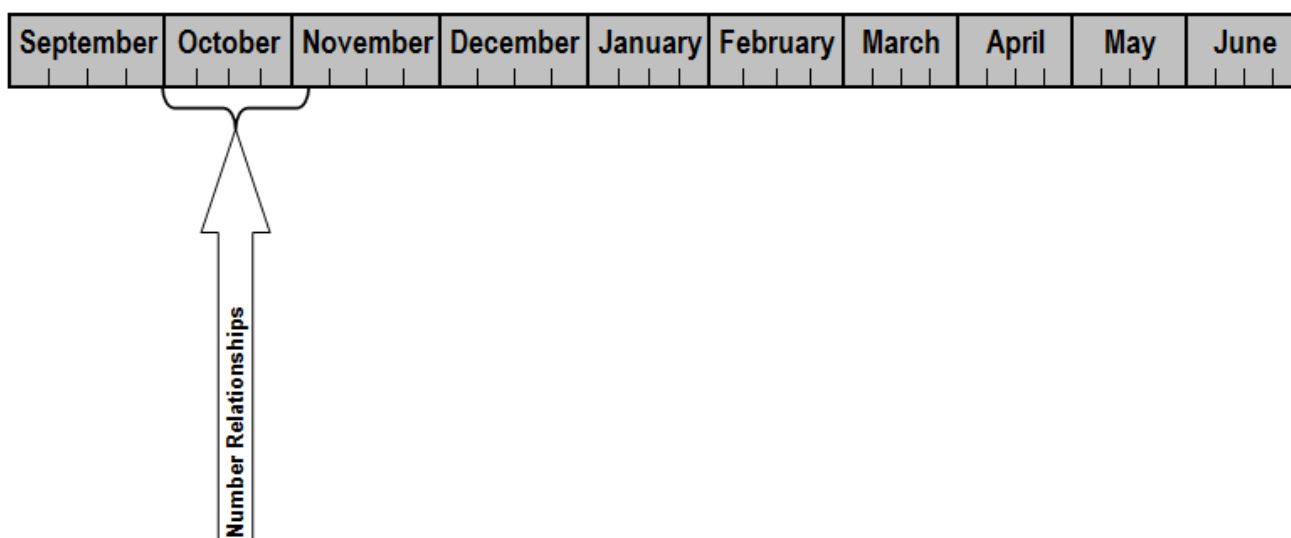


# Number Relationships

Suggested Time:  $4\frac{1}{2}$  Weeks



# Unit Overview

## Focus and Context

This unit focuses on finding factors and multiples of whole numbers to 100, classifying numbers as being prime or composite, exploring integers and using the order of operations. Previously, students have been working with multiplication, which will help them quickly find factors and multiples of numbers once this concept is explored. Students who have not mastered their basic multiplication facts may be able to use this work on factors and multiplies to help strengthen and solidify the skills and understanding of multiplication. Working with positive and negative integers will further develop students place value knowledge by extending the number line to the left of zero. Students need to recognize that negative numbers are the mirror reflection of their positive counterpart on the number line. Grade 6 students are formally introduced to the rules of the Order of Operations. This is merely an introduction as further work will be completed in Grade 7. It should be noted that students need to apply these rules in a problem solving situation to discover why they exist.

## Math Connects

Connecting student learning with life outside the classroom, providing meaningful contexts where math can be used in students' lives and encouraging students to explore and begin to wonder about math should be the focus of any math program. Introducing factoring and prime numbers where students recognize that all numbers can be decomposed into prime numbers gives them a foundation of knowledge and skills which will be further developed in later grades. The introduction of integers allows students to extend their knowledge about the number system and begin to see that every situation that happens in life can be represented in some way with numbers. Keeping students curious about number and having them constantly question our number system will help keep students engaged and interested in mathematics. Practicing with the rules of the order of operations will only strengthen student understanding of how numbers are affected by operations and in turn, increase their number sense.

## Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

## Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Number	<p>6N3 Demonstrate an understanding of factors and multiples by:</p> <ul style="list-style-type: none"> <li>• determining multiples and factors of numbers less than 100</li> <li>• identifying prime and composite numbers</li> <li>• solving problems using multiples and factors.</li> </ul>	[CN, PS, R, V]
Number	<p>6N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically.</p>	[C, CN, R, V]
Number	<p>6N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).</p>	[C, CN, ME, PS, T]

Strand: Number

Outcomes

*Students will be expected to*

**6N3 Demonstrate an understanding of factors and multiples by:**

- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems using multiples and factors.**

[CN, PS, R, V]

**Achievement Indicator:**

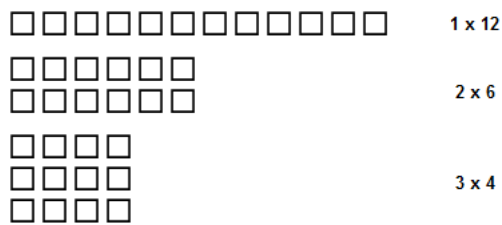
*6N3.1 Determine all the whole number factors of a given number, using arrays.*

Elaborations—Strategies for Learning and Teaching

Begin this unit by introducing students to factors. Factors are numbers that when multiplied, produce a product. Factors always come in pairs. For example, the factors of 12 are 1 and 12, 2 and 6, 3 and 4. A review of basic multiplication facts may be necessary for students experiencing difficulty, multiplication tables can be used to help identify factors of numbers. Students may also begin working with factors of numbers they are comfortable with.

