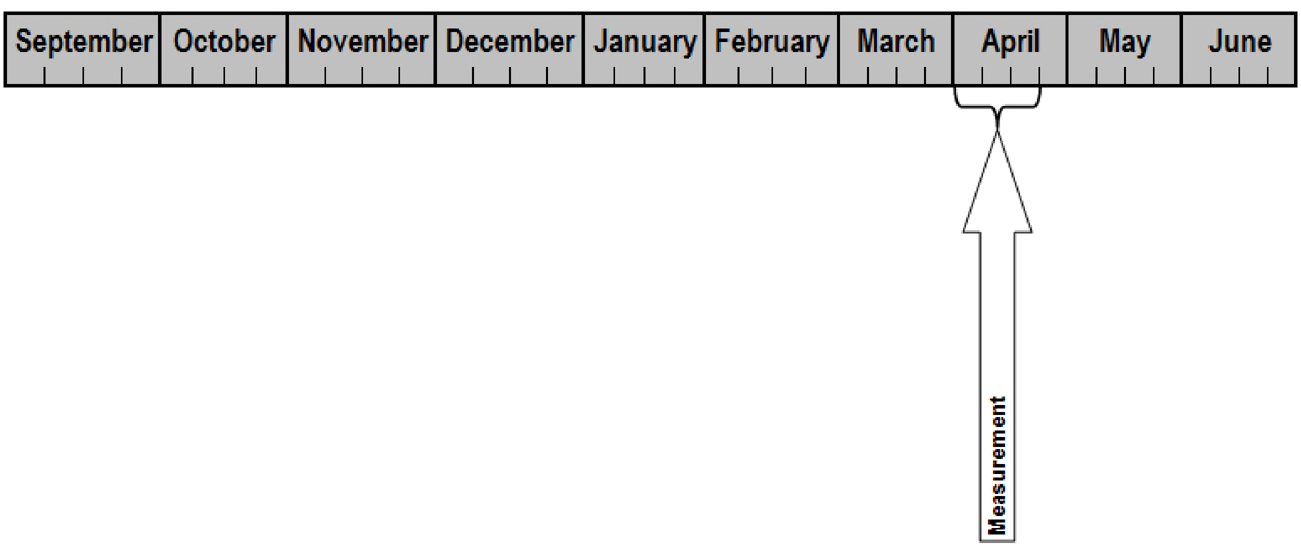
Measurement

Suggested Time: 3 Weeks



Unit Overview

Focus and Context

The focus of this unit is to develop an understanding of measurement within the tangible realm of linear distance, two-dimensional area, three-dimensional volume and degree of rotation. Students have likely attained a degree of familiarity with measurement through real life experience. This background experience and knowledge is a good foundation on which to develop and further concepts in measurement. The nature of the material also provides ample opportunity for group based, hands on learning activities which can increase student interest and motivation and thus greatly enhance learning.

In Grade 6, students are expected to:

- extend previous experience with rotational benchmarks such as quarter, half, three quarter and full turns to angle classification and measurement in degrees.
- identify, classify and measure angles in their environment.
- estimate angle measures in degrees using benchmarks as reference angles.
- measure angles using a 180° and 360° protractor.
- draw angles and angle approximations with and without the use of a protractor.
- explore relationships within the interior angles of triangles and quadrilaterals.
- derive formulae for the perimeter of polygons, area of rectangles and volume of right rectangular prisms through exploratory activities.
- use derived formulae to solve problems involving perimeter of polygons, area of rectangles and volume of right rectangular prisms.
- solve complex measurement problems through a process of solving simpler problems.

Math Connects

Measurement involves the use of quantitative (numerical) values to describe a specific attribute. These measurable attributes may be tangible, or intangible (e.g., time, temperature etc.). In order for measurement to be meaningful a holistic understanding of units and their relation and conversion must be developed. A comprehensive understanding of measurement is fundamental to the development of concepts in other branches of mathematics such as Euclidian geometry, transformational geometry, algebra and statistics. It is also important to ensure that students have a sound understanding of the metric system and appropriate usage of units.

Measurement is an essential link between mathematics, sciences, the arts and other disciplines and has an endless variety of applications in real life situations. Encourage students to research how measurement is involved in careers and hobbies in which they are interested. Providing students with opportunities to engage in measurement based activities with real world application will enhance student learning, making it meaningful and relevant.

Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Shape and Space (Measurement)	6SS1 Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles, using 45°, 90° and 180° as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified.	[C, CN, ME, V]
Shape and Space (Measurement)	6SS2 Demonstrate that the sum of interior angles is: • 180° in a triangle • 360° in a quadrilateral.	[C, R]
Shape and Space (Measurement)	6SS3 Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.	[C, CN, PS, R, V]
Patterns and Relations (Variables and Equations)	6PR3 Represent generalizations arising from number relationships, using equations with letter variables.	[C, CN, PS, R, V]

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Demonstrate an understanding of angles by:

- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles, using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

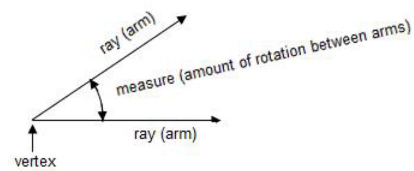
[C, CN, ME, V]

Achievement Indicator:

6SS1.1 Provide examples of angles found in the environment.

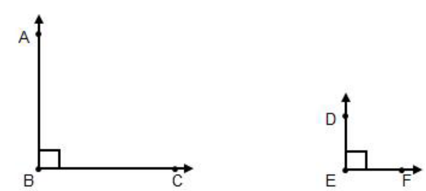
Elaborations—Strategies for Learning and Teaching

In Grade 5 students had experience with angles using fractions of a circle as benchmark angles. Up to this point they have been exposed to quarter, half, three-quarter and full turns within a circle. The measuring of angles in degrees and the subsequent use of a protractor will be new concepts in Grade 6. The concept of a circle being comprised of 360° will serve as a convenient starting point for measuring angles in degrees.



It may be necessary to review the concept of an angle as being the amount of rotation between two rays (arms) joined at a shared point (called the vertex). The amount of rotation required to get from one arm of the angle to the other is the angle's measure. The length of the arms does not affect the angles measure.

E.g., if students were asked which of the angles below has the greatest measure, some might respond that angle ABC is larger than DEF. In actual fact, both angles have the same measure (the amount of rotation between the arms is the same).



Initial exploration of angles will simply involve students identifying two line segments connected at a shared point and thus forming an angle in their surrounding environment. Examples in the classroom may include door and window frames (right angles), adjacent floor tiles (right or straight angles) or hands of a clock (any angle depending upon the given time).

Description of angles at this point will be based on the quarter turn (right angle), half turn (straight angle) and three-quarter turn benchmarks. For example, if a student identifies an obtuse angle they will describe it as being between a quarter turn and a half turn. Terms such as acute, obtuse and reflex may be unfamiliar to students at this point.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Ask students to identify angles in their surrounding environment. They may do this over the course of a class or a whole day. Keep a log of where and on what object the angle was identified. Create a sketch of the object, highlighting the identified angle in a different colour. Include a brief description of the angle measure relative to the quarter-turn (right angle), half-turn (straight angle) and three-quarter turn benchmarks. (6SS1.1)

- Using the log of angles that they previously identified from their surroundings ask students to label each of the angle they recorded as acute, right, obtuse, straight or reflex. (6SS1.2)

Resources/Notes

Math Focus 6

Lesson 1: Identifying Angles

6SS1

TG pp. 13 – 17

Children's Literature
(not provided)**:**
Alder, David A. *Shape Up: Fun With Triangles and other Polygons*.
ISBN: 0-8234-1638-0

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

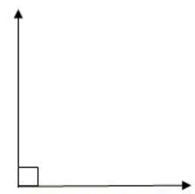
Achievement Indicator:

6SS1.2 Classify a given set of angles according to their measure; e.g., acute, right, obtuse, straight, reflex.

