Shape and Space

General Curriculum Outcome E:

Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *identify, draw, and build physical models of geometric figures*

iv) *solve problems using geometric relationships and spatial reasoning*

SCO: By the end of grade 6, students will be expected to

E1 **describe and represent the various cross-sections of cones, cylinders, pyramids and prism**s

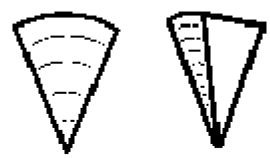
Students should be challenged to analyze their thought processes and explanations. They should be allowed sufficient time to discuss the quality of their answers and to ponder such questions as, Could it be another way? What would happen if ... ? (Curriculum and Evaluation Standards for School Mathematics, p. 113)

Elaboration–Instructional Strategies/Suggestions

E1 A cross-section is the 2-D face produced when a plane cut is made through a solid 3-D shape. For example, consider a right-circular cone.

a) If it is cut in any plane parallel to its base, the face produced is a circle.

b) If it is cut down through its vertex, the exposed face is a triangle.



c) If the cone is cut obliquely—not parallel to its base—the face produced is an oval shape. (In later grades students will learn to call this an ellipse.)

While other cross sections of the cone can be examined, they should not be considered core material at this level.

Cross-sections should be investigated by actually cutting shapes or by observing water surfaces in models of shapes. Students will come with prior experiences with cross-sections of cubes, square prisms, and rectangular prisms.

It is not intended at this stage for students to remember the various cross-sections for each 3-D shape without the presence of the shape; however, they should try to draw their predictions of the cross-sections before they cut the shape. Using elastic bands on the shapes to represent cuts is a way to help students visualize.

This should be done in conjunction with E7.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E1.1 Ask students to make some triangular prisms with plasticine. Ask them to predict and draw the polygonal cross-section(s) that would result with each of the following cuts and to check their predictions by making the appropriate cuts:

(a) cut parallel to its bases
(b) cut parallel to one of its rectangular faces
(c) cut obliquely (not parallel) to its bases
(d) cut obliquely to a rectangular face

E1.2 Ask students to explain and demonstrate how a square pyramid could be cut to produce each of the following cross-sections:

(a) a triangle (b) a rectangle (c) a trapezoid

Presentation

E1.3 Ask students to stack four hexagonal pattern blocks to make a hexagonal prism. Ask them to discuss with a partner some ways this prism could be cut and what shapes would be produced. Ask them to present their ideas to the class, including their pictures of the different cross-sections.

Interview/Journal

E1.4 Provide a variety of 3-D shapes for students to examine. A mystery 3-D shape has been cut to make a triangular cross-section. Think of four possibilities for this mystery shape. Describe the cuts that would have been made to produce this cross-section.

E1.5 Ask students to describe how a cylinder could be cut to produce each of the following cross-sections:

(a) a circle (b) a rectangle (c) an oval

Note: This outcome does not lend itself to pencil and paper assessment without the aid of concrete materials to physically slice or at least models to look at while deciding how various cross sections would appear.

Suggested Resources

3-D Shapes

Pattern Blocks

Plasticine or Play Dough

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i) *identify, draw, and build physical models of geometric figures*

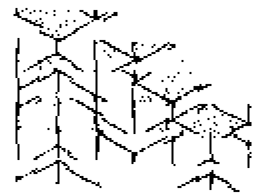
iv) *solve problems using geometric relationships and spatial reasoning*

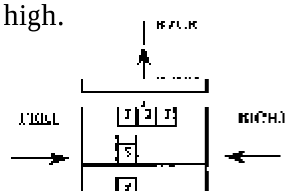
SCO: By the end of grade six, students will be expected to

E2 **make and interpret orthographic drawings of 3-D shapes made with cubes**

Elaboration–Instructional Strategies/Suggestions

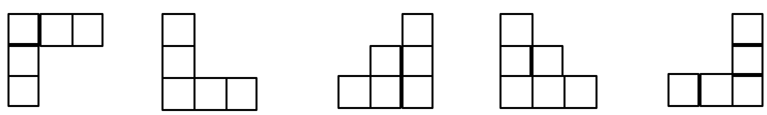
E2 Orthographic drawings are a series of 2-D views of a 3-D shape drawn by looking at the shape straight down (to get a top view) and straight on (to get front, back, right, and left views). The figure at right could be interpreted in a mat plan that shows its base with numbers indicating the number of cubes high.