The term ‘factor’ can be added to the class math word wall with students defining it in their own words, giving examples to illustrate their understanding.

When students are introduced to factors, it would be a good idea to use various manipulatives such as square tiles or snap cubes to form arrays or rectangles to identify the factors of a given number. To find the factors of 12, give each student 12 tiles or snap cubes and ask them to form a rectangle(s) or an array using only these 12 tiles. Ask students to find other ways to arrange the 12 tiles to make complete rectangles. Students should conclude that they can make a rectangle of  $1 \times 12$ ,  $2 \times 6$  and  $3 \times 4$ . These numbers then, would represent the factors of 12.



The factors of 12 are: 1, 2, 3, 4, 6, 12

Students sometimes forget to list 1 and the number itself, as factors of a given number. Remind students they are to find all whole numbers factors.

*(continued)*

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Finding Factors – Present numbers that have several factors – E.g., 12, 18, 24, 30 or 36. With counters and snap cubes ask students to attempt to find a way to separate them into equal sets. With arrays, students build rectangles that have the given number of squares. For each arrangement, a multiplication equation should be written. These equations would represent the factors of that number.

Van de Walle (2006). *Teaching Student Centered Mathematics. Grade 3-5* p. 63

(6N3.1)

- Ask students to find a number that has exactly 4 factors and another one that has 5 factors. (6N3.1)
- Ask students to draw one or more rectangles to show that 8 is a factor of 16 and 24. (6N3.1)
- Factor Patterns - Tell students that they are going to look for multiplication expressions and the corresponding rectangular array for several numbers. Their task includes trying to find all the multiplication expressions and rectangular arrays for each number. Have square tiles available that students can use to explore possible arrays. Once they have created an array, they should draw it on grid paper. Ask students to write a multiplication sentence for each array. Students should group together all arrays with the same number of squares. After identifying the multiplication expression and the rectangular arrays, students are to look for patterns in the factors and rectangular arrays. E.g., which numbers have the least number of arrays and therefore, the least number of factors? Which numbers have arrays that form a square? Which numbers have a factor of 2? What can you say about the factors for even numbers? Do even numbers always have as a factors? What do you notice about the factors of odd numbers? Encourage students to think about why different patterns occur. (6N3.1)

## Resources/Notes

*Math Focus 6***Lesson 1: Identifying Factors****6N3**

TG pp. 13 – 17

**Additional Reading (provided):**

Small, Marion (2008) *Making Math Meaningful to Canadian Students K-8. pp.149-151*

Strand: Number

Outcomes

*Students will be expected to*

**6N3 Continued**

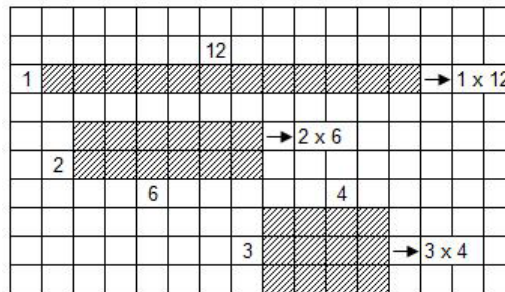
**Achievement Indicator:**

*6N3.2 Identify the factors and multiples for a given number, and explain the strategy used; e.g., concrete or visual representations, repeated division by prime numbers, factor trees.*

Elaborations—Strategies for Learning and Teaching

Manipulative use is helpful for students as they learn to identify the factors of numbers. Allowing students to use manipulatives to break the numbers apart into arrays will give them a visual of the factors for a given number. This hands-on method then becomes a strategy. Using manipulatives will allow students to become comfortable with identifying factors and be ready to create visual representations of the number.

One or two centimetre grid paper can be used to create arrays or rectangular shapes to represent a given number and its factors. When finding the factors of a number, students may color squares on the grid paper to form different rectangles. The width and the length of these rectangles will represent the factors of that number.



Factors of 12 are 1, 2, 3, 4, 6, 12

Factorize - In this game students are able to create rectangles of a given area on a grid that represent the number they are finding factors for. The dimensions of the rectangle can then be entered as factors for that number.

Students can also create an organized list of the factors of a given number where students identify the number and then list the factors in a row. Suggest that students, when listing the factors of a number, start on the left and list them in ascending order to ensure they do not omit any.

<p><b>Factors of 12</b></p> <p>(6 factors)</p> <table style="margin: auto;"> <tr><td>1</td><td>12</td></tr> <tr><td>2</td><td>6</td></tr> <tr><td>3</td><td>4</td></tr> </table> <p>Factors of 12 are 1,2,3,4,6,12</p>	1	12	2	6	3	4	<p><b>Factors of 16</b></p> <p>(5 factors)</p> <table style="margin: auto;"> <tr><td>1</td><td>16</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>4</td><td>4</td></tr> </table> <p>Factors of 16 are 1,2,4,8,16</p>	1	16	2	8	4	4
1	12												
2	6												
3	4												
1	16												
2	8												
4	4												

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General Outcome: Develop Number Sense

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Suggested Assessment Strategies

*Journal*

- Give students the following journal prompt:
  - Find a number which has 4, 7, 28 and 12 as factors. Ask them if there is a smaller number which will meet the conditions and to explain why or why not. (6N3.1, 6N3.2)

