Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number theory concepts.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, the students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals

iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 6, students will be expected to

A1 represent, estimate, and order numbers to billions using fraction and decimal notation

Elaboration - Instructional Strategies/Suggestions

A1 Students at this level may still need practice, or further experience, in reading and recording very large numbers, including fractions and decimals (e.g., $3\frac{1}{2}$ million, 3.5 million).

In grade 6, an emphasis should be placed on representing large whole numbers using rounding and decimal notation. For example, rather than writing 34 518, students should be encouraged to approximate it as 34.5 thousands. Although 12 340 000 could technically be written as 123.4 hundred thousand it is normally written as 12.34 million using the million as the unit.

Ask students to practise recording numbers that are presented to them orally and rounding each one to the nearest tenth or hundredth of a million. For example, one hundred eight million, ninety-three thousand, forty-six might be estimated as 108.1 million or 108.09 million.

Students should also be able to recognize and represent fractional parts of large numbers.

43 489 784 is about 43 million

247 986 is about million

8 762 154 375 is about 8 billion

- ☐ Use real data when possible. For example: The population of Atlantic Canada, as recorded in the 1991 census, is two million, three hundred seventy-eight thousand two hundred ninety-seven. (Newfoundland - 568 474, Prince Edward Island - 129 765, Nova Scotia - 920 000 and New Brunswick - 760 058.) Ask students to work in groups to approximate the populations of each of the four provinces.
- ☐ Ask students to find various representations for large numbers in newspapers and magazines. Encourage discussion on the need for accuracy in reporting and the appropriate use of rounded numbers.
- ☐ Ask students to prepare a report on populations of Canadian cities. Ask that the report include a graph of the five most populous cities, comparing them to the five most populous cities of another country.

This should be taught in conjunction with A7 and A8.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A1.1 Prepare and shuffle 5 sets of number cards (0-9 for each set). Ask students to select nine cards and ask him/her to arrange them to make the greatest possible and least possible whole number. Ask students to read each of the numbers. Consider extending the activity by asking the student to determine

- the five largest and the five smallest whole numbers that could be made using all nine of the digits selected
- the number of $1000 bills one would get if the greatest and least numbers represented money amounts.

Interview

A1.2 Discuss the idea of counting to 100 by 10s. Ask: How many numbers are named? Then ask: How many numbers are named when counting to 1 000 000 by thousands? by hundreds? by tens? Then ask: How many numbers are named when counting to 10 000 000 000 by hundred thousands? by thousands? by hundreds?

A1.3 Tell the student that light from a star takes 7000 centuries to reach earth. Ask: How many years is that?

A1.4 Present this library information to students

Metropolitan Toronto Library	3 068 078 books
Bibliotheque de Montreal	2 911 764 books
North York Public Library	2 431 655 books

Ask them to rewrite the numbers in a format such as __.__ million or __.__ __ million books. Then ask them to make comparison statements about the number of books.

Portfolio

A1.5 Ask students to collect newspaper and magazine clippings in which large numbers are used. Discuss the type of situations in which one is most likely to encounter large numbers and why that might be.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, the students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals

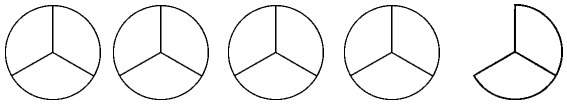
iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 6, students will be expected to

A2 represent, compare and order fractions and decimals

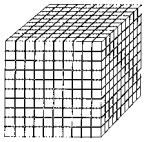
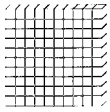
Elaboration - Instructional Strategies/Suggestions

A2 Students should fluently move between mixed number and improper fraction formats of a number. Rather than only applying a rule to move from one format to the other, students should be encouraged to focus on the meaning. For example, since means 14 thirds and it takes 3 thirds to make 1 whole, represents 4 wholes and another 2 thirds or 4 . (It is important to change the shape of the "whole", so that students experience modeling of fractions in a variety of forms)



Often it is easier for students to grasp the size of mixed numbers than improper fractions. Students may use mixed numbers to develop understanding of the size of improper fractions. For example, a student may think of as 4 (or a little more than 4).

Students should be familiar with base ten block representations of decimals.

If  = 1, then  =0.1,  =0.01 and  =0.001

These representations help students visualize the relative sizes of decimals.