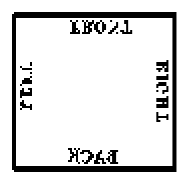


Students could build this shape with cubes, place them on mat plans, and draw the various orthographic views on square dot paper. The following are the views for this shape:

Top View **Front View** **Left View** **Right View** **Back View**



The use of mats often helps students with these drawings. A square of plain paper appropriately marked with directions would be a simple mat for this purpose. The students could then place a 3-D shape on the mat and move the mat to make the drawing of each view.



Note: Left and right are always relative to the front.

Some students might find it helpful to close one eye and place themselves so that they are at eye level with the shape; they should then see only one face of the 3-D shape.

- ❑ Provide students with directional mats and 3-D shapes made of eight cubes. Ask them to draw mat plans and make top, front, and right orthographic views of these shapes.
- ❑ Provide students with top, front, and left orthographic views of various 3-D shapes. Ask them to use cubes to build the shapes with these views and to draw mat plans.

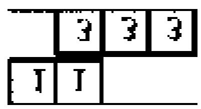
Note: Teachers should familiarize themselves with the grade 4 and 5 outcomes relative to this topic.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

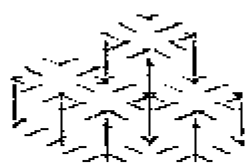
Performance

E2.1 Ask students to use cubes to construct the shape that has this mat plan.

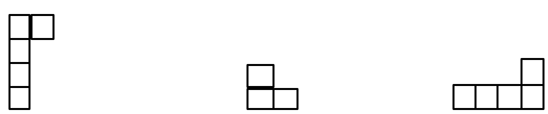


Ask them to place it on a mat and to draw the various orthographic views, labelling each.

E2.2 Ask students to use cubes to construct the shape shown at right. Ask them to draw the mat plan for this shape and the top, front, and left orthographic views, labelling each. Ask: Would the back and right orthographic views of this shape be needed for someone to be able to build this shape or would the three views you have be sufficient?



E2.3 Ask students to construct a building with cubes that would have these orthographic views.

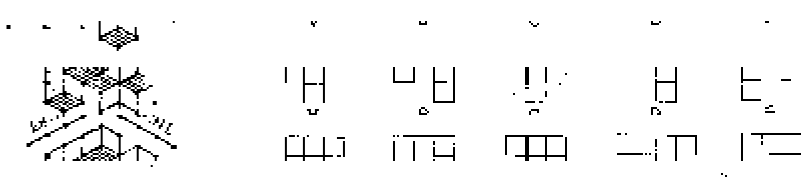


TOP **FRONT** **RIGHT**

Ask them to draw its mat plan.

Paper and Pencil

E2.4 Provide students with this picture of a building drawn from its front-right corner. Ask them which one of A–E is the orthographic view from the right.



Suggested Resources

Interlocking Cubes

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) describe, model, and compare 2- and 3-D figures and shapes, explore their properties and classify them in alternative ways

SCO: By the end of grade 6, students will be expected to

E3 make and apply generalizations about the sum of the angles in triangles and quadrilaterals

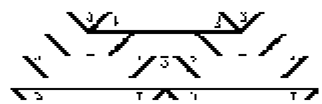
E4 make and apply generalizations about the diagonal properties of trapezoids, kites, parallelograms and rhombi

E5 sort the members of the quadrilateral "family" under property headings

Modeling, mapping, and engaging in activities and spatial experiences organized around physical models can help students discover, visualize, and represent concepts and properties of geometric figures in the physical world. (Geometry in the Middle Grades, p. 1)

Elaboration–Instructional Strategies/Suggestions

E3 Students should investigate these generalizations in a variety of ways. For example, if each student cuts out a triangle, tears off its three angles, and places them together, the three angles will form a straight line (180^o); if each student cuts out three copies of a triangle, labels the three angles, and starts a tessellation, they can see that the sum of the measure of the angles is 180^o; if students measure the three angles in a variety of triangles and add them, the 180^o relationship might be revealed, although measurement often produces only approximate results.



