Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

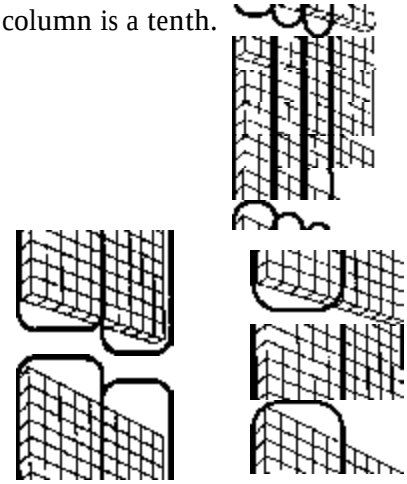
i) model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures

SCO: By the end of grade 6, students will be expected to

B1 compute products of whole numbers and decimals to thousandths

Elaboration - Instructional Strategies/Suggestions

B1 Students should be able to compute products of whole numbers using an algorithm. In grade 5, they worked with 2-digit by 2-digit situations. This can be extended to 3-digit by 2-digit situations in grade 6. **They should also know when it is appropriate to use a paper/pencil algorithm, a mental procedure, or a calculator for the operation.** Students require practice estimating products and a knowledge of the multiplication facts is essential. They should continue to use base ten blocks and money models to make sense of the multiplication algorithm involving decimals. It is not enough to tell students to multiply, estimate and decide where to put the decimal point; they need to see why the procedure works.
Base ten materials are effective models for calculations involving whole numbers and decimals. If a flat represents one unit, each column is a tenth.

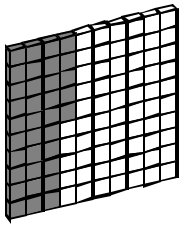


3 x 0.5 = 15 tenths or 1.5 (one and five tenths)

Students might also rewrite the multiplication. For example,

$$\begin{array}{r} 5.4 \\ \times \quad 2 \\ \hline \end{array} \qquad \begin{array}{r} 54 \\ \times \quad 2 \\ \hline 108 \text{ tenths} \end{array} \text{or } 10.8$$

Similarly, base ten materials can be used to represent the multiplication of hundredths by a whole number. If the flat represents one, each of the hundred squares would represent 0.01.



7 x 0.05 = 7 times 5 hundredths

Note: It is important to read 0.05 as five hundredths as opposed to zero point zero five.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B1.1 Ask the student to draw or build a model to illustrate 4 x 3.453.

B1.2 Ask students to describe how to calculate 3 x 4.23 using a money model.

Paper and Pencil

B1.3 Ask students to determine how much more five cans of juice cost at $1.29 each than six cans at $0.99 each.

B1.4 Ask students to identify a decimal which, when multiplied by 500, will produce a result of 200. (This could be checked using a calculator.)

B1.5 Ask students to find the missing digits:

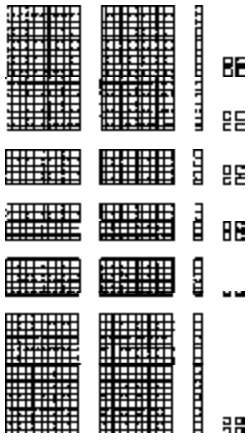
$$\begin{array}{r} 5.\square 3 \\ \times \quad \square \\ \hline 3\square.58 \end{array}$$

Ask them to "think about their thinking" and be prepared to explain what steps they took, and why, in finding the missing digits.

Interview

B1.8 Ask students to if the result of multiplying a decimal by a whole number can be a whole number. Show examples to support your answer.

B1.6 Present the arrangement of blocks shown at right. Ask: What multiplication is shown if the flat is assumed to represent one whole?



Journal

B1.7 Ask students to respond to the following: Jane said 3.45 x 4 must be 1.380. There is only one digit before the decimal place in 3.45, so there must be one digit before the decimal place in the product.

Suggested Resources

Base Ten Materials

Decimal Squares

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures

SCO: By the end of grade 6, students will be expected to

B2 model and calculate the products of two decimal numbers

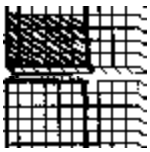
Elaboration - Instructional Strategies/Suggestions

B2 Patterns can be used to help students understand the placement of the decimal in the product of two decimal amounts. For example, consider:

$$\begin{array}{r} 42 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4.2 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4.2 \\ \times\ 0.4 \\ \hline \end{array}$$

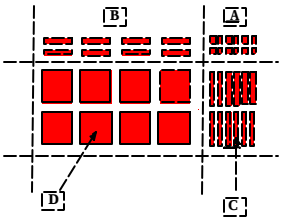
Decimal placement can be explored using the calculator but it is very important that students practice applying pencil and paper techniques.

An area approach can also be used to explain the placement of the decimal. For example, 0.4 x 0.6 is $\frac{4}{10}$ of $\frac{6}{10}$.