Measurement contexts continue to be natural ones for decimal situations. For example, a student might consider how many kilograms of ground beef one would need for four hamburgers or how many kilometres one could walk in a minute.

- As a class activity, ask students to make a place value chart that is divided into 6 sections representing numbers to the thousandths. Select number cards, one at a time, (or toss a 10-sided die) and ask each of the students to decide in which section to place the digit as it is selected (or tossed), in order to try to make the greatest (least) possible number. This activity encourages students to take risks and to think about probability. After all six digits are called, ask students to compare their numbers. Extensions might include estimating how far each student's number is from a target number or determining how the number would be rounded for a newspaper article and what it might represent.

This should be taught in conjunction with outcome A9 and B5.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A2.1 Ask students to colour squares to show why 3 = .

A2.2 Ask students to place the given numbers on the number line (could use string and clothes pins) in the most appropriate location.

$$\frac{13}{3},\ 4.57,\ 4.057,\ \frac{14}{3},\ 4.17$$

Ask students to explain their placement of the numbers.

Paper and Pencil

A2.3 Contexts that lend themselves to using large numbers include astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data and metric measurements. An interesting activity involving decimals might require students to complete a chart such as:

In 0.1 years from now, I could...
In 0.01 years from now, I could...
In 0.0001 years from now, I could...

A2.4 Ask students to determine the number of whole numbers between 2.03 million and 2.35 million.

A2.5 Provide thousandths grids. Ask students to shade the grids, one at a time, to show the following decimals:

0.004
0.203
0.023
1.799

Ask them to tell which was easiest to do and why.

Portfolio

A2.6 Ask students to write a report on what he/she has learned about decimals and what questions he/she may now have concerning the topic.

Suggested Resources

Fraction Circles

Fraction Factory Pieces

Base Ten Materials

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore integers, ratios and percents in common, meaningful situations

SCO: By the end of grade 6, students will be expected to

A3 write and interpret ratios, comparing part-to-part and part-to-whole

Both part-to-whole and part-to-part ratios are comparisons of two measurements of the same type of thing. The measuring unit is the same for each value. (Elementary School Mathematics, p. 275)

Elaboration - Instructional Strategies/Suggestions

A3 Use the students themselves, counters, or other simple models to introduce the concept of ratio as a comparison between two numbers. For example, in a group of 3 boys and 2 girls,

3:2 tells the ratio of boys to girls (part-to-part)
3:5 tells the ratio of boys to the total group (part-to-whole)
2:5 tells the ratio of girls to the total group (part-to-whole)
2:3 tells the ratio of girls to boys (part-to-part)

Students should read "3:2" as "3 to 2" or "3 __ for every 2 ___."

Exploring ratios can productively take place in everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or "3 to 1") or in relation to other topics in mathematics.

For example, students might examine the ratio of

- the perimeter of a square to its side length
- the corresponding sides of similar shapes (see E9)

Ratios and fractions are both comparisons. Sometimes the fraction or ratio compares a part to a whole. For example, if $\frac{3}{5}$ of a rectangle is shaded, the ratio of shaded part to the whole is 3:5. Sometimes the fraction describes a multiplicative relationship. For example, if there are 4 red circles and 8 white circles, there are as many red circles as white circles and the ratio of red to white is 4:8.

To further illustrate, consider that there are fourteen boys and eleven girls in a class. The ratio (fraction) of boys to all the students (part-to-whole) is . This can be expressed as "fourteen twenty-fifths of the students are boys." The ratio of boys to girls is 14:11 (read "fourteen to eleven"), can be written , and describes the relationship between the number of boys and the number of girls.

Note: While the concept of ratio (in which the units are identical) may be contrasted with the concept of rate (for which each quantity has a different unit (e.g., km/h)), rate is not an outcome in grade six. However, rate was explored informally in grade five.

This should be taught in conjunction with A4, A5, C5, and C6.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A3.1 Ask students to model two situations which could each be described by the ratio 3:4. Specify that the situations must involve a different total number of items.

A3.2 Ask students to find the following body ratios, comparing results with others:

wrist size:ankle size
wrist size:neck size
hand width:hand length
arm span:body height

A3.3 Ask students to select 20 tiles of four different colours so that pairs of colours show the following ratios: 4 to 3, 2:1, .

Paper and Pencil

A3.4 Give students the following information and ask them to write/read ratio comparisons and to identify those that can be expressed as fractions.