Similarly, students could be led to generalize the 360^o sum of the measures of the angles of any quadrilateral by drawing one diagonal in a quadrilateral and observing that this divides the quadrilateral into two triangles.

E4 Generalizations about diagonal properties should result from guided investigations. These properties should not be treated as things to memorize. Diagonal properties of squares and rectangles were studied in grade 5.

a) For a rhombus, the diagonals are perpendicular bisectors of each other, form four congruent right triangles, bisect the angles of the rhombus, and are its two lines of reflective symmetry.

b) For a parallelogram, the diagonals bisect each other and form two pairs of congruent triangles.

c) For a kite, the diagonals are perpendicular and form two pairs of congruent right triangles; one of the diagonals is bisected, and the other diagonal is a line of reflective symmetry and bisects two opposite angles of the kite.

d) For a trapezoid, there are no special properties of its diagonals.

These properties should be developed for each figure, applied in a variety of ways, compared to the others, and combined with the side and angle properties of the figures.

E5 Students should be able to sort pictures or cutouts of the various quadrilaterals into sets according to one or more properties. These properties include: diagonals that bisect each other; opposite sides that are congruent; four right angles; diagonals that are perpendicular to each other; opposite angles that are congruent; has reflective symmetry; and diagonals form two pairs of congruent triangles.

E4, E5 and E8 should be addressed together.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E3.1 Tell students that you heard someone say, "Since any quadrilateral can be divided into two triangles and a triangle has a sum of angles of 180^o, it is obvious that the sum of the angles of a quadrilateral is 360^o. Ask students to draw pictures of various quadrilaterals to verify the truth of this statement. Ask students to extend this thinking to find the sum of the angles of a pentagon.

E4.1 Ask students to draw a scalene right triangle. Use a mira to draw the reflected images of this triangle in both arms of the right angle to make a pentagon. Ask them to join two vertices to produce a quadrilateral with its two diagonals showing. Ask them to name this quadrilateral. Ask if everyone in the class has the same type of quadrilateral. Ask them to list the properties of this quadrilateral that they can confirm from the way they drew it.

E5.1 Make a set of cards with a variety of pictures of different members of the quadrilateral family. Distribute them to the students. Choose an attribute card (e.g., opposite sides parallel). Ask students to put their shape cards under this attribute, if appropriate, and to discuss why or why not. Choose another attribute card (e.g., diagonals bisect each other) and place it with the first card. Ask students to discuss which shapes should now be removed and why.

Paper and Pencil

E3.2 Ask students to find the size of the missing angle(s) for each triangle and to draw the triangle if (a) two of its angles are 70^o and 45^o, (b) two of its angles are each 75^o, (c) it is a right triangle with a 60^o angle, and (d) it is an isosceles triangle with an angle of 102^o.

E3.3 Ask students to draw parallelograms on square dot paper. Ask them to measure one of the angles of the parallelograms and determine the other three angles by using known relationships.

E4.2 Ask students to list the properties of a rhombus that are the same as those of a kite and the ones that are different.

E4.3 Ask students to draw a segment that is 5-8 cm long. Ask them to use only a Mira to draw a rhombus that has this segment as one of its diagonals, explaining the process used. [Note: They might use the fact that the diagonals are perpendicular bisectors of each other to draw a second diagonal and then draw the rhombus by joining the end points of the diagonals.]