Resources/Notes

*Math Focus 6*

**Lesson 1 (Cont'd): Identifying Factors**

**6N3**

TG pp. 13 – 17

## Strand: Number

## Outcomes

*Students will be expected to*

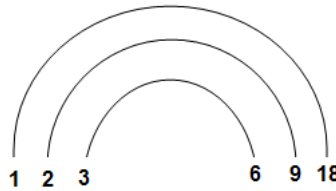
**6N3 Continued****Achievement Indicators:**

*6N3.2 Continued*

*6N3.3 Solve a given problem involving factors or multiples.*

## Elaborations—Strategies for Learning and Teaching

Factor Rainbows - Another visual representation that students can use to help factor numbers. It is recommended that students have prior experience factoring numbers before moving on to using factor rainbows. In a factor rainbow, students begin listing the factors of the number, horizontally, starting at 1. When the factor pairs are connected, the coloured, curved lines create a rainbow shape.

**A factor rainbow for 18**

**The factors of 18 are: 1, 2, 3, 6, 9, 18**

Posing real life problems that involve factors will allow students opportunities to see how finding factors of numbers are used outside of the math classroom. These real world connections help put a value on student work and understanding. Helping students understand the concept of factors can be done through problem solving where a problem can be posed and once it is solved the term “factor” can be introduced.

Consider the following example: Farmer Joe is trying to figure out how to plant a new potato garden. He has a plot of land that can cover a maximum of 100 m<sup>2</sup> and a minimum of 10 m<sup>2</sup>. He needs to decide how big to have his garden, but wants some choice on the dimensions of the area. He needs to know which area would give him the most choice of dimensions, but still have enough room to plant his stock of potatoes. What could you suggest?

Here, students are asked to find areas for Farmer Joe’s potato garden that would yield different dimensions, or factors of that area. They should see that numbers like 13, 37 and 59 only have one way to plant the area. Other areas, such as 36, 48 and 54 have many. They should also see that the larger the area, the more potatoes that can be planted.

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General Outcome: Develop Number Sense

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Suggested Assessment Strategies

*Journal*

- Tell students: Jill has an ice cream sandwich which measures 10cm by 10cm. She wants to cut the ice cream into squares. What possible sizes could the squares be? How many squares of each size would be cut? (6N3.1, 6N3.2, 6N3.3)
- Ask students to solve the following problem by using words, pictures and numbers:

Harry's dad had 36 Halloween treats that he wanted to share evenly among treat bags. What are the different possibilities of the number of bags that he could fill? (6N3.2, 6N3.3)

Resources/Notes

*Math Focus 6*

**Lesson 1 (Cont'd):** Identifying Factors

**6N3**

TG pp. 13 – 17

Strand: Number

Outcomes

*Students will be expected to*

**6N3 Continued**

**Achievement Indicator:**

*6N3.4 Identify multiples for a given number, and explain the strategy used to identify them.*

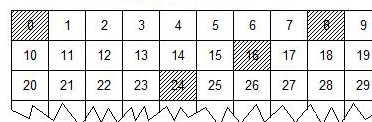
Elaborations—Strategies for Learning and Teaching

Students have worked previously with factors and will now be introduced to multiples.

Discuss with students that when we multiply two factors, the product is a multiple of those two factors. E.g.,  $2 \times 5 = 10$ . 2 and 5 are the factors of 10, whereas 10 is a multiple of 2 and 5. Students sometimes do not recognize 0 as a multiple of any number. Small (2008) states that there are two ways to approach this. One is to observe that, for example,  $0 = 0 \times 3$ , so 0 is a multiple of 3. The other is to use patterns. The multiples of 4 are 4 apart. So going down from 4 you get to 0. E.g., 24, 20, 16, 12, 8, 4, 0. (p.155)

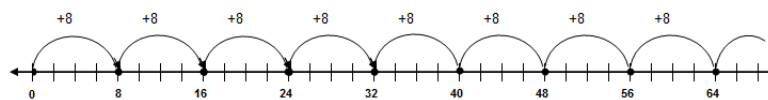
Multiples of a number can be identified using a hundred chart. Students can start at 0 and then skip count the specified number. For example, when asked to find the multiples of 8, students can start at 0, and shade in every 8 numbers. The shaded numbers are the multiples of 8. To emphasize 0 as a multiple of every number, it is suggested that a hundred chart that includes 0 be used.

Hundred chart illustrating multiples of 8



Students can also use a number line to skip count by the specified number.

Number Line illustrating multiples of 8



Students can also use an organized chart to identify multiples of a number. Here they can use multiplication of the number by various factors to determine multiples of the number.

Organized Chart Multiples of 8

<b>Multiply by 8</b>	0	1	2	3	4
<b>Product</b>	0	8	16	24	32

The multiples of 8 are: 0, 8, 16, 24, 32...

*(continued)*

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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Performance/Paper and Pencil*

- Present the following situations to students and ask them to find the answer:

Joe bought some \$10 computer games. Damian bought some \$15 computer games. They each spent less than \$200, but they both spent the same amount. How much could they have spent?

(6N3.4, 6N3.3)

- Tell students you are thinking about a number that is a multiple of 2 and 6. Ask them to identify some possible numbers that you could be thinking about.