Using a hundredths grid, shade 0.6, then shade 0.4 of the 0.6 previously shaded. What fraction of the whole grid does the area shaded twice represent?

Use the area model as developed in grade 5 for multiplication of whole numbers.



The dotted lines divide the rectangle into four sections.

$$\begin{array}{r} 4.6 \\ \times\ 2.2 \\ \hline 0.12 \\ 0.8 \\ 1.2 \\ 8 \\ \hline 10.12 \end{array}$$

(12 hundredths) - A
(8 tenths) - B
(12 tenths or 1.2) - C
(8 ones) - D

The same rectangle can be divided into two sections where sections A and B are one rectangle and C and D are the second rectangle.

$$\begin{array}{r} 4.6 \\ \times\ 2.2 \\ \hline 0.92 \\ 9.2 \\ \hline 10.12 \end{array}$$

(A & B together)
(C & D together)

This is closer to the standard algorithm

Students should interpret the symbolism in meaningful ways. For example, 0.6 x 34.5 is $\frac{6}{10}$ of 34.5, so the product is more than half of 34.5, but not much more. Rather than simply providing a rule about "counting decimal places," a rule which students often mix up, it is better if students understand why and how the whole number calculation can be used and adjusted for different decimal multipliers. Students must be encouraged to estimate products before calculating. For example, one might round each of the decimal numbers 2.86 x 8.153 for an estimate of 24 (3 x 8). When estimation is an automatic response students will not depend solely on the "counting back decimal places" rule.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B2.1 Tell students that two decimals are multiplied. The product is 0.48. Ask: What might they have been? Give two other pairs.

B2.2 Ask students to work in pairs sharing strategies for estimating and calculating in situations such as finding the cost of:

- 6.15 m of material at $4.95 a metre
- area of a rectangular plot of land 24.78 m x 9.2 m
- 0.5 of a length of rope 20.6 m long.

Ask them to write similar questions to share with their classmates.

Interview/Journal

B2.3 Ask: Why is the answer to 0.6 x 0.4 represented as hundredths?

B2.4 Ask: Is it possible to multiply 2 decimals and get the same result as if you had multiplied 2 whole numbers?

B2.5 Ask: When multiplying two decimals, how does the result compare in size to the numbers multiplied?

Presentation

B2.6 Ask students to find their own heights in metres. Ask them to research some animal sizes and prepare a report:

An animal that is about 0.1 of my height is _____.
An animal that is about 0.2 of my height is _____.
Etc.

Portfolio

B2.7 The product of two decimal numbers can be modeled using 13 base ten pieces. Determine what numbers might have been multiplied.

Suggested Resources

Base Ten Materials

Demical Squares

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

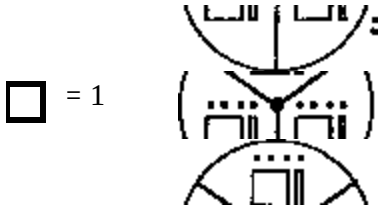
i) model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures

SCO: By the end of grade 6, students will be expected to

B3 compute quotients of whole numbers and decimals using up to 2-digit whole number divisors

Elaboration - Instructional Strategies/Suggestions

B3 Students should continue to use manipulative materials to model division of a decimal by a whole number.
For example: 3.44 ÷ 3

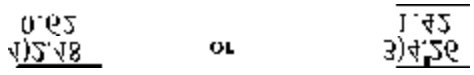


Common contexts in which this type of calculation would emerge, and to which the students would relate, are the sharing of money and unit pricing. Other possible contexts are sharing metres of ribbons, litres of juice or kilograms of meat. Students should be expected to estimate quotients. For example, 4.28 ÷ 3 will be a bit more than 1, but 4.28 ÷ 5 will be close to $\frac{4}{5}$ or 0.8. Some may think of 4.28 ÷ 5 as 428 hundredths divided by 5 (about 85 hundredths), or as 42.8 tenths ÷ 5 (about 8 tenths).

Students should understand that the "remainder" when they perform the division of a decimal number is different than with whole numbers. For example, when dividing 3.44 by 3, the remainder "2" at the end of the algorithm is really 0.02, not 1. They should recognize that, for more accuracy, they could continue the process.

$$
\begin{array}{r}
1.12 \\
3\overline{)3.44} \\
\underline{3} \\
4 \\
\underline{3} \\
14 \\
\underline{12} \\
2
\end{array}
$$

Many students will be ready to use short division when dividing by a single digit number. For example,



To illustrate division with 2-digit whole number divisors, start with an example such as 42.9 ÷ 11. Ask students to model 42.9 using base ten materials; then share the materials into 11 equal groups using base-ten language to explain the process (in this example the rod can represent ones). Move the students from here to the standard division algorithm.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B3.1 Ask students to draw a model to show how to find 5.28 ÷ 4.

