Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) describe, extend and create a wide variety of patterns and relationships to model and solve problems involving real-world situations and mathematical concepts

SCO: By the end of grade 6, students will be expected to

C1 solve problems involving patterns

C2 use patterns to explore division by 0.1, 0.01 and 0.001

Note: C2 may be addressed concurrently with B10

. . . Students can explore patterns that involve a progression from step to step. In technical terms these are called "sequences". We will simply call them "growing patterns". With these patterns, students not only extend patterns but look for a generalization or algebraic relationship that will tell them what the pattern will be at any point along the way. (Elementary School Mathematics, p. 376)

Elaboration - Instructional Strategies/Suggestions

C1 Students should continue to use patterns to help them solve mathematical problems. For example:

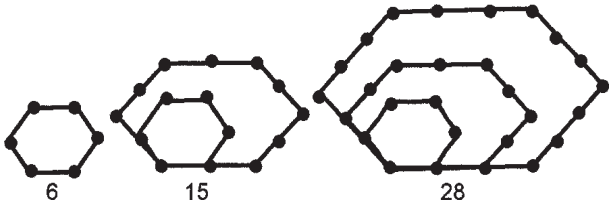
- Ask students to use a *calculator* to compute answers to the following pairs of questions: $\frac{1}{2} + \frac{1}{4}$ (and 1- $\frac{1}{4}$)

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ (and 1- $\frac{1}{8}$)

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ (and 1- $\frac{1}{16}$)

What do you think $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$ will be ?

- ❒ Ask students to continue the pattern to find the next three hexagonal numbers.



- ❒ Find the numbers of factors of 10, 20, 40, and 80. Predict the number of factors for 640.
- ❒ Two grade 6 students started an environmental club. They agreed to each get a new member every month. Each new member would recruit a new member by the end of their first month and every month thereafter. How many members will there be at the end of one year?

C2 By the end of grade 5 students have mentally multiplied and divided numbers by 10, 100 and 1000; multiplied numbers mentally by 0.1, 0.01 and 0.001; and examined the patterns connected with these operations. In grade 6 (SCO B10) students are expected to mentally divide numbers by 0.1, 0.01 and 0.001. Outcome C2 encourages students to recognize that the pattern of changes produced by dividing by 0.1, 0.01 and 0.001 is the same as that produced by multiplying by 10, 100 and 1000. For example:

4.71 x 10 = 47.1	4.71 ÷ 0.1 = 47.1
4.71 x 100 = 471	4.71 ÷ 0.01 = 471
4.71 x 1000 = 4710	4.71 ÷ 0.001 = 4710

It is important that students be able to describe these patterns with respect to place value changes, not just in terms of a rule involving moving the decimal point a certain number of places.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

C1.1 Ask students to explore the potential difficulty in using a calculator to multiply very large numbers, for which the product is greater than the display capability. For example, in lieu of using a calculator to directly find 999 999 999 x 999 999 999, students might use it to study a pattern and generalize:

99 x 99 =
999 x 999 =
9999 x 9999 =
....

Paper and Pencil

C1.2 Ask pairs of students to calculate:

38 x 32	48 x 42
36 x 34	46 x 44
37 x 33	47 x 43

Ask: What do you notice? Predict what 58 x 52 will be. Explain your prediction and verify.

C1.3 Ask students to find the sum of the first 30 even numbers (2 + 4, 2 + 4 + 6, etc.). Check to see if they are able to detect the multiplying pattern.

C1.4 Ask students to begin with 1, then add the next odd number for a sum, followed by the sum of the first three odd numbers, etc. Ask them to predict what the sum of the first twelve odd numbers would be.

Interview/Journal

C2.1 Tell students that Frank divided 42.8 by 0.1 and got an answer between three and four hundred. Ask the student to explain how he/she knows that Frank's answer must be incorrect.

C2.2 Ask students to explain why dividing a number by 0.01 results in a greater number than he/she originally started with.

Interview

C2.3 Ask students to predict (in terms of change in place value) what the effect of dividing by 0.00001 would be.