Elaborations—Strategies for Learning and Teaching

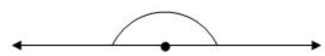
As students are familiar with the quarter, half and three-quarter turn benchmarks it will now be necessary to introduce names of other angles that fall between these benchmarks.

- Right Angle (Quarter-Turn) – Two rays that connect to form a square corner.



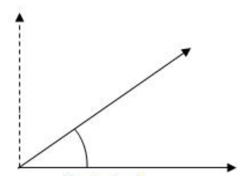
Right Angle (half-turn)

- Straight Angle (Half Turn) – Two rays that connect to form a straight line.



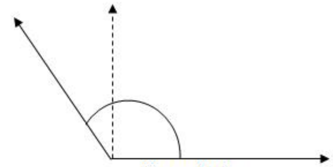
Straight Angle

- Acute Angle – An angle with a measure less than a right angle (half turn)



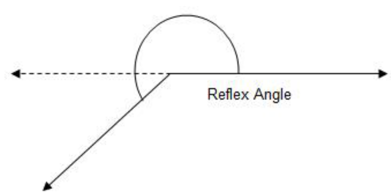
Acute Angle

- Obtuse Angle – An angle with a measure greater than a right angle but less than a straight angle.



Obtuse Angle

- Reflex Angle – An angle with a measure greater than a straight angle.



Reflex Angle

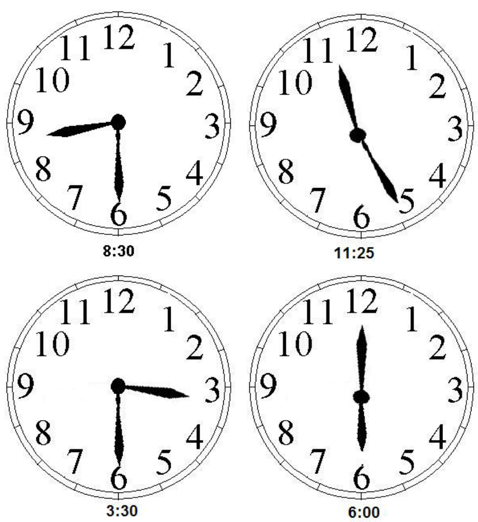
Some students will likely make the extension that an acute or obtuse angle also has a reflex angle on the outside of its arms. It may be appropriate to address this point as a class. Any given angle has a second angle outside of its arms (the remainder of the circular rotation).

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Journal

- Ask students to identify the type of angle formed by the hands of a clock at various times of the day. Identify the type of angle created by each time in the clocks shown below.



At what time will each type of angle be formed again?

Note: Reflex angles may also be possible answers for 8:30, 11:25 and 3:30. (6SS1.1, 6SS1.2)

Performance

- Ask students to draw a picture of a familiar object using only one type of angle, acute or obtuse. Check the angles you drew with a square corner (page, ruler etc.) to compare them to a right angle and ensure that they are all the type of angle you chose. (6SS1.2)

- Provide students with printed paper copies of flags from various provinces, states and countries that have linear designs or are composed of various shapes. The Newfoundland and Labrador flag would be a good example. Ask students to identify and record all of the angles they can find on the flag and classify them as either acute, right, obtuse, straight or reflex. (6SS1.2)

Resources/Notes

Math Focus 6

Lesson 1 (Cont'd): Identifying Angles

6SS1

TG pp. 13 – 17

Additional Reading (provided):

Small, Marion (2008) *Making Math Meaningful to Canadian Students K-8*. pp. 455-466

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.3 Estimate the measure of an angle, using 45°, 90° and 180° as reference angles.

Elaborations—Strategies for Learning and Teaching

Introduce the concept of the degree. Students will be familiar with the use of metric units such as cm, m, and mm for measuring linear distance. A degree is the unit used to measure rotation within a circle and subsequently can be used to measure angles.

Some students will likely attempt to make an association between degrees of a circle and degrees Celsius or Fahrenheit used for measuring temperature. It may be necessary to differentiate between the two. Although the word ‘degree’ is used in both cases, they are units of measurement for two entirely unrelated quantities.

Establish the fact that the measure of rotation for a full turn within a circle will always be 360°. This is true regardless of the size (diameter) of the circle. Students will likely have heard “360” as a stunt performed by snow boarders, skateboarders, ice skaters, etc., where the athlete makes a complete spin and ends up facing forward in his/her original position. This may be a good starting point for the introduction of this concept. Students will also likely be familiar with the stunt called a “180”, where the athlete performs a half spin and ends up facing opposite the original position. Hence, the term “180” is used because the athlete rotated his/her body in a half circle, and 180° is half of 360°.

Once it has been established that a full turn has a measure of 360° and a half turn measures 180°, the benchmarks of 90° and 45° can be easily introduced. Since a half turn has a measure of 180°, a quarter turn (half of 180°) would thus have a measure of 90° and an eighth of a turn (half of 90°) would measure 45°.

These benchmarks may also be established through student construction of a protractor.

Once the 45°, 90° and 180° benchmarks have been established students can use these to estimate the measure of other angles.

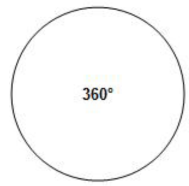
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

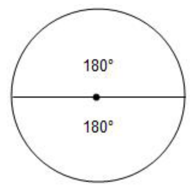
Performance

- Ask students to construct their own 360° protractor using a circular piece of paper. Through this process 45°, 90° and 180° benchmarks may be established.

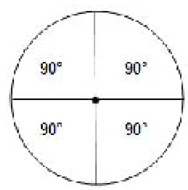
  - Students must first be aware that a circle (full turn) has a measure of 360°.



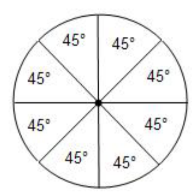
  - Fold the circle in half. If a full circle measures 360°, a half of a circle must measure 180°.



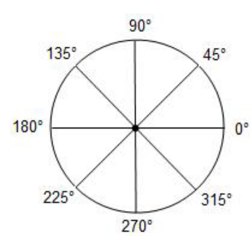
  - Fold the half circle in half again. This represents a quarter of the whole circle. If half of the circle measures 180°, a quarter must measure 90°.



  - Fold the quarter circle in half again. This represents an eighth of the whole circle. If a quarter of the circle measures 90°, an eighth must measure 45°.



  - Once the circle is unfolded it will be evident that there are eight sets of 45° in the whole. Choose a fold line as 0°; label each fold line in the counter-clockwise direction as a consecutive multiple of 45°, as shown below.



Students now have a guide by which they can estimate the measure of any angle. (6SS1.3)

Resources/Notes

Math Focus 6

Lesson 2: Constructing a Protractor (optional)

6SS1

TG pp. 18 - 21

Students will need to learn to measure given angles using a protractor. However, construction of a protractor is optional.

