify and describe regular and irregular polygons in the environment.

Elaborations—Strategies for Learning and Teaching

Students were exposed to sorting regular and irregular polygons in Grade 3. During the sorting of polygons, there are instances when students can rely on visual cues when it clear that a polygon is irregular. However, when determining if a figure is regular, students should be encouraged to check angles and/or side lengths.

Initiate a discussion about examples of regular and irregular polygons in the environment. Provide students with a placemat template (regular paper or chart paper to display in the classroom.)

Ask students to brainstorm and record (by drawing or writing) as many examples of polygons that they see or use in the world around them. Each group can share their “placemat” findings with the rest of the class.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Performance

• Provide students with several regular polygons and ask them to measure the angles with a protractor and side lengths with a ruler. Ask them to label them with appropriate hatch marks. (6SS1.5, 6SS5.2)

• Provide students with triangular dot paper. Ask them if they can draw a regular pentagon. Once they conclude it is not possible discuss with them why this is the case. (6SS5.2)

Student-Teacher Dialogue

• Ask students to tell you about the characteristics of a regular polygon. Then ask them which characteristic they prefer to use to check whether a polygon is regular or irregular. Ask why they prefer using this characteristic. (6SS5.3, 6SS5.4)

Paper and Pencil

• Provide students with a set of shapes including several regular polygons, irregular polygons, one non-polygon, and a Venn diagram similar to the one below. Ask them to place the shapes where they belong.

Resources/Notes

Math Focus 6
Lesson 5 (Cont'd): Sorting Polygons
6SS1
6SS5
TG pp. 34 - 37
Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS4 Continued

Achievement Indicator:

6SS4.4 Replicate a given triangle in a different orientation, and show that the two are congruent.

Elaborations—Strategies for Learning and Teaching

Students will build upon their existing knowledge of transformational geometry and congruency. During the Measurement unit, students were exposed to congruent line segments and congruent sides of regular polygons.

Using geoboards and geobands, students could work in pairs to construct the same triangle on each of their geoboards. Students should notice that after rotating one of the geoboards a \( \frac{1}{4} \) turn (90°), the rotated triangle has not changed but its orientation is different. If students have not already suggested that the triangles are congruent, review the concept of congruency with them, referring to their triangles. Congruent figures have exactly the same size and the same shape. They can have different orientations and still be congruent.

For teacher reference: For two triangles to be congruent, they must meet one of the following conditions:

- Side-Side-Side (SSS)
- Side-Angle-Side (SAS)
- Angle-Side-Angle (ASA)
- Angle-Angle-Side (AAS)

There are a number of ways students can demonstrate the congruency of regular polygons. This concept can include congruency of angles and side lengths within a single polygon or between sets of polygons. One way is to superimpose an image. This can be done using tracing paper, cut outs or a Mira in conjunction with transformations.

Provide students with two congruent regular polygons of differing orientations, and tracing paper. Students can trace one of the polygons and then place the tracing on top of the other polygon. The two shapes will match. The polygons coincide and one shape is superimposed on the other.

Another way to demonstrate congruency is to measure the side lengths and angles using a ruler and a protractor.
General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

**Performance**
- Provide students with dot paper. Ask them to draw a triangle. Students may use any transformation to change the triangle's orientation. (6SS4.4)

- Provide students with a set of regular polygons. Include several sets of congruent and similar polygons. Ask students to identify pairs of congruent polygons and to explain how they found them. (6SS5.5, 6SS5.6)

**Paper and Pencil**
- Provide students with triangular dot paper. Ask them to draw a regular hexagon. Ask them to show that all sides and all angles are congruent by measuring or superimposing. (6SS5.5, 6SS5.6)

- Provide students with a copy of two congruent regular polygons of different orientations. Label the vertices of both polygons. On one, indicate the side lengths and angle measures, but not on the other. Ask students to write the measure of each angle in the second polygon without using a protractor, and the measure of each side length without using a ruler.

Journal
- Ask students to answer a question such as: What does it mean when two regular polygons are congruent? Use words and pictures to explain your understanding. (6SS5.5, 6SS5.6)

**Resources/Notes**

* Math Focus 6
  Lesson 6: Congruent Polygons
  6SS4
  6SS5
  TG pp. 38 - 41

  Lesson 7: Communicating about Polygons
  6SS4
  6SS5
  TG pp. 42 – 44

* Math Game: Matching Cards
  6SS1
  6SS4
  6SS5
  TG pp. 45 - 46

* Math Contest: Polygon Contest
  SS1
  SS4
  SS5
  TG pp. 51- 52
Probability

Suggested Time: 2 Weeks
Unit Overview

Focus and Context
Probability involves the use of mathematics to describe the level of certainty that an event will occur.

Students have been exposed to experimental probability in grade 5. In grade 6, students will be exposed to theoretical as well as experimental probability.

The theoretical probability of an outcome is determined by analyzing what could happen, whereas the experimental probability reports what did happen. The theoretical and experimental probability of an outcome can differ, although they tend to be closer with more trials.

The theoretical probability of an outcome can be expressed as a ratio, a fraction, a percent and a decimal, which relates the number of favourable outcomes to the number of possible outcomes.

