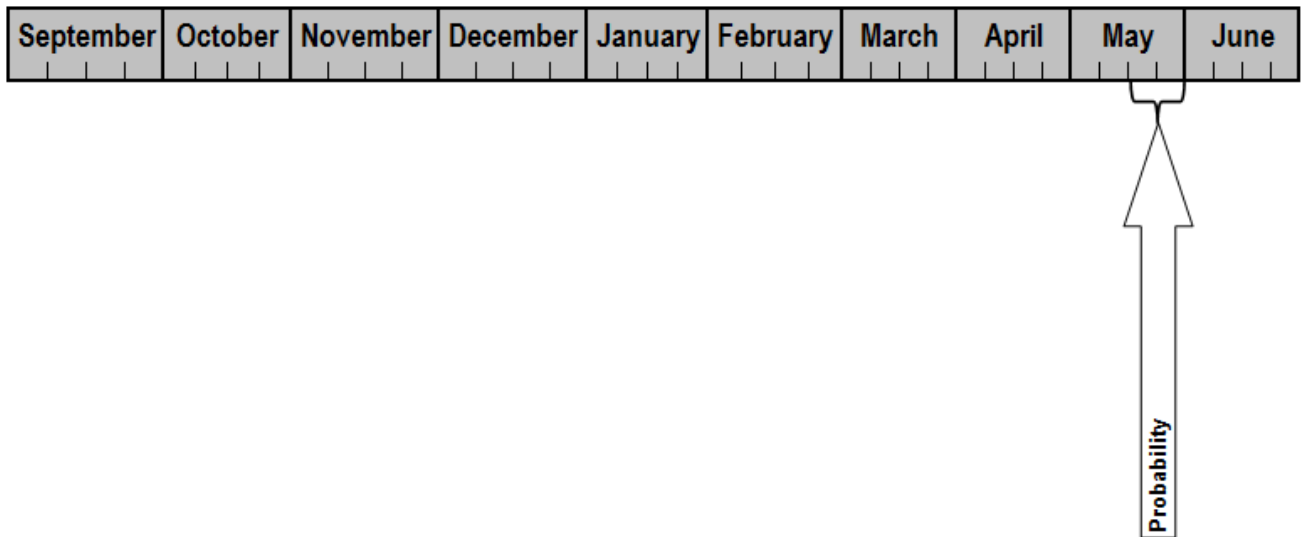


Probability

Suggested Time: 2 Weeks



Unit Overview

Focus and Context

Probability involves the use of mathematics to describe the level of certainty that an event will occur.

Students have been exposed to experimental probability in grade 5. In grade 6, students will be exposed to theoretical as well as experimental probability.

The theoretical probability of an outcome is determined by analyzing what could happen, whereas the experimental probability reports what did happen. The theoretical and experimental probability of an outcome can differ, although they tend to be closer with more trials.

The theoretical probability of an outcome can be expressed as a ratio, a fraction, a percent and a decimal, which relates the number of favourable outcomes to the number of possible outcomes.

This unit focuses primarily on identifying possible outcomes of a probability experiment, determining the theoretical probability of an outcome or event in a probability experiment, comparing experimental and theoretical probabilities, as well as communicating about probability. Students will also learn that in any probability situation, you can never be sure what will happen next.

Math Connects

Students should be aware that there are many references to probability all around us. Brainstorming the ways that probability is used in everyday life will focus the student's attention to this topic as well as help them realize that references to probability are all around us. The weather forecaster predicts a 60% chance of showers, medical researchers predict people with certain diets have a high chance of heart disease or you have a one in five million chance in winning the lottery this week. Examples of probability are all around us, and probability has become more visible in our Mathematics curriculum. Students need to collect and analyze data from experiments that they will conduct. One of the advantages of teaching probability is that students naturally have some understanding of the topic from their real life experiences - from hearing weather reports, to rolling a die to try to win a game. This unit will build on students' past experience and engage them in a variety of games, activities and investigations in which they're encouraged to predict outcomes and then test their predictions through their first hand experiences.

Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Statistics and Probability (Chance and Uncertainty)	<p>6SP4 Demonstrate an understanding of probability by:</p> <ul style="list-style-type: none"> • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment. 	[C, ME, PS, T]

Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Demonstrate an understanding of probability by:

- identifying all possible outcomes of a probability experiment
- differentiating between experimental and theoretical probability
- determining the theoretical probability of outcomes in a probability experiment
- determining the experimental probability of outcomes in a probability experiment
- comparing experimental results with the theoretical probability for an experiment.

[C, ME, PS, T]

Elaborations—Strategies for Learning and Teaching

Students have conducted probability experiments in Grade 5 and it will be helpful to review, with students, the meaning of probability - the chance of an event happening out of all possible outcomes. Revisit the concepts of ‘more likely’, ‘less likely’, ‘equally likely’, ‘possible’, ‘impossible’ or ‘certain’. Give students opportunities to experiment with probability and provide them with meaningful explorations. The materials students use to conduct experiments should be familiar. This becomes important when students are determining all the possible outcomes of an experiment.

Refer to R. Dahl’s novel, *Charlie and the Chocolate Factory*. Make a connection between the chances of winning the golden ticket and what was needed to better the chances of winning! Remind them that probabilities can be expressed as a fraction, a decimal, a ratio or a percent. Students may remember hearing percent used frequently on a weather report. E.g., there is a 60% chance of rain tomorrow.

Carry out basic experiments using spinners, dot cubes or coins to help students gain a better understanding of probability. E.g., What is the likelihood of flipping a coin and having it land on heads? What is the likelihood of rolling a dot cube and having it land on a 6?

Students should be able to make predictions of the outcome of an experiment based on what they know about the materials (coins, dot cubes, spinners, etc) they are using. Students can be encouraged to create their own tools for determining probability. Creating spinners is motivating for many students.

Engage students in a discussion about how probability is used in advertising. Ask students to look at newspapers and magazines for examples of how numbers are used in advertisements. E.g., it is not unusual to see something like “four out of five dentists recommend Brand T gum for their patients who chew gum.”

Ask students questions such as the following:

Why do advertisers use numbers like these? What information are they trying to convey? Do you think that the numbers give accurate information about a product? Why or why not?

Discuss, with your students, how probability is used to influence choices.

(continued)

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Presentation

- Discuss the following terms with students:

impossible

possible

less likely

equally likely

more likely

certain

Ask them to choose from the above terms to describe the probability of each of the following events:

- School will start in September
- Trees shed their leaves in July
- If you run a lap around the track, you can run a marathon
- If you toss a coin, it will land on heads (6SP4)

Resources/Notes

Math Focus 6

Lesson 1: Predicting Probabilities

6SP4

TG pp. 13 - 16

Math Game: Take A Chance

6SP4

TG pp. 17 - 18

Children's Literature

(not provided):

Dalh, D. *Charlie and the Chocolate Factory*

Additional Reading (provided):

Small, M. (2008). *Making Math Meaningful to Canadian Students, K - 8*. pp.543 - 564

Additional Reading (provided):

Van de Walle, John A. and Lovin, LouAnn H. (2006). *Teaching Student-Centered Mathematics Grade 3-5*. pp. 339 - 355

Strand: Statistics and Probability (Chance and Uncertainty)

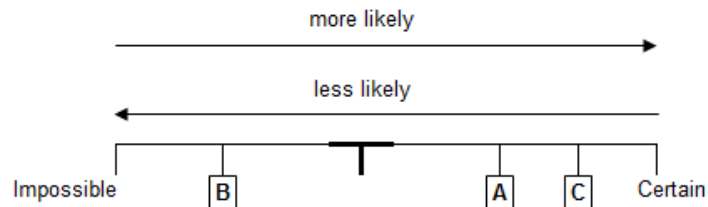
Outcomes

Students will be expected to

6SP4 Continued

Elaborations—Strategies for Learning and Teaching

Emphasize that probabilities range from 0 (impossible) to 1 (certain). Probabilities less than 0 and greater than 1 do not make sense because events either happen or they do not. Guide students to create their own ‘Probability Line’ (shown below) to describe and compare probabilities. Students should place events on the line and justify their choices for placing the events where they did. Group work or paired work is suggested as they can once again, get used to using the proper terminology and they can analyze what they are doing.



