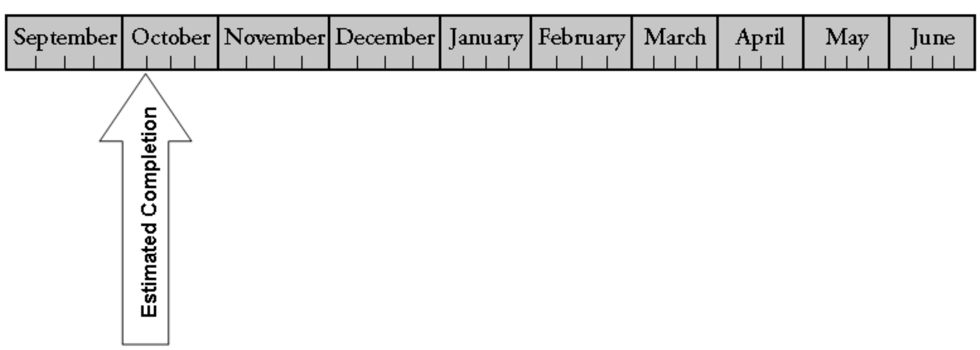
Square Roots and the Pythagorean Theorem

Suggested Time: 4 Weeks



Unit Overview

Focus and Context

In this unit, students will explore perfect squares and square roots. They will relate the side lengths of squares to square roots and the areas to perfect square numbers. They will determine if numbers are perfect squares through the use of prime factorization and an examination of the factors for the given numbers. If a number is not a perfect square, they will use a variety of methods to estimate the square root to the nearest tenth. This will lead to an examination of the Pythagorean Theorem. Students will discover the theorem through exploration and investigation, and use their previously learned knowledge of squares and square roots to determine the lengths of sides of right triangles.

Math Connects

The Pythagorean Theorem is used by people from many walks of life on a daily basis and some students may have already been exposed to its use without being aware of it. By using actual examples to introduce and explore this theorem it will be easier to develop the students' understanding of this ubiquitous and important theorem.

It can be argued that the Pythagorean Theorem is the most powerful mathematical equation used in the construction industry. It permits us to enlarge drawings, lay foundations and create perfect right angles. We can use it to calculate inaccessible distances like the height of a mountain, the width of a river, the distance to Mars, or the diameter of solar systems. Carpenters can use it to keep their work square. Draftsmen use it to make sure their architectural drawings are accurate. Its use is pervasive and powerful.

Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Curriculum

STRAND	OUTCOME	PROCESS STANDARDS
Number	Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [**8N1**]	C, CN, R, V
Number	Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [**8N2**]	C, CN, ME, R, T
Shape and Space (Measurement)	Develop and apply the Pythagorean theorem to solve problems. [**8SS1**]	CN, PS, R, T, V

Strand: Number

Outcomes

Students will be expected to

8N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

[C, CN, R, V]

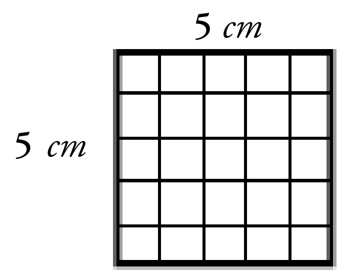
Achievement Indicator:

8N1.1 *Represent a given perfect square as a square region using materials, such as grid paper or square shapes.*

Elaborations—Strategies for Learning and Teaching

Students should recognize that a square is a quadrilateral with four equal sides and four right angles. They may also recognize that a polygon of this type is called a regular polygon. In grade 7, students were exposed to formulas for finding the areas of certain quadrilaterals (squares, rectangles and parallelograms). They should also be familiar with cm^2, m^2, mm^2, etc. for the units of area.

A perfect square is the product of two identical factors. Perfect squares, or square numbers, can be specifically connected to the areas of squares. In the figure below, students should be encouraged to view the area as the perfect square, and either dimension of the square as the square root.



The area of this square is $5cm \times 5cm = 25cm^2$. Therefore, 25 is a perfect square.

The use of exponents is introduced here. It should be noted that exponent notation is not a component of the grade 7 curriculum.

$5^2 = 25$ is expressed as "5 to the power of 2 is 25" or "5-squared is 25".

