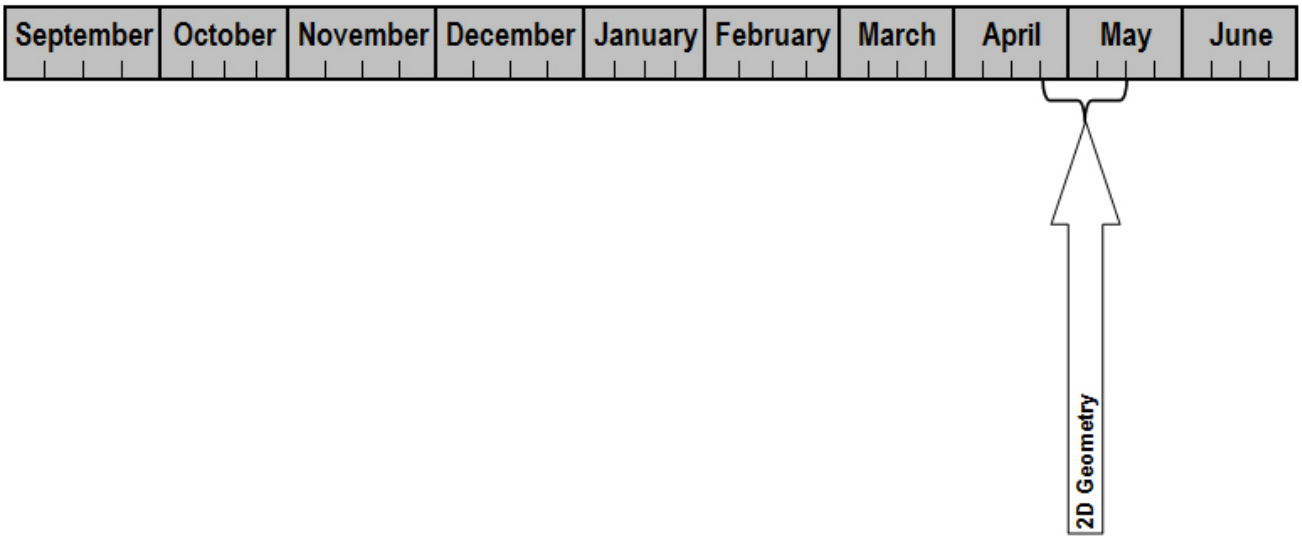
2-D Geometry

Suggested Time: 3 Weeks



Unit Overview

Focus and Context

Students will use their knowledge of 2-D geometry and measurement to sort triangles and other polygons. They will further develop their knowledge of congruency by using transformational geometry and both formal and informal measurement strategies to establish congruent angles and line segments of polygons.

The use of appropriate mathematical language to enhance communication should be emphasized. Teachers should be aware of the importance of modelling for students the mathematical language necessary to demonstrate their understanding. Students should recognize that using this language will help them effectively communicate their understanding.

Math Connects

Students develop their spatial sense by making connections to their everyday life and environment. Recognizing 2-D shapes in the world around them will provide students the opportunity to develop the concepts they will learn in the classroom and apply them to these shapes. Spatial sense will encourage students to visualize objects, to understand the effects of changes made to objects, and to help with problem solving situations.

Process Standards Key

[C] Communication
[CN] Connections
[ME] Mental Mathematics and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Shape and Space (3-D Objects and 2-D Shapes)	6SS4 Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations.	[C, PS, R, V]
Shape and Space (3-D Objects and 2-D Shapes)	6SS5 Describe and compare the sides and angles of regular and irregular polygons.	[C, PS, R, V]

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS4 Construct and compare triangles, including:

- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.

[C, PS, R, V]

Achievement Indicator:

6SS4.1 *Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.*

Elaborations—Strategies for Learning and Teaching

In Grade 5, students categorized quadrilaterals according to side length and pairs of parallel lines. Students will expand their knowledge of properties to categorize triangles based on side length and interior angles. Begin by exploring side lengths of triangles and naming the triangles as scalene, isosceles and equilateral.

