Shape and Space

General Curriculum Outcome D:

Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations

SCO: By the end of grade 6, students will be expected to

D1 use the relationship among particular SI units to compare objects

D2 describe mass measurements in tonnes

The purpose of an interview is to uncover how students think about mathematics, to provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (<u>Mathematics Assessment</u>, Stenmark, ed. NCTM, 1991, p.29)

Elaboration - Instructional Strategies/Suggestions

D1 Students should review the meaning of the SI prefixes, i.e.,

milli ($\frac{1}{1000}$ of) centi ($\frac{1}{100}$ of)
deci ($\frac{1}{10}$ of) kilo (1000 of)

Using these meanings, students can compare amounts. For example:

- Which is greater, 3.45 L or 345 mL?
- How many milligrams make a kilogram?
- How many metres is 45.2 cm?

Students should realize that the relationship between linear SI units is not the same as the relationship between corresponding SI area and volume units. For example, 100 cm = 1m, but $100\ cm^2 \neq 1\ m^2$ and $100\ cm^3 \neq 1\ m^3$.

Students should explore the relationships between SI area and volume units by comparing values in a series of concrete situations. For example, the linear dimensions in cm for the figure below, are $\frac{1}{10}$ the magnitude of the same dimensions in mm. The area, in cm^2, however, is the magnitude of the area in mm^2.

$A = 60\ cm^2 = 6000\ mm^2$ 5 cm = 50 mm

12 cm = 120 mm

D2 Students should be introduced to the "tonne". The tonne is equivalent to 1000 kg. (This unit should be distinguished from the "ton" which is used in the U.S. to represent 2000 pounds.)

Students should

- be aware of the types of items which might have masses measured in tonnes
- relate the tonne to other mass units (e.g., 456 kg = 0.456 tonnes)
- solve problems involving tonnes using real life examples.

❐ Students might investigate to determine the number of children (based on average mass) it would take to balance an elephant, a rhino, a blue whale, a brontosaurus, etc.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

D1.1 Ask: How many cubic decimetres are there in a cubic metre?

D1.2 Describe an object as being 0.003 dm long. Ask students whether or not it would be a whole number of centimetres (millimetres) long.

D1.3 Tell students that the area of a rug is 9000 cm^2 and ask how many square metres that is.

Interview

D1.4 Ask: What do you think "kilosecond" should mean (i.e., how long would one last)?

Journal

D1.5 Tell students that Sue said "10dm = 1m, so then 10dm^2 = 1m^2." Ask: Do you agree? Why or why not?

D1.6 Tell students that the area of a rectangular rug is 10 000 cm^2. Ask: What are some possible dimensions, in metres?

Presentation

D2.1 Ask students to work in pairs and decide whether or not items with the following masses would be easy to lift:

0.001 tonnes	0.001 kg
10 000 tonnes	10 000 g

Ask them to share their conclusions and reasoning with the class.

Suggested Resources

Base Ten Materials

Measuring Tapes

Metre Sticks

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter, and angle*

ii) *communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations*

SCO: By the end of grade 6, students will be expected to

D3 demonstrate an understanding of the relationship between capacity and volume

Elaboration - Instructional Strategies/Suggestions

D3 It is important to have students explore the relationship between the cubic units of volume and capacity. Students need to solve problems involving both capacity and volume and to understand the following relationships between the two:

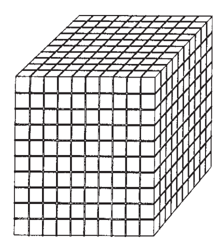
$1\ cm^3 = 1\ mL$ $1\ dm^3 = 1\ L$ $1\ m^3 = 1\ kL$

❒ Ask students to investigate the capacity, or volume, of moving trucks of various sizes. Let them determine how much and what furniture could be moved using trucks of various sizes.

Base ten blocks serve as good models. The small (units) block has a volume of 1 cubic centimetre and would displace 1 mL of liquid in a container; the 10 x 10 x 10 block (dm^3) has a volume of 1000 cubic centimetres and would displace 1 L of liquid. (Some may be interested in pursuing the relationship among volume, capacity, and mass. A cubic centimetre of water, equivalent to 1 mL, has a mass of about 1 g; 1000 cm^3 of water, equivalent to 1 L, has a mass of about 1 kg.)

Capacity and volume are both measures of the size of a 3-D region of space. Capacity is usually associated with measuring fluids and/or the containers which hold fluids (e.g., 2 L bottle). Volume is more commonly associated with solid objects and is an expression built in three dimensions upon length units.