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

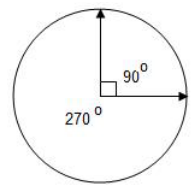
6SS1.4 Sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them.

Elaborations—Strategies for Learning and Teaching

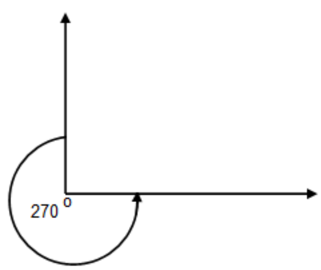
In addition to estimating the measure of an angle using the 45°, 90° and 180° benchmarks, students will be sketching these angles without the use of a protractor and describing relationships between them.

Having estimated angle measures previously students should be able to visualize each of the 45°, 90° and 180° benchmarks individually. They can now use combinations of these benchmarks to sketch angles without the use of a protractor.

It will be necessary here to introduce the correct method for labelling angles. As previously discussed, any angle can be viewed as two different angles, the smaller angle between the arms and a larger angle outside as shown below.



To avoid confusion as to which angle is being referred to in a particular drawing, an arc (hatch mark) is drawn between the two arms. Note that a special mark, a small square, is used to indicate a right angle. E.g.



The arc between the arms of the angle above indicates that this drawing is showing a 270° angle and not a 90° angle.

(continued)

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Angle "I Spy" Game – Ask students to identify an angle in the classroom. Write down the type of angle (acute, right, obtuse, reflex or straight), an estimate of its measure and draw a sketch of its orientation. Trade angles information with a partner and attempt to find a representation of each other's angle. (E.g. Floor, wall, corner, etc.) (6SS1.1, 6SS1.2, 6SS1.3, 6SS1.4)

Resources/Notes

Math Focus 6

Lesson 3: Estimating Angle Measures

6SS1

TG pp. 22 - 25

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

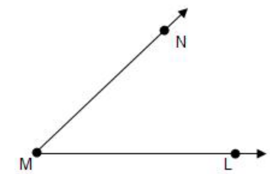
6SS1 Continued
Achievement Indicator:

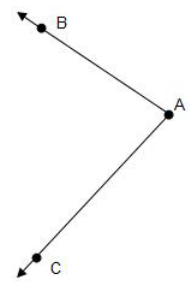
6SS1.3 Continued

Elaborations—Strategies for Learning and Teaching

This would also be an appropriate time to introduce the correct method of naming angles. When an angle is presented with three labelled points, the angle would be named using the angle symbol ∠ and the three points written with the vertex in the middle. E.g.

This angle could be correctly named ∠LMN or ∠NML, because in each case the vertex point is in the middle of the name. Emphasize that writing the points in alphabetical order is not a requirement and will not necessarily be correct in all cases, depending on the letter name of the vertex.

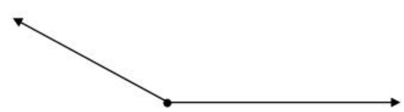




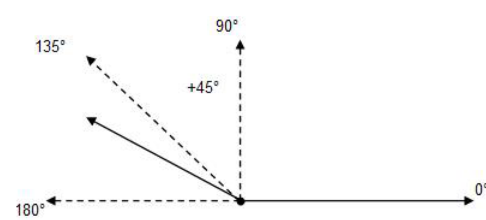
This angle could be named ∠BAC or ∠CAB. Writing the points in alphabetical order (∠ABC) in this case would be incorrect, as the vertex would not be in the middle.

Visualizations of the 45°, 90° and 180° benchmarks and their combinations can now be used to estimate the measures of other angles.

E.g. Estimate the measure of the angle below.



Students may visualize or sketch benchmark angles on a separate sheet of paper or directly on the given angle, as shown.The angle in question is between 135° and 180°. Its measure might be estimated at somewhere between 150° to 160°.



General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Ask students to sketch each of the following angles without using a protractor. Label each angle correctly and use an arc to indicate the direction or rotation.

  (i) ∠ ABC is 135°

  (ii) ∠ DOG is 275°

  (iii) ∠ LMN is 88°

  (iv) ∠ ZYX is 190°

  (v) ∠ PRQ is 100°

  (v) ∠ GEF is 290°

  (vi) ∠ CAT is 376°

  Ask students to explain which benchmarks they found helpful in sketching each angle and how they used these to produce their sketch. Focus on the reasoning students used to arrive at the sketches they produced. (6SS1.4)

- Using the log of angles previously created from their surroundings, ask students to estimate the measure of the angle in each of their drawings and write the estimated measure between the arms of each angle they sketched. (6SS1.3)

- Provide students with various flags of provinces, states and countries that have linear designs or include various shapes. The Newfoundland and Labrador Flag would be a good example. Ask them to identify as many different angles as they can find. Ask students to estimate the measure of each angle they identified using benchmarks and explain how they arrived at each estimation.

  (6SS1.3)

Resources/Notes

Math Focus 6

Lesson 3 (Cont'd): Estimating Angle Measures

6SS1

TG pp. 22 - 25

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

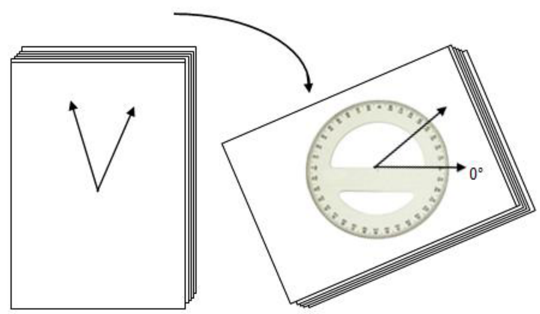
6SS1.5 Measure, using a protractor, given angles in various orientations.

Elaborations—Strategies for Learning and Teaching

Building upon the benchmarks and estimation skills developed previously, students will now use an actual protractor to measure angles. The initial use of a circular (360°) protractor rather than a semi-circular (180°) protractor is recommended. This will reinforce students' understanding of an angle as a rotation within a full circle.

Students will now use a protractor showing all 360 degrees of a circle (full turn) to measure angles to the nearest degree. As this is the first time students have used a protractor, it will be necessary to explain that the degrees on the protractor are only labelled in multiples of ten, however each unlabeled tick represents 1°, thus allowing them to measure angles between the labelled values.

Not all angles will be oriented with one horizontal arm. This may present a challenge for some students. In any case, the centre point of the protractor must be placed on the vertex of the angle and the zero degree mark of the protractors baseline must be lined up with one of the angle's arms. If this orientation of the angle presents a problem, the page or book may be turned so that one arm becomes horizontal from the students point of view.



(continued)

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Provide students with illustrations from books. Ask them to identify and estimate the measure of angles found in each picture. Students can create a list of the angles and their approximated measures. Another possibility would be to provide photocopies of the illustrations so that students can highlight or colour the identified angles and write their estimated measures on the picture. (6SS1.5)

Journal

- Engage students in a journal entry activity using the following prompt:

    Pretend you work for a company that makes and sells protractors. You have been given the job of writing instructions for the protractor package. Write a detailed step-by-step set of instructions (with diagrams if needed) to explain how to measure any angle using the protractor. Assume the reader has never seen a protractor before.

  Trade instructions with a partner and follow them to measure a given angle.

  What changes would you make to your partner's instructions?