To begin classifying triangles, focus on side length. There are three classifications when naming triangles according to their side lengths:

- Scalene – no equal sides
- Isosceles – two equal sides
- Equilateral – three equal sides

The use of concrete models will help students visualize side length and classify triangles. It will also help students realize that orientation does not affect a triangle's classification. Provide students with manipulatives (toothpicks, string, geoboards and elastic bands, etc.) and ask them to make several triangles. Ask students to measure each side of the triangles they have constructed, and discuss what types of triangles they have made. When using geoboards, students commonly have the misconception that the diagonal distance between points is equal to the sum of the vertical and horizontal distances. It is important to ask students to measure the diagonal distances to see that this is not true.

Display an equilateral triangle. Ask students to replicate that triangle with any choice of manipulative and identify the type of triangle that is presented. Review the concept of perimeter and ask students to calculate the perimeter of their triangle. As a class, use the manipulatives and brainstorm to come up with other triangles with the same perimeter. Ask students to identify the type of triangle.

Students should come to realize that not just any combination of three lengths can become the side lengths of a given triangle. The sum of the two shorter sides of a triangle must be greater than the length of the longest side; otherwise the three sides could never connect. If students struggle with remembering the names for each triangle, a strategy such as the following may help. Place the three triangle names in alphabetical order. Then apply the 3, 2, 1 rule: the first triangle (equilateral) has 3 equal sides, the second (isosceles) has 2 equal sides and the third (scalene) has 1, or no equal sides.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Performance

- Provide students with a geoboard, elastic bands and square dot paper. Ask them to create 3 different scalene triangles on the geoboard and then reproduce the triangles on the dot paper. Ask students to explain how they know they have created scalene triangles. Repeat the activity for isosceles and equilateral triangles. (6SS4.1)

- Provide students with a copy of a number of triangles. Be sure to include at least one of each of the three types. Ask students to sort the triangles. Rulers may be used. (6SS4.1, 6SS4.2)

- Ask students to create their own 'What am I?' cards (e.g., I am a triangle with one angle measuring 120° and two equal sides. What am I? Answer: obtuse isosceles triangle) and use them to play with a friend. (6SS4.1)

Journal

- Ask students to answer the following questions:
  (i) Can you create a triangle with more than one obtuse angle? Explain using words, pictures and/or numbers. (6SS4.1)
  (ii) Can an obtuse triangle be an equilateral triangle? Explain using words, pictures and/or numbers. (6SS4.1)
  (iii) Can a right triangle be an isosceles triangle? Explain using words, pictures, and/or numbers. (6SS4.1)

Resources/Notes

Math Focus 6

Lesson 1: Classifying Triangles by Side Length

6SS4

TG pp. 13 – 16

Lessons 1 and 2 may be combined

Lesson 2: Exploring Triangles

6SS4

TG pp. 17 - 20

Additional Reading (provided)**:**

Small, Marian. *Making Math Meaningful to Canadian Students* K-8. pp. 292 – 296

Van de Walle, John and Lovin, LouAnn (2006) *Teaching Student Centered Mathematics Grades 3–5* pp. 220 - 225

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS1 Demonstrate and understanding of angles by:

- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles, using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified .**

[C, CN, ME, V]

Achievement Indicator:

6SS1.3 *Estimate the measure of an angle, using 45°, 90° and 180° as reference angles.*

6SS4 Continued

Achievement Indicator:

6SS4.2 *Sort a given set of triangles and explain the sorting rule.*

Elaborations—Strategies for Learning and Teaching

Students will focus on identifying the interior angles of a triangle as right, obtuse or acute, using 90° as a benchmark. Ask students to identify objects in the classroom that could be used as a 90° reference point (e.g., the corner of a sheet of paper, a textbook, the corner of a ruler, etc.). Students can use the object to identify the angles of a triangle as acute (smaller than the 90° angle), obtuse (larger than the 90° angle) or right (equal to the 90° angle).