Suggested Resources

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore how a change in one quantity in a relationship affects another

SCO: By the end of grade 6, students will be expected to

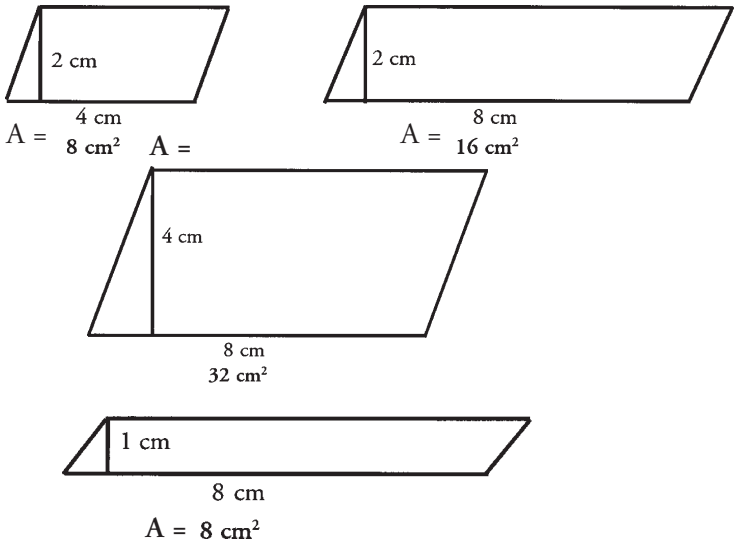
C3 recognize and explain how changes in base or height affect areas and rectangles, parallelograms and triangles

C4 recognize and explain how changes in height, depth or length affect volumes of rectangular prisms

C5 recognize and explain how a change in one term of a ratio affects the other term

Elaboration - Instructional Strategies/Suggestions

C3 As students explore the formulas for areas of rectangles, parallelograms and triangles, they need to interpret what the formulas actually mean, not just what to do with the numbers. For example, the formula for area of a parallelogram is A = bh; that should mean, to the student, that if b is doubled, then so is A; if b and h are both doubled, then the area is quadrupled; if b is doubled but h is halved, the area remains the same. This is connected to the "doubling and halving" strategy for mental multiplication explored in Grade 5. Explore these relationships through models such as:



C4 Similarly, with volumes of rectangular prisms, students should be aware of the impact of changes, to each of the variables in the formula.

C5 Just as with equivalent fractions, students should be aware that as one term of a ratio is multiplied or divided by a particular quantity, so is the other if the original ratio is to be maintained. Students should also be aware that, usually, when the same quantity is added to or subtracted from each term, the resulting ratios are not equivalent. Exploration should take place to confirm this Ex:
but $\frac{12-4}{16-4}=\frac{8}{12}\neq\frac{2}{3}$ (ratio maintained in the first case but not in the second)

Note: Outcomes A3, A4, A5, C5 and C6 should be addressed together.

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Worthwhile Tasks for Instruction and/or Assessment

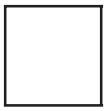
Performance

C4.1 Provide students with multilink cubes. Ask them to construct rectangular prisms with the dimensions 3 x 5 x 2 and 6 x 5 x 2. Ask students to find the volume of each. Ask: How could you have anticipated that the second volume would be twice the first? How do you think a 6 x 5 x 4 prism would compare to one 3 x 5 x 2? Note: Actually building a 6 x 5 x 4 prism may be problematic due to the large number of cubes required.

C3.1 Ask students to construct three different triangles on a geoboard (optionally, the triangles may be drawn on grid paper, but a geoboard may allow greater ease of exploration), all of which have a base of 4 units and an area of 8 square units. Ask them to choose one of their triangles and change its base so that the triangle's area will be 16 square units.

Paper and Pencil

C3.2 Ask students to draw a rectangle with an area exactly four times the area of the one shown and indicate the dimensions. Then ask the student to compare the factors and tell how one could have predicted the area relationship.



