

# **Grade 7 Mathematics**

## **Unit 7**

### **Data Analysis**

*Estimated Time: 18 Hours*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization



# Unit 7 Overview

## Introduction

Students will learn beginning techniques of data management and analysis. They will also learn the introductory concepts of statistics and probability. The big ideas in this unit are:

- There are three measures of central tendency (sometimes referred to as averages); they are the mean, median and mode.
- When deciding which measure of central tendency best represents a set of data, the presence (and effects) of outliers must be considered
- Probability ranges between two values, 0 and 1; or 0% and 100%. A probability of 0 means an event is impossible, while a probability of 1 means an event is certain.
- There are two types of probability to consider; experimental and theoretical. As the number of trials in an experiment increases the experimental probability of an event occurring will get closer to the theoretical probability of that event occurring.

## Context

The students will examine sets of data in order to determine the three measures of central tendency. They will use computational methods to accomplish this and also use technology to assist them. The students will be required to determine the existence of outliers and analyze the effects of any outliers on the measures of central tendency. The students will learn to determine which measure is the best representative of a set of data.

Using skills learned in earlier units (shifting between fractions, decimals, and percents) the students will express the probability of an event in several formats. They will learn what it means, statistically speaking, to say that an event is certain or impossible.

Students will develop sample spaces (lists of all possible outcomes) for events. They will utilize tree diagrams and tables to accomplish this, and then use those sample spaces to determine the probability of two independent events occurring simultaneously, or sequentially. The students will then compare those theoretical probabilities to experimentally determined probabilities for the same independent events.

## Why are these concepts important?

Understanding data analysis is a valuable skill for any person living in a modern 21<sup>st</sup> century society. The concepts taught in data management and probability are those used everyday to make important decisions in many industries. Industries such as marketing, research, sports, medicine, law making and law enforcement, business, and government. Being familiar with these ideas will allow the student to make intelligent and relevant decisions as they go through life.

*Then there is the man who drowned crossing a stream with an average depth of six inches.*

*~W.I.E. Gates*

***General Outcome: Collect, display and analyze data to solve problems.***

Specific Outcome	Elaborations: Suggested Learning and Teaching Strategies
<p><i>It is expected that students will:</i></p> <p><b>7SP1. Demonstrate an understanding of central tendency and range by:</b></p> <ul style="list-style-type: none"> <li>• <b>determining the measures of central tendency (mean, median, mode) and range</b></li> <li>• <b>determining the most appropriate measures of central tendency to report findings.</b></li> </ul> <p>[C, PS, R, T]</p>	<p><b>Measures of central tendency</b> allow us to describe a set of data with a single meaningful number. The study of mean, median and mode as measures of central tendency is entirely new to these students in grade 7.</p> <p>The focus of this outcome is to determine mean, median and mode and to understand that situational context will determine which measure is most meaningful. It may be appropriate to use one, two or three of these measures to represent a given data set.</p> <p>The <b>mean</b> is the sum of the numbers in a set of data divided by the number of items of data (arithmetic average). It is the number that most people are describing when they talk about average. The calculation of the mean describes a set of data by identifying a value obtained from combining all values of the data set and distributing them equally.</p> <p>The <b>median</b> is the middle number when data are arranged in numerical order. Half of the data values are above the median and half are below. If there are two numbers in the middle of a data set, the median is the mean of those two values. The median may be the same as the mean or, it may be different.</p> <p>The <b>mode</b> is the number that occurs most often in a set of data. It is possible that the data set have one mode, several modes or no mode at all. Ordering, bar graphs and stem and leaf plots are useful data displays to easily identify the mode of a given data set. Students have studied these displays prior to grade 7.</p> <p>When considering the data as a whole it is often of value to consider the spread of the data. One strategy for examining this is to consider the <b>range</b> of the data. Students will calculate the range by subtracting the smallest data value from the greatest. Range may be used in combination with one of the other measures of central tendency to create a better representation of the data in a set.</p>
<p><b>Achievement Indicators</b></p>	
<p>7SP1.1 Determine mean, median and mode for a given set of data, and explain why these values may be the same or different.</p>	
<p>7SP1.2 Determine the range for a given set of data.</p>	
<p>7SP1.3 Solve a given problem involving the measures of central tendency.</p>	