Paper and Pencil

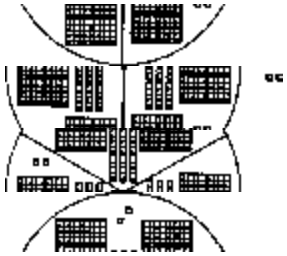
B3.2 A can of pop holds 0.355 L. Ask students to determine the number of cans of pop required to total 5 L.

B3.3 537 hot dog buns are needed for the school fun day. The buns come in packages of 12. Ask how many packages are needed.

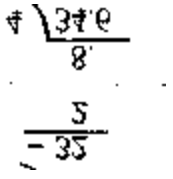
B3.4 Ben's mom opened a new restaurant. She decided to put lace trim around all the tablecloths. Each cloth requires 5 m of lace. She bought 4 rolls of lace with 22.5 m of lace on each roll. How many tablecloths can be trimmed?

Interview

B3.5 Present the display shown at right and ask what division is being modelled, assuming that a flat represents 1.



B3.6 Ask students to complete the calculation at right. Observe how the remainder is handled.



Ask students to create a problem for the computation and to explain what to do with the remainder.

Presentation

B3.7 Ask students to use a store flyer to find items that are sold in twos, threes or other groupings, and provide unit prices for the various items. Ideally, these unit prices will be compared to prices of the same or similar products from another store's flyer.

Journal

B3.8 How are the results of 423 ÷ 3 and 42.3 ÷ 3 related?

B3.9 Why is the remainder not really 1 when you divide 2.1 by 4?

Suggested Resources

Base Ten Materials

Decimal Squares

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures

SCO: By the end of grade 6, students will be expected to

B4 model and calculate the quotients of two decimals up to 2-digit divisors

Elaboration - Instructional Strategies/Suggestions

B4 As with multiplication, students should relate the division of a decimal by a decimal to the corresponding division of a decimal by a whole number. Approaches include

- considering the unit - For example, 43.2 ÷ 0.5 = 432 tenths ÷ 5 tenths. How many 5 tenths are in 432 tenths? (432 ÷ 5) Similarly, 43.25 ÷ 0.5 = 432.5 tenths ÷ 5 tenths. How many 5 tenths are in 432.5 tenths? (432.5 ÷ 5)

  Students may notice that since 43.2 ÷ 0.5 = 432 ÷ 5, they could physically change the problem before solving it, i.e.,

  $$0.5\overline{)43.2} \quad \rightarrow \quad 5\overline{)432}$$

- money model - Students might find it productive to use a money model. For example, 43.2 ÷ 0.4 might be interpreted as determining the number of sets of 4 dimes in $43.20. Since 10 sets of 4 dimes each is $4, 100 sets of 4 dimes is $40. Another eight sets (8 x 40¢) would be needed to make the extra $3.20. Therefore, 43.2 ÷ 0.4 is 108.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B4.1 Ask students to use a number line or base ten blocks to show why 4.2 ÷ 0.2 is the same as 42 ÷ 2.

B4.2 Ask students to use a metre stick to explain why 3.4 ÷ 0.2 is the same as 34 ÷ 2 (3.4 dm ÷ 0.2 dm = 34 cm ÷ 2 cm).

Paper and Pencil

B4.3 To practise division, ask students to compete in paper airplane races. Each student flies his/her plane 3 times, measuring each distance travelled in metres. The score is determined for each by finding the average distance.

For example: 2.43 m (or 2 m, 43 cm)
1.89 m
2.25 m
Average distance = 6.57 m ÷ 3
= 2.19 m

B4.4 Ask students to fill in the missing digits. The boxes may represent different amounts.

4.□ ÷ 0.□ = 14.□

B4.5 Ask students to describe the situation by referring to coins.

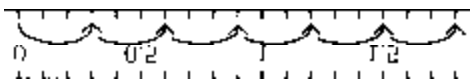
2.40 ÷ 0.1 = 24

Interview

B4.6 Ask: Which question has an answer different from the others? How could you know in advance of complete calculations?

42.5 ÷ 0.5 425 ÷5 85 ÷ 1 0.425 ÷ 0.05

B4.7 Ask students to explain how the diagram shows that 1.8 ÷ 0.3 = 6



Journal

B4.8 Why might someone find it easier to divide 8.8 by 0.2 than 1.1 by 0.3?

B4.9 Jack wanted to calculate 4.84 ÷ 1.2. he said, "To make it simplier I will simplify, 484 ÷ 12." Will he get the same answer?

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) model problem situations involving the addition and subtraction of simple fractions

SCO: By the end of grade 6, students will be expected to

B5 add and subtract simple fractions using models

When area models are used for addition and subtraction, common denominators are frequently not involved at all. (Elementary School Mathematics, p. 244)

Elaboration - Instructional Strategies/Suggestions

B5 Usually students have no difficulty adding or subtraction fractions with like denominators. For example:

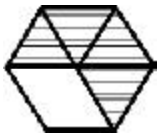
$$\frac{3}{5}+\frac{2}{5}=3 \text{ fifths}+ 2\text{fifths}$$
$$=5\text{fifths}$$
$$=\frac{5}{5}$$

Students should realize that fractions with unlike denominators can be added and subtracted.