Event A could be any event that is fairly likely but not certain, such as rain during the third week of April

Event B could be an event that is less likely but not impossible, such as an animal coming into the classroom.

Event C could be an event that is more likely but not certain, such as school happening tomorrow.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Resources/Notes

Math Focus 6

Lesson 1: Predicting Probabilities

6SP4

TG pp. 13 - 16

Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued**Achievement Indicators:**

6SP4.1 List the possible outcomes of a probability experiment, such as:

- *tossing a coin*
- *rolling a die with a given number of sides*
- *spinning a spinner with a given number of sectors.*

6SP4.2 Determine the theoretical probability of an outcome occurring for a given probability experiment.

Elaborations—Strategies for Learning and Teaching

Theoretical probability is based on what SHOULD happen. Explain to students that, in order to find theoretical probability, they must first determine the total number of possible outcomes. E.g., Show a spinner with numbers 1 to 5. The possible outcomes are 1, 2, 3, 4 or 5. Students should observe that each section of the spinner is of equal size and each number shows up only once. Therefore, each digit will have an equal chance of being spun. To find the theoretical probability of spinning a 3, students should see that 3 shows up once on the spinner where there are 5 numbers in total. Therefore, the probability of spinning a 3 would be 1 out of 5, or 20%. So, if the spinner is spun 100 times, theoretically it will land on “3” 20 times.

$$\text{Probability of Event A} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Students should be given opportunities to identify the possible outcomes of an experiment, and they need to be able to relate the experimental probability to the theoretical probability.

Provide examples of other experiments that could be carried out. Ask students to identify the experimental and theoretical probabilities for each experiment. For example, ask students to conduct the following experiment:

Take 10 two coloured counters and drop them on a table. Record how many land with the red side up and how many land with the white side up. Do at least 10 trials and record the results. What can you conclude? What is the theoretical probability of the counter landing with the white side up? What is the experimental probability?

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Resources/Notes

Performance

- Mental Math - If you are rolling a regular die, what is the probability of rolling an even number? (6SP4.2)

- Ask the students to watch as you put linking cubes into a bag. First place 20 blue, 10 yellow and 5 green. Ask students questions about which colour they think you will pull from the bag. Why is a certain colour more likely or less likely? What are the chances of pulling an orange cube out? Empty the bag and fill it again with 1 green, 1 blue, 1 brown and 1 red. Discuss the possible outcome with this combination of colours in the bag. Describe the probability using fractions. (6SP4.1, 6SP4.2)

- Tell students that there are 10 yogurt bars in a bag. 3 bars are caramel flavoured, which Dave likes, 2 are strawberry flavoured, which he dislikes, and 5 are orange flavoured, which he also likes. Dave puts his hand in the bag without looking, and pulls out a bar.
 Ask: • What are his chances of pulling out a yogurt bar that he dislikes?
 • Write another probability question about the yogurt bars that were in the bag. (6SP4.2)

- Tell students that you are visiting a kennel that has 3 German Shepherds, 5 Labrador Retrievers, 3 Chihuahuas, 4 Poodles, and 5 West Highland Terriers. When you arrive, the dogs are taking a walk. What is the probability of seeing a German Shepherd first? (6SP4.2)

Math Focus 6

Lesson 2: Theoretical Probability
6SP4

TG pp. 19 - 23

Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued**Achievement Indicators:**

6SP4.3 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.

6SP4.4 Distinguish between theoretical probability and experimental probability, and explain the differences.