4 cats 3 goldfish 2 hamsters

$\frac{1}{3}$

Interview

A3.5 Ask students whether or not he/she believes that the ratio of the population of any city in Canada to the total population of Canada could be 1:2. Students should explain their responses.

A3.6 Ask: Why might you describe the ratio below as 4:1? as 1:4? Are there other ratios to describe the boys and girls?

B B B B G

B= boy G=girl

Presentation/Research

A3.7 Students might investigate how ratios are used in various sports (e.g., batting averages, goals against).

Portfolio

A3.8 Students might create a report on ratios found in the classroom. They could include such ratios as boys:girls; teacher:pupils; desks:students; tables:students; pencils:students; and square metres of classroom:student.

Suggested Resources

Coloured Tiles

Multilink Cubes

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore integers, ratios and percents in common, meaningful situations

SCO: By the end of grade 6, students will be expected to

A4 demonstrate an understanding of equivalent ratios

Elaboration - Instructional Strategies/Suggestions

A4 Students should understand why, for example, the ratios 2:3 and 4:6 represent the same relationship, i.e., if for every 2 of one item, there are 3 of another, then, for every 4 of the first item, there are 6 of the second.

Many students will recognize the similarity between the concept of equivalent ratios and the concept of equivalent fractions. For example, in the diagram above, $\frac{2}{5}$ of the counters in the top row are white, but of the counters in total are white, so = . Also, the ratios 2:5 and 4:10 are equivalent, since if 2 of every 5 are white, then 4 of every 10 would also be white.

Students should use the concept of equivalent ratios primarily to make interpretation of situations easier. For example, in a large bag of marbles, the ratio of blue marbles to the total number of marbles is 4:10 (i.e., 4 out of every 10 marbles are blue). Use this to predict the number of blue marbles you would expect in 100 selections.

Students should recognize that there are many equivalent ratios but should also be able to determine the simplest form for a given ratio.

- ☐ Ask students to work in pairs or small groups to discuss equivalent ratios if Sue received 36 votes and Sam received 9 votes.
  36:9 or 4:1 (Sue received 4 votes for every 1 vote Sam received.)
  9:36 or 1:4 (Sam received 1 vote for every 4 Sue received.)
  36:45 or 4:5 (Sue received 4 votes for every 5 votes cast.)
  9:45 or 1:5 (Sam received 1 vote for every 5 cast.)
- ☐ Ask students to work in pairs writing situations for which classmates would practise dealing with equivalent ratios.
- ☐ Ask students to use Cuisenaire rods to find rods in the ratio of 1:2 (e.g., white to red). By writing the values of the rods, equivalent ratios are generated (e.g., 1:2 = 2:4).

This should be taught in conjunction with A3, A5, C5, and C6

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A4.1 Ask students to set up a display of counters which shows the ratio 5:6. Set up another display to show an equivalent ratio so that the reason for the equivalence is clear.

Paper and Pencil

A4.2 Present the diagram below.

x x x o

x x x o

x x x o

Ask students to write equivalent ratios from this diagram. Ask them to explain their thinking.

A4.3 For each of the following, ask students to find an equivalent ratio in which one of the terms is 20.

4:6 10:30 3:5 4:5 3:6

A4.4 Ask the student to list all the ratios that are equivalent to 1:2 in which the second term is less than 50.

Interview

A4.5 Ask students to explain how a place value chart can be used to generate equivalent ratios.

A4.6 Ask: Why do you get an equivalent ratio by multiplying both terms of a ratio by 3?

A4.7 Ask: Can the ratio 4:5 be equivalent to any other ratio in which the two terms are only one apart? Why or why not?

Presentation

A4.8 758 people were surveyed and 248 responded that they used Brighto detergent. Ask students (working in pairs) to estimate to find what ratio would best describe the number of people who use Brighto. Ask students to explain their reasoning. Ask them to make up similar situations for their classmates to solve.

Journal

A4.9 Tell students that in a class of 30 students, there are 20 girls. Ask them to explain why the ratio of boys to girls is 1:2.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore integers, ratios and percents in common, meaningful situations

SCO: By the end of grade 6, students will be expected to

A5 demonstrate an understanding of the concept of percent as a ratio

Elaboration - Instructional Strategies/Suggestions

A5 Percent should be viewed as a special ratio, a ratio for which the second term is 100.