C5.1 Tell students that two ratios are equivalent. The first term of the first ratio is 10 and the first term of the second ratio is 25. Ask: How are the second terms of the ratios related?

Interview/Journal

C3.3 Tell students that a particular parallelogram has the same height as another, but the base is three times as long. Ask: How are the areas related and why?

C5.2 Ask students to explain why halving one term of a ratio necessitates halving the other term as well in order to preserve the ratio.

Portfolio

C4.2 Ask students to design a number of different rectangular boxes for fudge. Each design must have a volume of 1200 cm^3. Have the students write a report on their favourite designs, providing reasons for their choices.

Suggested Resources

Geoboards

Milti-link Cubes

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) represent mathematical patterns and relationships in a variety of ways (including rules, tables and one-dimensional and two-dimensional graphs)

SCO: By the end of grade 6, students will be expected to

C6 represent equivalent ratios using tables and graphs

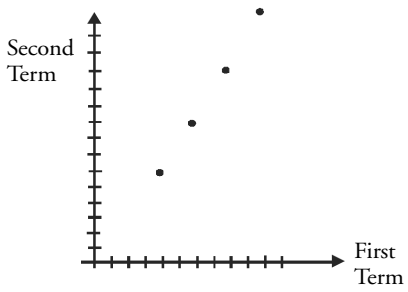
Elaboration - Instructional Strategies/Suggestions

C6 By representing equivalent ratios in tables and graphs, students will more clearly see both the relationship between the two elements of the ratio and the relationship between the two equivalent pairs.

For example, the ratio 2:3 is equivalent to:

First Term	4	6	8	10
Second Term	6	9	12	15

This can be graphed as shown:



Students should see from the table that, in each case, the ratio of all the number pairs is $\frac{2}{3}$ and that in each case, the same factor used to multiply by both terms to create the new terms. It is also possible to notice from the graph that a ratio such as 6.66:10 is approximately 2:3.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

C6.1 Ask students to draw a graph to show some (four or five) equivalents of the ratio 4:5.

Paper and Pencil

C6.2 There are about 11 seniors in Canada for every 7 children under age 4. In the following table, list the approximate number of seniors for the given numbers of children:

Children under 4	7000	14 000	28 000	31 500
Seniors	11 000			

Interview/Journal

C6.3 Tell students that a certain ratio is listed as 11:32. Ask: What simpler ratio would be a good estimate for this? How do you know?

Suggested Resources

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iii) represent mathematical patterns and relationships in a variety of ways (including rules, tables and one- and two-dimensional graphs)

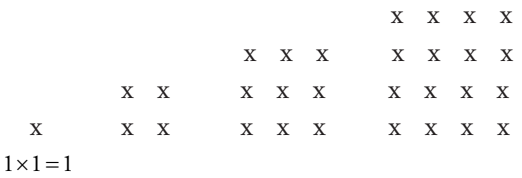
SCO: By the end of grade 6, students will be expected to

C7 represent square and triangular numbers concretely, pictorially and symbolically

Elaboration - Instructional Strategies/Suggestions

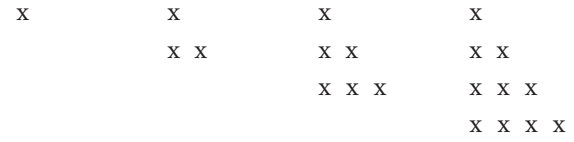
C7 Students may have had informal exposure previously to square and triangular numbers. These special numbers have both geometric and numerical significance. It would be worthwhile to ensure that they are familiar with these patterns of numbers.

Square numbers:



Square numbers may be represented in square arrays and are the products of numbers multiplied by themselves.

Triangular numbers:



Each triangular number is half the number in an array with dimensions that differ by one unit. For example, 6, the third triangular number, is half the number in a 3 row x 4 column array.

Likewise 10, the fourth triangular number, is half the number in a 4 x 5 array.

Triangular numbers may be displayed in triangle-like shapes and are calculated by adding the consecutive whole numbers beginning at 1.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

C7.1 Ask students to draw a picture of the 8th triangular number and tell what it is.