***General Outcome: Collect, display and analyze data to solve problems.***

Suggested Assessment Strategies	Resources/Notes								
<p><u>Graphic Organizer</u> Create a tri-fold foldable to define and create examples of each of the measures of central tendency. On each of the outside panels, name and define mean, median or mode. On the corresponding inside panel, create and solve an example of a problem using the measure of central tendency on the front. (Refer to Appendix 4-D for the tri-fold foldable.)</p> <p><u>Journal</u></p> <ol style="list-style-type: none"> <li>Create a set of data for each of the following. Each set must have at least 6 pieces of data.               <ol style="list-style-type: none"> <li><u>Situation 1</u>: The mean, median and mode are the same.</li> <li><u>Situation 2</u>: The mean, median and mode are different.</li> </ol> </li> <li>The following data was collected to represent the progress of two students in science class. Each of the students would have the same mark based on the calculation of the mean. Find the range of the data for each student and explain how the range can add valuable information to the representation of the progress of each student.               <ol style="list-style-type: none"> <li>Student 1: 76% 78% 80% 82% 84%</li> <li>Student 2: 60% 70% 80% 90% 100%</li> </ol> </li> </ol> <p><u>Paper and Pencil</u> Between January and March one year school was cancelled in Snowytown seven times due to blizzards. The following data gives the number of days each blizzard lasted.</p> <table border="1" data-bbox="428 1409 764 1598"> <tr> <td>1 day</td> <td>6 days</td> </tr> <tr> <td>4 days</td> <td>2 days</td> </tr> <tr> <td>2 days</td> <td>3 days</td> </tr> <tr> <td>3 days</td> <td></td> </tr> </table>	1 day	6 days	4 days	2 days	2 days	3 days	3 days		<p><i>Math Makes Sense 7</i> <b>Lesson 7.1</b> <b>Lesson 7.2</b> Unit 7: Data Analysis TR: ProGuide, pp. 4–7 &amp; pp. 8–12 Master 7.11, 7.19 Master 7.12, 7.20 CD-ROM Unit 7 Masters</p> <p>ST: pp. 258–261 ST: pp. 262–266 Practice and HW Book pp. 154–155 pp. 156–157</p>
1 day	6 days								
4 days	2 days								
2 days	3 days								
3 days									
<p>Find the mean, median and mode for this data.</p>									

***General Outcome: Collect, display, and analyze data to solve problems.***

Specific Outcome	Elaborations: Suggested Learning and Teaching Strategies
<p><i>It is expected that students will:</i>  <b>7SP2. Determine the effect on the mean, median and mode when an outlier is included in a data set.</b>  <b>[C, CN, PS, R]</b></p>	<p>In a set of data, we often find values which are significantly different from the others. These values are called <b>outliers</b>. The presence of outliers may affect which measure of central tendency best represents the data.</p>
<p><b>Achievement Indicators</b></p>	<p>Outliers are often identified from a numbered set but can also be identified in different data displays.</p>
<p>7SP2.1 Analyze a given set of data to identify any outliers.</p>	<p>Consider each of the following scenarios.</p> <p>For the data set 3, 4, 5, 5, 6, 7, the mean <math>\frac{3+4+5+5+6+7}{6} = \frac{30}{6} = 5</math> is a good representation because some of the data is slightly greater and some of the data is slightly smaller than the mean.</p>
<p>7SP2.2 Explain the effect of outliers on the measures of central tendency for a given data set.</p>	<p>For the data set 3, 4, 5, 5, 6, 7, a median of 5 is also appropriate. Half of the data are above 5 and half are below.</p> <p>If, however, the data contained just one or two extreme pieces of data, the mean may be less representative. For the data set 3, 4, 5, 5, 6, 19, the mean <math>\frac{3+4+5+5+6+19}{6} = \frac{42}{6} = 7</math> is greatly influenced by the outlier 19 and does not represent the data as well as the median (3, 4, 5, 5, 6, 19) which remains unchanged at 5.</p> <p>In the case of a mode, outliers may or may not have an effect on its value.</p> <p>For a data set of 4, 6, 5, 5, 5, 5, the mean <math>\frac{4+6+5+5+5+5}{6} = \frac{30}{6} = 5</math>, the median of 5 (4, 6, 5, 5, 5, 5) and the mode of 5 (4, 6, 5, 5, 5, 5) are all representative.</p>
	<p>But, in the case of a data set such as 9, 9, 2, 1, 4, 5, the outlying value of 9 is the mode, but is not representative of the entire set.</p>

