Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers," and they advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers and Department of Education officials to plan and develop cooperatively the curricula in mathematics, science, language arts and social studies in both official languages.

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. This

foundation document is supplemented by others that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO.

This curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback; and (v) learners benefit, both socially and intellectually, from a variety of learning experiences, both independently and in collaboration with others.

Program Design and Components

A. Program Organization

As already indicated, the mathematics curriculum is designed to support the six Essential Graduation Learnings (EGLs). While the curriculum contributes to students' achievement of each of these, the communication and problem-solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then presents outcomes at four key stages of the student's school experience.

This particular curriculum guide presents specific curriculum outcomes for each grade level. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.

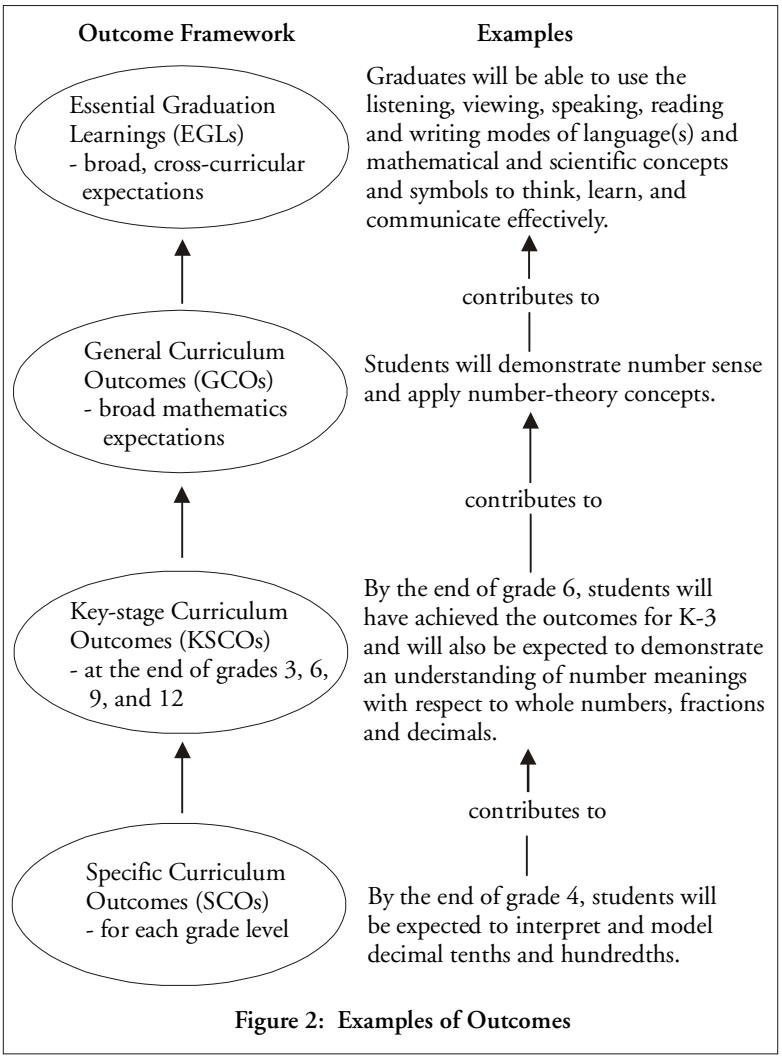

Figure 2: Examples of Outcomes

It is important to emphasize that, while the grade level outcomes (SCOs) provide a framework on which educators will base decisions regarding instruction and assessment, they are not intended to limit the scope of learning experiences. Although it is expected that most students will be able to attain the outcomes, some students' needs and performance will range across grade levels. Teachers will need to take this variation into consideration as they plan learning experiences and assess students' achievement.

The presentation of the specific curriculum outcomes follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and **does not represent a suggested teaching sequence**. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes, such as those pertaining to patterns and data management, may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province, such as an election, an exhibition, or a fair.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)

These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Meeting the Needs of All Learners

The F*oundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student development as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction, to accommodate differences in student learning styles. Different instructional modes are clearly appropriate; for example, some students are primarily visual learners while others learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies. This implies the use of a variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Nevertheless, other resources will be significant in the mathematics classroom in as much as they support achieving the specific curriculum outcomes. Many manipulative materials need to be readily at hand, and technological resources, e.g., software and videos, should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM), the *Addenda Series,* the *Navigations Series* and *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* or *Elementary and Middle School Mathematics: Teaching Developmentally* (John van de Walle), *Developing Number Concepts Using Unifix Cubes* (Kathy Richardson), and *About Teaching Mathematics: A K-8 Resource* (Marilyn Burns)

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through learning centres, teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

H. Career Connections

Mathematics plays a major role in many career options available to students. Teachers should take every opportunity to point out to students the variety of career options that utilize mathematics in a major way. All strands of the mathematics curriculum can be directly connected with careers. For example, engineering programs require understanding of patterns and relationships; pharmacists, optometrists, carpenters, electricians, and surveyors use measurement on a day-to-day basis; business draws largely upon data management, and meteorologists use probability to report on weather data.