Students should not be computing with percentages at this time and need not work with percentages greater than 100, but should recognize

- situations in which percent is commonly used
- diagrams that represent various percentages (e.g., 2%, 35%)

- the relationship between the percent and decimal names of ratios (e.g., 48% and 0.48)
- the percent equivalents for common ratios like
- that finding a percentage is the same as finding an equivalent ratio out of 100

☐ Students might explore a variety of geographic or social studies data expressed in terms of percentages. For example:
  - about 70% of the earth is water
  - about 68% of Canadian households own microwaves
  - over 80% of car passengers wear seat belts

☐ Students might cut sheets of paper and/or lengths of string to show 50%, 10%, 25%, etc.

☐ Ask students to predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of
  - red counters when fifty 2-coloured counters are shaken and spilled
  - each colour of Bingo chips, if a total of 100 blue, red, and green chips are shown on an overhead for 10 seconds (use 2-term ratios only)
  - a hundredths grid that is shaded to make a picture.

This should be taught in conjunction with A3, A4, C5, and C6.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A5.1 Ask students to draw a design in a hundredths grid (or partially cover a flat) and describe the percentage of the grid covered. Ask further questions such as: How many more squares would you have to shade (or cover) to cover $\frac{1}{2}\left(\frac{1}{4}, 0.68, 80\%\right)$ of the squares?

A5.2 Ask the student to shade hundredths grids to show particular percentages such as 20%, 60%, etc..

Paper and Pencil

A5.3 Describe a family of 5 which includes 3 children. Ask students to indicate the percentage of the family the children represent and the percentage each child represents. Ask the student to describe a family of a different size with the same percentage of children.

Interview

A5.4 Ask: Which is least? most? Explain your answer.

$\frac{1}{20}$ 20% 0.020

A5.5 Ask: What percent of a metre stick is 37 cm? Ask: How do you know?

A5.6 Ask the students to name percents that indicate
- almost all of something
- very little of something
- a little less than half of something (ask students to explain their thinking)

A5.7 Ask: Why might teachers use percentages to indicate marks on tests rather than just indicating the number right? Why is it not necessary to have 100 marks on a test to use percent?

Portfolio

A5.8 Ask students to create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages of each colour within the patch and then estimate the percent of the total quilt that is each colour.

A5.9 Ask students to collect (from newspapers, flyers, magazines) examples of situations in which percent is used and ask them to make a collage for a class display.

Journal

A5.10 Ask each student to write a letter to a friend/relative/teacher telling what he/she learned about ratio.

Suggested Resources

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ii) explore integers, ratios and percents in common, meaningful situations

SCO: By the end of grade 6, students will be expected to

A6 demonstrate an understanding of the meaning of a negative integer

Negative numbers are an important set of numbers. They can and should be explored before they are encountered in algebra. In fact, students almost every day either have some interaction with negative numbers or experience a phenomenon that negative numbers can model. (Elementary School Mathematics, p. 411)

Elaboration - Instructional Strategies/Suggestions

A6 Students have encountered negative integers informally, as in dealing with winter temperatures. To build on this informal understanding, it might be useful to start with a vertical number line (which resembles a thermometer).

The main ideas for students to understand are that

- each negative integer is the mirror image of a positive integer with respect to the 0 mark
- 0 is neither positive nor negative
- negative integers are all less than any positive integer.

Students should be encouraged to read -5 as “negative 5” rather than “minus 5,” to minimize confusion with respect to the operation of subtraction.

Other useful contexts for considering negative integers are

- golf scores above and below par
- money situations involving debits and credits
- height above and below sea level.

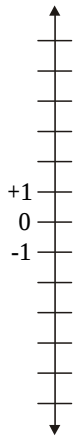
Note: Addition and subtraction situations involving integers should only be dealt with informally.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A6.1 Ask students to place the numbers -4, -3, and +2 at the appropriate places on the number line.



Paper and Pencil

A6.2 How many negative integers are greater than -7? Explain how you know.

A6.3 A number is 12 jumps away from its opposite on a number line. Ask: What could the number be? How do you know?

Interview

A6.4 Explain why -4 and +4 are closer to each other than -5 and +5.

A6.5 Ask: In what situations might you encounter negative numbers?

A6.6 Ask students to explain ways in which -4 is like +4.

A6.7 Ask: Why is an integer never 11 away from its opposite on a number line?