This unit focuses primarily on identifying possible outcomes of a probability experiment, determining the theoretical probability of an outcome or event in a probability experiment, comparing experimental and theoretical probabilities, as well as communicating about probability. Students will also learn that in any probability situation, you can never be sure what will happen next.

Math Connects
Students should be aware that there are many references to probability all around us. Brainstorming the ways that probability is used in everyday life will focus the student’s attention to this topic as well as help them realize that references to probability are all around us. The weather forecaster predicts a 60% chance of showers, medical researchers predict people with certain diets have a high chance of heart disease or you have a one in five million chance in winning the lottery this week. Examples of probability are all around us, and probability has become more visible in our Mathematics curriculum. Students need to collect and analyze data from experiments that they will conduct.

One of the advantages of teaching probability is that students naturally have some understanding of the topic from their real life experiences - from hearing weather reports, to rolling a die to try to win a game. This unit will build on students’ past experience and engage them in a variety of games, activities and investigations in which they’re encouraged to predict outcomes and then test their predictions through their first hand experiences.
**Process Standards Key**

<table>
<thead>
<tr>
<th>C</th>
<th>Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>CN</td>
<td>Connections</td>
</tr>
<tr>
<td>ME</td>
<td>Mental Mathematics and Estimation</td>
</tr>
<tr>
<td>PS</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>R</td>
<td>Reasoning</td>
</tr>
<tr>
<td>T</td>
<td>Technology</td>
</tr>
<tr>
<td>V</td>
<td>Visualization</td>
</tr>
</tbody>
</table>

---

**Curriculum Outcomes**

<table>
<thead>
<tr>
<th>STRAND</th>
<th>OUTCOME</th>
<th>PROCESS STANDARDS</th>
</tr>
</thead>
</table>
| Statistics and Probability (Chance and Uncertainty) | 6SP4 Demonstrate an understanding of probability by:  
• identifying all possible outcomes of a probability experiment  
• differentiating between experimental and theoretical probability  
• determining the theoretical probability of outcomes in a probability experiment  
• determining the experimental probability of outcomes in a probability experiment  
• comparing experimental results with the theoretical probability for an experiment. | [C, ME, PS, T] |
Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Demonstrate an understanding of probability by:

- identifying all possible outcomes of a probability experiment
- differentiating between experimental and theoretical probability
- determining the theoretical probability of outcomes in a probability experiment
- determining the experimental probability of outcomes in a probability experiment
- comparing experimental results with the theoretical probability for an experiment.

Elaborations—Strategies for Learning and Teaching

Students have conducted probability experiments in Grade 5 and it will be helpful to review, with students, the meaning of probability - the chance of an event happening out of all possible outcomes. Revisit the concepts of 'more likely', 'less likely', 'equally likely', 'possible', 'impossible' or 'certain'. Give students opportunities to experiment with probability and provide them with meaningful explorations. The materials students use to conduct experiments should be familiar. This becomes important when students are determining all the possible outcomes of an experiment.

Refer to R. Dalh's novel, Charlie and the Chocolate Factory. Make a connection between the chances of winning the golden ticket and what was needed to better the chances of winning! Remind them that probabilities can be expressed as a fraction, a decimal, a ratio or a percent. Students may remember hearing percent used frequently on a weather report. E.g., there is a 60% chance of rain tomorrow.

Carry out basic experiments using spinners, dot cubes or coins to help students gain an better understanding of probability. E.g., What is the likelihood of flipping a coin and having it land on heads? What is the likelihood of rolling a dot cube and having it land on a 6?

Students should be able to make predictions of the outcome of an experiment based on what they know about the materials (coins, dot cubes, spinners, etc) they are using. Students can be encouraged to create their own tools for determining probability. Creating spinners is motivating for many students.

Engage students in a discussion about how probability is used in advertising. Ask students to look at newspapers and magazines for examples of how numbers are used in advertisements. E.g., it is not unusual to see something like “four out of five dentists recommend Brand T gum for their patients who chew gum.”

Ask students questions such as the following:

Why do advertisers use numbers like these? What information are they trying to convey? Do you think that the numbers give accurate information about a product? Why or why not?

Discuss, with your students, how probability is used to influence choices.

(continued)
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Presentation</strong></td>
</tr>
<tr>
<td>• Discuss the following terms with students:</td>
</tr>
<tr>
<td>impossible</td>
</tr>
<tr>
<td>possible</td>
</tr>
<tr>
<td>less likely</td>
</tr>
<tr>
<td>equally likely</td>
</tr>
<tr>
<td>more likely</td>
</tr>
<tr>
<td>certain</td>
</tr>
<tr>
<td>Ask them to choose from the above terms to describe the probability of each of the following events:</td>
</tr>
<tr>
<td>• School will start in September</td>
</tr>
<tr>
<td>• Trees shed their leaves in July</td>
</tr>
<tr>
<td>• If you run a lap around the track, you can run a marathon</td>
</tr>
<tr>
<td>• If you toss a coin, it will land on heads (6SP4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Focus 6</strong></td>
</tr>
<tr>
<td>Lesson 1: Predicting Probabilities</td>
</tr>
<tr>
<td>6SP4</td>
</tr>
<tr>
<td>TG pp. 13 - 16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Math Game: Take A Chance</th>
</tr>
</thead>
<tbody>
<tr>
<td>6SP4</td>
</tr>
<tr>
<td>TG pp. 17 - 18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Children's Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>(not provided):</td>
</tr>
<tr>
<td>Dalh, D. Charlie and the Chocolate Factory</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>(provided):</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>(provided):</td>
</tr>
</tbody>
</table>
Outcomes