  (6SS1.5)

Resources/Notes

Math Focus 6

Lesson 4: Measuring Angles

6SS1

TG pp. 26 - 30

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

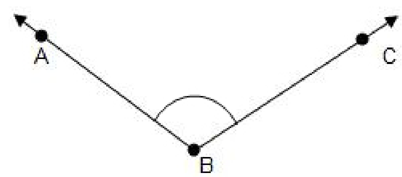
6SS1.5 Continued

Elaborations—Strategies for Learning and Teaching

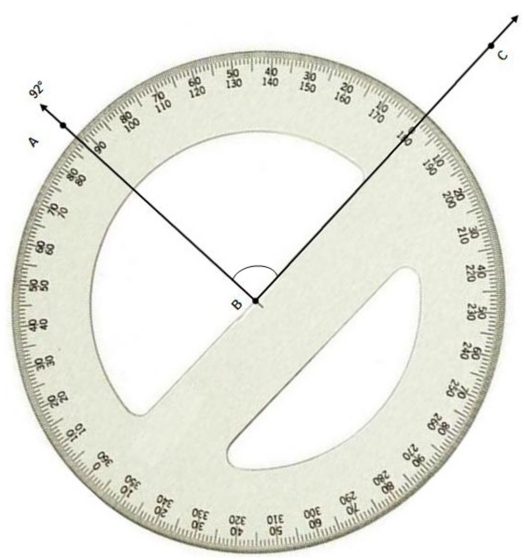
When measuring angles using a protractor it is important that the reading always begins at zero. Incorrect measures often result from students beginning at the 180° mark. Emphasize to students that they must always line up one arm of the angle with the zero degree and then begin counting up from zero until they reach the next arm. This will avoid incorrect scale readings.

It is also important to recognize that the direction of rotation between angle arms will determine which scale is used on the protractor. Again, lining up the zero degree with the first arm of the angle and counting up to the next arm will avoid any confusion with regard to which scale should be used. Remind students that the arc (hatch mark) between the arms of the angle indicates which angle is being measured.

Estimating angle measures before using the protractor is important in helping students avoid mistaken scale readings. E.g., A student might estimate angle ∠ABC to have a measure of a little more than 90°.



If they measure the angle and mistakenly read the protractor scale as 88° as opposed to the correct measurement of 92°, they should realize that this measurement does not agree with their estimation and therefore is probably incorrect.



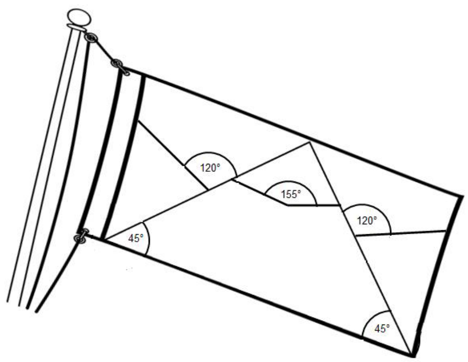
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Create a Flag - Give students a list of angle measures that must be incorporated in an original flag design. Students can create their own flag design that incorporates the specified angle measures. A colour or line type key can be used to indicate the specified angle measures. E.g. “Create a flag design that contains at least one 45°, 120° and 155° angle.”

  This activity can be extended by having students measure any additional angles that may have been formed in their flag creation.



(6SS1.3)

Resources/Notes

Math Focus 6

Lesson 4 (Cont’d): Measuring Angles

6SS1

TG pp. 26 - 30

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS1 Continued

Achievement Indicator:

6SS1.6 Draw and label a specified angle in various orientations, using a protractor.

Elaborations—Strategies for Learning and Teaching

Previously students have sketched angle approximations using benchmarks. At this point they will construct angles with a specified angle measure using a protractor and straightedge.

After students have become proficient measuring angles with a protractor ask them to reverse the process to construct angles with a specified measure using a ruler and protractor. When completed, students should measure their angle using the protractor to ensure that they completed the construction correctly. Discuss the process that was used to construct the angles with the class.
Students may find it easier to use a semi-circular (180°) protractor to construct angles to an accurate degree. It is important to note that as with some circular protractors, there is an inner and outer scale, and that to avoid confusion between the two, students must always start measuring from the 0° mark. Ask students to sketch an approximation of the angle using benchmarks before attempting the actual construction. A mental visualization of this approximation may be sufficient for some students at this point.

When constructing angles, students should always use a straight edge to construct the first arm. This arm may be horizontal unless otherwise specified. The centre mark of the protractor would then be placed on the endpoint of the first arm and the baseline rotated so that the 0° mark is in line with that arm. Keep in mind that the direction of rotation will determine whether the inner or outer scale of the protractor will be used and indicate from which 0° mark (left or right) measurement will begin. Students should then count up from zero to the desired measure, mark that degree and connect it to the vertex using a straight edge. The angle must be indicated by drawing an arc between the two arms and any given points should be labelled correctly remembering that the vertex is always found in the middle of the angle name. Students need lots of opportunities to practice drawing angles of different measures and orientations to attain proficiency.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Use a straightedge and protractor to draw the following angles:

  (i) 54 °

  (ii) 135 °

  (iii) 75 °

  (iv) 156 °

  Label each of the constructed angles and the specified measure.

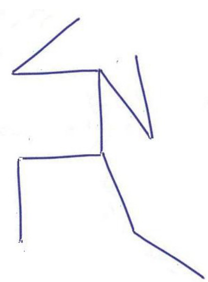
(6SS1.6)

Performance

- Angle Art – Ask students to create pieces of art that incorporate a number of specified angle measures. E.g., "Draw a scene that contains a 46°, 125°, 270° and a 285° angle." Use a protractor to construct these angles. Label each of the constructed angles using an arc and the specified measure. If students do not want to write the angle measures on their artwork they may wish to use a colour key. This would involve colouring the arms of each specified angle a different colour and using a key/legend to indicate its measure.

(6SS1.6)

- Ask students to make a stick person out of pipe cleaners or wire and pose it so that the assigned angles are displayed somewhere on its body. (6SS1.6)



Resources/Notes

Math Focus 6

Lesson 5: Drawing Angles

6SS1

TG pp. 31 – 34

Curious Math: Strange Buildings

6SS1

TG pp. 37 - 38

Math Game: Buried Treasure

6SS1

TG pp. 39-40

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS2 Demonstrate that the sum of interior angles is:

- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

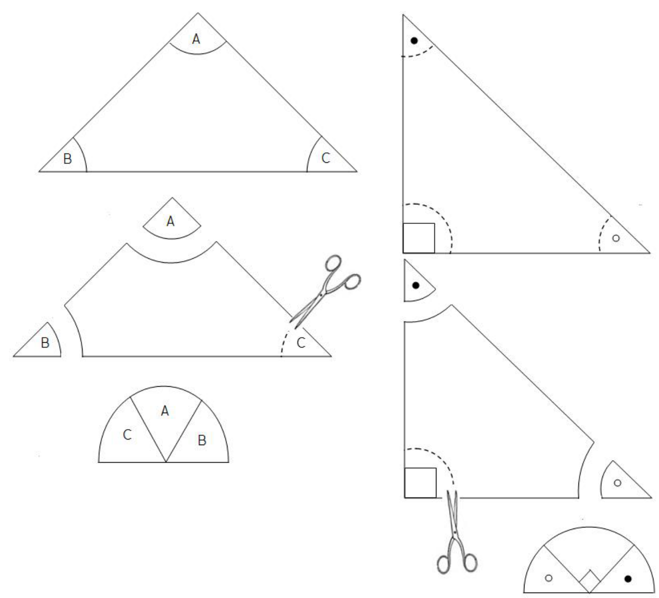
Achievement Indicator:

6SS2.1 Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

Elaborations—Strategies for Learning and Teaching

To determine the sum of the interior angles of triangles and quadrilaterals it is recommended that an exploratory approach be employed. It may also be necessary to review the concept of an interior angle compared to an exterior angle.