It is important that students at all grade levels understand the significance of working hard and doing as well as possible in mathematics in order to keep career options open. A strong mathematics program can keep doors to future choices open to students.

Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are not only critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessment and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment would include i) the use of a wide variety of assessment strategies and tools, ii) aligning assessment strategies and tools with the curriculum and instructional techniques, and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborates good assessment practices and it serves as a guide for student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

Designing an Instructional Plan

It is important to design an instructional plan for the school year. This plan should reflect the fact that specific curriculum outcomes (SCOs) falling under any given general curriculum outcome (GCO) should not be taught in isolation. There are many opportunities for connections and integration across the various strands of the mathematics curriculum.

Consideration should be given to the relative weighting for outcomes under each GCO so that this can be reflected in the amount of time devoted to each aspect of the curriculum. Naturally, time spent must be sensitive to the background of students as well as to cross-curricular issues. Without an instructional plan, it is easy to run out of time in a school year before all aspects of the mathematics curriculum have been addressed. A plan for instruction that is sufficiently comprehensive to cover all outcomes and strands will help to highlight the need for time management.

It is often advisable to use pre-instructional assessment to determine what students have retained from previous grades relative to a given set of outcomes. In some cases, pre-instructional assessment may also identify students who have already acquired skills relevant to the current grade level. Pre-instructional assessment is often most useful when it occurs one to two weeks prior to the start of a set of outcomes. In this case, a set of outcomes may define a topic or unit of work, such as fraction concepts and operations. When assessment is done early enough and exposes deficiencies in prerequisite knowledge/skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of development or coverage in the previous grades. This may imply that an adjustment is required to the starting point for instruction. As well, it may be necessary to meet with previous grade level teachers to address these concerns so the identified concerns are addressed in subsequent class groups.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in overall savings in time for both disciplines. The most obvious of these connections relates to the use of measurement in science and the use of a variety of data displays in social studies.

Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for each grade. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled "Elaboration–Instructional Strategies/ Suggestions," provides a clarification of the specific curriculum outcome(s), as well as suggestions for possible strategies/activities which could be used to help students achieve the outcome(s). While the strategies/activities presented are not intended to be rigidly applied, they will help to further clarify depth and breadth of treatment of the specific curriculum outcome(s). They will also illustrate ways to work toward the achievement of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol □ .

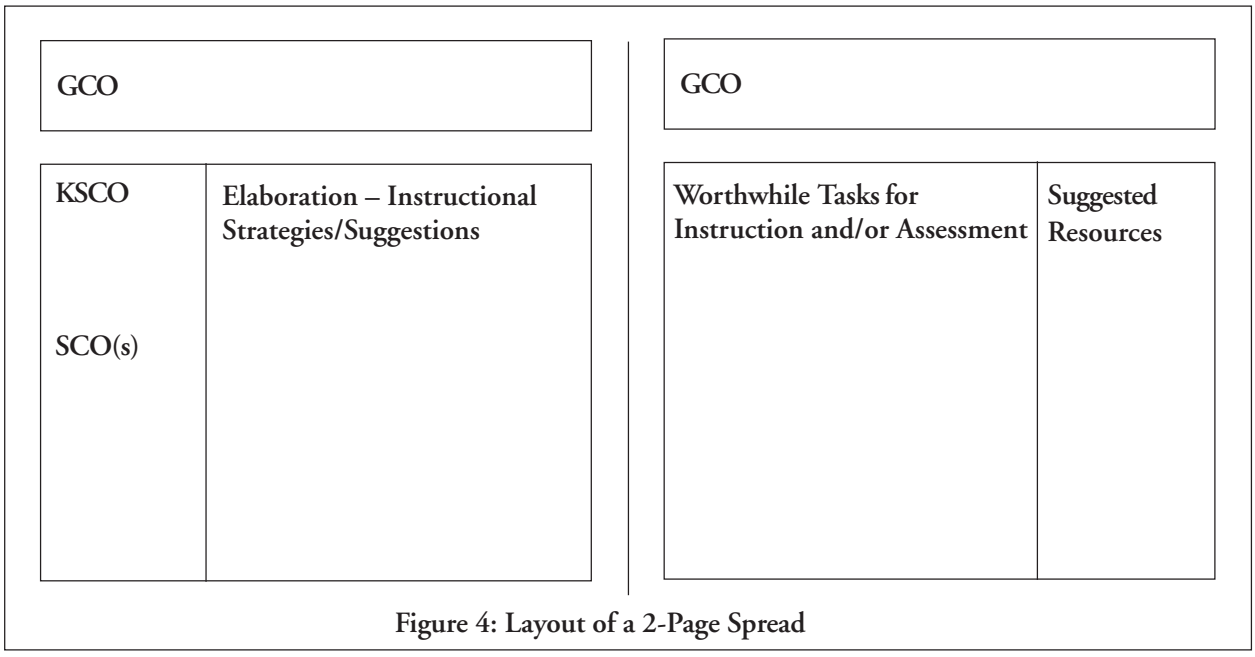

Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled "Worthwhile Tasks for Instruction and/or Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Suggested Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).