Students will be expected to

6SP4 Continued

Elaborations—Strategies for Learning and Teaching

Emphasize that probabilities range from 0 (impossible) to 1 (certain). Probabilities less than 0 and greater than 1 do not make sense because events either happen or they do not. Guide students to create their own ‘Probability Line’ (shown below) to describe and compare probabilities. Students should place events on the line and justify their choices for placing the events where they did. Group work or paired work is suggested as they can once again, get used to using the proper terminology and they can analyze what they are doing.

Event A could be any event that is fairly likely but not certain, such as rain during the third week of April.

Event B could be an event that is less likely but not impossible, such as an animal coming into the classroom.

Event C could be an event that is more likely but not certain, such as school happening tomorrow.
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

<table>
<thead>
<tr>
<th>Suggested Assessment Strategies</th>
<th>Resources/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Math Focus 6</em></td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 1:</strong> Predicting Probabilities</td>
</tr>
<tr>
<td></td>
<td><strong>6SP4</strong></td>
</tr>
<tr>
<td></td>
<td>TG pp. 13 - 16</td>
</tr>
</tbody>
</table>
Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued

Achievement Indicators:

6SP4.1 List the possible outcomes of a probability experiment, such as:
- tossing a coin
- rolling a die with a given number of sides
- spinning a spinner with a given number of sectors.

6SP4.2 Determine the theoretical probability of an outcome occurring for a given probability experiment.

Elaborations—Strategies for Learning and Teaching

Theoretical probability is based on what SHOULD happen. Explain to students that, in order to find theoretical probability, they must first determine the total number of possible outcomes. E.g., Show a spinner with numbers 1 to 5. The possible outcomes are 1, 2, 3, 4 or 5. Students should observe that each section of the spinner is of equal size and each number shows up only once. Therefore, each digit will have an equal chance of being spun. To find the theoretical probability of spinning a 3, students should see that 3 shows up once on the spinner where there are 5 numbers in total. Therefore, the probability of spinning a 3 would be 1 out of 5, or 20%. So, if the spinner is spun 100 times, theoretically it will land on “3” 20 times.

\[
\text{Probability of Event A} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}
\]

Students should be given opportunities to identify the possible outcomes of an experiment, and they need to be able to relate the experimental probability to the theoretical probability.

Provide examples of other experiments that could be carried out. Ask students to identify the experimental and theoretical probabilities for each experiment. For example, ask students to conduct the following experiment:

Take 10 two coloured counters and drop them on a table. Record how many land with the red side up and how many land with the white side up. Do at least 10 trials and record the results. What can you conclude? What is the theoretical probability of the counter landing with the white side up? What is the experimental probability?
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Performance

- Mental Math - If you are rolling a regular die, what is the probability of rolling an even number? (6SP4.2)

- Ask the students to watch as you put linking cubes into a bag. First place 20 blue, 10 yellow and 5 green. Ask students questions about which colour they think you will pull from the bag. Why is a certain colour more likely or less likely? What are the chances of pulling an orange cube out? Empty the bag and fill it again with 1 green, 1 blue, 1 brown and 1 red. Discuss the possible outcome with this combination of colours in the bag. Describe the probability using fractions. (6SP4.1, 6SP4.2)

- Tell students that there are 10 yogurt bars in a bag. 3 bars are caramel flavoured, which Dave likes, 2 are strawberry flavoured, which he dislikes, and 5 are orange flavoured, which he also likes. Dave puts his hand in the bag without looking, and pulls out a bar.

  Ask: • What are his chances of pulling out a yogurt bar that he dislikes?
  • Write another probability question about the yogurt bars that were in the bag. (6SP4.2)

- Tell students that you are visiting a kennel that has 3 German Shepherds, 5 Labrador Retrievers, 3 Chihuahuas, 4 Poodles, and 5 West Highland Terriers. When you arrive, the dogs are taking a walk. What is the probability of seeing a German Shepherd first? (6SP4.2)

Resources/Notes

Math Focus 6
Lesson 2: Theoretical Probability
6SP4
TG pp. 19 - 23
Strand: Statistics and Probability (Chance and Uncertainty)

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Elaborations—Strategies for Learning and Teaching</th>
</tr>
</thead>
</table>
| Students will be expected to
6SP4 Continued
Achievement Indicators: |

6SP4.3 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.

To have a strong understanding of probability, students must be able to differentiate between experimental probability and theoretical probability. Coin flipping experiments can be used to facilitate this. For example, students could be asked to predict the probability of a coin landing on heads. The theoretical probability of this is \( \frac{1}{2} \). Ask students to flip a coin 100 times and record the results. When they carry out the experiment, they may discover that the result is not necessarily heads being tossed 50 times out of 100, and therefore, the experimental probability may not be \( \frac{1}{2} \). This is a great way to demonstrate the difference between experimental and theoretical probability.