To illustrate that the sum of the interior angles of a triangle is the same for all triangles in a visual/tactile manner, create several large triangles of varying shapes and sizes on construction paper or card stock. Use a protractor to trace an arc between the arms of all three angles of each triangle. Ask students to cut the corners off each triangle along the curved arc. Arrange the three corners so that they are all joined at the vertex forming a straight angle. Students will observe that when the three corners are arranged at a common vertex the three angles form a half circle (180°).



By using triangles of various sizes and shapes it becomes evident that the sum of the three interior angles will always be 180° for all triangles.

Cutting the corners of the triangles in an arc and labelling the vertices with letters or symbols will avoid confusion as to which vertices need to be adjacent when aligning the angles. The curved edges also provide a clearer visual for students that the three interior angles of any given triangle form a half circle.

It may be worthwhile to ask students to measure the interior angles of the triangles they used in the above activity using a protractor and then find the sum. They may notice that in some cases their sum does not equal 180° exactly but is very close. This would be a good opportunity to discuss sources of human error in measurement.

(continued)

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Resources/Notes

Math Focus 6

Lesson 5 (Cont'd): Drawing Angles

6SS1

TG pp. 31 – 34

Math Focus 6

Lesson 6: Angle Relationships in Triangles

6SS2

TG pp. 41 - 45

Strand: Shape and Space (Measurement)

Outcomes

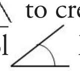
Students will be expected to

6SS2 Continued

Achievement Indicator:

6SS2.1 Continued

Elaborations—Strategies for Learning and Teaching

This concept can also be reinforced using FX Draw and Interactive Whiteboard technology computer software. Use the triangle shape tool  to create a triangle of any size or shape. Using the angle measure tool  label the measure of each interior angle. The program will automatically place the correct measurement within the angle. Ask students to drag each vertex of the triangle to a different position on the Interactive Whiteboard to create a triangle of a new shape and size. The angle measure tool will automatically change the measure of the angle as the student drags the vertex. Students will observe that no matter how they change the shape and size of the triangle, the sum of the interior angles will always be 180°.

Extend this concept to find a missing angle in a given triangle when two other angle measures are known. By subtracting the sum of the two known angles from 180°, the unknown angle measure can be determined.

6SS2.2 Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

After establishing that the sum of interior angles for any triangle is 180°, students could use this information to explore the sum of the interior angles of any quadrilateral. They must first recognize that any given quadrilateral is composed of two triangles. This can be reinforced using tangram pieces. By combining two tangram triangles to form a quadrilateral students should deduce that since the sum of the interior angles of a triangle is 180°, then the sum of the interior angles of the quadrilateral would be 360° (180° + 180° = 360°).

Perhaps the most straightforward manner in which to demonstrate this concept is to ask students to draw a diagonal connecting opposite vertices of various quadrilaterals. Students can be provided with these quadrilaterals on construction paper or card stock or they may create their own. Students can then cut the quadrilateral along the diagonal they drew producing two triangles. Having already learned that the sum of the interior angles of a triangle is 180°, they can make the deduction that because any quadrilateral can be deconstructed into two triangles, the sum of the interior angles of any quadrilateral must be 180° + 180° = 360°.

(continued)

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Journal

- Give students a measure of one angle in a triangle. Ask them to derive three other pairs of possible angle measures for the remaining two angles. Using a protractor draw these three triangles. E.g., "A triangle has a 45° angle. What are three possible sets of measures for the remaining two unknown angles? Draw these triangles using a protractor and a straight edge."

  Three possible angle sets are:

  45° (given) + 45° + 90° = 180°

  45° (given) + 20° + 115° = 180°

  45° (given) + 30° + 105° = 180°

  Etc… (6SS2.1, 6SS1.6)

Resources/Notes

Math Focus 6

Lesson 6 (Cont'd): Angle Relationships in Triangles

6SS2

TG pp. 41 - 45

Math Focus 6

Lesson 7: Angle Relationships in Quadrilaterals 6SS2

TG pp. 46-50

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

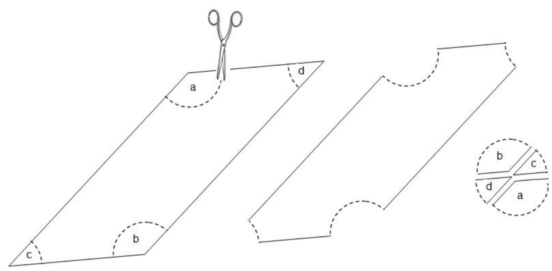
6SS2 Continued

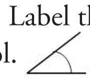
Achievement Indicator:

6SS2.2 Continued

Elaborations—Strategies for Learning and Teaching

This concept may be approached in a similar fashion as triangles were previously. Ask students to cut out quadrilaterals of various shapes and sizes. Use a protractor to draw arcs between the arms of each interior angle. Cut out each angle along the arc and arrange them so that their vertices are adjacent. Students will observe that the four angles of any triangle form a full circle (360°).



FX Draw may also be used to illustrate this concept along with Interactive Whiteboard technology. Use the quadrilateral shape tool  to draw a quadrilateral of any shape and size on the interactive whiteboard. Label the measure of the interior angles using the angle measure tool.  Ask students to physically change the position of the vertices to create quadrilaterals of different shapes and sizes. The angle measure tool will automatically show the change in each angle, and students will observe that the sum of the four angles of any quadrilateral will always be 360°.

Extend this concept to find unknown angles within quadrilaterals given the measure of any three angles. By subtracting the sum of the three known angles from 360°, the unknown angle measure can be determined.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Organize stations within the classroom. Each station will have a quadrilateral or triangle cut-out. All interior angles would be labelled on the cut out except for one angle which has been torn off each shape. Assign students in small groups or pairs to a station. Ask them to find the missing angle measure for their given quadrilateral or triangle. Groups will rotate from station to station until they have found all the missing angle measures.

  (6SS2.2)

- Another way to approach this activity would be to have each group of students construct their own triangle or quadrilateral using a protractor, cut it out and tear off one angle and label the remaining angles. This could be the first station. Groups will then rotate, finding everyone else's missing angle measures. When they arrive back at their own shape the activity is over. (6SS2.2, 6SS1.6)

Resources/Notes

Math Focus 6

Lesson 7 (Cont'd): Angle Relationships in Quadrilaterals

6SS2

TG pp. 46 - 50

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Develop and apply a formula for determining the:

- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms.

  [C, CN, PS, R, V]

Achievement Indicator:

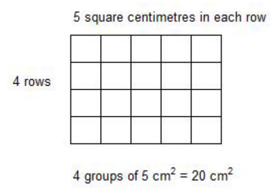
6SS3.1 Explain, using models, how the area of any rectangle can be determined.

Elaborations—Strategies for Learning and Teaching

It is important to approach this outcome as an investigation rather than providing a rote formula. Students should be given the opportunity to discover how to find the area of a rectangle independently.