At this point, introduce another three classifications of triangles, based on their interior angles:

- A right triangle has one 90° angle.
- An acute triangle has all angles less than 90°.
- An obtuse triangle has one angle greater than 90°.

Students can investigate the remaining angles in a right triangle and an obtuse triangle to determine what types of angles they must be.

Exploration should lead to the discovery of the angle relationships in equilateral triangles and isosceles triangles. The definition of equilateral triangles, isosceles triangles and scalene triangles should then be expanded to include interior angles.

- Equilateral triangle – has all sides equal and all angles equal
- Isosceles triangle – has two equal sides and two equal angles
- Scalene triangle – has no equal sides or equal angles

Give students the opportunity to sort a set of triangles and complete a table similar to the one below:

Type / Triangle	Equilateral Triangle	Isosceles Triangle	Scalene Triangle	Right Triangle	Acute Triangle	Obtuse Triangle
A						
B						
C						
D						

During this activity, students should discover that the triangles will have two classifications. They can be classified according to their side lengths and angles. For example, a right triangle can also be an isosceles triangle but an equilateral triangle can never be right or obtuse. Students should be allowed time to explore which combinations of right, acute and obtuse triangles and scalene, isosceles and equilateral triangles are possible. Those that exist are right isosceles, right scalene, acute isosceles, acute equilateral, acute scalene, obtuse isosceles and obtuse scalene.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a Venn diagram template and ask them to sort given triangles according to chosen properties, such as isosceles triangles vs. right triangles. (6SS4.2)

Journal

- Ask students to discuss the statement, "An equilateral triangle is a special type of isosceles triangle." (6SS4.2)

Performance

- Ask students to create any triangle with construction paper. Put students in groups and ask them to sort their triangles into two groups using a sorting rule of their choice. (6SS4.2)

- Provide students with a sheet of paper. Using a ruler, ask them to tear off each of the four corners to form four different triangles. Ask them to use a ruler and protractor to name the triangles according to both their side lengths and angle measures. (6SS4.1, 6SS4.2)

Resources/Notes

Math Focus 6

Lesson 3: Classifying Triangles by Interior Angles

6SS1

6SS4

TG pp. 21 – 24

Children's Literature (not provided)**:**

Adler, David A. *Shape up! Fun with Triangles and Other Polygons.* ISBN 0-8234-1638-0

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **6SS1 Continued** **Achievement Indicators:** 6SS1.5 *Measure, using a protractor, given angles in various orientations.* 6SS1.6 *Draw and label a specified angle in various orientations, using a protractor.*	Review the use of a protractor. Students have been exposed to using a protractor to measure angles in the Measurement unit. While reviewing, ask questions such as: "What steps would you take to draw a 45° angle?" and "What steps would you take to draw a 120° angle?" Ask students to draw specified angles as practice. Discuss with them that in this unit they will be drawing angles in order to create triangles.
6SS4 Continued **Achievement Indicator:** 6SS4.3 *Draw a specified triangle, e.g., scalene.*	Students need step by step instruction when they are first learning to draw triangles. It may be beneficial to ask students to begin by drawing a line segment and adding a specified angle to one of its ends. Then progress to asking students to draw a triangle with one angle specified. As students become more comfortable with this, add more specifications to the instructions. For example, give them two line segments and one angle, or two angles and one line segment, to include in the triangle. When drawing triangles, students should be able to identify the unspecified angles and line segments. They should recognize that being given two angles and one side length results in a unique triangle. For example, if two students are asked to draw a triangle with a side length of 3 cm and angles measuring 40° and 70°, their resulting triangles will be congruent acute isosceles triangles. The orientation may be different, and students may need to be reminded that a change in orientation does not make it a different triangle. Unique triangles are also produced when two sides and the contained angle are specified, or when three sides are specified. This idea will be further developed later in the unit. It is important that students be able to draw triangles of any specified type (e.g., acute scalene).

(continued)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to draw 3 different obtuse isosceles triangles.