6SP4.4 Distinguish between theoretical probability and experimental probability, and explain the differences.

To help students understand the difference between theoretical and experimental probability is to explain to students that theoretical probability deals with what ‘should’ happen in an ideal world within a given event. E.g., “in theory”, given there are two equal colors on a spinner, each color should have a 50% chance of being spun. When experiments are completed, the resulting experimental probability may state that red, for example, was spun 65% of the time whereas blue was spun only 35% of the time. Experimental probability, then, is telling what exactly happened within a given event. Reinforce that theoretical probability occurs BEFORE the experiment happens and experimental probability occurs once the experiment is completed.

Experimental probability differs from theoretical probability in that it depends on the actual results of experiments. Sometimes the experimental probability is not the same as the theoretical probability even though, theoretically, the likelihood of the events happening is equal. (getting heads or tails when flipping a coin).
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Journal

- Bring a Farmer's Almanac to show the students. Ask students how they think authors of The Farmer's Almanac make their predictions about weather for a year? How do they use probability?

Paper and Pencil

- Ask students to create spinners with paper, pencils and paper clips. Sections of the spinners should be equal in size and coloured different colours. Have student pairs calculate the theoretical probability (expressed as a fraction, a decimal a ratio or a percent) of each outcome of their own spinners and those of their partners. Then, ask students to calculate the experimental probability of spinning one colour. Demonstrate spinning the spinner 10 times. Then, ask students to record the results of 25 spins of their own spinners in a frequency table.

Performance

- Prepare three paper bags, one with 2 blue and 8 yellow cubes, one with 5 blue and 5 yellow cubes and one with 8 blue and 2 yellow cubes. Tell students the content of the three bags, but not which bag is which. Pull a cube from one of the bags, allow students to see the colour and then return the cube to the bag. Repeat the draw 10 times. Ask students how many yellow cubes are in the bag – 2, 5 or 8. Ask them to justify their answer.
Outcomes

Students will be expected to

6SP4 Continued

Achievement Indicator:

6SP4.5 Conduct a probability experiment, with or without technology, and compare the experimental results with the theoretical probability.

Elaborations—Strategies for Learning and Teaching

It is important that students conduct experiments to help them make the connection between experimental and theoretical probability. If they are aware of the theoretical probability of an experiment, they will be better able to make predictions as to what will happen once they carry out an experiment.

Experimental probability is determined by the results of an experiment that has already occurred. In contrast, theoretical probability involves analyzing possible outcomes in advance and using logic and reason to predict what is likely to happen.

Give students examples of experiments that could be carried out. Ask them to identify the experimental and the theoretical probabilities for each experiment.

After reading the book, Martha Blah Blah by Susan Meddaugh ask students to work in pairs to investigate which alphabet letters would be the best ones for Granny Flo to eliminate from the alphabet and tell why. Ask students to work together to find the frequency with which each letter in the English alphabet is used. To do this, ask pairs of students to choose any book, select a short paragraph in that book, and analyze the frequencies of the letters used in that paragraph. Based on their findings, students present to the class if they think it would be less disruptive to remove certain letters, consonants or vowels from the alphabet. Ask them how they came to this conclusion.
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Performance

- Rock-Paper-Scissors - Students play this game in groups of 3 with a number (1, 2 or 3) assigned to each person. Play 20 times and record the results of each game.
  
  Player 1 gets a point if all players show the same sign
  Player 2 gets a point if two players show the same signs
  Player 3 gets a point if all players show the different signs

  Ask students to compare the experimental results with the theoretical probability. (6SP4.4, 6SP4.5)

- Coin Flip – Ask students to experiment with flipping a coin a specified number of times and record the results. If they flip a penny two times, they may get 2 heads, or two tails or one of each. If it is flipped 10 times, they may find that the results will be different. Likewise, if they flip the coin 100 times, the results will look different than if it had been rolled only ten times. Draw students’ attention to this as they flip their coins. Students should share their results once they have completed their trials. (6SP4.5)

- Ask students what the probability is of rolling ‘doubles’ using two number cubes? Make a chart and then roll the number cube 20 times. Record the results of your rolls.

  Ask:
  
  • What is the theoretical probability of rolling doubles?
  • What is the theoretical probability of not rolling doubles?
  • What patterns do you see? Record your observations in your journal or exercise book.

  Make a chart to list all possible outcomes of rolling two number cubes:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(6SP4.5)

Resources/Notes

* Math Focus 6
  Lesson 3: Comparing Experimental and Theoretical Probability

* 6SP4
  TG pp. 27 – 31

* Related Literature (provided): by Meddaugh, Susan. *Martha Blah Blah*
Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued

Achievement Indicator:

6SP4.6 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.

Elaborations—Strategies for Learning and Teaching

Students should know that the more often they carry out the trials for an experiment, the better they will be able to predict a particular outcome. When making important decisions based on an experiment, it is important to carry out an experiment enough times so that you can make a firm prediction of the outcome. As the number of trials increase, the better their predictions will be.