Elaborations—Strategies for Learning and Teaching

To have a strong understanding of probability, students must be able to differentiate between experimental probability and theoretical probability. Coin flipping experiments can be used to facilitate this. For example, students could be asked to predict the probability of a coin landing on heads. The theoretical probability of this is $\frac{1}{2}$. Ask students to flip a coin 100 times and record the results. When they carry out the experiment, they may discover that the result is not necessarily heads being tossed 50 times out of 100, and therefore, the experimental probability may not be $\frac{1}{2}$. This is a great way to demonstrate the difference between experimental and theoretical probability.

To help students understand the difference between theoretical and experimental probability is to explain to students that theoretical probability deals with what ‘should’ happen in an ideal world within a given event. E.g., “in theory”, given there are two equal colors on a spinner, each color should have a 50% chance of being spun. When experiments are completed, the resulting experimental probability may state that red, for example, was spun 65% of the time whereas blue was spun only 35% of the time. Experimental probability, then, is telling what exactly happened within a given event. Reinforce that theoretical probability occurs BEFORE the experiment happens and experimental probability occurs once the experiment is completed.

Experimental probability differs from theoretical probability in that it depends on the actual results of experiments. Sometimes the experimental probability is not the same as the theoretical probability even though, theoretically, the likelihood of the events happening is equal. (getting heads or tails when flipping a coin).

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Journal

- Bring a Farmer's Almanac to show the students. Ask students how they think authors of The Farmer's Almanac make their predictions about weather for a year? How do they use probability?

(6SP4.3)

Paper and Pencil

- Ask students to create spinners with paper, pencils and paper clips. Sections of the spinners should be equal in size and coloured different colours. Have student pairs calculate the theoretical probability (expressed as a fraction, a decimal a ratio or a percent) of each outcome of their own spinners and those of their partners. Then, ask students to calculate the experimental probability of spinning one colour. Demonstrate spinning the spinner 10 times. Then, ask students to record the results of 25 spins of their own spinners in a frequency table.

(6SP4.3)

Performance

- Prepare three paper bags, one with 2 blue and 8 yellow cubes, one with 5 blue and 5 yellow cubes and one with 8 blue and 2 yellow cubes. Tell students the content of the three bags, but not which bag is which. Pull a cube from one of the bags, allow students to see the colour and then return the cube to the bag. Repeat the draw 10 times. Ask students how many yellow cubes are in the bag – 2, 5 or 8. Ask them to justify their answer.

(6SP4.3, 6SP4.5)

Resources/Notes

Math Focus 6

Lesson 2 (Cont'd): Theoretical Probability

6SP4

TG pp. 19 - 23

Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued**Achievement Indicator:**

6SP4.5 Conduct a probability experiment, with or without technology, and compare the experimental results with the theoretical probability.

Elaborations—Strategies for Learning and Teaching

It is important that students conduct experiments to help them make the connection between experimental and theoretical probability. If they are aware of the theoretical probability of an experiment, they will be better able to make predictions as to what will happen once they carry out an experiment.

Experimental probability is determined by the results of an experiment that has already occurred. In contrast, theoretical probability involves analyzing possible outcomes in advance and using logic and reason to predict what is likely to happen.

Give students examples of experiments that could be carried out. Ask them to identify the experimental and the theoretical probabilities for each experiment.

After reading the book, *Martha Blah Blah* by Susan Meddaugh ask students to work in pairs to investigate which alphabet letters would be the best ones for Granny Flo to eliminate from the alphabet and tell why. Ask students to work together to find the frequency with which each letter in the English alphabet is used. To do this, ask pairs of students to choose any book, select a short paragraph in that book, and analyze the frequencies of the letters used in that paragraph. Based on their findings, students present to the class if they think it would be less disruptive to remove certain letters, consonants or vowels from the alphabet. Ask them how they came to this conclusion.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Performance

- Rock-Paper-Scissors - Students play this game in groups of 3 with a number (1, 2 or 3) assigned to each person. Play 20 times and record the results of each game.