Suggested Resources

Mira (transparent mirror)

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ii) describe, model, and compare 2-D and 3-D figures and shapes, explore their properties and classify them in alternative ways

SCO: By the end of grade 6, students will be expected to

E6 recognize, name, describe and represent similar figures

Elaboration–Instructional Strategies/Suggestions

E6 Many students have an intuitive sense of similar shapes—shapes that are enlargements or reductions of each other. Students' experiences with negatives of photographs that can be developed in different sizes, with maps or pictures that are drawn to scale, and with images produced by magnifying glasses provide natural connections for this concept. Overhead projectors, photocopiers, and film projectors are other sources of real-world contexts to relate to similar figures.

Students should discuss what the word *similar* might mean to them in everyday contexts. Compare these meanings to the specific meaning of the word similar in mathematics (i.e., corresponding angles equal and pairs of corresponding sides have the same ratio).

- ❑ Prepare pairs of shapes, some of which are similar and some which are not. Tape the larger onto the board and place the smaller on the overhead projector. Ask students to move the projector until the image coincides, or does not, with the one taped on the board. They are similar if a match can be made.
- ❑ Place a red pattern block on the overhead projector. Ask students to compare the projected image to the actual block, asking them what is the same and what is different. Ask a student to place the pattern block in the corresponding angles of the projected image. (This should emphasize the role angles play in making shapes similar.) Informally compare the lengths of corresponding sides of the block and of the projected image, seeing approximately how many times longer the image sides are than the sides of the block. (This will be easier if you move the projector in advance so the sides will be a whole number times larger rather than a fractional times larger.)

All dilatation images of a shape (see SCO E9) are similar; however, not all similar figures are merely dilatation images of one another—similar figures can be on different planes and/or be the result of a dilatation in combination with other transformations.

Students are likely to recognize the similarity of different sizes of regular polygons (e.g., equilateral triangles and squares). Also, because all triangles with corresponding pairs of angles equal are similar, they may be more likely to easily recognize similar triangles than most similar quadrilaterals.

Note the connection to E9.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E6.1 Ask students to examine the 3 different-sized triangles in a tangram set for similarity. Ask them if they think these triangles are similar. Ask them to compare the angles and the lengths of the corresponding sides of these triangles to confirm or refute their answer.

E6.2 Provide students with a sheet of rectangles, most of which are similar. Ask them to cut out the rectangles they think are similar and to draw in the diagonals. Ask them to lay the similar rectangles on top of one another, starting with the largest and in such a way that they share one common vertex. Ask them what they notice about the diagonals of the similar rectangles. Ask them to check one of the rectangles that is not similar to see the difference in the diagonals.

E6.3 Ask each student to make a triangle on a geoboard using the bottom left peg and the pegs directly above and to the right of it. [to make room for more triangles] Ask them to make four different triangles, all similar to first one. Explain why they are similar.

E6.4 Using a flashlight and a shape in a dimmed classroom, move the flashlight to cast shadows on the wall asking students to identify if the shadow is similar, or not, to the shape. Repeat using other shapes. Ask: Where does the flashlight need to be held to produce a similar shape?

E6.5 Ask students to use only the triangles in the pattern blocks to make other larger triangles. Ask: Are these larger triangles similar to the green pattern block? Ask them to hold a green block close to one of their eyes, stand over one of the larger triangles staring down at it using the eye with the block in front of it, and move the block until it coincides with the larger triangle. (This is another way that students can test for similarity.) Ask them to compare the sizes of the angles in the triangles and the lengths of the corresponding sides. Repeat these tasks using the other pattern blocks.

Paper and Pencil

E6.6 Ask students to draw scalene triangles, cut them out, and use these triangles to help draw smaller and larger similar triangles.

E6.7 Provide students with shapes or pictures drawn on one-sized grid paper. Ask them to use a different-sized grid paper to draw similar shapes or pictures.