It is important to emphasize the importance of a reasonably large sample size. The results of a small number of trials or just one experiment, can be misleading, but when multiple trials are used and/or experiments repeated many more times, the experimental probability will gradually approach the theoretical probability. A good way to do this is to carry out an experiment where students record results individually, and then combine results from the entire class. The larger number of trials should result in a probability closer to the theoretical probability. E.g., Ask students to determine the theoretical probability of rolling a 3 on a six-sided die. Then ask them to roll a six-sided die ten times and record the result each time. What is the experimental probability of rolling a 3? Combine all the trials for the class, and again ask what the experimental probability of rolling a 3 is. Which is closer to the theoretical probability?

Students have learned about experimental probabilities and theoretical probabilities. They have solved problems throughout. Encourage students to use appropriate mathematical language. Upon completion, they should be able to talk about the experiments that they have completed and clearly explain the results. The students will gain experience in using the words likely, most likely, less likely, impossible and certain. Give students the opportunity to work in pairs or in small groups to use the language and to carry out these simple experiments.
General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Student-Teacher Dialogue

- Use everyday life events that the students can talk about probability for. E.g.,
  - The sun will rise tomorrow
  - A parent will be home when you get home from school
  - You will have pizza for dinner
  - A moose will come to the schoolyard on Friday. (6SP4.1)

Using previously conducted experiments, ask students to select one or two, and present what the results were. Encourage the use of appropriate terminology in explaining their analysis. (6SP4.3)

Journal

- How would you explain experimental and theoretical probability to someone? Record your answer, drawing pictures that would help you accurately describe the types of probability. (6SP4.2)

Resources/Notes

Math Focus 6
Lesson 3 (Cont’d): Comparing Experimental and Theoretical Probability
6SP4
TG pp. 27 – 31

Lesson 4: Communicating about Probability
6SP4
TG pp. 32 - 35

Curious Math: Fair or Unfair?
6SP4
TG pp. 36 – 37

Math Game: Predict and Score
6SP4
TG pp. 39 - 40
Appendix A
Outcomes by Strand
(with page references)
### Strand: Number

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>General Outcome: Develop number sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is expected that students will:</td>
<td>The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
</tbody>
</table>

**6N1** Demonstrate an understanding of place value, including numbers that are:
- greater than one million
- less than one thousandth.

[C, CN, R, T]

*(pp. 34-39, 46-47, 50-51)*

- **6N1.1** Explain how the pattern of the place value system, i.e., the repetition of ones, tens and hundreds within each period, makes it possible to read and write numerals for numbers of any magnitude.
- **6N1.2** Provide examples of where large and small numbers are used; e.g., media, science, medicine, technology.

**6N2** Solve problems involving whole numbers and decimal numbers.

[ME, PS, T]

*(pp. 40-45, 48-49)*

- **6N2.1** Identify which operation is necessary to solve a given problem, and solve it.
- **6N2.2** Determine the reasonableness of an answer.
- **6N2.3** Estimate the solution to, and solve, a given problem.
- **6N2.4** Determine whether the use of technology is appropriate to solve a given problem, and explain why.
- **6N2.5** Use technology when appropriate to solve a given problem.

**6N3** Demonstrate an understanding of factors and multiples by:
- determining multiples and factors of numbers less than 100
- identifying prime and composite numbers
- solving problems using multiples and factors.

[CN, PS, R, V]