The number 5 is called the base, 2 is called the exponent, and 5^2 is called a power.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Refer to the NL government website for *Perfect Square Investigation.* Have them follow the directions to discover the first four perfect squares.

  www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/
  (8N1.1)

- Have students model perfect squares using 2-D square tiles, and identify the perfect square and its factors. (8N1.1)

Technology/Web Resources

- Play the game *Find the Matching Squares* at the following link: *http://www.quia.com/mc/65631.html* (8N1.1)

Resources/Notes

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

ProGuide: pp. 4-8, Master 1.15

CD-ROM: Master 1.24

Student Book (SB): pp. 6-10

Practice and HW Book: pp. 4-6

Strand: Number

Outcomes

Students will be expected to

8N1 Continued

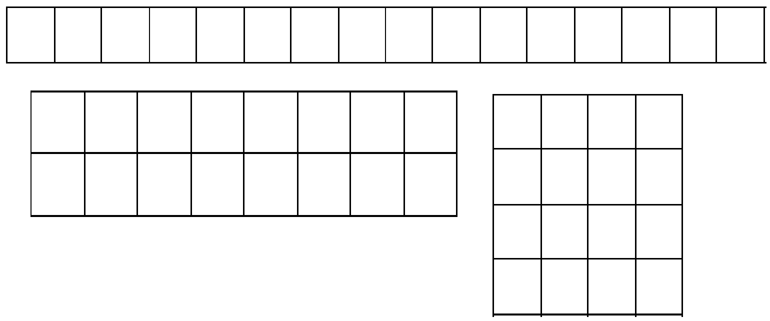
Achievement Indicator:

8N1.2 *Determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning.*

Elaborations—Strategies for Learning and Teaching

Grid paper can also be used to identify perfect squares. Give students a set area and have them construct as many rectangular regions as possible.

Consider an area of $16\,cm^2$.

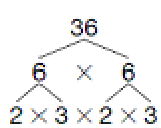


Students should decide if any of their rectangles are squares. It can then be determined if the number is a perfect square. Continue to make the connection between perfect squares and square roots by encouraging students to view the area as the perfect square and the dimensions as the square root.

Examples should not be limited to perfect squares. Have students complete this exercise for non-square numbers as well. This will help avoid the misconception that all numbers are square numbers.

Prime factorization is a method used to find the square root of perfect squares. A prime number is a whole number greater than 1 that has exactly two factors, 1 and itself. A composite number is a whole number greater than 1 that has more than two factors. Every composite number can be written as the product of prime numbers in exactly one way (if the order of the factors is ignored). This product is called the prime factorization of the number.

Consider the prime factorization to determine whether or not 36 is a perfect square. A factor tree can be used to list the prime factors.



Since $36 = 2 \cdot 3 \cdot 2 \cdot 3$ we can group factors into two equal groups, giving $36 = (2 \cdot 3) \times (2 \cdot 3) = 6 \times 6$. $\therefore \sqrt{36} = 6$

Continued

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Create a factor list for each of these numbers. Decide if the original number is a perfect square, and if so, identify the square root.

  (i) 2 (ii) 5 (iii) 9 (iv) 12 (v) 16 (vi) 20 (vii) 81

  (8N1.2)

Interview/Journal

- 361 has only 3 factors: 1, 19, and 361. Explain how you can use this information to show that 361 is a perfect square.

  (8N1.2)

Resources/Notes

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

Lesson 1.2: Squares and Square Roots

ProGuide: pp. 4-8, 9-14

CD-ROM: Master 1.24

SB: pp. 6-10, 11-16

Practice and HW Book: pp. 7-8

Strand: Number

Outcomes

Students will be expected to

8N1 Continued

Achievement Indicator:

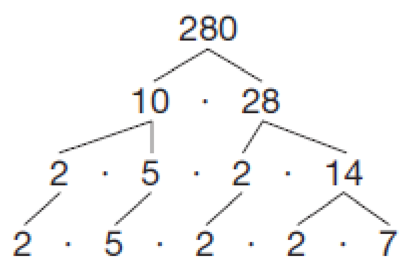
8N1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Alternatively, since each of the distinct prime factors occur an even number of times, they can be arranged in pairs.

$$36 = 2\cdot 3\cdot 2\cdot 3 = (2\cdot 2)\times(3\cdot 3)$$
$$\therefore \sqrt{36} = \sqrt{(2\cdot 2)\times(3\cdot 3)}$$
$$\sqrt{36} = \sqrt{(2\cdot 2)}\times\sqrt{(3\cdot 3)}$$
$$\sqrt{36} = 2\times 3$$
$$\sqrt{36} = 6$$

The prime factorization method can also be used to demonstrate that a number is not a perfect square. From the factor tree below, notice that none of the prime factors of 280 are present an even number of times.



$$280 = 2\cdot 2\cdot 2\cdot 5\cdot 7$$

A perfect square has each distinct prime factor occurring an even number of times.