Suggested Resources

Tangrams:

Geoboards:

Pattern Blocks:

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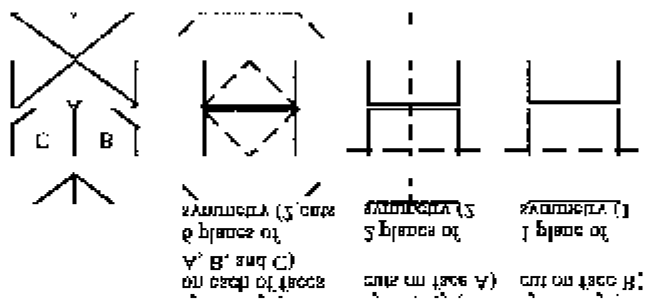
iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

SCO: By the end of grade 6, students will be expected to

E7 make generalizations about the planes of symmetry of 3-D shapes

Elaboration–Instructional Strategies/Suggestions

E7 The 3-D shapes explored relative to this outcomes should be limited to prisms and pyramids. In the same way that some 2-D shapes have lines of reflective symmetry, some 3-D shapes have planes of reflective symmetry, i.e., planes that bisect 3-D shapes such that all points in one-half are mirror images of the corresponding points in the other half. These planes of symmetry should be connected to cross-sections as examples of special cuts. A cube, for example, has nine different planes of symmetry as shown in the figure below. (Note: Although faces B and C could also be cut by perpendicular planes as shown on face A below, only one cut on face B would produce a different result.)



Students should investigate planes of symmetry of triangular, square, rectangular, pentagonal, and hexagonal pyramids. Students should discover the pattern that the number of planes of reflective symmetry for these pyramids is equal to the number of lines of reflective symmetry of their bases (e.g., a square pyramid has 4 planes of symmetry and its square base has 4 lines of symmetry).

- ❑ Provide students with triangular, rectangular, square, pentagonal, and hexagonal prisms. Ask them to investigate the number of lines of reflective symmetry of the bases of these prisms. Ask: Are planes through these lines of symmetry planes of symmetry of these prisms? Do these prisms have other planes of symmetry? Ask students to explain how to find the number of planes of symmetry of a prism.

Note: The prisms and pyramids used should be right prisms and pyramid such as the ones typically found in basic sets of solids.

This should be done in conjunction with E1.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

E7.1 By stacking pattern blocks, construct a 3-D configuration that has two planes of symmetry. Ask students to show where its two planes of symmetry are.

E7.2 Ask students to use a set of pyramids that have regular polygonal bases to help complete this table.

Pyramid	No. of Lines of Reflective Symmetry in the Base	No. of Planes of Symmetry
triangular (one equilateral face) square pentagonal hexagonal		

Ask: Do you see a pattern that you could use to predict how many planes of symmetry an octagonal pyramid has?

E7.3 Ask students to examine real-world objects of a variety of shapes (e.g., boxes, containers, toys, candies, and candles) for planes of reflective symmetry.

E7.4 Ask students to use 12 multi-link cubes to build a shape that has two planes of reflective symmetry.

Interview/Journal

E7.5 Provide pictures of houses, garden sheds, gazebos and other structures. Ask students to choose which structures have plane(s) of symmetry and describe the location(s) of these planes.

E7.6 Ask students to compare the number of planes of symmetry of a square prism (non-cubic) and a rectangular prism. Ask them to explain why the square prism has more planes of symmetry than the rectangular prism. Ask them to explain why they both have fewer planes of symmetry than a cube.

Suggested Resources

Geometric solids

Pattern Blocks

Interlocking Cubes

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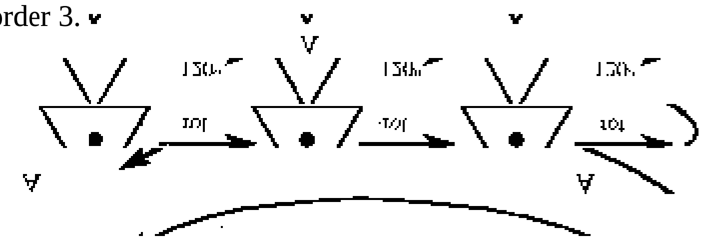
iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