***General Outcome: Collect, display, and analyze data to solve problems.***

**Suggested Assessment Strategies**

*Paper and Pencil*

Tanya received the following scores on her first five math quizzes:

75%	75%	80%	77%	82%
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- A. What is the mean, median and mode.
- B. Unfortunately, Tanya did not do very well on her next quiz. She only achieved a mark of 25%. What affect, if any, did this mark have on the measures of central tendency calculated in a)?

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.3**  
Unit 7: Data Analysis  
TR: ProGuide, pp. 13–16  
Master 7.13, 7.21  
CD-ROM Unit 7 Masters

ST: pp. 267–270  
Practice and HW Book  
pp. 158–160

***General Outcome: Collect, display, and analyze data to solve problems.***

**Specific Outcome**

*It is expected that students will:*  
**7SP2. Determine the effect on the mean, median and mode when an outlier is included in a data set.**  
**[C, CN, PS, R]**  
**(Cont'd)**

**Achievement Indicators**

7SP2.3 Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.

7SP2.4 Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.

**Elaborations: Suggested Learning and Teaching Strategies**

In any data collection process it is possible for errors to occur. These errors may be due to human measurement problems or in the recording of information.

Students must learn to distinguish between an error that appears as an outlier and a legitimate data value which is significantly different from the others. Data directly attributable to error are omitted from the calculation of measures of central tendency. If no error has occurred, the data value must be included in the calculation of the measures of central tendency.

When choosing the best measure of central tendency to represent a data set, the presence of outliers and their effect on the mean, median and mode must be considered.

Example:

Players on the grade 7 basketball team were asked to record their height in cm on a chart in the classroom. The data obtained was to be used to represent the height of the team. Here is the data.

155 cm	153 cm	150 cm	167 cm
164 cm	182 cm	170 cm	159 cm
185 cm	19 cm	182 cm	174 cm

- Does this data set contain any outliers? How can you tell?
- Suggest a reason for this outlier. Should this outlier be included in the calculations for the measures of central tendency? Why or why not?
- Calculate the mean, median and mode for these heights.
- Which measure(s) of central tendency would you use to represent the height of the team? Why?

(The solutions to these questions are **continued** on the next two page spread.)

***General Outcome: Collect, display, and analyze data to solve problems.***

**Suggested Assessment Strategies**

Journal

Define the term outlier. Provide an example of a situation in which an outlier must be excluded from the data before calculating the measures of central tendency. Why would you exclude the outlier in this case?

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.3**  
**(continued)**

***General Outcome: Collect, display, and analyze data to solve problems.***

<b>Specific Outcome</b>	<b>Elaborations: Suggested Learning and Teaching Strategies</b>
<p><i>It is expected that students will:</i>  <b>7SP2. Determine the effect on the mean, median and mode when an outlier is included in a data set.</b>  <b>[C, CN, PS, R]</b>  <b>(Cont'd)</b></p>	<p><i>Solutions</i> to the questions on the previous two page spread:</p>
<p><b>Achievement Indicators</b></p>	<p>a) When responding to this question, students should be able to determine that 19 cm is significantly different from all of the other values. It is over 130 cm in difference from the next closest value.</p>
<p>7SP2.3 Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.  <b>(continued)</b></p>	<p>b) Perhaps the student made an error and should have written 190 cm. Unfortunately we cannot know for certain. This outlier is, however, clearly an error. No grade 7 student would reasonably measure 19 cm tall. Consequently, it should be excluded when calculating the measures of central tendency.</p>
<p>7SP2.4 Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.  <b>(continued)</b></p>	<p>c) <math>\text{mean} = \frac{1841}{11} = 167.\overline{36}</math> which is 167 cm when rounded.</p>
	<p>median : 150, 153, 155, 159, 164, 167, 170, 174, 182, 182, 185  thus median = 167 cm</p>
	<p>mode: From the ordered list, we can easily see that 182 cm is the mode.</p>
	<p>d) Both the mean and the median are good representations for the height of the team in this case. The mode, however, is not a good choice as it represents a height greater than most of the players on the team.</p>