Player 1 gets a point if all players show the same sign
 Player 2 gets a point if two players show the same signs
 Player 3 gets a point if all players show the different signs

Ask students to compare the experimental results with the theoretical probability. (6SP4.4, 6SP4.5)

- Coin Flip – Ask students to experiment with flipping a coin a specified number of times and record the results. If they flip a penny two times, they may get 2 heads, or two tails or one of each. If it is flipped 10 times, they may find that the results will be different. Likewise, if they flip the coin 100 times, the results will look different than if it had been rolled only ten times. Draw students’ attention to this as they flip their coins. Students should share their results once they have completed their trials. (6SP4.5)

- Ask students what the probability is of rolling ‘doubles’ using two number cubes? Make a chart and then roll the number cube 20 times. Record the results of your rolls.

Ask:

- What is the theoretical probability of rolling doubles?
- What is the theoretical probability of not rolling doubles?
- What patterns do you see? Record your observations in your journal or exercise book.

Make a chart to list all possible outcomes of rolling two number cubes:

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2				
3						
4						
5						
6						

(6SP4.5)

Resources/Notes

Math Focus 6

Lesson 3: Comparing Experimental and Theoretical Probability

6SP4

TG pp. 27 – 31

Related Literature (provided): by Meddaugh, Susan. *Martha Blah Blah*

Strand: Statistics and Probability (Chance and Uncertainty)

Outcomes

Students will be expected to

6SP4 Continued**Achievement Indicator:**

6SP4.6 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.

Elaborations—Strategies for Learning and Teaching

Students should know that the more often they carry out the trials for an experiment, the better they will be able to predict a particular outcome. When making important decisions based on an experiment, it is important to carry out an experiment enough times so that you can make a firm prediction of the outcome. As the number of trials increase, the better their predictions will be.

It is important to emphasize the importance of a reasonably large sample size. The results of a small number of trials or just one experiment, can be misleading, but when multiple trials are used and/or experiments repeated many more times, the experimental probability will gradually approach the theoretical probability. A good way to do this is to carry out an experiment where students record results individually, and then combine results from the entire class. The larger number of trials should result in a probability closer to the theoretical probability. E.g., Ask students to determine the theoretical probability of rolling a 3 on a six-sided die. Then ask them to roll a six-sided die ten times and record the result each time. What is the experimental probability of rolling a 3? Combine all the trials for the class, and again ask what the experimental probability of rolling a 3 is. Which is closer to the theoretical probability?

Students have learned about experimental probabilities and theoretical probabilities. They have solved problems throughout. Encourage students to use appropriate mathematical language. Upon completion, they should be able to talk about the experiments that they have completed and clearly explain the results. The students will gain experience in using the words likely, most likely, less likely, impossible and certain. Give students the opportunity to work in pairs or in small groups to use the language and to carry out these simple experiments.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty.

Suggested Assessment Strategies

Student-Teacher Dialogue

- Use everyday life events that the students can talk about probability for. E.g.,
 - The sun will rise tomorrow
 - A parent will be home when you get home from school
 - You will have pizza for dinner
 - A moose will come to the schoolyard on Friday. (6SP4.1)

Using previously conducted experiments, ask students to select one or two, and present what the results were. Encourage the use of appropriate terminology in explaining their analysis. (6SP4.3)

Journal

- How would you explain experimental and theoretical probability to someone? Record your answer, drawing pictures that would help you accurately describe the types of probability. (6SP4.2)

Resources/Notes

Math Focus 6

Lesson 3 (Cont'd): Comparing Experimental and Theoretical Probability

6SP4

TG pp. 27 – 31

Lesson 4: Communicating about Probability

6SP4

TG pp. 32 - 35

Curious Math: Fair or Unfair?

6SP4

TG pp. 36 – 37

Math Game: Predict and Score

6SP4

TG pp. 39 - 40