$1\ cm^3 = 1\ mL$



$1000\ cm^3 = 1\ L$

Note: Students may be interested in exploring and using the relationship between capacity and volume to find the volume of irregularly shaped objects. For example, immerse a small rock in 100 mL of water in a 200 mL graduated cylinder. The difference between the new water level and the original 100 mL level can be used to determine the volume of the rock.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D3.1 Ask students to use base ten materials to make a structure with a volume of 1256 cm^3. Ask: What would be the capacity of this structure?

D3.2 Ask students to place a 20 cm^3 structure into a full container of water. Ask: How many millilitres of water spill out?

Interview/Journal

D3.3 Ask students to decide whether volume or capacity units would be used to describe

- amount of water in a pool
- amount of wheat in a barrel
- living space in a house

Students should justify all choices.

Portfolio

D3.4 Ask students to design a lesson plan for a grade four class in which they address the following:

- What does volume mean?
- What does capacity mean?
- How are they similar? different?

Invite them to teach their lesson to small groups of children and write a report on this experience.

Suggested Resources

Base Ten Materials

Volumetric Set

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter, and angle*

iii) *estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units*

SCO: By the end of grade 6, students will be expected to

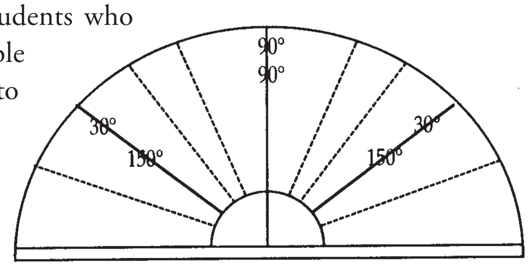
D4 estimate and measure angles using a protractor

D5 draw angles of a given size

The protractor is one of the most poorly understood measuring instruments found in schools . . . By making a protractor with a large unit angle, all of [the] mysterious features can be understood. Then, a careful comparison with a standard protractor will permit that instrument to be used with understanding. (<u>Elementary School Mathematics</u>, p. 305)

Elaboration - Instructional Strategies/Suggestions

D4 Students should learn how to use a protractor to measure angles reasonably accurately. Students who use protractors with double scales on them will need to learn how to determine which set of numbers to use in a given situation. This is best accomplished by first having students estimate the size of the angle and then decide which reading makes the most sense. Students should also use "benchmark" angles such as 30^0, 45^0, 60^0, 90^0, 120^0, 135^0, and 150^0 to help them estimate angle sizes visually before measuring and to check reasonableness of measurements.

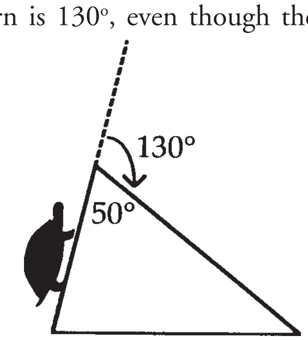


❒ Ask students to find the measures of each of the angles in various quadrilaterals. They should observe that the sum is always 360°. Similarly, they might find the sums of the angles in other types of polygons.

D5 Students need to learn how to use a protractor to draw an angle. Most work should focus on angles between 0° and 90°. There should be some discussion, however, of how to draw larger angles (e.g., 120° or 180°).

Students should be aware of the importance of positioning the 0° line on the protractor so that it coincides with the first arm of the angle in order to produce an accurate drawing.

❒ If available, use the computer programme Logo to ask students to examine the effect of various angle turns. In situations like the one illustrated below, students should note that the size of the required turn is 130°, even though the angle of the triangle is 50°.

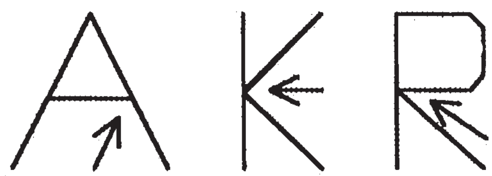


GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D4.1 Ask students to measure the angles found in various letters of the alphabet.



D4.2 Ask students to estimate, and show with their hands or two straight objects, the size of a 120° angle.

D5.1 Ask students to make a 135° angle without using a protractor.

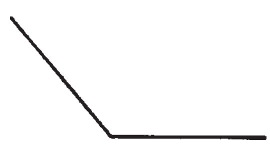
Pencil and Paper

D5.2 Draw an angle (e.g., 60º). Ask students to draw an angle 90˚ greater without using a protractor.

D5.3 Ask students to draw an angle they think might measure 150º. Ask: How close were you?

Interview/Journal

D4.3 Tell students that Jeff measured this angle and said it measured 50º. Ask: Do you agree? Why or why not?



D4.4 Tell students that Marc said he could make an angle bigger by extending both angle arms. Ask: What do you think of this plan? Explain.