***General Outcome: Collect, display, and analyze data to solve problems.***

**Suggested Assessment Strategies**

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.3**  
**(continued)**

***General Outcome: Collect, display, and analyze data to solve problems.***

Specific Outcome	Elaborations: Suggested Learning and Teaching Strategies																
<p><i>It is expected that students will:</i></p> <p><b>7SP1. Demonstrate an understanding of central tendency and range by:</b></p> <ul style="list-style-type: none"> <li>• <b>determining the measures of central tendency (mean, median, mode) and range</b></li> <li>• <b>determining the most appropriate measures of central tendency to report findings.</b></li> </ul> <p><b>[C, PS, R, T]</b></p> <p><i>(Cont'd)</i></p> <p><b>Achievement Indicators</b></p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>7SP1.4 Provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.</p> </div>	<p><b>Classroom discussion at this point should be focussed upon realistic situations and making decisions among measures of central tendency.</b></p> <p>The mean is most meaningful when the data values contain few outliers. For instance, when considering student progress, a teacher calculating a student’s mark uses the mean when the set of marks does not contain any extreme values.</p> <p>The median is most meaningful when there are a small number of significantly different pieces of data. In this case, the median is often a better representation of the data. For example, when considering the progress of a student who regularly performs well on tests but had one failure, the teacher may wish to use the median of the student’s scores when discussing test results with parents on interview night.</p> <p>Certain life situations exist in which the mode is the only acceptable measure of central tendency. A store selling shoes or dresses will not find it useful to know that the average size of shoe or dress is 8.32.</p> <p><b>Example:</b>                  Restaurant staff was required to prepare 400 meals per day. After a 15 day period, they noted that some food was always left over. The manager knew she wanted to prepare enough food for her customers, but did not want to be wasteful. In an effort to determine how many meals the staff should prepare each day, she analysed the sales receipts to determine the number of meals sold. She recorded the number of meals sold each day.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td>320</td><td>325</td><td>299</td><td>298</td><td>326</td><td>315</td><td>320</td><td>311</td> </tr> <tr> <td>295</td><td>272</td><td>120</td><td>265</td><td>278</td><td>288</td><td>296</td><td></td> </tr> </table> </div> <p>Analyse this information to determine how many meals the staff should prepare each day.</p> <p><i>(This elaboration is <b>continued</b> on the next two page spread...)</i></p>	320	325	299	298	326	315	320	311	295	272	120	265	278	288	296	
320	325	299	298	326	315	320	311										
295	272	120	265	278	288	296											

***General Outcome: Collect, display, and analyze data to solve problems.***

**Suggested Assessment Strategies**

Interview

For each situation, would mean, median, or mode be most helpful to know. Justify your choice.

- A. You are ordering bowling shoes for a bowling alley.
- B. You want to know if you read more or fewer books per month than most people in your class.
- C. You want to know the “average” amount spent per week on junk food in your class.

Journal

Darryl, Gordon and Joan are captains of the school math teams. Their contest results are recorded in the table below.

	Darryl	Gordon	Joan
Contest 1	82	84	85
Contest 2	82	84	85
Contest 3	88	90	85
Contest 4	100	71	81
Contest 5	77	78	81
Contest 6	81	87	85
Contest 7	87	89	82
Contest 8	83	88	85
Contest 9	83	86	83

- A. Based on the mean, whose math team is the best?
- B. Based on the median, whose math team is the best?
- C. Based on the mode, whose math team is the best?
- D. Which measure would you choose to determine whose team is the best? Why?
- E. Why might someone disagree with you?