SCO: By the end of grade 6, students will be expected to

E8 make generalizations about the rotational symmetry property of all members of the quadrilateral "family" and of regular polygons

Elaboration–Instructional Strategies/Suggestions

E8 If a shape can be turned about a point so that it exactly coincides with its original position at least once in less than a complete rotation, it is said to have rotational symmetry. The number of times it appears in the identical position during one complete rotation is the order of rotational symmetry. For example, if an equilateral triangle is turned clockwise 120° about its centre point, the image is identical; if it is turned another 120°, again the image is identical. It is said to have rotational symmetry of order 3.



If a shape has to be rotated 360 degrees before it fits its traced image, it does not have rotational symmetry.

- ❑ Ask students to use a blue block from the pattern blocks, tracing it on paper. Ask them to place the block on its traced image and lightly put a pencil mark in the upper left corner of the block. Ask them to turn the block within its traced image clockwise until it fits its traced image again. Ask them to notice where the mark on the block is now. Ask them to continue turning the block to the right until it fits its traced image again, noticing where the mark is on the block. (Through this activity, students should conclude that a rhombus has rotational symmetry of order 2. In other words, it fits its traced image twice within a 360⁰ rotation.)

Through activities such as the one above with other members of the quadrilateral family of shapes, students should generalize that a square has rotational symmetry of order 4; a rhombus, parallelogram, and rectangle each have rotational symmetry of order 2; in general a kite and a trapezoid do not have rotational symmetry.

Note: There are many handy contexts for exploring rotational symmetry. For example, consider the common toddler toy which involves fitting blocks through openings. In how many ways can the hexagonal block be fitted through the hexagonal opening?

E4, E5 and E8 should be addressed together.

Symmetry in two and three dimensions provides rich opportunities for students to see geometry in the world of art, nature, construction, and so on. (Curriculum and Evaluation Standards for School Mathematics, p.115)

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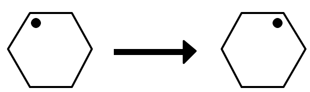
Worthwhile Tasks for Instruction and/or Assessment

Performance

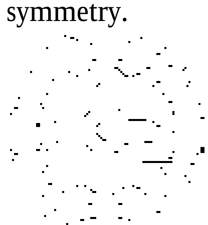
E8.1 Have students trace around a hexagonal pattern block and mark a dot at one vertex on the block. Ask them to rotate the hexagon clockwise until it fits the traced hexagon exactly.

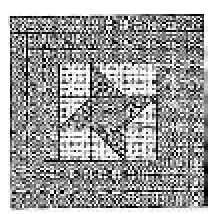


Ask them to continue until the marked vertex returns to its original position. Ask them how many times the block appeared in the identical position. Ask them to describe its rotational symmetry.

E8.2 Repeat E8.1 for square, rhombus, rectangle, parallelogram, kite, and trapezoid.

E8.3 Provide students with pictures of designs and quilt patterns such as the ones below. Ask them to predict whether they have rotational symmetry. Have them use tracing paper (or wax paper) to confirm their predictions by tracing the patterns and rotating the tracing paper on top of the pictures. Have them check as well for reflective symmetry.





E8.4 Ask students to use geoboards to make shapes that have rotational symmetry of order 2.

Interview/Journal

E8.5 Ask students to explain how someone would know the order of rotational symmetry for any regular polygon.

Portfolio

E8.6 Ask students to examine newspapers and magazines for pictures and logos that have rotational symmetry. Ask them to select four of their favourite ones, paste them on paper, and write short descriptions of their symmetry, including comments on their reflective symmetry if they have it. (Many companies (e.g., Chrysler and Mercedes Benz) have logos that are symmetric.)

Suggested Resources

Pattern Blocks

Computer software drawing packages sometimes allow rotations, given a centre. Examples of such software include, but are not limited to:

1 Wordperfect 9 or higher

2 Microsoft Word Drawing Tools

3 Geometric Sketchpad (if available)

4 FX Draw V2 (if available)

Of these, (1) and (2) are perhaps the simplest to use for rotations.