Portfolio

A6.8 Ask students to design a simple game for which positive and negative points might be awarded. Have the students play and keep track of their total scores.

Journal

A6.9 John is standing at +3 on a number line on the floor. Two other children are standing at different places on the same number line, each exactly 6 jumps away from John. Where are they? Explain how you found their location.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) read and write whole numbers and decimals and demonstrate an understanding of place value

SCO: By the end of grade 6, students will be expected to

A7 read and write whole numbers to billions using standard and expanded form

A8 demonstrate an understanding of the place value system

Elaboration - Instructional Strategies/Suggestions

A7 Students have already had opportunities to work with numbers in the millions. Whole numbers should be written in

- standard form (e.g., 345 321 400, read as 345 million, 321 thousand, 4 hundred)
- rounded form using decimal notation (e.g., 345.3 million)
- expanded form (e.g., 3 x 100 000 000 + 4 x 10 000 000 + 5 x 1 000 000 + 3 x 100 000 + 2 x 10 000 + 1 x 1 000 + 4 x 100)

Opportunities should be provided in which students record the numerical form of a number that is either spoken or written out in word form. Conversely, students should have the opportunity to both write out and say the word form of a number expressed in its symbolic form. Special attention should be given to numbers for which the numerical expression includes a number of internal zeroes, since these tend to cause the most difficulty for students (e.g., nine hundred two million thirty thousand three).

A8 Students should understand that the place value system follows a pattern such that

- each position represents 10 times as much as the position to its right
- each position represents $\frac{1}{10}$ as much as the position to its left
- place values are grouped in threes for purposes of reading them. This is done both before and after the decimal point.

All students should be aware that numbers extend to the left at least into the billions group and to the right into the hundred thousandths. If students inquire about these extensions, a discussion is in order.

Since “billions” refers to numbers rarely found in students' experiences, it is important to have students investigate numbers of this magnitude as they relate to real world situations such as national debt, personal fortunes, populations, pieces of trivia (e.g., "How long is a billion millimetres?"), etc.

This should be taught in conjunction with A1.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A7.1 Ask students to read these numbers:

105 020 003
64 203 006
920 000 029

A7.2 Ask students to arrange the cards shown below in at least 3 ways and record the numeral for each.

4	2	million	5	3	thousand	billion

A7.3 Provide students with "pretend" cheques for which the dollar amounts have been listed. Ask them to write out the word form of each amount.

Paper and Pencil

A8.1 Ask students to use only the digits 2, 3, and 4 to create three numbers with values between 42 million and 43 million. (Each digit can be used more than once.)

A7.4 Ask students to write the number 3 thousand as millions.

Interview

A8.2 Ask students to explain the difference(s) in how these numbers are written:

two thousandths
two thousand
twenty thousand
twenty thousandths

Portfolio

A7/8.1 Ask students to prepare a "lesson plan" to teach a grade 5 student what a billion means. They may wish to actually do the teaching and report on the experience.

Suggested Resources

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iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 6, students will be expected to

A9 relate fractional and decimal forms of numbers

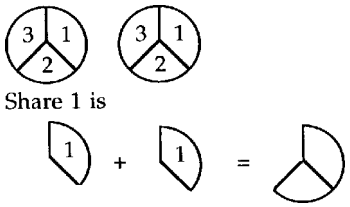
To connect the two numeration systems, fractions and decimals, students should make concept-oriented translations from one system to another . . . The calculator can also play a significant role in decimal concept development. (Elementary School Mathematics, p. 262)

Elaboration - Instructional Strategies/Suggestions

A9 Student have worked with decimal equivalents of some simple fractions (e.g., = 0.5, = 0.25, = 0.2) as well as any fraction with a denominator of 10, 100, or 1000. For example, to locate 8.75 on a number line, many students think of 0.75 as being three quarters of the way from 8 to 9.

By building on the connection between fractions and division, students should be able to represent any fraction in decimal form. Calculators may be used as an aid when working with "less common fractions".

For example, means 2 wholes shared among 3, so = 2 ÷ 3. Physically, the demonstration could be 2 pizzas shared by 3 people.



or of one whole

Students should recognize when a decimal repeats, but need not deal with the symbolism for handling repeating decimals at this time.

They should, however, be able to record a decimal such as as 0.666... to show the pattern continues.