(6N3.4, 6N3.3)

*Journal*

- Tell students you were trying to figure out the multiples of 8 and here is the list you came up with: 0, 8, 16, 23, 32 and 40. Ask if they agree and have them explain their thinking.
- Ask students to choose a number from 2 – 10 and ask them to list at least 5 or 6 multiples of that number. Ask them to explore any patterns they see in the multiples and discuss why this may happen.

(6N3.4, 6N3.3)

## Resources/Notes

*Math Focus 6***Lesson 2:** Identifying Multiples

**6N3**

TG pp. 18 – 21

**Curious Math:** String Art

6N3

TG pp. 22-23

Strand: Number

Outcomes

*Students will be expected to*

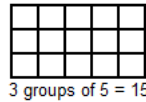
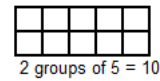
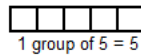
**6N3 Continued**

**Achievement Indicators:**

*6N3.4 Continued*

Elaborations—Strategies for Learning and Teaching

Students may use various manipulatives such as snap cubes, counters, buttons, etc, to create equal groups of the specified number to find its multiples. The total amount of items would be the multiple of that number. E.g., if a student was asked to find the multiples of 5, they could create one group of 5 and list 5 as a multiple, two groups of five would be 10 and 10 can be listed as a multiple of 5. Five groups of five would be 25 and 25 would be multiple. Remind students that 0 is a multiple of every number and it should be included as a multiple here.



Therefore, 5, 10, 15 are multiples of 5

*6N3.3 Continued*

Encourage students to develop their own strategies in determining multiples of numbers. They can use these strategies to help solve various problems involving multiples.

Ask students to figure out how many packs of wieners with 12 wieners in a pack and how many packs of hot dog buns with 8 buns in a pack would be needed to fill an order of 72 hot dogs.

Spiders have 8 legs and ants have 6 legs. There is a container on the table that contains both spiders and ants. The number of spider legs equal the number of ant legs. What are some possible numbers of spiders and ants that would produce this result?

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**General Outcome: Develop Number Sense**

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**Suggested Assessment Strategies***Performance/Paper and Pencil*

- Present the following situations to students and ask them to find the answer:

The cafeteria is having a promotion. Every second student receives free milk and every sixth student receives a slice of pizza. If 60 students are served at the cafeteria that day, which students received free milk? A slice of pizza? Both?

(6N3.4, 6N3.3)

*Journal*

- Tell students that Olivia was in a class of 24 people. Her teacher told the class to line up and that every second student would receive a pencil and every sixth student would receive a smelly sticker. If Olivia wanted both a pencil and a smelly sticker, ask students which position in the line Olivia should be in. (6N3.4, 6N3.3)

**Resources/Notes***Math Focus 6*

**Lesson 2 (Cont'd):** Identifying Multiples

**6N3**

TG pp. 18 – 21

## Strand: Number

## Outcomes

*Students will be expected to*

**6N3 Continued****Achievement Indicators:**

*6N3.5 Provide an example of a prime number, and explain why it is a prime number.*

*6N3.6 Provide an example of a composite number, and explain why it is a composite number.*

*6N3.7 Sort a given set of numbers as prime and composite.*

## Elaborations—Strategies for Learning and Teaching

The work that students have completed on factors will help them as they work on prime and composite numbers. Students see how factors help determine prime and composite numbers. Arrays and rectangular arrangements provide students with a visual and concrete representation of how a number can be broken down. Numbers that can only be arranged in one array are prime numbers. Numbers that can be arranged in more than one array are composite. Give students several numbers to explore. E.g., 3, 6, 9, 13 and 16. Ask them to find the factors of the numbers. Ask students if they notice any similarities or differences with the factors of the numbers. Discuss with students the fact that some numbers only have 2 factors, which are prime and others have more than 2 factors, which are considered composite.

Students continue to work with prime and composite numbers identifying whether a number is prime or composite, factoring a number into its prime factors and continuing to develop personal strategies to solve problems. Students need opportunities to apply these skills and ideas in problem solving situations where they are thinking about and reasoning through their work. Students, through working with factoring numbers, are now asked to demonstrate an understanding that composite numbers can be created by multiplying prime numbers together.

An interesting way to help students determine whether a number is prime or composite is a technique called “The Sieve of Eratosthenes”.

Use a hundred chart to find the primes from 1-100:

Step 1: Color the number 1 blue.

Step 2: Color every multiple of 2 red but not 2 itself.

Step 3: Color every uncoloured multiple of 3 yellow but not 3 itself.

Step 4: Color every uncoloured multiples of 5 green but not 5 itself.

Step 5: Color every uncoloured multiple of 7 orange but not 7 itself

The remaining uncoloured numbers are prime numbers!

After completing “The Sieve of Eratosthenes” activity, ask students to explore the result of multiplying two prime numbers. Students should realize that when they multiply any two prime numbers the resulting product will always be a composite number. Ask students to explain why this happens. If students work with smaller prime numbers as factors, they will be able to identify on the hundred chart that the resulting product is a composite number. (Small, 2008)

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Student-Teacher Dialogue*

- Ask students: Is it possible for an even number, other than 2, to be prime? Explain. (6N3.5, 6N3.6)

*Performance*

- Factor Game - Using a number chart, ask students to work in groups. Students would need one game board and different colored markers/crayons. The object of the game is to circle the factors of a number. Player One chooses a number on the number board and scores that many points. Player Two crosses out all the factors of that chosen number that are not already circled. Player Twos' score is the sum of the circled factors. E.g., Player One chooses the number 16. They score a total of 16 points. Player Two would then circle the available factors of 16, which are 1, 2, 4, 8. (The number 16 would already be circled by Player One). Player Two would add the factors together to get a total score of 15. Player Two would then choose a number and score that many points, for example, 21. Player One would circle all available factors of 21 which are 3 and 7 and score a total of 10. As play continues, it may be possible for a player to select a number where all possible factors are circled, therefore they would get that many points with the opponent not scoring any. Play continues until all numbers are circled where students would then total their points. The player with the most points, wins.