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iii) *investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts*

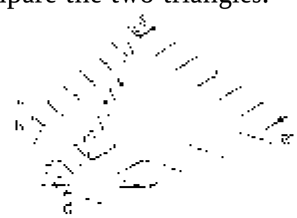
SCO: By the end of grade 6, students will be expected to

E9 **recognize and represent dilatation images of 2-D figures and connect to similar figures**

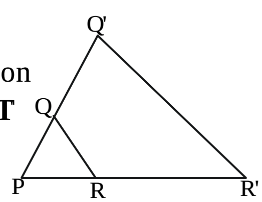
Elaboration–Instructional Strategies/Suggestions

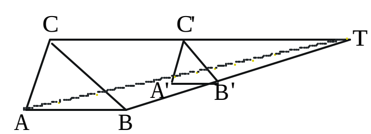
E9 Introduce dilatations by asking students to participate in activities such as the one below.

- ❑ Ask each student to draw a scalene triangle, labelled JKL, on a sheet of paper and select a point C outside this triangle. Ask them to measure the distance from C to J, and triple this distance for CP; measure the distance from C to K, and triple this distance for CQ; and measure the distance from C to L, and triple this distance for CR. Ask them to draw a triangle by joining P, Q, and R and ask them to compare the two triangles.



Explain to students that if lines through all corresponding vertices of two shapes on a plane converge at a single point (as they do at point C in the activity above), these shapes are dilatation images of one another; the point of convergence is the centre of dilatation; and the two shapes are similar.





$\Delta A'B'C'$ is the dilatation image of ΔABC with **T** as the centre of dilatation; $\Delta P'Q'R'$ is the dilatation image of ΔPQR with P, as the centre of dilatation. Comparing the distances from P the corresponding image points are twice as far from P as the original points. Likewise, comparing the distances from T of the image points, it is evident that the image points are half as far from T as the original points.

The centre of dilatation could be connected to the concept of vanishing point if students have done perspective drawing in art.

Students should be encouraged to compare dilatation images and to see if they can see any relationships between them. Some students might notice that corresponding sides are parallel and that each pair of corresponding sides have the same ratio.

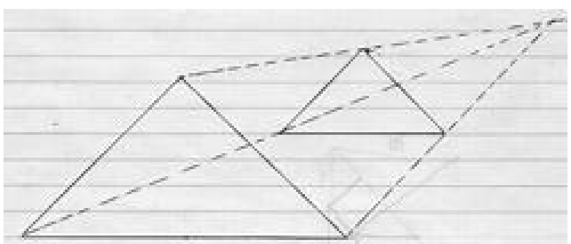
Note the connection to E9.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

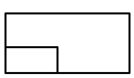
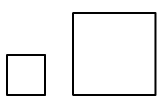
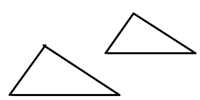
Worthwhile Tasks for Instruction and/or Assessment

Performance

E9.1 Ask students to each trace the large triangle from tangram sets on lined paper so that its longest side lies along a line (see below). Ask them to also trace one of the smaller triangles from the sets so that its longest side lies along a different line. Ask them to draw lines between corresponding vertices of the two triangles and extend them. Ask: What is the point where the three lines intersect called? Ask them to investigate whether lined paper could be used in the same way to set up any two similar figures to be dilatation images.



E9.2 Ask students to try to visualize the existence of a centre of dilatation for each of the following pairs to predict if they are dilatation images of one another. Ask them to check by actually locating the centre using a straight edge.

a)  b)  c) 

E9.3 Ask students to each trace a red pattern block on a sheet of plain paper. Ask them to draw for this pattern block a dilatation image of their choice. Ask them to compare the angles and sides of their two trapezoids.