This can be done using 1 cm^2 grid paper. Students must first understand that each square unit on the grid paper has a 2-D area of 1 cm^2 meaning it has linear dimensions of 1 cm in height and 1 cm in width.

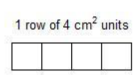
Ask students to draw rectangles with various dimensions on 1 cm^2 grid paper. They will initially find the area by counting the number of 1 cm^2 units enclosed within the rectangle. Ask students to find the length and width of each rectangle. Prompt students to view these dimensions as rows of a given number of square centimetres. From this reasoning, students should deduce that by viewing the dimensions of a rectangle as groups of square units (width x length) the area of the rectangle can be found. E.g.,



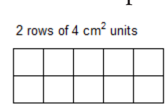
Students should realize that a change in the unit of measurement (mm^2, cm^2, m^2) simply means that the uniform size of each unit within the rectangle has changed but the area can still be determined in the same way. This point can be illustrated by finding the area of rectangles on grid paper with units of varying dimensions.

This can be addressed using grid paper as previously mentioned. Students may also use fraction strips to model rectangles and squares of different areas. The strip format of the units will help students visualize area in terms of groups of units. It will first be important to establish the dimensions and area of each unit within the strip. E.g.,

A student uses a single strip of 4 units each with an area of 1cm^2, to model a rectangle with dimensions of 1 × 4.



This rectangle consists of one row of four 1cm^2 units, thus the area of the rectangle is 4cm^2. The student might add another strip of 4 units to create the following rectangle.



This rectangle consists of two rows of four 1cm^2 units, thus the area of the rectangle is 8cm^2 (2 × 4 = 8). Other rectangles can be modelled and the area found through this method.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Resources/Notes

Math Focus 6

Lesson 8: Area of a Rectangle

6PR3

6SS3

TG pp. 51 - 54

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued

Achievement Indicator:

6SS3.2 Generalize a rule (formula) for determining the area of rectangles.

Elaborations—Strategies for Learning and Teaching

Students have not had exposure to the concept of a formula. A formula is a rule (process) that can be used to find a desired quantity or measurement. From the activity completed previously, students derived the general rule that the area of any rectangle can be found by multiplying its length by its width. Students should be able to express this rule in words and as a formula written using variables to represent the changing quantities.

In words: The area of a rectangle can be found by multiplying its length by its width.

As a formula: $A = l \times w$, where A = area, l = length and w = width

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Designing a Dream House - Provide students with 1cm^2 grid paper. Ask them to design a floor plan for their dream house. Rooms can be rectangles of various dimensions. Students may even design two or three floors on separate sheets of grid paper. Once the floor plans are complete, ask students to use the formula they derived for area of a rectangle to find the area for each room in their floor plan. Once the floor area of each room is found, students can then find the combined floor area of the entire house. (6SS3.2)

- This activity could be extended to incorporate outcomes from other units. For instance students could be asked to develop a scale for their floor plan. What would each square centimetre represent in a real house? Students could also be given prices for various square areas of flooring and asked to calculate the cost of flooring their dream house. This could be part of a cumulative project, incorporating outcomes from multiple unit. (6SS3.2)

- Ask students to draw / construct rectangles of the same dimensions using cm^2 and pattern block grid paper. Using the formula developed previously ask them to find the area of each rectangle. Ask:

  Did the size of the unit affect the numerical value of the area in each case?

  Which area is actually larger? Explain how this is possible even though the numerical value of each area is the same.

  (6SS3.2)

Resources/Notes

Math Focus 6

Lesson 8 (Cont'd): Area of a Rectangle

6PR3

6SS3

TG pp. 51 - 54

Math Focus 6 Masters Booklet p.40 and p.47.

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

6PR3 Represent generalizations arising from number relationships, using equations with letter variables.

[C, CN, PS, R, V]

Achievement Indicators:

6PR3.3 Write and explain the formula for finding the area of any given rectangle.

Elaborations—Strategies for Learning and Teaching

The focus here is on deriving expressions to represent perimeter and area of various polygons as opposed to actually calculating these values given known measurements.

At this point it may be necessary to review area as the amount of flat (2-dimensional) surface within an enclosed shape. Students may find it easier to think of area in terms of what they would be colouring in if asked to colour a shape. This may be illustrated using computer drawing programs. Ask students to construct any enclosed shape and then use the “fill” or “paint” feature to colour in the shape.

It may be necessary to review the use of square units when measuring area. Students should realize that square units are used to measure 2-dimensional shapes. They will not be expected to understand $n \times n = n^2$, as they will not be dealing with exponents until junior high. However, at this level students may view the exponent of 2 on any squared unit as meaning 2-dimensional. Once students have derived an expression for the area of a rectangle, they will be able to use this formula to find the area of any given rectangle.

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Resources/Notes

Math Focus 6

Lesson 8 (Cont'd)**:** Area of a Rectangle

6PR3

6SS3

TG pp. 51 - 54

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued
Achievement Indicator:

6SS3.3 Explain, using models, how the perimeter of any polygon can be determined.

Elaborations—Strategies for Learning and Teaching

Students have been introduced to perimeter in previous grades as the distance around a shape. Perimeter can be illustrated by having a student walk around the edge (perimeter) of the classroom, or trace the edges of their desk or textbook cover with their finger.

It may be beneficial to review the concept of a polygon as being enclosed shapes with three or more sides (polygon meaning "many sided figure").

The use of variable *l* (length) and *w* (width) will be familiar to students at this point. However, in the case of squares, hexagons or other regular polygons having equal side lengths there will be no actual length and width dimensions as all side measures are equal. In this case the variable *s* (side) may be used.

Students have had experience finding perimeter by measuring the sides of a polygon and finding the sum. The focus here however is on deriving a formula that can be used to generalize the process by which the perimeter of any given polygon can be found.

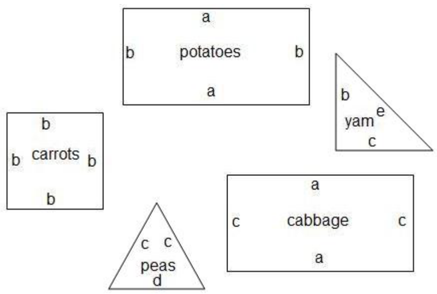
Perimeter can be initially modelled and determined by using drawings of various polygons on 1 cm^2 grid paper and using the number of units along the combined sides to find the perimeter. However, modelling of this nature would be restricted to squares, rectangles or combinations of these.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

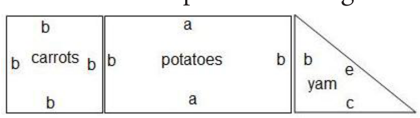
Suggested Assessment Strategies

Performance

- Farmer's Fence - Provide students with a variety of polygons (squares, rectangle, triangles, parallelograms etc.) cut out from card stock or construction paper and having common side lengths labelled with a variable. Each polygon represents a different shaped field on a farm. Each different variable indicates the length of each side.



  In groups students can arrange the polygons to create various combinations of different shaped farms. E.g.