Player One	Player Two
16	15 (1+2+4+8)
10 (7+3)	21

(6N3.5, 6N3.6, 6N3.7, 6N3.3)

## Resources/Notes

*Math Focus 6***Lesson 3:** Prime and Composite Numbers**6N3**

TG pp. 24 – 28

**Math Game:** Colouring Factors**6N3**

TG pp. 29 - 30

**Additional Reading** (provided):

Small, Marion (2008) *Making Math Meaningful to Canadian Students K-8*. pp.154-156

## Strand: Number

## Outcomes

*Students will be expected to*

**6N3 Continued****Achievement Indicators:**

*6N3.3 Continued*

*6N3.8 Explain why 0 and 1 are neither prime nor composite.*

*6N3.2 Continued*

## Elaborations—Strategies for Learning and Teaching

Students can employ personal strategies to solve problems involving factors and prime and composite numbers. Solving problems becomes a vehicle which allows students to apply the knowledge and skills they are learning in a meaningful way.

Ask students to think about the following situation: If you had 17 tissue boxes, how many different ways could you design a carton to package your tissue boxes with no empty space in the carton? How would this change if there were 34 tissue boxes? (students may want to model this problem using snap cubes to create a rectangular prism)

It may be necessary to point out that the number 1 is neither prime nor composite, since it does not fit the definition of a prime or composite number. To be prime, the number must have only two factors, one and itself. The number 1, only has one factor. Therefore, it cannot be composite (it does not have more than 2 factors).

0 is another special number. 0 cannot be a prime number because every number is a factor of zero.  $0 \times 1$  does equal 0, but 0 multiplied by anything equals 0.

0 is not a composite number because it cannot be written as a product of 2 factors, neither of which is itself.

Students have had opportunities to work on factoring numbers using various strategies that involved using concrete and visual representations. Students should begin to understand that our whole number system can be built up by prime numbers and that any one number can be broken down into a series of prime numbers. This is called prime factorization.

E.g.,  $24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$

Students may also see that if they use repeated division of prime numbers, they will end up with prime factors.

E.g.,  $36 \div 3 = 12$        $12 \div 3 = 4$        $4 \div 2 = 2$

so the prime factors would be 3, 3, 2, 2.

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**General Outcome: Develop Number Sense**

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**Suggested Assessment Strategies***Performance*

- Ask students to use colored tiles or grid paper to determine whether the numbers 7, 10, and 18 are prime or composite numbers. Ask them to explain their thinking. (6N3.5, 6N3.6)

*Student-Teacher Dialogue*

- Ask the student how he/she can determine, without factoring, that certain large numbers such as 17932 and 19875 are not prime. (6N3.7)

*Journal*

- Give students 6 – 8 numbers and ask them to sort them as either prime or composite numbers. Ask students to justify their reasoning. (6N3.7)

**Resources/Notes***Math Focus 6***Lesson 3 (Cont'd): Prime and Composite Numbers****6N3**

TG pp. 24 – 28

*Math Focus 6***Lesson 4: Identifying Factors by Dividing****6N3**

TG pp. 31 - 34

Strand: Number

Outcomes

*Students will be expected to*

**6N3 Continued**

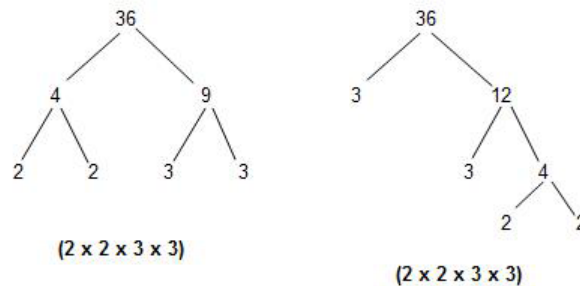
**Achievement Indicators:**

*6N3.2 Continued*

Elaborations—Strategies for Learning and Teaching

Ask students to work in groups to factor the number 36. Give students time to employ different strategies to find the prime factors. Have each group present their strategies to the class, looking at similarities and differences in each groups approach and their starting point to factor the number. It is important that students discover that regardless of where they start the factorization process, they will always end with the same prime factors.

**Factor Trees**



In Grade 6, students should realize they can use division to find the factors of a given number. For example, when asked to find all the factors of 12, students can look for different numbers that can divide evenly into 12. These are the factors of 12.  $12 \div 1 = 12$ , so 1 and 12 are factors.  $12 \div 2 = 6$ , so 2 and 6 are factors and  $12 \div 3 = 4$ ; 3 and 4 are factors.