C7.2 Ask students to draw a number pattern in which the numbers might be called 'doublesquares'. Ask: How would you test to determine whether or not a number is doublesquare?

Paper and Pencil

C7.3 Ask students to find out (a) if two triangular numbers added together could ever make a square number [yes, e.g., 1 + 3 = 4] and (b) if two square numbers could be added together to make a triangular number [yes, e.g., 1 + 9 = 10].

C7.4 Ask students to find out (a) if the sum of two square numbers is ever a square number [yes, e.g., 36 + 64 = 100] and (b) whether the sum of two triangular numbers is ever triangular [yes, e.g., 6 + 15 = 21].

Interview/Journal

C7.5 Sara said that 100 is not a square number since, if you draw a 25 x 4 array, it's not square. Ask: How would you respond to Sara?

C7.6 Ask: Why is 8 not a square number?

C7.7 Ask students if 144 is a square number and to give reasons for the answer.

C7.8 Ask students to decide whether 55 or 56 is a triangular number and give reasons to support the choice.

Suggested Resources

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KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) solve linear equations using informal, non-algebraic methods

SCO: By the end of grade 6, students will be expected to

C8 solve simple linear equations using open frames

C9 demonstrate an understanding of the use of letters to replace open frames

Elaboration - Instructional Strategies/Suggestions

C8 Students should be able to solve simple linear open frame equations. Example: Suppose that in a class of 23, 8 students are working independently and the others are in groups of 3. How many groups are there? The information can be represented as

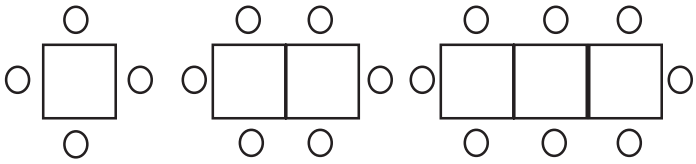
. The number of groups, 5, is the missing number.

Model the language used to express the meaning of equations. The above example might be read, "Three times a number and 8 more is 23." Encourage students to regularly read equations in meaningful ways.

C9 Having had experience for several years using open sentences, students should be introduced to the use of letters to represent variable (unknown) quantities. Students can be shown the parallel between sentences such as . They also need to realize that the particular letter used is irrelevant.

Students should understand that this use of a letter is simply a convention and no more or less meaningful than the use of the open frame. They must also be taught that the multiplication sign is often not written when a letter is used (e.g., instead of writing . Many students misinterpret 3n to mean a number in the thirties, so it is very important that this convention be made clear.

There are many patterns students could investigate that may be conveniently expressed using variables. For example, consider how many people can be seated at "n" tables in the following situation:



4 people at 1 table	6 people at 2 tables	8 people at 3 tables	2n + 2 people at n tables

Note: Students may well benefit from explicitly describing their counts as follows:

(1 table)	2 at the ends	+	2	on the sides	=	4 people
(2 tables)	2 at the ends	+	2	2 on the sides	=	6 people
(3 tables)	2 at the ends	+	2	3 on the sides	=	8 people
(40 tables)	2 at the ends	+	2	40 on the sides	=	82 people
(n tables)	2 at the ends	+	2	n on the sides	=	2 + 2n people

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Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

C8.1 Provide a few simple equations using open frames for students to solve such as

(a) $\square \times 2 + 8 = 37$ and $72 \times \square + 4 = 31$
(b) $3 \times \square + 4 = 25$
(c) $18 \div \square + 3 = 6$

C8.2 Ask students to create three different equations containing open frames for which the solution is 5.

Interview/Journal

C8.3 Tell students that $\square$ represents a certain number. Ask: Why must the solutions to be the same?

C9.1 Ask students to tell what each phrase means:

$5 + n$
$3 - n$
$2n$
$n \div 2$

C9.2 Ask students which would have the larger value, "n" or "y," and to explain why.

$2n + 16 = 32$ $\qquad$ $2y + 16 = 36$

Suggested Resources