It is important to continue to provide concrete experiences to help students build an understanding of simple fraction operations.

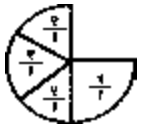
Models to use include

- pattern blocks

$\frac{4}{6}-\frac{1}{3}$  $\frac{2}{6}$ or $\frac{1}{3}$ remaining

(Let the yellow hexagon represent the whole; each green triangle will be $\frac{1}{6}$ of the whole and the blue rhombus will be $\frac{1}{3}$ of the whole.)

- fraction circles

$\frac{1}{4}+\frac{3}{6}$  $=\frac{3}{4}$

☐ Students might enjoy finding many sums for 1 by covering the hexagon pattern block in as many different ways as possible.

For example:

$1=\frac{2}{3}+\frac{2}{6}(2\text{blue}+2\text{green})$ 

$=\frac{1}{2}+\frac{3}{6}(1\text{red}+3\text{green})$ 

$=\frac{4}{6}+\frac{1}{3}(4\text{green}+1\text{blue})$ 

Note: The intent at this level is to perform operations using models, not algorithms. Therefore, denominators should be limited to those typically found in fraction model kits (e.g., 2, 3, 4, 5, 6, 8, 10, and 12).

Some students may be ready to explore the repeated addition model of multiplication via an activity such as: Ask the student to show $\frac{1}{4}$. Show 4 one-quarters. How might that be written? How much is 4 one-quarters? Show six one-quarters. What can you say about this? How might this be written?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

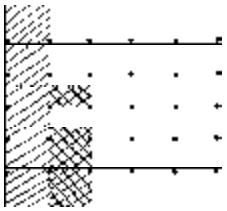
Worthwhile Tasks for Instruction and/or Assessment

Performance

B5.1 Ask students to model for $\frac{2}{3}+\frac{1}{2}$.

Paper and Pencil

B5.2 Ask students to state the fraction addition modelled on dot paper, assuming that the portion shown is the whole.



B5.3 Ask students to explain how the diagram at right shows that $\frac{4}{12}+\frac{5}{12}+\frac{3}{12}=1$.

B5.4 Ask students to list groups of 3 fractions that add to equal $\frac{1}{2}$.

Interview/Journal

B5.5 Ask students to explain why $\frac{2}{3}+\frac{4}{5}$ has to be greater than 1.

B5.6 Tell students that Jane said: $\frac{4}{8}+\frac{2}{8}=\frac{6}{16}$. Ask: Is she right or wrong? Why?

B.7 Tell students that you have subtracted two fractions and the result is less than $\frac{1}{2}$. Ask: Can both fractions be greater than $\frac{1}{2}$? less than $\frac{1}{2}$? Explain your thinking.

Portfolio

B5.8 Using triangular grid paper, ask students to copy the models they create to show the different ways that a hexagon can be covered. For each model, students are to write the appropriate fraction addition that goes with it.

Suggested Resources

Pattern Blocks

Fraction Circles

Fraction Factory Pieces

Geoboards

Colored Tiles

Grid Paper

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

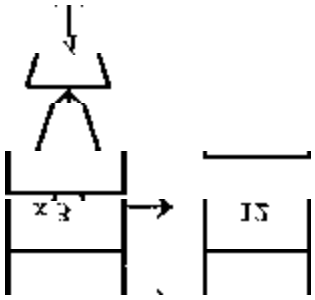
iii) explore algebraic situations informally

SCO: By the end of grade 6**,** students will be expected to

B6 demonstrate an understanding of the function nature of input-output situations

Elaboration - Instructional Strategies/Suggestions

B6 In introducing the concept of a function, the "function machine" works well. Show students both how inputs are acted upon by functions and the resulting outputs.



Grade six students can try to decide what the function machine does if they are given a series of inputs and outputs. For example, if 4 → 12, 6 → 16, and 10 → 24, what did the machine do? (Answer: Doubled and added 4)

Number tricks provide an enjoyable context for students to practise the concept of function, where a given input results in a specified output.

For example, consider this "trick:"	Explanation
Choose a number.	□
Add 8.	□ + 8
Multiply by 2.	2 x □ + 16
Subtract 14.	2 x □ + 2
Divide by 2.	□ + 1
Subtract 1.	□

Students should recognize that the result depends on the original number chosen. After discussing the "trick" and seeing why it works, students will enjoy making up their own, thereby further exploring the concept of function.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B6.1 Tell students that you intend to create a number "trick" in which you perform a sequence of calculations. In each case, ask what one calculation could be substituted for the sequence of calculations. (Students can explore by trying examples since most are not ready for the algebra of the situation.)