  You (The Farmer) want to build a fence around your farm. Write a formula for the perimeter of your farm. E.g. The formula for the above farm would be

  $P = 3b + 2a + c + e$

  Once students have found the formula for the perimeter of their farm assign each variable a number value and ask students to use their formula to find the perimeter. For example: if $a = 5$ metres, $b = 2$ metres, $c = 3$ metres, $e = 4$ metres. The perimeter of the farm above would be

  $P = 3(2) + 2(5) + 3 + 4$

  $P = 6 + 10 + 3 + 4 = 23$ metres

  (6SS3.3, 6SS3.4, 6SS3.7)

- This activity could also be extended to incorporate other outcomes involving multiplication with decimals by giving students a set price per metre of fencing and asking them to calculate the total cost of fencing their farm. (6SS3.7)

Resources/Notes

Math Focus 6

Lesson 9: Perimeter of a Polygon

6PR3

6SS3

TG pp. 55 - 58

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued

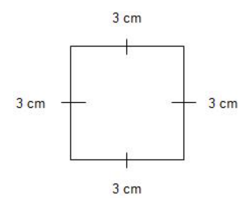
Achievement Indicator:

6SS3.4 Generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares.

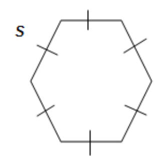
Elaborations—Strategies for Learning and Teaching

Provide students with a variety of polygons on paper or cardstock. Ask them to identify which dimensions of that particular polygon are congruent (the same). Illustrate that the sum of sides having equal measures can be found through repeated addition and multiplication.

For example, in the square below the perimeter may be written as 3 cm + 3 cm + 3 cm + 3 cm or, through multiplication, 4 × 3cm (Four groups of 3 cm) = 12 cm.



In the case where the measure of a side is not specified, students should be able substitute a variable for the unknown amount and use this to derive their formula. E.g. In the regular hexagon below the side length is unknown so we use the variable *s* to represent it.

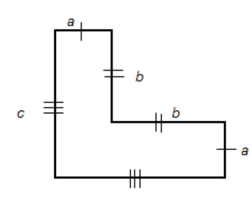


Thus, the expression for its perimeter may be written in repeated addition as $s + s + s + s + s + s$ or more conveniently as 6s.

For polygons with two or more different side measures addition of the combined congruent dimensions will be required. For example;

The expression for perimeter would be:

$P = 2a + 2b + 2c$.



Problems of this nature will not always necessarily involve regular polygons. However, as long as students can recognize congruent sides, a formula should be easily derived.

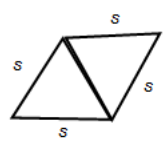
General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

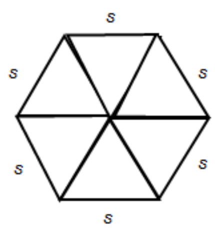
Performance

- Provide students with pattern blocks and other manipulatives of varying shape and size. Ask them to identify which dimensions (sides) on the pattern blocks are of equal length. Trace these patterns on paper. Assign the equal sides a common variable or colour to represent the unknown length that these sides share. Write an expression to represent the perimeter of each pattern block.

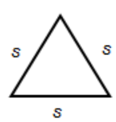
  E.g.,



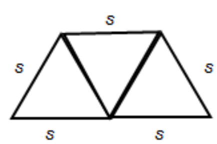
$P = s + s + s + s$
or
$P = 4s$



$P = s + s + s + s + s + s$
or
$P = 6s$



$P = s + s + s$
or
$P = 3s$



$P = s + s + s + s + s$
or
$P = 2s + s + s + s$
or
$P = 5s$
or
$P = 3s + 2s$

(6SS3.4)

Resources/Notes

Math Focus 6

Lesson 9 (Cont'd): Perimeter of a Polygon

6PR3

6SS3

TG pp. 55 - 58

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

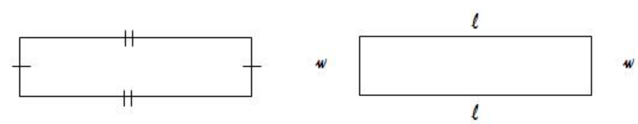
6PR3 Continued
Achievement Indicator:

6PR3.4 Write and explain the formula for finding the perimeter of any given rectangle.

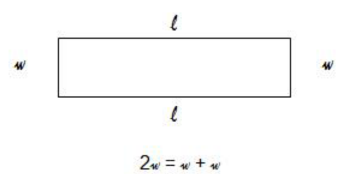
Elaborations—Strategies for Learning and Teaching

Remind students that perimeter is a measure of linear distance and is thus written in one dimensional units (e.g., mm, cm, m, km etc…).

The focus here is not on actually finding the perimeter of a given rectangle but on deriving a rule that can be used to find the perimeter of any given rectangle. Present students with a variety of rectangles. Students must recognize that sides with same measure are indicated using the same hatch mark or variable.



Ask them to identify which sides are congruent (equal). The sum of these congruent sides can be found through repeated addition and therefore, by multiplication. For example, in the rectangle below students may write the combined width as $w + w$ or $2 \times w$ (Two sets of the width).



The combined width and lengths can be expressed as $2w$ and $2l$ respectively. Thus, the perimeter would be expressed as Perimeter = $2w + 2l$. Students should recognize that this formula is the same as Perimeter = $w + w + l + l$.

It will also be necessary to discuss the convention of writing multiplication involving a variable and a number without using the multiplication symbol. For example, $2 \times w$ would be written as $2w$. This is the form most commonly used in Grades 7, 8 and 9 when writing expressions.

6PR3.5 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication; e.g., $a + b = b + a$ or $a \times b = b \times a$.

The commutative property simply shows that the order in which terms are added or multiplied does not affect the final outcome.

This can be illustrated to students using real number examples such as:

$2 + 4 = 6$ and $4 + 2 = 6$

$5 \times 2 = 10$ and $2 \times 5 = 10$

In terms of developing formulae to find perimeter or area, the same would apply. E.g.,

$P = 2w + 2l$ would yield the same results as $P = 2l + 2w$

$A = w \times l$ would yield the same results as $A = l \times w$

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Performance

- Ask students to measure the length and width of a rectangular room in the school using a measuring tape. If the floor has square tiles on it, students could find the length of one tile and find the length of each wall by determining the number of tiles that run along it. Partial tiles can be rounded off to the nearest whole. Once the length and width of the room has been determined ask students to use the formula they derived to calculate the room's perimeter.

  Extension:

  "Approximately how many times would you have to walk around this room to have walked 1 km?"

  "What is the area of this room?" (6SS3.1, 6PR3.4, 6SS3.7)

Resources/Notes

Math Focus 6

Lesson 9 (Cont'd): Perimeter of a Polygon

6PR3

6SS3

TG pp. 55 - 58

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Develop and apply a formula for determining the:

- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

Achievement Indicator:

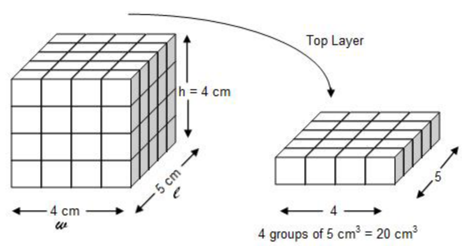
6SS3.5 Explain, using models, how the volume of any right rectangular prism can be determined.