**Resources/Notes**

*Math Makes Sense 7*

**Lesson 7.4**

Unit 7: Data Analysis

TR: ProGuide, pp. 17–21

Master 7.14, 7.22

CD-ROM Unit 7 Masters

ST: pp. 271–275

Practice and HW Book  
pp. 161–163

***General Outcome: Collect, display, and analyze data to solve problems.***

<p><b>Specific Outcome</b></p> <p><i>It is expected that students will:</i></p> <p><b>7SP1. Demonstrate an understanding of central tendency and range by:</b></p> <ul style="list-style-type: none"> <li>• <b>determining the measures of central tendency (mean, median, mode) and range</b></li> <li>• <b>determining the most appropriate measures of central tendency to report findings.</b></li> </ul> <p>[C, PS, R, T]</p> <p><i>(Cont'd)</i></p> <p><b>Achievement Indicators</b></p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>7SP1.4 Provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings. <b>(continued)</b></p> </div>	<p><b>Elaborations: Suggested Learning and Teaching Strategies</b></p> <p><i>Solution</i> to the question on the previous two page spread:</p> <p>A reasonable suggestion would be to prepare 320 meals per day. The following analysis is suggested.</p> <p>Students should think about finding the measures of central tendency for this data.</p> <table border="1" style="width: 100%; margin-top: 10px;"> <tr> <td data-bbox="656 701 1060 1434"> <p>Mean = <math>\frac{4328}{15} = 288.5\bar{3}</math>.</p> <p>Therefore the mean number of meals is 289.</p> <p>120 265 272 , 278 288 295, 296, 298, 299 311, 315 320, 320, 325, 326</p> <p>The median is 298.</p> <p>The mode is 320.</p> </td> <td data-bbox="1060 701 1463 1434"> <p>Students can use a calculator to find the mean.</p> <p>Using an ordered list, students can easily find the median and the mode.</p> <p>***If students chose to remove the outlier of 120, there is no significant change in the measures of central tendency.***</p> </td> </tr> </table> <p>Considering all of the measures of central tendency, it can be easily seen that both the mean and the median are inappropriate in this context. If the manager prepares only 289 or 298 meals each day the restaurant would have run out of prepared meal on seven days. The sensible conclusion would seem to be a regular preparation of 320 meals to reduce the amount of food wasted and almost always meet the customer orders. From time to time they may have to prepare a few extra meals on demand.</p>	<p>Mean = <math>\frac{4328}{15} = 288.5\bar{3}</math>.</p> <p>Therefore the mean number of meals is 289.</p> <p>120 265 272 , 278 288 295, 296, 298, 299 311, 315 320, 320, 325, 326</p> <p>The median is 298.</p> <p>The mode is 320.</p>	<p>Students can use a calculator to find the mean.</p> <p>Using an ordered list, students can easily find the median and the mode.</p> <p>***If students chose to remove the outlier of 120, there is no significant change in the measures of central tendency.***</p>
<p>Mean = <math>\frac{4328}{15} = 288.5\bar{3}</math>.</p> <p>Therefore the mean number of meals is 289.</p> <p>120 265 272 , 278 288 295, 296, 298, 299 311, 315 320, 320, 325, 326</p> <p>The median is 298.</p> <p>The mode is 320.</p>	<p>Students can use a calculator to find the mean.</p> <p>Using an ordered list, students can easily find the median and the mode.</p> <p>***If students chose to remove the outlier of 120, there is no significant change in the measures of central tendency.***</p>		

***General Outcome: Collect, display, and analyze data to solve problems.***

**Suggested Assessment Strategies**

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.4**  
**(continued)**

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*

**7SP4. Express probabilities as ratios, fractions and percents.**

[C, CN, R, T, V]

**Achievement Indicators**

7SP4.1 Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction and percent.

**Elaborations: Suggested Learning and Teaching Strategies**

In grade 6, students were exposed to probability as a measure of how likely an event is to occur. They have been exposed to identifying possible outcomes and determining theoretical and experimental probabilities.

**Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory,  $\frac{1}{2}$ . Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several trials (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**.

It is important for students to acquire an understanding that probability can be represented in multiple forms. The probability of an event occurring is most often represented by using a fraction, where the numerator represents the number of favourable outcomes and the denominator represents the total possible outcomes.

$$P(\text{event}) = \frac{\# \text{ of favorable outcomes}}{\# \text{ of possible outcomes}}$$

This representation has many advantages since it often maintains the original numbers in simple situations. Probability can similarly be represented as a ratio. However, probability can just also be represented in decimal form. Likewise, students will often hear in news/weather reports various probability data presented as percents. For example, the likelihood of rainfall for a given day is almost always provided in percent form. In order for all situations encountered to be meaningful to the student, they should work with all representations of probability (fractions/decimals, ratios and percents).

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Suggested Assessment Strategies**

Paper and Pencil

Chris noted the results of spinning a spinner in the table below.