(i) add 5, add 20, subtract 10
(ii) multiply by 5, multiply by 20, divide by 10
(iii) add 4, double, add 6, divide by 2

Interview

B6.2 Tell students to that you put 25 into the function machine and 77 came out. Ask for four possible functions that would do this.

B6.3 Ask pairs of students to each provide an explanation as to why the following number "trick" works:

Pick a number between 0 and 10
Add 7 to the number.
Double the new number.
Add 11.
Subtract 25
Divide by 2.

Presentation

B6.4 Ask students to prepare a set of five "number tricks," the trickier the better. Each trick should be based on doing various computations with an initial number that the other person chooses. Students should prepare a "manual" of the tricks with explanations for the user.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) apply computational facts and procedures (algorithms) in a wide variety of problem situations involving whole numbers and decimals

SCO: By the end of grade 6, students will be expected to

B7 solve and create relevant addition, subtraction, multiplication and division problems involving whole numbers

Elaboration - Instructional Strategies/Suggestions

B7 Students should continue to use the four operations to solve mathematical and real-world problems. They should also have the opportunity to create problems for others to solve.

Students should be encouraged to estimate answers to test for reasonableness and whenever the calculation can be done mentally, children should do so.

There are many interesting sources of data, both on the Internet and in reference books. Some of the most useful print resources include the *Canadian Global Almanac*, the *Guiness Book of World Records* and the *Top Ten of Everything*.

Internet searches can be done for data relating to any topic of student interest, such as sports, populations or food.

When solving and creating problems, number size and type should be those studied at the grade 6 level. While addition and subtraction still require some practice, grade appropriate problems should incorporate addition and subtraction with another operation. For example.

- In the magazine subscription fund raiser, John's class sold 104 magazine subscriptions and Jane's class sold 108. If the profit on each subscription was $11.00, how much money did the two grade 6 classes make altogether?
- When the fundraiser money for the whole school was counted, they found that $2612 was raised by grade 7s, $2204 by grade 8s, and $1880 by grade 9s. What was the average amount raised per grade?
- Make up your own fundraising problem that uses more than one operation.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B7.1 Provide students with appropriate data and ask them to determine how much farther away Jupiter is from Earth than from Mars. This might be reported as a ratio or in terms of absolute distance.

B7.2 Subtract the sum of the three smallest numbers in this set from the sum of the two largest numbers. 389, 243, 301, 332, 91.

Interview

B7.3 Ask: When might one multiply to find the perimeter of a shape?

Presentation

B7.4 Ask students to pretend to work in a post office. Institute a "rule" that states that the total of the length, height, and width of a package has to be less than 100 cm to mail it. The students determine whether or not various packages will work. In addition, ask them to create a list of package sizes that would "just make it."

B7.5 Ask students to plan a trip with various stops. Using distance charts or maps, they find the distances between the stops. They then determine the length of the entire round trip. Alternatively, students could plan a trip of a given length (between 1200 and 1500 km, for example). An activity such as this could extend to calculations which could include total cost of trip (gas, lodging, meals, etc.).

Portfolio

B7.6 Ask students to create a variety of "outlandish" problems involving lengths. For example:

- How many toothbrushes are required to make a line that is 2 km long?
- How many pennies must be lined up to make a kilometre?

B7.7 Ask students to create problems based on the information provided.

Urban/Rural Population 1991

	RURAL	URBAN
Canada	6 389 724	20 909 135
NL	264 023	304 451
NS	418 434	481 508
PEI	77 952	51 813
NB	378 686	345 214

Suggested Resources

Problem Solving Experiences in Mathematics - Grade 6

Statistics Canada Website, *www.statscan.ca*

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) apply computational facts and procedures (algorithms) in a wide variety of problem situations involving whole numbers and decimals

SCO: By the end of grade 6, students will be expected to

B8 solve and create relevant addition, subtraction, multiplication and division problems involving decimals

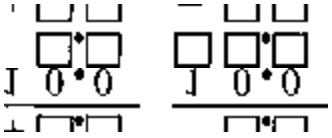
Elaboration - Instructional Strategies/Suggestions

B8 Provide problems involving decimals. Contexts might include money and measurement problems.

Demographic data can be a good source of material for problem solving and creation. Provide students with information about the populations of various places in Canada. Ask them to create authentic problems based on data provided in decimal form. For example:

- How many people in B.C. are living in urban areas?
- How many more people live in Edmonton than Saskatoon?
- How many more people live in Halifax than Saint John?
- How many people/km^2 are found in Canada's largest cities?

Some problems can be strictly mathematical challenges. For example, students might be asked to find as many combinations of digits as they can to fill in the templates below so that the sum or difference is 10.0.



Pieces of children's literature often provide interesting contexts for problems. A good example is *Code Red at the Supermall* by Eric Wilson. The picture book *Counting on Frank,* by Rod Clement, is a favourite of students, as well.

Example

- Pat's mom is making costumes for the school play. Each costume takes 2.4 m of fabric. She has 27.8 m. Create two questions which can be answered from the given information. Answer the questions showing all workings.