*(pp. 58-75)*

- **6N3.1** Determine all the whole number factors of a given number, using arrays.
- **6N3.2** Identify the factors for a given number, and explain the strategy used; e.g., concrete or visual representations, repeated division by prime numbers or factor trees.
- **6N3.3** Solve a given problem involving factors or multiples.
- **6N3.4** Identify multiples and factors for a given number, and explain the strategy used to identify them.
- **6N3.5** Provide an example of a prime number, and explain why it is a prime number.
- **6N3.6** Provide an example of a composite number, and explain why it is a composite number.
- **6N3.7** Sort a given set of numbers as prime and composite.
- **6N3.8** Explain why 0 and 1 are neither prime nor composite.
<table>
<thead>
<tr>
<th>Strand: Number</th>
<th>General Outcome: Develop number sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcomes</td>
<td>Achievement Indicators</td>
</tr>
<tr>
<td>It is expected that students will:</td>
<td>The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
<tr>
<td>6N4 Relate improper fractions to mixed numbers. [CN, ME, R, V] (pp. 202-223)</td>
<td>6N4.1 Demonstrate, using models, that a given improper fraction represents a number greater than 1</td>
</tr>
<tr>
<td></td>
<td>6N4.2 Translate a given improper fraction between concrete, pictorial and symbolic forms.</td>
</tr>
<tr>
<td></td>
<td>6N4.3 Express improper fractions as mixed numbers.</td>
</tr>
<tr>
<td></td>
<td>6N4.4 Translate a given mixed number between concrete, pictorial and symbolic forms.</td>
</tr>
<tr>
<td></td>
<td>6N4.5 Express mixed numbers as improper fractions.</td>
</tr>
<tr>
<td></td>
<td>6N4.6 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.</td>
</tr>
<tr>
<td>6N5 Demonstrate an understanding of ratio, concretely, pictorially and symbolically. [C, CN, PS, R, V] (pp. 174-183, 190-191, 196-197)</td>
<td>6N5.1 Provide a concrete or pictorial representation for a given ratio.</td>
</tr>
<tr>
<td></td>
<td>6N5.2 Write a ratio from a given concrete or pictorial representation.</td>
</tr>
<tr>
<td></td>
<td>6N5.3 Express a given ratio in multiple forms, such as 3:5, or 3 to 5.</td>
</tr>
<tr>
<td></td>
<td>6N5.4 Identify and describe ratios from real-life contexts, and record them symbolically.</td>
</tr>
<tr>
<td></td>
<td>6N5.5 Explain the part/whole and part/part ratios of a set; e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8 and 5:8.</td>
</tr>
<tr>
<td></td>
<td>6N5.6 Demonstrate an understanding of equivalent ratios.</td>
</tr>
<tr>
<td></td>
<td>6N5.7 Solve a given problem involving ratio.</td>
</tr>
<tr>
<td>6N6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically. [C, CN, PS, R, V] (pp. 184-189, 192-195)</td>
<td>6N6.1 Explain that “percent” means “out of 100.”</td>
</tr>
<tr>
<td></td>
<td>6N6.2 Explain that percent is a ratio out of 100.</td>
</tr>
<tr>
<td></td>
<td>6N6.3 Use concrete materials and pictorial representations to illustrate a given percent.</td>
</tr>
<tr>
<td></td>
<td>6N6.4 Record the percent displayed in a given concrete or pictorial representation.</td>
</tr>
<tr>
<td></td>
<td>6N6.5 Express a given percent as a fraction and a decimal.</td>
</tr>
<tr>
<td></td>
<td>6N6.6 Identify and describe percents from real-life contexts, and record them symbolically.</td>
</tr>
<tr>
<td></td>
<td>6N6.7 Solve a given problem involving percents.</td>
</tr>
<tr>
<td>Strand: Number</td>
<td>General Outcome: Develop number sense</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>Specific Outcomes</td>
<td>It is expected that students will:</td>
</tr>
</tbody>
</table>
| 6N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically. | [C, CN, R, V] (pp. 76-81) | 6N7.1 Extend a given number line by adding numbers less than zero, and explain the pattern on each side of zero.  
6N7.2 Place given integers on a number line, and explain how integers are ordered.  
6N7.3 Describe contexts in which integers are used; e.g., on a thermometer.  
6N7.4 Compare two integers; represent their relationship using the symbols <, > and =; and verify the relationship, using a number line.  
6N7.5 Order given integers in ascending or descending order. |
| 6N8 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors). | [C, CN, ME, PS, R, V] (pp. 232-253) | 6N8.1 Predict products and quotients of decimals, using estimation strategies.  
6N8.2 Solve a given problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.  
6N8.3 Place the decimal point in a product, using front-end estimation; e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m.  
6N8.4 Correct errors of decimal point placement in a given product or quotient without using paper and pencil.  
6N8.5 Place the decimal point in a quotient, using front-end estimation; e.g., for $26.83 ÷ 4, think$24 ÷ 4, so the quotient is greater than $6. |
| 6N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). | [C, CN, ME, PS, T] (pp. 80-83) | 6N9.1 Explain, using examples, why there is a need to have a standardized order of operations.  
6N9.2 Apply the order of operations to solve multistep problems with and without technology; e.g., a computer, a calculator. |
### Strand: Patterns and Relations
(Patterns)

**General Outcome:** Use patterns to describe the world and to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>Achievement Indicators</th>
</tr>
</thead>
</table>
| **6PR1** Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R] (pp. 88-101, 104-106, 112-113) | **6PR1.1** Create a concrete or pictorial representation of the relationship shown in a table of values.  
**6PR1.2** Identify errors in a given table of values.  
**6PR1.3** Describe the pattern within each column of a given table of values.  
**6PR1.4** Create a table of values to record and reveal a pattern to solve a given problem.  
**6PR1.5** Generate values in one column of a table of values, given values in the other column and a pattern rule.  
**6PR1.6** State, using mathematical language, the relationship in a given table of values.  
**6PR1.7** Predict the value of an unknown term, using the relationship in a table of values, and verify the prediction.  
**6PR1.8** Formulate a rule to describe the relationship between two columns of numbers in a table of values.  
**6PR1.9** Identify missing elements in a given table of values. |
| **6PR2** Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V] (pp. 136-137, 142-145) | **6PR2.1** Create a table of values from a given pattern or a given graph.  
**6PR2.2** Describe, using everyday language, orally or in writing, the relationship shown on a graph.  
**6PR2.3** Translate a pattern to a table of values, and graph the table of values (limited to linear graphs with discrete elements). |
<table>
<thead>
<tr>
<th>Strand: Patterns and Relations (Variables and Equations)</th>
<th>General Outcome: Represent algebraic expressions in multiple ways.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcomes</td>
<td>Achievement Indicators</td>
</tr>
<tr>
<td>It is expected that students will:</td>
<td>The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
</tbody>
</table>

6PR3 Represent generalizations arising from number relationships, using equations with letter variables.  
[C, CN, PS, R, V]  
(pp. 102-107, 284-285, 290-291)

- 6PR3.1 Describe the relationship in a given table, using a mathematical expression.
- 6PR3.2 Represent a pattern rule, using a simple mathematical expression such as 4d or 2n + 1.
- 6PR3.3 Write and explain the formula for finding the area of any given rectangle.
- 6PR3.4 Write and explain the formula for finding the perimeter of any given rectangle.
- 6PR3.5 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication; e.g., a + b = b + a or a \( \times b = b \times a \).