Suggested Resources

Protractors

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units

SCO: By the end of grade 6, students will be expected to

D6 solve measurement problems involving length, capacity, area, volume, mass and time

Elaboration - Instructional Strategies/Suggestions

D6 Students should regularly solve problems involving a variety of types of measurement. Many problems can and should be coordinated with the teaching of other curricular areas (e.g., using map scales and making scientific observations).

Time problems should include the use of a variety of time units, exploration of the recording of time using the 24-hour clock, and exploration of the idea of time zones. This provides opportunities for students to plan "worldwide" trips using travel schedules and taking time zones into account.

There are many opportunities to link measurement with relevant problem solving. For example:

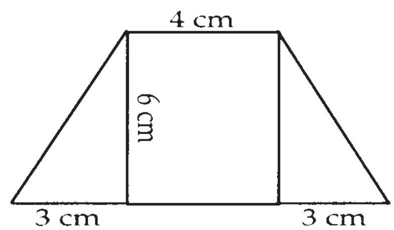
- time and length measures to determine speed
- area and number measures to calculate population densities
- area and length measures to find ratios in similar figures
- mass and capacity measures to conclude that the mass of 1L of water is about 1kg.
- capacity and volume (see Note under elaboration for D3).

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D6.1 Ask students to find the area of the trapezoid and explain their method(s).



D6.2 Tell students a flight leaves St. John's at 11:20 a.m. (Newfoundland Time) and arrives in Toronto 1:50 p.m. (Eastern Time). How long was the flight? (Assume direct flight.)

Interview/Journal

D6.3 Ask students how they might estimate the volume of an inflated beach ball (or basketball, volleyball, banana, apple, etc.).

Portfolio/Project

D6.4 Ask students to plan a trip (car and ferries only) starting on a Monday morning at Saint John, N.B. They can assume that they will average 90 km/h on land. Ask: When would you expect to arrive in St. John's, Newfoundland? Include all times and schedules.

D6.5 Ask pairs of students to design pens for 6 gorillas in a zoo. The animals need an exercise area and a watering hole. Encourage them to be creative in their designs. Ask the students to calculate the cost of tiling the floors of their pens.

Project

D6.6 Provide students with airline schedules and ask that they work in pairs to discuss the times of flight arrivals and departures. They may wish to plan a cross-Canada trip. Ask them to visit at least 6 major cities. What is the quickest way to go to Victoria, B.C.?

Suggested Resources

Time zone chart from telephone book.

24 hour clock (can be in the form of a blackline master, or teacher-made). Note: many ordinary digital watches have a 24-hour time function.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter, and angle*

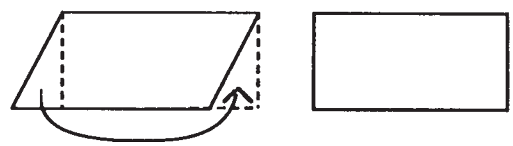
iv) *develop and apply rules and procedures for determining measures (using concrete and graphic models)*

SCO: By the end of grade 6, students will be expected to

D7 **demonstrate an understanding of the relationship among the bases, heights and areas of parallelograms and use this relationship to solve problems**

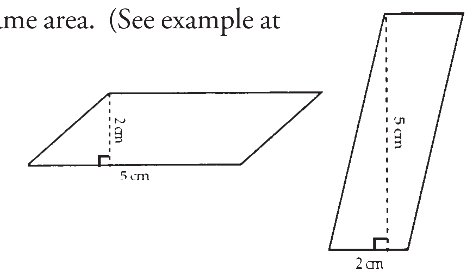
Elaboration - Instructional Strategies/Suggestions

D7 Students should recognize that the area of a parallelogram is the same as the area of a related rectangle, i.e., one with the same base and height.



Students should be able to determine the base or height, given the area and the other dimension.

Students should recognize that a variety of parallelograms can have the same area. (See example at right.)



❒ Try the following "trick." Make a flexible rectangle using geostrips or cardboard strips and brads. Begin to "deform" (tilt) the rectangle. Ask the students whether or not the area has changed. Keep tilting until students see that the area has decreased. Discuss how with each additional deformation, a new parallelogram was created with the same base, but less height; therefore, the area decreased. Eventually, the rectangle will be "deformed" to a straight line segment which encloses no area.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

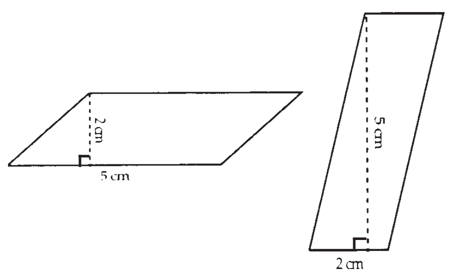
D7.1 Ask students to draw on grid paper (or create on a geoboard) a parallelogram with an area of 24 cm^2. Then ask the student to create three other parallelograms with the same area.