This should be taught in conjunction with B1-B4.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

B8.1 Ask students to draw three quadrilaterals with a perimeter of 16.3 cm. Class Discussion: Did anyone draw a regular quadrilateral? What difficulties would be encountered?

Paper and Pencil

B8.2 Ask students to create a measurement problem involving the addition or subtraction of hundredths of metres.

B8.3 Ask students to create addition, subtraction, multiplication and division word problems, each with an answer of 4.2.

B8.4 Ask students to describe a situation involving both addition and multiplication of decimals.

B8.5 Mr. Brown drives 15.9 km to work and the same distance returning home.

a) How many kilometres does he drive in a 5-day work week
b) If he buys the house he is considering, it will save him 133 km of driving per week. How far is the house from his work?
c) Create another question which can be solved based on this situation and solve it.

B8.6 Ask students to create and solve a multiplication problem involving a particular item (e.g., bananas) using decimals (not in a money context).

Portfolio

B7/8.3 Ask students to each create a "Problem Sheet of the Week" based on a predetermined topic or holiday. Select one or more to assign to the class for homework. Be sure to credit the author. They may wish to publish a class problem booklet.

B8.7 Ask students to create problems involving this data:

Population by Official Language 1991 (%)

	English	French	Bilingual
NL	96.5	0.04	3.3
NS	91.1	0.02	8.6
PEI	89.6	0.2	10.1
NB	57.9	12.5	29.5

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply estimation techniques to predict, and justify the reasonableness of results in relevant problem situations involving whole numbers and decimals

SCO: By the end of grade 6, students will be expected to

B9 estimate products and quotients involving whole numbers only, whole numbers and decimals, and decimals only

Number sense is a critical component of our students' education. Encouraging students to estimate and check answers is an integral part of any numerical exercise, discussing common measurement situations with them, and asking them to justify their mathematical choices will help students develop this crucial ability. (Curriculum and Evaluation Standards, Addenda Series, Sixth-Grade Book, p.10)

Elaboration - Instructional Strategies/Suggestions

B9 Estimation should precede all calculations so results can be tested for reasonableness.

When considering multiplication by a decimal, students should recognize that, for example, 0.9 of a quantity or object will almost be the whole and 2.4 multiplied by an amount will be double the amount with almost another half of it added on.

It is important for students to realize estimation is a useful skill in their lives; therefore, regular emphasis on real-life contexts should be provided. On-going practice in computational estimation is a key to developing understanding of number and number operations and increasing mental process skills. Although rounding has often been the only estimation strategy taught, there are others (many of which provide a more accurate answer) that should be part of a student's repertoire.

- Rounding:
  - Multiplication: 688 x 79 is easily rounded to 700 x 80 to give 56 000, which is a good estimate. Consider, however, 653 x 45. If one were to round according to the "rounding rule," the estimate (35 000) would not be close to the actual answer (29 385). Multiplying 700 by 40 would give a much more accurate estimate (28 000) and 600 x 50 provides an even closer one (30 000). It is important that students explore possible rounding combinations with their calculators and discuss the reasons for the variances.
  - Division: 789.6 ÷ 89 Think: "90 multiplied by what number would give an answer close to 800?"
- Front-end:
  - Multiplication: 6.1 x 23.4 might be considered to be 6 x 20 (120) plus 6 x 3 (18) plus a little more for an estimate of 140, or 6 x 25 = 150.
  - Division: Pencil and paper division involves front-end estimation. The first step is to determine in which column the first digit of the quotient belongs. For 8)424.53, the first digit is a 5 and is placed over the 2 of the 42 tens. The front-end estimate is, therefore, 5 tens or 50.

☐ Ask students to estimate each of the following and tell which of their estimates is closer to the actual amount and how they know:

9.7 kg of beef at $4.59/kg,
4.38 kg of fish at $12.59/kg.

This should be taught in conjunction with B1-B4.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B9.1 Ask students to compute the approximate number of hours in 10 000 seconds or 100 000 seconds.

B9.2 A decimal is multiplied by a whole number and the estimate is 5.5. Ask: What might the numbers be?

B9.3 Provide students with a supermarket checkout slip on which the total has been removed. Ask students to estimate the total amount.

B9.4 Tell students that it takes about 0.08 kg of beef to make a hamburger patty. Sue checks the label on the package and finds she has 2.456 kg of beef. Ask: About how many patties can she make?

Interview/Journal

B9.5 Tell the student that two numbers have been multiplied together. The result is about 40 000. Ask: What might the numbers be?

B9.6 Ask: Which is the best estimate for 37 x 94 and why?

30 x 90 40 x 100 30 x 100 40 x 90

B9.7 Ask students why someone might estimate 516 x 0.48 by taking half of 500.

B9.8 Tell students that the cashier told Samantha her total for 3 kg of grapes at $3.39/kg was $11.97. Ask: How did Samantha know right away that the cashier had made a mistake?