*6N3.3 Continued*

Review with students that they can find the factors of numbers in different ways:

- forming arrays
- factor rainbows
- repeated division
- factor trees (factorization)

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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Paper and Pencil*

- Tell students that a number has exactly 4 prime factors. Ask them to think about what could this number be. (6N3.2, 6N3.3)
- Ask students to draw two different factor trees for 56 and for 32. Ask them why is it possible to draw two different factor trees for each number. (6N3.2, 6N3.3)
- Ask students if they can name a composite number for which you can draw only one factor tree. How many factor trees can they draw for the number 13? Explain. (6N3.2, 6N3.3)

*Performance*

- There are 84 students in four grades and they are arranged into teams with the same number on each team. How many teams are there and how many students might there be on each team? How many possible solutions can you find to this problem? (6N3.4, 6N3.2, 6N3.3)

## Resources/Notes

*Math Focus 6*

**Lesson 4 (Cont'd):** Identifying Factors by Dividing

**6N3**

TG pp. 31 - 34

*Math Focus 6*

**Lesson 5:** Creating Composite Numbers

**6N3**

TG pp. 35 – 38

*This lesson does not require a lot of time.*

Strand: Number

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## Outcomes

**Problem Solving Strategy:**  
Using an Organized List

## Elaborations—Strategies for Learning and Teaching

As students continue to develop their own problem solving strategies, they may wish to include creating an organized list. Continue to build on various problem solving strategies and when students use one of the strategies, ask them to present how they used the strategy and how it helped them solve the problem.

An organized list helps students organize their thinking about a problem. Students may wish to use an organized list to begin a problem, or to organize given information in a problem. Students can record their thinking in an organized list which makes it easy for them to review what has been done, the stage of the problem solving process they are in and what is left to be done. Using an organized list provides a good way to record computations and any other important information needed to solve the problem.

Ask students to create a problem whereby they could use an organized list to help them solve the problem. Ask them to write this problem on a large sheet of paper and then show how they would solve it using the list. Students could then add this strategy to their math wall.

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**General Outcome: Develop Number Sense**

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**Suggested Assessment Strategies***Performance*

- Tell students the following stories and ask them to find the answers:
  - (i) Michael and his sister, Rebecca, both play soccer. They are on different teams and both have a tournament this weekend. Michael and Rebecca's parents are trying to figure out if there is a time when they are playing against each other. Michael plays every three games and Rebecca is playing every two games. If there are 12 games in the tournament, is it possible for Michael and Rebecca to play against each other? If so, how many times will they play against each other? (6N3.4, 6N3.2, 6N3.3)
  
  - (ii) Both Craig and John bought packages of granola bars. Craig had 24 bars in total and John had 30 bars in total. Find all the possible sizes of packages for each student.
  
  - (iii) If both Craig and John bought packages with the same number of bars in them, what would be the number of bars in each packages? (6N3.4, 6N3.1, 6N3.2, 6N3.3)

**Resources/Notes***Math Focus 6***Lesson 6: Solving Problems Using an Organized List**

TG pp. 43 - 46

## Strand: Number

## Outcomes

*Students will be expected to*

**6N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically.**

[C, CN, R, V]

**Achievement Indicators:**

*6N7.1 Extend a given number line by adding numbers less than zero, and explain the pattern on each side of zero.*

*6N7.2 Describe contexts in which integers are used; e.g., on a thermometer.*

## Elaborations—Strategies for Learning and Teaching

Students are familiar with numbers 0 and above. This unit introduces them to integers which includes numbers above and below 0. The main ideas for students to understand are that:

- each negative integer is the mirror image of a positive integer with respect to the 0 mark
- 0 is neither positive nor negative
- negative integers are all less than any positive integer

Using the word ‘negative’ and not ‘minus’ is important when describing negative integers. This will avoid confusing the sign of the integer with the operation of subtraction.

Negative integers are the opposites of natural numbers. Each integer is the reflection of its opposite across the number line. A number line is a useful tool in helping students see the relationship between positive and negative integers. Tape a number line on the floor, use cash register tape or string, whereby the middle, or center of the number line is marked with a 0. Ask students to think about what numbers would go to the left, or below the 0. Ask students to explore these numbers asking them to think about the placement of the numbers that are ‘less’ than 0. Show students that numbers such as +1 and -1 are the same distance away from 0.

Students, almost daily, have experiences that can be modelled with negative numbers. For example, slowing down a car is negative acceleration. Students may also gain an understanding of negative integers by putting them into meaningful context such as driving in reverse, describing floors below the main floor in a building, talking about below sea level or golf scores that are below par. It could also be described in terms of owing money. Discuss some of these situations where it can be represented by negative numbers. E.g., losing \$15 or when trying to walk up an icy hill and you fall backward a few steps.

Using a number line (painter tape works well), ask students to stand on a negative integer and tell where they would see this number in real life, or describe a situation that could be represented by this number.

Use play money to represent various situations where students can see a gain or loss, emphasizing a loss would represent a negative situation.

It may be necessary to help students understand that a situation can be represented by a negative number without the result being a negative number. For example, if you have \$5.00 in your pocket and you gave \$3.00 to your friend for his birthday, you would still have \$2.00 left, but it would be a loss of \$3.00.