Paper and Pencil

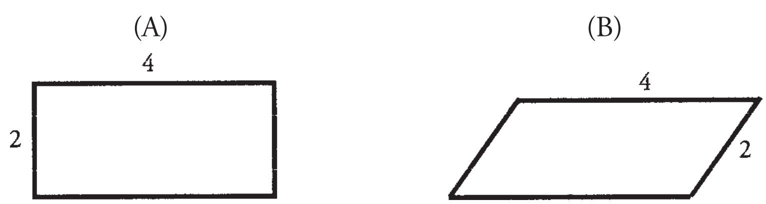
D7.2 Ask: If two parallelograms have the same area, do they have to be similar? [Note: See also outcome E6]

Interview

D7.3 Ask: Do these parallelograms have the same area? How do you know?



D7.4 Ask students to determine which of the two shapes below has the greater area and to provide an explanation.



Suggested Resources

Geostrips

Geoboards

Grid paper (1 cm grid or larger)

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter, and angle*

iv) *develop and apply rules and procedures for determining measures (using concrete and graphic models)*

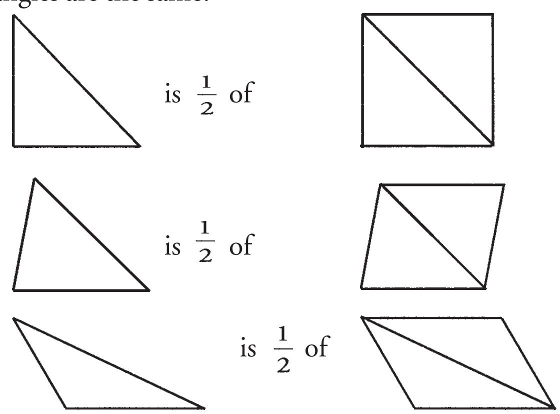
SCO: By the end of grade 6, students will be expected to

D8 demonstrate an understanding of the relationship between the area of a triangle and the area of a related parallelogram and use this relationship to solve problems

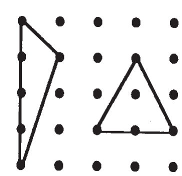
D9 demonstrate an understanding of the relationships between the three dimensions of rectangular prisms and volume and surface area and use these relationships to solve problems

Elaboration - Instructional Strategies/Suggestions

D8 Students should recognize that any triangle is one half of a parallelogram. Thus, students should see that the area of the triangle is just one-half of the area of the related parallelogram. Students can use this relationship to find areas of simple triangles. Students should understand that, as long as the base and height are the same, the areas of visually-different triangles are the same.



❐ Have students locate on a geoboard as many triangles as possible which have an area of 2 cm^2. Students should understand that any triangle with base 4 and height 1, or base 2 and height 2, will qualify.



D9 To determine volumes and/or surface areas, students should build structures or fill containers with centimetre cubes, always estimating before calculating.

Although students need not commit the symbolic form of the formulas to memory, their experiences should indicate to them that each of the three dimensions of the prism, i.e., the height, depth, and width, affects the volume and surface area. For example, in a 3 cm × 6 cm 2 cm box, 3 6 or 18 cubes fill a layer (the base). Since there are 2 layers, the volume must be 2 18 = 36 cm^3. In general, the volume of any rectangular prism is equal to the area of the base multiplied by the height of the prism.

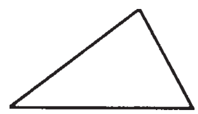
❐ Milk cartons can be cut and fashioned into cubes with side lengths of 10 cm and used as building blocks. Each block has a volume of 1 dm^3 (or 1000 cm^3, or 0.001 m^3) and each face a surface area of 100 cm^2 (or 0.01m^2).

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D8.1 Ask students to draw a parallelogram with twice the area of the triangle at right.



Paper and Pencil

D9.1 Ask students to write an explanation for why a prism with a rectangular base that is 5 cm 3 cm and a height of 4 cm has to have a volume of 60 cm^3.

D9.2 Ask: Which has the greater effect on the volume of a prism, doubling the area of the base or doubling the height? Ask them to explain their thinking.

Portfolio/Journal

D8.2 Present the following scenario: The area of a certain triangle is found. The area of another triangle is 2 units less and its height is 1 unit less than the height of the first triangle. Ask: What do you know about the bases? Explain your thinking.

Suggested Resources

Geoboards

Base Ten Materials