B9.9 Tell students that Sandy's marks for English, Math, Science and French totaled of 319. Ask the student to estimate Sandy's average mark.

B9.10 Ask students for an estimate of the total cost of 25 pens at $0.79 each. Ask what estimating strategy he/she used and if there is another way to easily estimate the answer.

Suggested Resources

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 6, students will be expected to

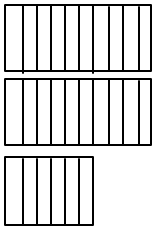
B10 divide numbers by 0.1, 0.01 and 0.001 mentally

In fact, there are relatively few workable divisions that can be done mentally compared with the other three operations . . . That does not mean that division is less important as a mental computation skill. However, mental division is more of a tool for estimation. (Elementary School Mathematics, p. 209)

Elaboration - Instructional Strategies/Suggestions

B10 By the end of grade 5, students have mentally multiplied and divided numbers by 10, 100 and 1000, and multiplied numbers mentally by 0.1, 0.01 and 0.001. Students will now add division by 0.1, 0.01, and 0.001 to their mental repertoires.

Since students generally expect the division process to result in a quotient which is smaller than the dividend, it is important that students understand why that is not the case here. One way to illustrate this is by analogy. For example, students should understand that one way to illustrate 12 ÷ 3 is to consider how many 3s there are in 12. Obviously, there are 4. Similarly, to illustrate 2.6 ÷ 0.1, consider how many 0.1s (i.e., one-tenths) there are in 2.6. Clearly, there are 10 in each unit and another 6 in 0.6, for a total of 26 one-tenths. (See diagram below.)



Q. How many one-tenths in two and six-tenths?

A. 26

Ultimately, students should see that dividing by 0.1 (one-tenth) increases the number of parts (and, hence, the answer) by a factor of 10 (ten times). Similarly, they should understand that dividing by 0.01 (one-hundredth) results in increasing the answer by a factor of 100, and dividing by 0.001 (one-thousandth) increases the answer by a factor of 1000. Students should be able to describe these changes in terms of place value.

- ☐ Include dividing by 0.1, 0.01 and 0.001 as part of regular mental math activities. As students become comfortable with questions of this type, include a mix of questions involving multiplication and division by 1000, 100, 10, 0.1, 0.01 and 0.001.

This should be taught in conjunction with C2.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B10.1 Ask students to divide 0.0034 by 0.1. Tell them to divide the result by 0.1, that result by 0.1, and the final result by 0.1. Ask: Is the result greater than or less than 1? Will repeatedly dividing by 0.1 always lead to a number greater than 1? Why or why not?

Paper and Pencil

B10.2 Present the following:

4 □6.□ ÷ 0.1 = □ 5.□ 3 ÷ 0.01

Ask: What digits belong in the boxes?

B10.3 Ask: Which answer will have a 3 in the tens place?

42 345 ÷ 0.1 42.345 ÷ 0.01 42.345 ÷ 0.001

Interview/Journal

B10.4 Ask: What digit would be in the tens place after dividing 453.2 by 0.01. Why?

B10.5 Tell students that you have divided a decimal number by 0.001 and the answer is a decimal number also. Ask: What do you know about the original decimal number?

B10.6 Ask students to explain why dividing by 0.01 produces the same result as multiplying by 100.

B10.7 Ask: Why does multiplying a number by 0.1 usually give a lesser answer than dividing the same number by 0.1?

Enrichment

B10.8 0.7834 ÷ 0.00001 = 0.7834 x □ Explain your answer.

Suggested Resources

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vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 6, students will be expected to

B11 calculate sums and differences in relevant contexts by using the most appropriate method

Elaboration - Instructional Strategies/Suggestions

B11 Students should recognize the need for different approaches to computation depending on the problem situation. Estimation must be used with all computations, but when an exact answer is required, students need to decide whether it is more appropriate to use a mental strategy, a pencil-and-paper calculation, or some form of technology, most often the calculator.

Students should continue to practise mental math strategies. One objective is for them to use it in their daily lives, not just in math class. It is also important to point out that the benefits students gain from exploring number and number patterns while developing mental math strategies are immeasurable. It is recommended that regular, maybe daily, practice be provided. This could take the form of out-of-context exploration of number at times (drill/practice or strategies), but would be most effective when addressed in a problem- solving context. For example, the following question might be presented to see if students can not only solve problems, but recognize that a mental strategy could be employed:

Mason bought 32.5 m of fencing that was on sale for $3 a metre. How much change did he receive from a $100 bill?

Students should perform mental computations with facility using strategies as outlined in the grades 4 and 5 guides.

- Front-end (left-to-right):

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 24 345
  3214
+10 116
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Beginning at the front or ten thousands place, the reasoning may be applied mentally/orally as follows:
20 000 + 10 000 and 4 000 + 3 000 or
24 000 + 13 000 makes 37 000, then 660 and one more 10 makes 70 + 5, altogether makes 37675

a)
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 28164
-12052
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b)
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15347
-9579
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a) can easily be solved with the front-end strategy. A quick glance at the digits in b) would lead one to consider something other than a mental strategy.