6PR4 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.  
[C, CN, PS, R, V]  
(pp. 108-111)

- 6PR4.1 Model the preservation of equality for addition, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.
- 6PR4.2 Model the preservation of equality for subtraction, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.
- 6PR4.3 Model the preservation of equality for multiplication, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.
- 6PR4.4 Model the preservation of equality for division, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.
- 6PR4.5 Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials, e.g., 3b = 12 is the same as 3b + 5 = 12 + 5 or 2r = 7 is the same as 3(2r) = 3(7).
### Strand: Shape and Space (Measurement)

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>General Outcome: Use direct or indirect measurement to solve problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6SS1</strong> Demonstrate an understanding of angles by:</td>
<td><strong>Achievement Indicators</strong> The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
<tr>
<td>• identifying examples of angles in the environment</td>
<td>6SS1.1 Provide examples of angles found in the environment.</td>
</tr>
<tr>
<td>• classifying angles according to their measure</td>
<td>6SS12 Classify a given set of angles according to their measure; e.g., acute, right, obtuse, straight, reflex.</td>
</tr>
<tr>
<td>• estimating the measure of angles, using 45°, 90° and 180° as reference angles</td>
<td>6SS1.3 Estimate the measure of an angle, using 45°, 90° and 180° as reference angles.</td>
</tr>
<tr>
<td>• determining angle measures in degrees</td>
<td>6SS1.4 Sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them.</td>
</tr>
<tr>
<td>• drawing and labelling angles when the measure is specified.</td>
<td>6SS1.5 Measure, using a protractor, given angles in various orientations.</td>
</tr>
<tr>
<td><strong>[C, CN, ME, V]</strong> (pp. 258-273, 304-306)</td>
<td>6SS1.6 Draw and label a specified angle in various orientations, using a protractor.</td>
</tr>
</tbody>
</table>

**6SS2** Demonstrate that the sum of interior angles is:  
• 180° in a triangle  
• 360° in a quadrilateral.  
**[C, R]** (pp. 274-279)  

**6SS3** Develop and apply a formula for determining the:  
• perimeter of polygons  
• area of rectangles  
• volume of right rectangular prisms.  
**[C, CN, PS, R, V]** (pp. 280-283, 286-289, 292-297)  

6SS1.1 Explain, using models, how the area of any rectangle can be determined.  
6SS1.2 Generalize a rule (formula) for determining the area of rectangles.  
6SS1.3 Explain, using models, how the perimeter of any polygon can be determined.  
6SS1.4 Generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares.  
6SS1.5 Explain, using models, how the volume of any right rectangular prism can be determined.  
6SS1.6 Generalize a rule (formula) for determining the volume of right rectangular prisms.  
6SS1.7 Solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.
## Appendix A

### Strand: Shape and Space (3-D Objects and 2-D Shapes)

### General Outcome:
Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

### Specific Outcomes

**It is expected that students will:**

**Achievement Indicators**

The following set of indicators help determine whether students have met the corresponding specific outcome:

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>Achievement Indicators</th>
</tr>
</thead>
</table>
| 6SS4 Construct and compare triangles, including: | 6SS4.1 Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.  
6SS4.2 Sort a given set of triangles, and explain the sorting rule.  
6SS4.3 Draw a specified triangle; e.g., scalene.  
6SS4.4 Replicate a given triangle in a different orientation, and show that the two are congruent. |
| • scalene  
• isosceles  
• equilateral  
• right  
• obtuse  
• acute in different orientations. | [C, PS, R, V] (pp. 302-303, 306-309, 314-315) |
| 6SS5 Describe and compare the sides and angles of regular and irregular polygons. | 6SS5.1 Sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.  
6SS5.2 Demonstrate that the sides of a given regular polygon are of the same length and that the angles of a regular polygon are of the same measure.  
6SS5.3 Sort a given set of polygons as regular or irregular, and justify the sorting.  
6SS5.4 Identify and describe regular and irregular polygons in the environment.  
6SS5.5 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.  
6SS5.6 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring. |
<p>| [C, PS, R, V] (pp. 310-313, 314-315) |</p>
<table>
<thead>
<tr>
<th>Strand: Shape and Space (Transformations)</th>
<th>General Outcome: Describe and analyze position and motion of objects and shapes.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Achievement Indicators</strong></td>
</tr>
<tr>
<td><em>It is expected that students will:</em></td>
<td><em>The following set of indicators help determine whether students have met the corresponding specific outcome:</em></td>
</tr>
<tr>
<td>6SS6 Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V] (pp. 162-169)</td>
<td>6SS6.1 Model a given set of successive translations, successive rotations or successive reflections of a 2-D shape. 6SS6.2 Describe the transformations performed on a 2-D shape to produce a given image. 6SS6.3 Demonstrate that a 2-D shape and its transformation image are congruent. 6SS6.4 Model a given combination of two different types of transformations of a 2-D shape. 6SS6.5 Draw and describe a 2-D shape and its image, given a combination of transformations. 6SS6.6 Model a given set of successive transformations (translations, rotations and/or reflections) of a 2-D shape. 6SS6.7 Perform and record one or more transformations of a 2-D shape that will result in a given image.</td>
</tr>
<tr>
<td>6SS7 Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V] (pp. 168-169)</td>
<td>6SS7.1 Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape(s) and the transformations used to create the design. 6SS7.2 Create a design using one or more 2-D shapes, and describe the transformations used.</td>
</tr>
</tbody>
</table>
### Appendix A