2	3	3	1	4	1	2	5	1	2	2	3	1
1	2	4	5	2	4	1	1	2	3	3	1	

Find each of the following probabilities. Express your answer as a ratio, as a fraction and as a percent each time.

- P(spin of 2)
- P(spin of 5)
- P(spin of even number)

Interview

Use a scale with benchmarks 0 (0%),  $\frac{1}{4}$  (25%),  $\frac{1}{2}$  (50%),  $\frac{3}{4}$  (75%), and 1 (100%) to assess the reasonable probability of the events described below. Explain each choice.

- The next baby born in your town will be a boy.
- It will snow at least once in the month of June.
- Living 6 months without water.
- The sun will set tomorrow.

**Resources/Notes**

*Math Makes Sense 7*

**Lesson 7.5**

Unit 7: Data Analysis

TR: ProGuide, pp. 25–29

Master 7.10, 7.15, 7.23

CD-ROM Unit 7 Masters

ST: pp. 279–283

Practice and HW Book

pp. 164–166

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*

**7SP4. Express probabilities as ratios, fractions and percents.**

**[C, CN, R, T, V]**

*(Cont'd)*

**Achievement Indicators**

7SP4.2 Provide an example of an event with a probability of 0 or 0% (impossible) and an example of an event with a probability of 1 or 100% (certain).

**Elaborations: Suggested Learning and Teaching Strategies**

Students should understand that impossible events have a probability of 0 and events that are certain to occur have a probability of 1.

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Suggested Assessment Strategies**

*Paper and Pencil*

Chris was asked to find two more probabilities from his results of spinning a spinner.

2	3	3	1	4	1	2	5	1	2	2	3	1
1	2	4	5	2	4	1	1	2	3	3	1	

Express your answer as a ratio, as a fraction and as a percent each time.

- A.  $P(\text{spin of } 7)$
- B.  $P(\text{spin of } 1, 2, 3, 4 \text{ or } 5)$

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.5**  
**(continued)**

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*  
**7SP5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**  
**[C, ME, PS]**

**Achievement Indicators**

7SP5.1 Provide an example of two independent events, such as:

- spinning a four section spinner and rolling an eight-sided die
- tossing a coin and rolling a twelve-sided die
- tossing two coins
- rolling two dice and explain why they are independent.

7SP5.2 Identify the sample space (all possible outcomes) for each of two independent events, using a tree diagram, table or other graphic organizer.

**Elaborations: Suggested Learning and Teaching Strategies**

In grade 7, the study of sample space is limited to independent events. Events are considered to be independent if the result of one does not depend on the result of another. The sample space for a probability is the list of all possible outcomes for the events.

Students must understand that spinning a four sided spinner does not in any way affect the number an eight-sided die will land upon when tossed.

Students will explore several ways to organize the sample space for two independent events.

For example:

What is the sample space for spinning a four coloured spinner and rolling a six-sided die?

Using a table:

Spinner	Die	Spinner	Die
Red	1	Green	1
Red	2	Green	2
Red	3	Green	3
Red	4	Green	4
Red	5	Green	5
Red	6	Green	6
Yellow	1	Blue	1
Yellow	2	Blue	2
Yellow	3	Blue	3
Yellow	4	Blue	4
Yellow	5	Blue	5
Yellow	6	Blue	6

(This elaboration is **continued** on the next two page spread...)

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Suggested Assessment Strategies**

*Paper and Pencil*

Suzie is a creative girl who loves bright colors and interesting color combinations. In her closet, she has a variety of tops and bottoms to choose from. As tops, she has t-shirts that are blue, green, yellow, red, orange and pink. As bottoms, she chooses between a skirt, shorts, capri pants, jeans and casual pants.

- A. What are the two independent events in this example?
- B. Explain why these events are independent?
- C. Using an appropriate method, identify the sample space which describes all possible combinations of tops and bottoms Suzie can create.
- D. Suzie’s mom buys her a new purple shirt. How many different outfits can she now create?