Journal

E9.4 Ask students to draw a 9cm x 12cm rectangle on lined paper, with a 12cm length lying along one of the lines on the paper. Ask them to draw a 3cm x 4cm rectangle somewhere else on the paper with a 4cm length lying along a line. Ask: Are the two rectangles dilatation images of one another? How do you know?

Suggested Resources

Tangrams

Pattern Blocks

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

SCO: By the end of grade 6, students will be expected to

E10 predict and represent the result of combining transformations

Computer software allows students to construct two- and three-dimensional shapes on a screen and then flip, turn, or slide them to view them from a new perspective. (<u>Curriculum and Evaluation Standards for School Mathematics</u>, p. 114)

Elaboration–Instructional Strategies/Suggestions

E10 Students should understand that two congruent shapes on the same plane are images of one another under a translation, reflection, rotation, or any combination of these three transformations. Students should investigate a variety of combinations, each time trying to visualize the result to make a prediction before actually carrying out the transformations. These combinations should include a reflection followed by a translation, two translations, two reflections, a translation followed by a rotation, and two rotations.

- Place three geoboards side by side. Ask one student to make a scalene triangle on the first geoboard. Ask another student to construct on the second geoboard the image of this triangle if the right side of the first geoboard is used as a mirror line. Ask another student to construct on the third geoboard the image of the triangle on the second geoboard under a 90 degree counterclockwise rotation. (Repeat this activity using other shapes.)

- Provide each student with grid paper marked with a co-ordinate system and three pattern blocks of the same type. Ask students to place one block on the system so that one of its vertices is at (-5,3). Ask them to place a second block so that it would be the image of the first block under a horizontal translation of 10 units. Then ask them to place the third block so that it is the image of the second block under a reflection in the x-axis. Ask them to compare the first and third blocks. Repeat this activity using two other transformations. (Extension: Ask each student to carry out two transformations of their choice on the co-ordinate system and leave only the first and third blocks in place. Ask them to exchange co-ordinate systems with a partner and ask them to try to predict the two transformations that took place. Share their predictions and actual transformations.)

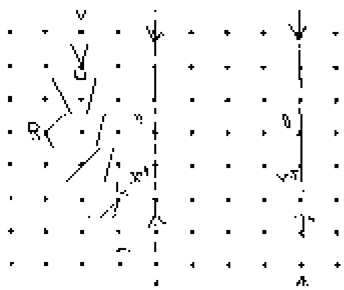
- Ask students to investigate such questions as:
  - If a shape undergoes 2 translations, does it matter in which order they take place?
  - Does a reflection followed by a translation produce the same result as the translation followed by the reflection?

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

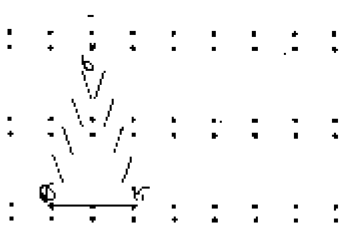
Worthwhile Tasks for Instruction and/or Assessment

Performance

E10.1 Ask students to locate the image of ΔABC after a reflection in line 1 followed by a reflection in line 2. Ask them what single transformation of ΔABC would have the same result. [Ans: Translation 8 units to the right.]



E10.2 Ask students to draw an isosceles triangle on dot paper and to translate it 4 units horizontally. Ask them to describe the reflection that would produce the same result.



Journal

E10.3 Present students with the pictures on grid paper of two congruent shapes—the first and the third (after two transformations were performed). Ask students to predict what two transformations were performed. Ask: Could this have been done in more than one way? Could this have been done by a single transformation?

Paper and Pencil

E10.4 Ask students to each draw a 4 x 4 square on grid paper and shade some of the small squares inside to create a design. Have them draw another 4 x 4 square attached under the first square. Ask them to draw, in this square, the image of their design under a reflection in the bottom side of the first square. In another 4 x 4 square attached under the second square, ask them to draw the half-turn rotation image of the design in the second square. Ask them to compare the designs in the first and third squares.

Suggested Resources

Geoboards

Pattern Blocks