Base ten blocks can be used to explain the decimal equivalents to fractions, even when these decimals repeat. For example, 1 ÷ 3 could be modelled as follows: The large cube represents one whole. It must be shared by 3 people. Trade the cube for 10 flats (10 tenths). Each of the 3 people gets 3 tenths, so the decimal begins with 0.3. Trade the leftover tenth for 10 rods (10 hundredths). Each of the 3 people gets 3 hundredths, so the next digit in the decimal is 3 (i.e., the decimal begins with 0.33). Continue the process.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A9.1 Ask students to use a calculator to find the decimal forms for and and then subtract. Ask: How might you have predicted the difference?

Paper and Pencil

A9.2 Ask students to explain how they know that the decimal form of cannot begin with 0.6.

A9.3 Ask students to describe a fraction which is slightly less than 0.4 and to justify the selection. Ask: Can you name another that is between these two?

Interview

A9.4 Ask students to identify how the decimal forms of and are related and explain what this tells about fractions. Have the student provide another pair of fractions with the same relationship.

A9.5 Ask: How does knowing that = 0.25 help you find the decimal form of ? of ?

$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

Portfolio

A9.6 Ask students to use calculators to find the decimal forms for a group of fractions and make as many observations as possible about the decimals obtained. A sample group is

Journal

A9.7 How are fractions and decimals alike and how do they differ?

Suggested Resources

Base Ten Materials

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals

SCO: By the end of grade 6, students will be expected to

A10 determine factors and greatest common factors of two numbers

Number theory is the study of relationships found among the natural numbers. At the elementary level, number theory includes the concepts of prime number, odd and even numbers, and the related notions of factor, multiple, and divisibility. (Elementary School Mathematics, p. 404)

Elaboration - Instructional Strategies/Suggestions

A10 Most students should have little difficulty with the concept of one number being a factor of another. Factors can be found by dividing a number by smaller numbers and looking for remainders of zero. This concept extends directly from previous work in multiplication and division. The concept of common factors, however, will be new to most students. It may be useful to ensure that students understand that "common" is used in the sense of "joint", rather than "ordinary"; this is a typical misunderstanding on the part of students.

To introduce the concept of common factors, ask students to compare the factors of 2 numbers (e.g., 16 and 18) and note any factors which are factors of both numbers, i.e., common factors.

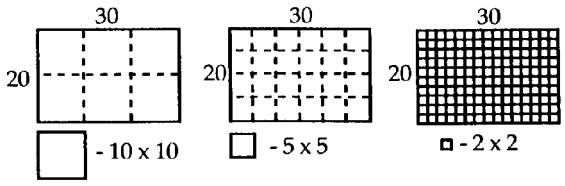
16- 1,2,4,8,16

18- 1,2,3,6,9,18

Upon examining these lists, the students will see that 1 and 2 are the only factors common to both 16 and 18.

Students should soon conclude that 1 is always a common factor of any two numbers.

As developed in grade 5, another way to find common factors of a pair of numbers is to begin by creating a rectangle, using the two numbers as the length and width. A common factor is the side length of any square which can be used to tile (or cover) the rectangle exactly. For example, for a 20 x 30 rectangle, 10, 2 and 5 are common factors since the following tilings are possible:



Another interesting approach to factors and common factors involves putting different coloured cubes on a hundreds chart as one skip counts by different amounts. For example, skip count by 2s, putting a red cube on each number; skip count by 3s, putting a blue cube on each number; skip count by 5s, putting a yellow cube on each number. Ask: What numbers have both a red and blue cube? All three cubes? What does this mean about their factors?

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A10.1 Ask each student to draw one or more rectangles to show that 8 is a common factor of 16 and 24.

Paper and Pencil

A10.2 Ask students to create a number which has 4, 7, 28 and 12 as factors. Ask them if there is a smaller number which will meet the conditions and to explain why or why not.

A10.3 Ask each student to find a number with exactly 6 factors.

A10.4 If 3 and 4 are common factors of a pair of numbers, what might the numbers be? List 3 possibilities and explain hyour answer.

A10.5 Mary is making a quilt for her little sister's doll. The quilt is 20 cm by 25 cm. She wants to use square blocks to make the quilt. What are some possible sizes for the squares need to make this quilt? In each case how many blocks would be required?