**Resources/Notes**

*Math Makes Sense 7*

**Lesson 7.6**

Unit 7: Data Analysis

TR: ProGuide, pp. 30–34

Master 7.10, 7.16, 7.24

CD-ROM Unit 7 Masters

ST: pp. 284–288

Practice and HW Book

pp. 167–170

**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

**Specific Outcome**

*It is expected that students will:*  
**7SP5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**  
 [C, ME, PS]  
 (Cont'd)

**Achievement Indicators**

7SP5.1 Provide an example of two independent events, such as:

- spinning a four section spinner and rolling an eight-sided die
- tossing a coin and rolling a twelve-sided die
- tossing two coins
- rolling two dice and explain why they are independent.

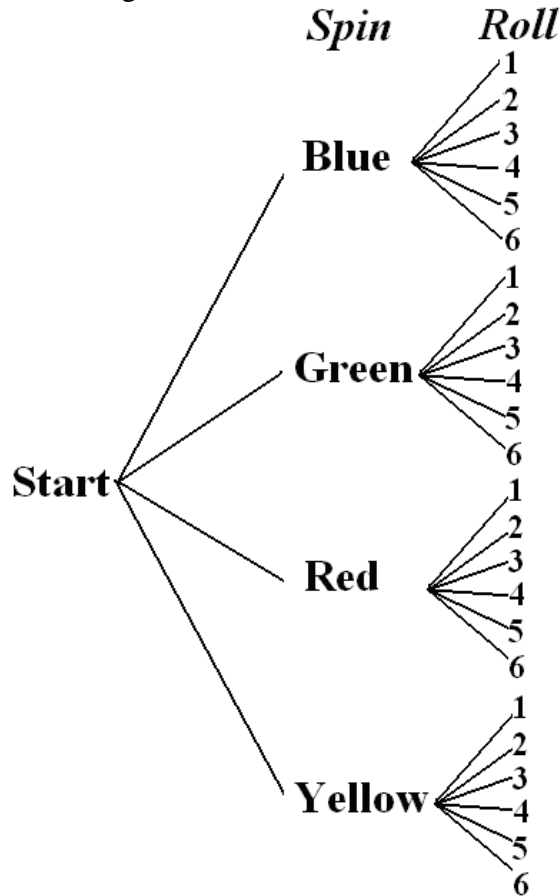
(continued)

7SP5.2 Identify the sample space (all possible outcomes) for each of two independent events, using a tree diagram, table or other graphic organizer.

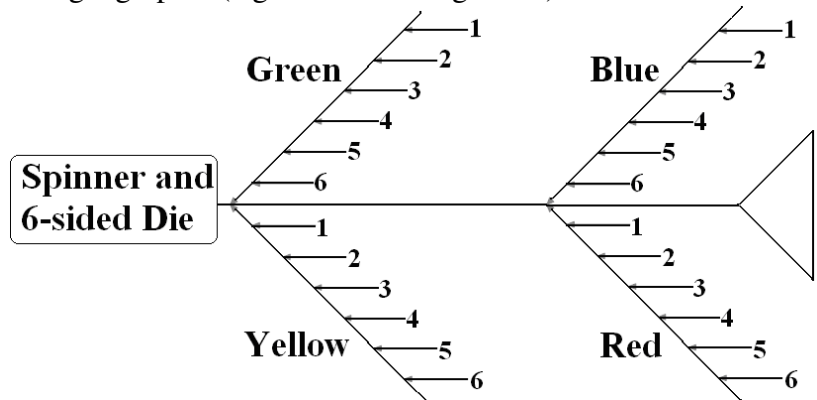
(continued)

**Elaborations: Suggested Learning and Teaching Strategies**

Using a tree diagram:



Using a graphic (e.g. a fishbone organizer):



***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

Suggested Assessment Strategies	Resources/Notes
	<p><i>Math Makes Sense 7</i> <b>Lesson 7.6</b> <b>(continued)</b></p>

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*

**7SP6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.**  
[C, PS, R, T]

**Achievement Indicators**

7SP6.1 Determine the theoretical probability of a given outcome involving two independent events.

7SP6.2 Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.