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Student-Teacher Dialogue*

- Students may illustrate / discuss a situation in which they have encountered something that could be represented with a negative number. (6N7.2)
- Joe and John are standing on a number line. Joe is 6 spaces away from John. Joe is standing on a negative number and John is on a positive number. Ask students to determine some possible numbers Joe and John may be standing on. Ask the students to explain the strategies used to solve this problem. (6N7.1, 6N7.3, 6N7.5)

*Performance*

- Provide students with a blank number line. Give them positive and negative integers to place on the line with them choosing their end points and benchmarks. (6N7.1, 6N7.3, 6N7.4, 6N7.5)
- Ask students to design a game for which positive and negative points may be awarded. Ask students to play and keep track of their score. (6N7.1, 6N7.3, 6N7.5)
- A number is 12 jumps away from its opposite on a number line. Ask students what the number could be and how do they know. (6N7.1, 6N7.3, 6N7.5)
- Ask students to choose two negative integers. Ask them to compare these numbers by describing a context in which they could be used (temperature) and use this context to compare using the less than/ greater than symbols. (6N7.2, 6N7.4)

*Paper and Pencil*

- Ask students how many negative integers are greater than -7. Ask them to explain how they know. (6N7.1, 6N7.3, 6N7.5)

*Journal*

- Ask students to explain, in writing why -4 and +4 are closer to each other than -6 and +6.
- Ask students to explain, in writing, why an integer is never an odd number of jumps away from its opposite on a number line. (6N7.1, 6N7.3, 6N7.5)

## Resources/Notes

*Math Focus 6***Lesson 7: Representing Integers**

6N7

TG pp. 47 – 50

**Curious Math: Countdown Clock**

6N7

TG pp. 51 - 52

**Children's Literature** (not provided):Murphy, Stuart, *Less than Zero*

ISBN 9780060001261

Strand: Number

Outcomes

*Students will be expected to*

**6N7 Continued**

**Achievement Indicator:**

*6N7.3 Place given integers on a number line, and explain how integers are ordered.*

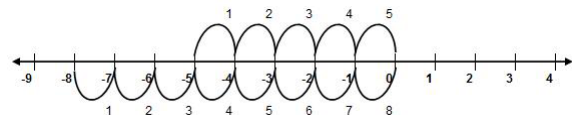
Elaborations—Strategies for Learning and Teaching

When comparing and ordering numbers, students should refer to a number line and look at the position of the number on the line in relation to 0 to determine its value. A common misconception students have is the greater the digit, the greater the value will be. They may see a number like -8 and think it is greater than -1 or +5 just because the digit 8 is greater than 1 or 5. Ask students to refer to the position of the number in relation to 0. Remind students that any number to the right is always greater than any number to the left. This applies to both positive and negative integers.

When placing integers on a number line, remind students to first look at the sign to see whether it is a positive or negative number. Then ask them to look at the digit to determine how far away from 0 the number should go. Ensure students understand that all negative numbers are less than 0 and placed to the left of the 0 on the number line.

Students can stand on number lines to help order integers. To illustrate the idea of whether a negative integer is greater than or less than another, ask students to stand on the number in question and ask them to hop to 0 counting how many times they needed to hop to get to 0. Ask them to do the same for the other number and then compare the number of hops it takes to get to 0.

E.g., When comparing -5 and -8, have a student stand on -5 and jump to 0. They would see it takes 5 jumps. Next, ask them to stand on -8 and jump to 0 where they will jump 8 times. They should see that because it took more jumps to get to 0 from -8, or -8 is farther away from 0, then -8 would be less than -5.



If students are struggling with this concept, ask them to refer to a thermometer or use a vertical number line to represent the thermometer.

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Ask students if any positive number is greater than any negative number. Ask them to use a number line to explain their thinking.  
(6N7.1, 6N7.3, 6N7.4)
- Provide students with incomplete number lines where they would fill in any missing numbers. (6N7.1, 6N7.3, 6N7.4, 6N7.5)
- Ask students to research cities in North America that are below sea level, at about sea level and some that are above sea level. Using a chart, ask them to list these cities from lowest elevation to highest.  
( 6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)
- Students may wish to research pro golf players databases where they compile data on scores. Explain that golf scores are reported in positive and negative numbers, where positive numbers show how many shots above par was needed to sink the ball. A negative number would be how many shots under par was needed to sink the ball. For example, on a hole that was par 5, it is suggested that it would take 5 shots to sink the ball. If a player took 3 shots, he/she would score -2 for that hole. If it took him 6 shots to sink the ball, his score would be +1. Ask students to rank players according to their scores.  
( 6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)
- There are 10 numbers between a set of integers. What are some possible numbers this set could be?  
(6N7.1, 6N7.2, 6N7.3, 6N7.4, 6N7.5)
- Create a number line on the board and incorrectly place a negative number on the positive side of the number line. Ask students to decide if this number line is correct and ask them to justify their thinking.  
( 6N7.1, 6N7.3, 6N7.4, 6N7.5)

## Resources/Notes

*Math Focus 6***Lesson 8:** Comparing Integers**6N7**

TG pp. 53 - 57

Strand: Number

Outcomes

*Students will be expected to*

**6N7 Continued**

**Achievement Indicators:**

*6N7.4 Compare two integers; represent their relationship using the symbols  $<$ ,  $>$  and  $=$ ; and verify the relationship, using a number line.*

*6N7.5 Order given integers in ascending or descending order.*

**6N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**

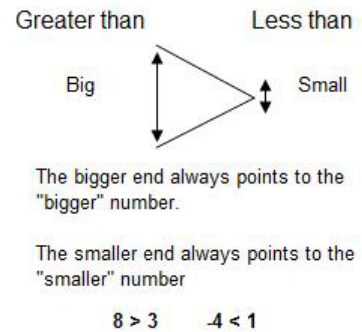
[C, CN, ME, PS, T]

**Achievement Indicator:**

*6N9.1 Explain, using examples, why there is a need to have a standardized order of operations.*

Elaborations—Strategies for Learning and Teaching

There are many different strategies to help students remember the symbols of less than and greater than. One way to help students remember the symbols would be to get students to look at the shape where they would see a sideways “V”. Tell students there are two sides to the symbol, a big side and a small side.