A10.6 The number pairs listed have more than one factor in common. For each pair, find the greatest common factor:

a) 12 and 18
b) 20 and 30
c) 30 and 45
d) 24 and 36

Interview/Journal

A10.7 Tell students that the common factors of a particular pair of numbers include 10. Ask them to explain how this guarantees that 2 and 5 are also common factors.

A10.8 Ask: Why is it not possible for a common factor of 38 and 90 to be greater than 20?

Portfolio

A10.9 Ask students to design a test they think could be used to determine students' understanding of common factors.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals

SCO: By the end of grade 6, students will be expected to

A11 distinguish between prime and composite numbers

Prime numbers can be viewed as fundamental building blocks of the other natural numbers. (Elementary School Mathematics, p. 405)

Elaboration - Instructional Strategies/Suggestions

A11 A prime number may be defined in two ways:

- a number which has only 2 factors, 1 and itself, (e.g., 29 - with factors of 1 and 29 is prime and 28 - with factors of 1, 2, 4, 7, 14 and 28 is not prime)
- a number for which that same number of squares can be arranged in a rectangle in only one way. For example, contrast 7 , which is prime, with 8 , or which is not.

☐ Ask the class to use up to 36 coloured tiles to explore the different rectangles that can be made for each number from 1 to 36. Pairs of students may be assigned 2 or 3 numbers each (e.g., one pair might explore the numbers 21, 22, and 23). Have the numbers (1-36) written horizontally across the front of the room or on the board. Ask each pair to cut out of grid paper all the rectangles that can be made for each of their numbers and to display them under the numbers. When the display is complete, only the prime numbers (except for 1) will have just one rectangle.

Students should recognize that the concept of prime numbers applies only to whole numbers. Although students should have strategies for determining whether or not a number is prime, it is not essential for them to be able to quickly recognize whether or not a number is prime. Exceptions would be that students should be able to identify numbers of composite i) if they are even (other than 2) or end in 5 (other than 5 itself). For other numbers, students should determine whether they are prime by searching for whether the numbers have factors other than themselves and 1.

Many students do not realize that 1 is neither a prime number nor a composite number. There are many explanations for this, but it is sufficient for students to realize that 1 has only 1 factor, whereas prime numbers have exactly 2 factors.

Students should be introduced to the term "composite" (for non-prime numbers other than one) and encouraged to accurately use language such as multiple, common multiple, factor, common factor, prime and composite. As well, encourage students to explore numbers and become familiar with their composition.

☐ Ask students to write about the number 36, describing it using “factor language” in as many ways as they can. Answers might include: 36 is a composite number with 9 factors (1, 2, 3, 4, 6, 9, 12, 18, 36). Two of its factors are prime numbers. Five rectangles can be made with 36 tiles, one of which is a square.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A11.1 Ask each student to draw a diagram to show why 10 is not a prime number.

A11.2 Ask students to express even numbers greater than 2 in terms of sums of prime numbers. (Sample answers include 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, ..., 48 = 43 + 5, 50 = 47 + 3, ...)

A11.3 Ask students to identify the prime numbers to 100 by exploring the sieve of Eratosthenes, using a hundreds chart. Before they start, ask students to review the concept that "1" is neither prime nor composite, but in a separate category by itself. Ask students to begin by circling the first prime number (2), and then crossing out every second number (the multiples of 2). These must be composites. Ask them to circle the next prime number (3) and cross out every third number thereafter (the multiples of 3, some of which have already been crossed off). The students then proceed to 5, 7, 11, etc. At the end of the process the circled numerals will be the prime numbers up to 100.

Paper and Pencil

A11.4 Are there more prime numbers between 50 and 60 or between 60 and 70?

A11.5 Find 3 pairs of prime numbers that differ by two (e.g., 5 and 7).

Interview

A11.6 Ask: Why is it easy to know that certain large numbers (e.g., 4 283 495) are not prime, even without factoring?

A11.7 Tell students that the numbers 2 and 3 are consecutive numbers, both of which are prime numbers. Ask: Why can there be no other examples of consecutive prime numbers?

Portfolio

A11.8 Ask students to use a computer or calculator to help them determine the prime numbers up to 100. Ask them to prepare a report describing as many features of their list as they can.

Journal

A11.9 Identify whether 17932 and 19875 are prime or composite. Explain how you know.

Enrichment

A11.10 Research why zero is neither prime nor composite.

Suggested Resources

Coloured Tiles