- Compensation:
  $25.95 + $3.99 + $11.98 is $26 + $4 + $12, or $42 subtract $0.08 for $41.92

For the following subtraction, one might use a combination of front-end and compensation:

$$\begin{array}{rcl} 7\,683 & \xrightarrow{+6} & 7\,689 \\ -5\,249 & \longrightarrow & -5\,249 \end{array}$$

2 440 - 6 = 2 434

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Interview

B11.1 Ask students to describe a way to compute 3000 - 2898 mentally. (The counting-on strategy would work well.)

B11.2 Ask students to explain how to solve each of the following using a mental strategy:

$75 + $12.25 + $5.75 =

2435.7
304.1
1050.2

470 + 1068 + 30 =

4579.25
-2134.17

B11.3 The numbers “75, 25,” “45, 55,” “340, 660” are examples of what are termed “compatible” or “nice” or “partner” numbers. Ask students to give the “partner” number for each of the following:

40 49 730 21

B11.4 Provide examples of computations and ask students which method they would use. Some examples are:

a) 2 x 22.3 b) 7.64 x $2.38 c) 100 - 12
d) $4.63 x 11 e) 24.8 x 0.5 f) $126.48 - 14.20
g) 1097.6 ÷ 39.2 h) $1.99 + $2.99 + $5.98 + $0.99

Many students at this grade should be able to mentally compute "a" (maybe using the front-end method), "c" (subtracting 10 and 2 more), "e" (finding half of the number), "f" (using front-end, although this is easier to see when written vertically) and "h." Examples “b,” “d,” and “g” would be solved with a paper/pencil algorithm or with a calculator.

Portfolio

B11.5 Ask students to make up an exercise sheet of computation examples that would lend themselves to mental strategies. Ask that they include a range of the strategies used in class. These could serve as practice sheets for the class.

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vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 6, students will be expected to

B12 calculate products and quotients in relevant contexts by using the most appropriate method

Elaboration - Instructional Strategies/Suggestions

B12 Students will have much experience with multiplication and division situations by grade 6. They should have mastered the mental strategies of multiplying and dividing whole and decimal numbers by multiples of ten and it is expected that they will know their multiplication facts. For students who still have not acquired their multiplication facts, work should still continue on fact recall strategies. See the grade 4 curriculum guide for strategies.

- Front-end

Multiplication:

3 x 325.15 Using the front-end strategy, one would say, 9 hundred (3 x 300) seventy-five (3 x 25) and 45 (3 x 15) hundredths (975.45).

Students very often proceed to find solutions using a paper/pencil algorithm without first checking to see if a mental strategy could be employed. *It is important that there is a mix (mental and paper/pencil) when problem situations are presented.*

Division:

Calculation has traditionally proceeded from left to right in division algorithms. Students should be able to divide using the "short" method. To provide practice, give students a mix of division exercises. Ask students to decide which examples can be calculated quickly using the mental front-end strategy, and which would require the use of the "short" division algorithm (which can also be a mental strategy).

$3\overline{)120.96}$ $5\overline{)176.28}$ $12\overline{)2400}$ $4\overline{)248.04}$

- Compensation

Multiplication: Students should be able to recognize that 9 x $4.95 is 9 x $5.00, or $45, less $0.45 (9 x 5) for a total of $44.55.

Division: Students should recognize that dividing by 5 can often be easier if the dividend is doubled and one divides by 10. For example
1632 ÷ 5 is the same as 3264 ÷ 10, or 326.4.

One other useful multiplication strategy is "double/halve," or "halve/double." An explanation must be provided as to how, for example, 22 groups of 15 gives the same product as 11 groups of 30.

It is important that students be taught to use some of the calculator features, other than the basic operations. However, because students need to practise paper/pencil calculations, the use of the calculator should be monitored.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B12.1 Ask students to give a written explanation for how the double/halve strategy works.

Interview/Journal

B12.2 Ask students to how to calculate 14 000 ÷ 50 mentally.

B12.3 Ask students to describe two numbers greater than 100 that would be easy to multiply mentally and to explain how.

B12.4 Give students a variety of computation questions and ask which could be easily computed mentally and which they would choose to calculate using a paper-and-pencil algorithm. Ask students to find the products that can be calculated mentally and estimate the product for the others.

2 x 315.2 35 x 18 99 x 85 47 x 58

B12.5 Ask students to how to use a calculator to help find 999 999 x 343 343.

B12.6 Ask students to describe two numbers greater than 100 that would be easy to divide mentally. Ask students to explain their choices.

B12.7 Ask: How would you calculate 90 316 248 x 10.1? (An 8-digit calculator display makes this question problematic.)

Suggested Resources