A number line is a useful tool to help students order integers.

The order of operations is a set of rules that tells you which order to compute numbers so you will get the same answer as everyone else. It is just merely a convention of mathematics and something that students will practice.

When applying these rules, multiplication and division will always come first unless there are brackets in the equation. Here, you can explain that sometimes you may not want or need to follow the rules and need to highlight a specific computation. It is here you need to use brackets.

For example, consider the following question: Each of 6 friends got a full box of bars that contained 10 bars in each box and then each of them got an extra 4 bars. Write an equation to show how many bars all 6 friends got.

Students may write the following:

$$6 \times 10 + 4$$

The order of operations would tell us to multiply  $6 \times 10$  and then add 4. But that would not make sense as  $6 \times 10$  would show there are 6 friends with 10 bars each totalling 60 bars and then if you added 4 there would only be a total of 64. We know from the problem there are 4

*(continued)*

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Create number cards where there are both positive and negative numbers. Play Integer War where students would pick a card and then compare each. The player with the greatest value would keep the cards. Play continues until all cards are used. The player with the most cards wins. (6N7.1, 6N7.3, 6N7.4, 6N7.5)

- Mrs. Smith bought 2 boxes of bars for her three children. Each box of bars has 6 bars in it. Ask students to determine how many bars each child receives. Ask students to write an expression to show the order of operations you would use to solve the problem.

(6N9.1, 6N9.2)

- Ask students to place a set of brackets into this equation to explore how many different solutions are possible.

$$10 + 2 \times 8 - 6 \div 2 = ?$$

(6N9.1, 6N9.2)

- Ask students to find some skill testing questions that can be found on contests. Ask students to answer the question and compare the answers when following the order of operations and the answers when you do not follow them. Discuss the importance, in terms of the contest, of following these rules. (6N9.1, 6N9.2)

## Resources/Notes

*Math Focus 6***Lesson 8 (Cont'd):** Comparing Integers**6N7**

TG pp. 53 - 57

*Math Focus 6***Lesson 9:** Order of Operations**6N9**

TG pp. 58 – 61

**Math Game:**

Four in a Row

**6N9**

TG pp. 62 - 63

## Strand: Number

## Outcomes

*Students will be expected to*

**6N9 Continued****Achievement Indicators:**

*6N9.1 Continued*

*6N9.2 Apply the order of operations to solve multistep problems with and without technology; e.g., a computer, a calculator.*

## Elaborations—Strategies for Learning and Teaching

extra bars for each of the 6 friends. Therefore you would have to add the 10 and 4 to show that all 6 friends would have a total of 14 bars each. Then you would multiply  $14 \times 6$  to get the total number of bars. So the expression then would look like:

$$6 \times (10 + 4)$$

From here, students would see that you would compute the numbers inside the brackets first, then continue with multiplication/division and end with addition or subtraction.

To introduce students to the idea of the order of operations, provide the class with a question similar to the following:  $4 + 8 \times 2 - 7$  and ask them to find the solution. Ask students to share their solutions and discuss if there are other possible solutions for this problem.

Discuss why people may have different answers for this question. Some may add 4 and 8 and then multiply by 2 and subtract 7 to get 14, whereas some others may add 4 to  $8 \times 2$  and then subtract 7 for an answer of 13. Explain that we need to have rules to make sure everyone is getting the same answer. Many times in real life people are in a situation where there are various operations to calculate in order to solve the problem. To ensure the correct solution is found, these rules must be followed.

To illustrate the necessity of the rule, pose this problem to students: Mac bought 6 pairs of socks for \$7.00 each and a scarf for \$4.00. How much money did Mac spend?

To find the amount of money spent the equation could be written as:  $6 \times 7 + 4$  or  $4 + 6 \times 7$ . Explain to students in order to find the correct answer we would have to multiply  $6 \times 7$  and then add 4 in order for the amount to make sense. If we have 6 pairs of socks and spent \$7.00 on each pair we would have to multiply these two numbers. It would not make sense to add 4 and 6 and then multiply this number by 7.

Students must be aware that most calculators will not use the order of operations to calculate equations automatically. Therefore they cannot rely on their calculator to solve problems involving multiple operations. Students will need practice entering the digits on the calculator in the order that the operation should be performed.

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Give students an equation with the operations missing. Ask them to fill in the missing mathematical symbols to make the statement correct. This could also be done in a riddle type activity.

(6N9.1, 6N9.2)

- Molly was doing her math homework when all of a sudden her pet mouse came along and began chewing her paper. When she looked, she noticed all the operation symbols were missing. Ask students to help Molly put these symbols and numbers back to make the statements true. Brackets should be included where necessary.

$$12 \ ? \ 8 \ ? \ 3 \ ? \ 2 = 26$$

$$8 \ ? \ 6 \ ? \ 4 \ ? \ 2 = 10$$

(6N9.1, 6N9.2)

## Resources/Notes

*Math Focus 6***Lesson 9 (Cont'd):** Order of Operations**6N9**

TG pp. 58 – 61