  (6SS1.5 , 6SS1.6, 6SS4.1, 6SS4.3)

- Ask students to construct a triangle with one angle measuring 65° and one angle measuring 40°. Ask questions such as:

  (i) What is the measure of the third angle?
  (ii) What kind of triangle did you make?
  (iii) How do you know?

  (6SS1.6, 6SS4.1, 6SS4.3)

Performance

- Provide students with a geoboard and geobands.

  Ask students to construct a triangle with two 45° angles. Ask students to make three additional triangles with the same attributes. (We are expecting the triangles to vary in size.) Ask students how their triangles are the same and how they are different. Ask them what kind of triangle they made. (6SS4.3)

Resources/Notes

Math Focus 6

Lesson 4: Drawing Triangles

6SS1

6SS4

TG pp. 25 – 28

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS4 Continued

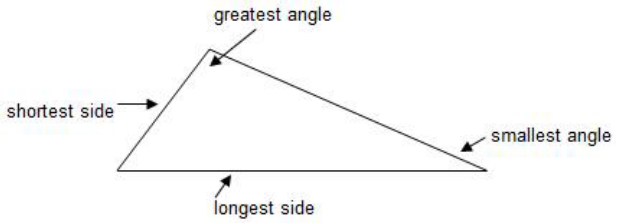
Achievement Indicator:

6SS4.3 *Continued*

Elaborations—Strategies for Learning and Teaching

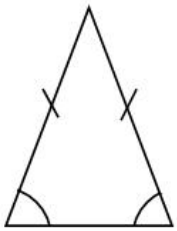
Having an understanding of the properties of triangles is beneficial when drawing triangles. These properties include:

- The greatest angle is opposite the longest side, and the smallest angle is opposite the shortest side.



- The sum of the two shorter sides must be greater than the longest side.

- When two angles of a triangle are congruent, then the sides opposite them are congruent (and vice versa).



Students should be encouraged to use appropriate hatch marks on triangles to indicate when sides and angles are equal.

- The sum of the interior angles of any triangle is 180°.
- A triangle can never have more than one obtuse angle or one right angle.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies	Resources/Notes
	Math Focus 6 **Lesson 4 (Cont'd):** Drawing Triangles **6SS1** **6SS4** TG pp. 25 – 28

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes

Students will be expected to

6SS5 Describe and compare the sides and angles of regular and irregular polygons.

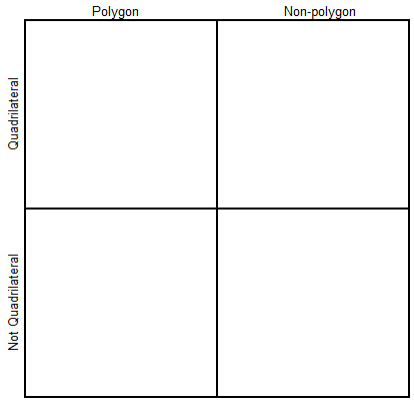
[C, PS, R, V]

Elaborations—Strategies for Learning and Teaching

Achievement Indicators:

6SS5.1 *Sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.*

Provide students with a template similar to the one below and a number of polygons and non-polygons that they can place into their template. Ask students to cut out polygons and glue them into the appropriate section of the template.



When defining polygons with your class, highlight examples of non-polygons as well.

6SS5.2 *Demonstrate that the sides of a given regular polygon are of the same length and that the angles of a regular polygon are of the same measure.*

It may be necessary to revisit what a polygon is - a closed 2-D figure bound by straight line segments that intersect at the vertices. Naming polygons may also require some review.

The definition of a regular polygon should be developed through a discovery approach. Display several regular polygons and ask students to measure the angles and side lengths, and discuss what they notice. This should highlight that, in regular polygons, all angles are equal and all side lengths are equal. Also include symmetry in the definition of a regular polygon. Students were exposed to symmetry in Grade 4.