Elaborations—Strategies for Learning and Teaching

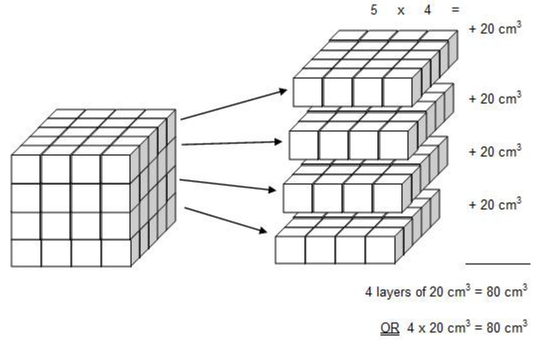
Students have been introduced to volume as the amount of 3-dimensional space occupied by an object. Students should be reminded that volume is measured in cubed units such as (mm^3, cm^3, m^3). Again, because students have not yet been introduced to exponents they may think of the exponent 3 as meaning 3-dimensional.

Use cubic centimetre blocks to model right rectangular prisms. As each cube has a volume of $1cm^3$, the total volume can initially be found by counting the number of cubes in the prism. As this will be very time-consuming for larger prisms it will be necessary to derive a formula to find its volume.

Begin by establishing the 3-dimensions of the prism. This is done simply by counting the number of cubes along the length, width and height. Keep in mind that these dimensions are linear distances and are not measured in cubed units. The height of the prism will indicate how many layers of cubes are in the prism. Students will be familiar with the concept of layers in a three dimensional solid from working with base-ten flats. First find the volume of one layer. For example in the prism below students would determine that the top layer is composed of four rows of five $1cm^3$ blocks. Therefore the volume of the top layer would be $20\ cm^3$(4 x 5 = 20).



Students should then draw the conclusion that because there are 4 layers and each has a volume of $20\ cm^3$, the total volume of the prism must be $80\ cm^3$ (4 groups of $20\ cm^3$)



General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Arrange students into small groups or pairs. Provide each group with a different number of cubic unit manipulatives. These could be cubic centimetre blocks, multi-link cubes or both depending on what is available. Have each group construct the largest right rectangular prism that they can, using the blocks they were given. Have each group determine the volume of their own prism. Students can then leave their prism at their desk as a station. Groups can then rotate around the room finding the volume of the prisms constructed by other groups. Once the activity is complete ask students to share their findings with those of other groups to compare their methods and verify their results. (6SS3.5)

Resources/Notes

Math Focus 6

Lesson 10: Volume of a Rectangular Prism

6PR3

6SS3

TG pp. 59 - 62

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued

Achievement Indicator:

6SS3.6 Generalize a rule (formula) for determining the volume of right rectangular prisms.

Elaborations—Strategies for Learning and Teaching

From work with finding volume from the models used previously, students should have come to the conclusion that the volume of a given right rectangular prism can be found by finding the number of cubes in one layer (length x width), and then multiplying the volume of one layer by the number of layers (height).

Thus, the general formula $V = l \times w \times h$ can be derived, where V = volume, l = length, w = width and h = height.

Students should also be able to interpret this formula in words as "The volume of a rectangular prism can be found by multiplying the length by the width by the height."

Remind students of the commutative property and that the formula may be written with the three dimensions being multiplied in any order.

For example,

$V = l \times w \times h$

$V = w \times h \times l$

$V = h \times l \times w$

Etc…

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Provide students with various base-ten blocks. Ask them to determine the volume of each different type of block (E.g. A unit is $1cm^3$, a rod is $10cm^3$, a flat is $100\ cm^3$, and the cube block is $1000cm^3$.).

(6SS3.6)

Resources/Notes

Math Focus 6

Lesson 10 (Cont'd): Volume of a Rectangular Prism

6PR3

6SS3

TG pp. 59 - 62

Strand: Shape and Space (Measurement)

Outcomes

Students will be expected to

6SS3 Continued

Achievement Indicator:

6SS3.7 Solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.

Elaborations—Strategies for Learning and Teaching

This will involve the application of concepts previously developed to solve problems.

It is important for students to realize that in many cases solving a complex problem involves the process of solving several simpler problems and that it may take several steps to solve the whole problem.

Flag Colour Areas - Provide students with a variety of flags from different countries that have rectangular patterns, E.g. Newfoundland Independent Flag, Ireland, Latvia, Costa Rica, France, Germany, Austria, Yemen, Poland, Spain, Ivory Coast, etc… A collection of flags could be acquired as a cross-curricular activity in Social Studies or students could create replications of the flags in art class. Make sure all flags are the same size. The flags may also be printed or drawn and coloured on 1 cm^2 grid paper to make the length and width of each rectangle in the flag easier to identify.

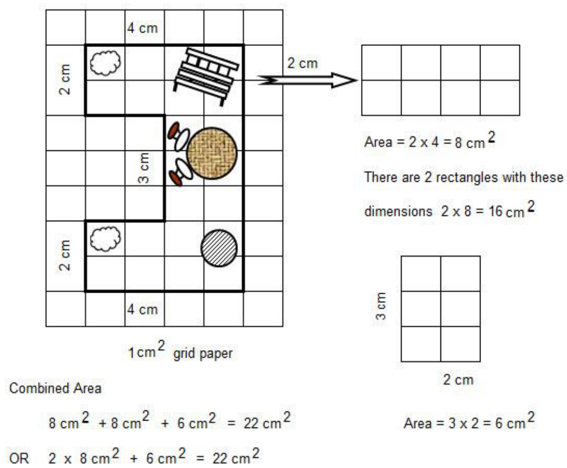
Distribute the various flags amongst the students. Ask them to identify the length and width of each rectangle in their flag's design. This could be done by measuring with a ruler or, if the flag is printed on grid paper, by counting the units along each edge. Using these dimensions students can determine the total area of each colour on the flag. E.g., If students are given the Newfoundland Independent flag they would find what area of the flag is pink, white and green. This activity could also be organized as a station activity with a different flag at each station.

General Outcome: Use Direct and Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Ask students to design a patio/deck floor plan for their Dream House on 1 cm^2 grid paper. Set the condition that the deck design must be composed of two or more rectangles. Ask students to answer the following questions :
  - What is the total area of their deck that would need to be stained? (This would be done by calculating the area of each individual rectangle and then finding the sum.)

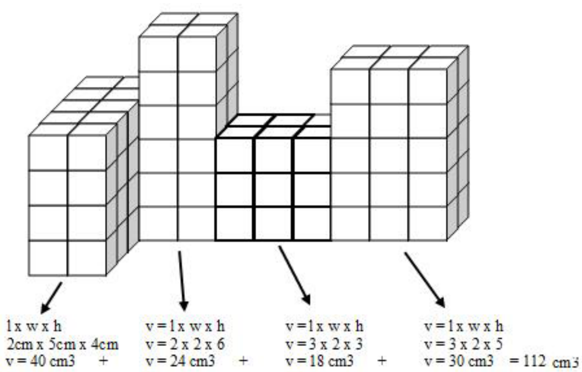


  - If you want to put patio lanterns around the perimeter of your deck. How much wire would you need?

(6SS3.3, 6SS3.4, 6SS3.1, 6SS3.2, 6SS3.7)

Performance

- 3-D Building - Ask students to construct right rectangular prisms of various dimensions using multi-link cubes or Lego/Duplo blocks. Combine these prisms to create a large building. Ask students to calculate the total volume of the building. This would be done be calculating the volume of each prism in the structure and finding the sum.



(6SS3.5, 6SS3.6, 6SS3.7)

Resources/Notes

Math Focus 6

Lesson 11: Solving a Problem by Solving a Simpler Problem

6SS3

TG pp. 63 - 66