**Elaborations: Suggested Learning and Teaching Strategies**

**Theoretical probability** of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. It can only be used to predict what will happen in the long run, when events are equally likely to occur. The theoretical probability of event Y is:

$$P(Y) = \frac{\text{total \# of favorable outcomes}}{\text{sample space (total \# of possible outcomes)}}$$

**Experimental probability** is the ratio of the number of favourable outcomes in an event to the number of possible outcomes (sample space) observed in simulations and experiments. Students should realize that the probability in many situations cannot be characterized as equally likely, such as tossing a thumb tack to see if it lands with the point up or down, and therefore, theoretical probability is more difficult to determine. In such cases, experiments may be conducted to identify the probability. The experimental probability of an event Y is:

$$P(Y) = \frac{\text{total \# of observed occurrences of Y}}{\text{Sample space (total \# of trials)}}$$

Before conducting experiments, students should predict the probability whenever possible, and use the experiment to verify or refute the prediction.

When conducting experiments, students may use a variety of materials. These may include spinners, dice, coins, coloured marbles as well as graphing calculator simulations (TI-84) or computer software (Winstats) or SmartBoard probability generators.

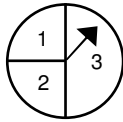
(This elaboration is **continued** on the next two page spread...)

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Suggested Assessment Strategies**

*Informal Observation*

Conduct an experiment of spinning the spinner twice, and finding the sum of the numbers from the two spins. Predict which sum will appear most often. Explain your thinking.



*\*\*\* Teachers may wish to have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain at least 100 trials. Have students compare the experimental results to their prediction and explain why there may be differences. \*\*\**

**Resources/Notes**

Free download of Winstats software is available from Peanut Software at <http://math.exeter.edu/rparis>

*Math Makes Sense 7*  
**Lesson 7.6**  
**(continued)**

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*

**7SP6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.**

[C, PS, R, T]

*(Cont'd)*

**Achievement Indicators**

7SP6.1 Determine the theoretical probability of a given outcome involving two independent events.  
**(continued)**

7SP6.2 Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.  
**(continued)**

**Elaborations: Suggested Learning and Teaching Strategies**

For example, tell students that an experiment of tossing two fair coins was conducted. About how many times in an experiment with 64 trials would you expect to get two heads? Explain your thinking.

Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain 64 trials and then add more trials as needed to show that experimental probability approaches theoretical probability as the number of trials increases. Have them calculate the experimental probability of getting two heads when two coins are tossed. Have students compare the experimental probability to the theoretical probability. (This can be simulated using the TI-84 graphing calculator.)

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

Suggested Assessment Strategies	Resources/Notes
	<p data-bbox="1122 617 1479 835">Free download of Winstats software is available from Peanut Software at <a href="http://math.exeter.edu/rparris">http://math.exeter.edu/rparris</a></p> <p data-bbox="1122 1094 1479 1199"><i>Math Makes Sense 7</i> <b>Lesson 7.6</b> <b>(continued)</b></p>

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Specific Outcome**

*It is expected that students will:*

**7SP6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.**

[C, PS, R, T]

*(Cont'd)*

**Achievement Indicators**

7SP6.3 Solve a given probability problem involving two independent events.

**Elaborations: Suggested Learning and Teaching Strategies**

It is essential that context be considered when solving problems related to probability.

The following example questions the fairness of game rules. (adapted from Van de Walle p. 335)

Three students play a coin toss game in which points are awarded depending on the following rules.

Player A gets a point if two tosses result in two heads.

Player B gets a point if two tosses result in two tails.

Player C gets a point if two tosses result in one head and one tail.

Students play the game for twenty times. The player who has the most points wins.

Discussion around this activity can focus upon questions like the following:

- Is there a favoured player? How do you know? Why is this player favoured? Is this player likely to win the next game? Is this player guaranteed to win the next game?
- How many ways can two heads occur? Two tails? One head and one tail?
- Is this game fair? It will be useful to consider both the theoretical and experimental probabilities.

***General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.***

**Suggested Assessment Strategies**

*Paper and Pencil*

Matthew's brand new MP3 player has only 5 songs in memory. They are all different. He hits the shuffle button to randomly select a song. Matthew's favourite song "Bright Light" begins to play. At the end of that song, he hits the shuffle button again to get another random selection.

- A. Organize the sample space (possible outcomes) for choosing two songs at random.
- B. What is the probability that Matthew will hear his favourite song two times in a row? Show clearly how you obtained your answer.

**Resources/Notes**

*Math Makes Sense 7*  
**Lesson 7.6**  
**(continued)**

Unit Problem:  
Page 296, 297

Strand: Statistics and Probability (Chance and Uncertainty)