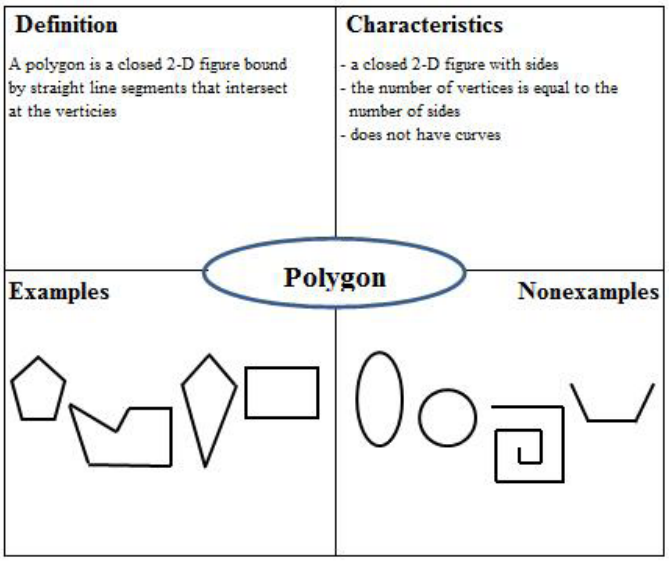
Discussion should lead students to conclude that if all angles of a polygon are equal then all sides are also equal, and vice versa.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a template for the Frayer Model. Ask them to complete the sections individually to demonstrate their understanding of the geometric concept "polygon". (6SS5.1)



Journal

- Ask students to draw a polygon and a non-polygon, and explain why one is a polygon and the other is not. (6SS5.1)

- Ask students to agree or disagree with the statement below and to explain their thinking:

  Because all the angles of a rectangle measure 90°, the angles are congruent. That means that rectangles are regular polygons. (6SS5.2)

Performance

- Ask students to sort these shapes into two groups; polygons and non-polygons.

(6SS5.1)

Resources/Notes

Math Focus 6

Lesson 5: Sorting Polygons

6SS1

6SS5

TG pp. 34 - 37

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **6SS5 Continued** **Achievement Indicators:**	
6SS5.3 *Sort a given set of polygons as regular or irregular, and justify the sorting.*	Students were exposed to sorting regular and irregular polygons in Grade 3. During the sorting of polygons, there are instances when students can rely on visual cues when it clear that a polygon is irregular. However, when determining if a figure is regular, students should be encouraged to check angles and/or side lengths.
6SS5.4 *Identify and describe regular and irregular polygons in the environment.*	Initiate a discussion about examples of regular and irregular polygons in the environment. Provide students with a placemat template (regular paper or chart paper to display in the classroom.) Ask students to brainstorm and record (by drawing or writing) as many examples of polygons that they see or use in the world around them. Each group can share their "placemat" findings with the rest of the class.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Performance

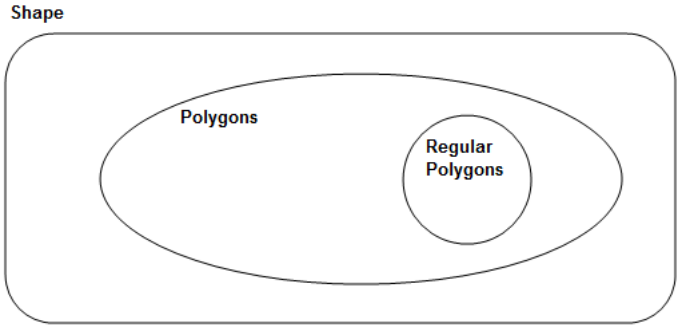
- Provide students with several regular polygons and ask them to measure the angles with a protractor and side lengths with a ruler. Ask them to label them with appropriate hatch marks. (6SS1.5, 6SS5.2)
- Provide students with triangular dot paper. Ask them if they can draw a regular pentagon. Once they conclude it is not possible discuss with them why this is the case. (6SS5.2)

Student-Teacher Dialogue

- Ask students to tell you about the characteristics of a regular polygon. Then ask them which characteristic they prefer to use to check whether a polygon is regular or irregular. Ask why they prefer using this characteristic. (6SS5. 3, 6SS5.4)

Paper and Pencil

- Provide students with a set of shapes including several regular polygons, irregular polygons, one non-polygon, and a Venn diagram similar to the one below. Ask them to place the shapes where they belong.