<table>
<thead>
<tr>
<th>Strand: Shape and Space (Transformations)</th>
<th>General Outcome: Describe and analyze position and motion of objects and shapes.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Achievement Indicators</strong></td>
</tr>
<tr>
<td><strong>It is expected that students will:</strong></td>
<td>The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
</tbody>
</table>

#### 6SS8 Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs.

[C, CN, V]

*(pp. 130-135)*

- 6SS8.1 Label the axes of the first quadrant of a Cartesian plane, and identify the origin.
- 6SS8.2 Plot a point in the first quadrant of a Cartesian plane, given its ordered pair.
- 6SS8.3 Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.
- 6SS8.4 Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5 or 10 on its axes, given whole number ordered pairs.
- 6SS8.5 Draw shapes or designs, given ordered pairs, in the first quadrant of a Cartesian plane.
- 6SS8.6 Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.
- 6SS8.7 Draw shapes or designs in the first quadrant of a Cartesian plane, and identify the points used to produce them.

#### 6SS9 Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).

[C, CN, PS, T, V]

*(pp. 150-161)*

- 6SS9.1 Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).
- 6SS9.2 Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image (limited to the first quadrant).
- 6SS9.3 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to the first quadrant).
<table>
<thead>
<tr>
<th><strong>Strand:</strong> Statistics and Probability (Data Analysis)</th>
<th><strong>General Outcome:</strong> Collect, display and analyze data to solve problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcomes</strong></td>
<td><strong>Achievement Indicators</strong></td>
</tr>
<tr>
<td><em>It is expected that students will:</em></td>
<td><em>The following set of indicators help determine whether students have met the corresponding specific outcome:</em></td>
</tr>
</tbody>
</table>
| 6SP1 Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V] (pp. 138-141) | 6SP1.1 Determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs.  
6SP1.2 Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data), and explain why.  
6SP1.3 Create a line graph from a given table of values or a given set of data.  
6SP1.4 Interpret a given line graph to draw conclusions. |
| 6SP2 Select, justify and use appropriate methods of collecting data, including:  
• questionnaires  
• experiments  
• databases  
• electronic media. [C, CN, PS, R, T] (pp. 118-120, 124-127) | 6SP2.1 Select a method for collecting data to answer a given question, and justify the choice.  
6SP2.2 Design and administer a questionnaire for collecting data to answer a given question, and record the results.  
6SP2.3 Explain when it is appropriate to use a database as a source of data.  
6SP2.4 Gather data for a given question by using electronic media, including selecting data from databases.  
6SP2.5 Answer a given question by performing an experiment, recording the results and drawing a conclusion. |
| 6SP3 Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T] (pp. 3120-123, 128-129) | 6SP3.1 Determine an appropriate type of graph for displaying a set of collected data, and justify the choice of graph.  
6SP3.2 Solve a given problem by graphing data and interpreting the resulting graph. |
**Strand:** Statistics and Probability (Chance and Uncertainty)  
**General Outcome:** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is expected that students will:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4 Demonstrate an understanding of probability by:</td>
</tr>
<tr>
<td>• identifying all possible outcomes of a probability experiment</td>
</tr>
<tr>
<td>• differentiating between experimental and theoretical probability</td>
</tr>
<tr>
<td>• determining the theoretical probability of outcomes in a probability experiment</td>
</tr>
<tr>
<td>• determining the experimental probability of outcomes in a probability experiment</td>
</tr>
<tr>
<td>• comparing experimental results with the theoretical probability for an experiment.</td>
</tr>
<tr>
<td>[C, ME, PS, T] (pp. 320-331)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
</tr>
<tr>
<td>The following set of indicators help determine whether students have met the corresponding specific outcome:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.1 List the possible outcomes of a probability experiment, such as:</td>
</tr>
<tr>
<td>• tossing a coin</td>
</tr>
<tr>
<td>• rolling a die with a given number of sides</td>
</tr>
<tr>
<td>• spinning a spinner with a given number of sectors.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.2 Determine the theoretical probability of an outcome occurring for a given probability experiment.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.3 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.4 Conduct a probability experiment, with or without technology, and compare the experimental results with the theoretical probability.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.5 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6SP4.6 Distinguish between theoretical probability and experimental probability, and explain the differences.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


Computation, Calculators, and Common Sense. May 2005, NCTM.


REFERENCES


Western and Northern Canadian Protocol (WNCP) for Collaboration in Education. The Common Curriculum Framework for K-9 Mathematics, 2006. Reproduced and/or adapted by permission. All rights reserved.