(6SS5.1, 6SS5.3)

Resources/Notes

Math Focus 6

Lesson 5 (Cont'd): Sorting Polygons

6SS1

6SS5

TG pp. 34 - 37

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **6SS4 Continued** **Achievement Indicator:** 6SS4.4 *Replicate a given triangle in a different orientation, and show that the two are congruent.*	Students will build upon their existing knowledge of transformational geometry and congruency. During the Measurement unit, students were exposed to congruent line segments and congruent sides of regular polygons. Using geoboards and geobands, students could work in pairs to construct the same triangle on each of their geoboards. Students should notice that after rotating one of the geoboards a $\frac{1}{4}$ turn (90°), the rotated triangle has not changed but its orientation is different. If students have not already suggested that the triangles are congruent, review the concept of congruency with them, referring to their triangles. Congruent figures have exactly the same size and the same shape. They can have different orientations and still be congruent. For teacher reference: For two triangles to be congruent, they must meet one of the following conditions: • Side-Side-Side (SSS) • Side-Angle-Side (SAS) • Angle-Side-Angle (ASA) • Angle-Angle-Side (AAS)
6SS5 Continued **Achievement Indicators:** 6SS5.5 *Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.* 6SS5.3 *Continued*	There are a number of ways students can demonstrate the congruency of regular polygons. This concept can include congruency of angles and side lengths within a single polygon or between sets of polygons. One way is to superimpose an image. This can be done using tracing paper, cut outs or a Mira in conjunction with transformations. Provide students with two congruent regular polygons of differing orientations, and tracing paper. Students can trace one of the polygons and then place the tracing on top of the other polygon. The two shapes will match. The polygons *coincide* and one shape is *superimposed* on the other. Another way to demonstrate congruency is to measure the side lengths and angles using a ruler and a protractor.

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Suggested Assessment Strategies

Performance

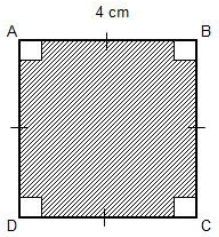
- Provide students with dot paper. Ask them to draw a triangle. Students may use any transformation to change the triangle's orientation. (6SS4.4)

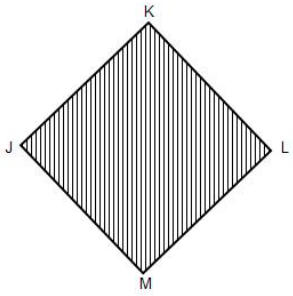
- Provide students with a set of regular polygons. Include several sets of congruent and similar polygons. Ask students to identify pairs of congruent polygons and to explain how they found them. (6SS5.5, 6SS5.6)

Paper and Pencil

- Provide students with triangular dot paper. Ask them to draw a regular hexagon. Ask them to show that all sides and all angles are congruent by measuring or superimposing. (6SS5.5, 6SS5.6)

- Provide students with a copy of two congruent regular polygons of different orientations. Label the vertices of both polygons. On one, indicate the side lengths and angle measures, but not on the other. Ask students to write the measure of each angle in the second polygon without using a protractor, and the measure of each side length without using a ruler.





Journal

- Ask students to answer a question such as: What does it mean when two regular polygons are congruent? Use words and pictures to explain your understanding. (6SS5.5, 6SS5.6)

Resources/Notes

Math Focus 6

Lesson 6: Congruent Polygons

6SS4

6SS5

TG pp. 38 - 41

Lesson 7: Communicating about Polygons

6SS4

6SS5

TG pp. 42 – 44

Math Game: Matching Cards

6SS1

6SS4

6SS5

TG pp. 45 - 46

Math Contest: Polygon Contest

SS1

SS4

SS5

TG pp. 51- 52