Students should be able to recognize each of the perfect squares from 1 through 144. This automatic recognition will be very useful when determining the reasonableness of results that involve square roots found using a calculator. It will also be valuable in later work with algebra and number theory. It is also valuable to use patterns to determine that since the square root of 25 is 5, then the square root of 2500 is 50.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Refer to the NL government website for a copy of *Prime Factorization Method for Finding Square Roots*.

  www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/

  (8N1.2)

Interview/Journal

- Explain why the Prime Factorization Method cannot be used to find a whole number square root for numbers that are not perfect squares. (8N1.2)

- Is there a perfect square between 900 and 961? Explain. Would you use prime factors to determine whether 900 is a perfect square? Why or why not? (8N1.2)

Resources/Notes

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

Lesson 1.2: Squares and Square Roots

ProGuide: pp. 4-8, 9-14

CD-ROM: Master 1.24

SB: pp. 6-10, 11-16

Practice and HW Book: pp. 7-8

Strand: Number

Outcomes

Students will be expected to

8N1 Continued

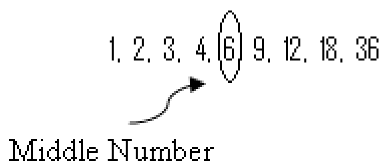
Achievement Indicator:

8N1.3 *Determine the factors of a given perfect square, and explain why one of the factors is a square root and the others are not.*

Elaborations—Strategies for Learning and Teaching

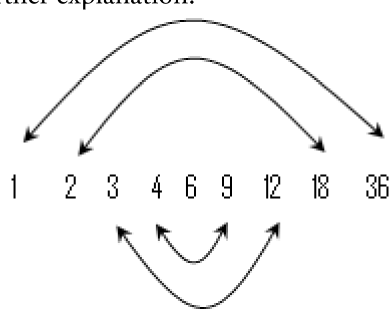
A square root is a number that when multiplied by itself equals a given value. Each perfect square has a positive and a negative square root. However, because this outcome is limited to whole numbers, the principal (positive) square root is the focus. Students will not be introduced to negative square roots in this course but directions are referring to “a” square root rather than “the” square root because the positive value is not the only square root of a number.

Students have determined the factors of a number in previous grades through systematic trial and the application of the divisibility rules. To find a square root by using a list of factors first arrange the factors in ascending order. Consider the factors of 36, a perfect square. Notice there is an odd number of factors. The middle factor is the square root.



This may require further explanation.

1 x 36 = 36
2 x 18 = 36
3 x 12 = 36
4 x 9 = 36
6 x 6 = 36



Since the 6 cannot be paired with another factor, it is the square root of 36. This is written as $\sqrt{36} = 6$. This notation, $\sqrt{x}$, is called radical notation and is new to students in grade 8. They must be introduced to the $\sqrt{\ }$ symbol to represent positive square roots.

This same method can also be used to determine when a number is not a perfect square. After examining the factors of a given number, students should conclude that if that number has an even number of factors, it is not a square number. For example, the factors of 35 are 1, 5, 7, 35. Since no factor multiplies by itself to give 35, it is not a perfect square.

Continued

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- List the factors of each perfect square, and use them to determine the square roots.

  (i) $\sqrt{9}$ (ii) $\sqrt{25}$ (iii) $\sqrt{81}$ (iv) $\sqrt{169}$

  (v) $\sqrt{36}$ (vi) $\sqrt{16}$ (vii) $\sqrt{64}$

(8N1.3)

Interview/Journal

- The factors of 81 are 1, 3, 9, 27, and 81. Use words and/or diagrams to explain how you know if 81 is a perfect sqaure and, if so, which factor is the square root of 81.

(8N1.3)

Resources/Notes

Math Makes Sense 8

Lesson 1.2: Squares and Square Roots

ProGuide: pp. 9-14

CD-ROM: Master 1.25

SB: pp. 11-15

Practice and HW Book: pp. 7-8

Strand: Number

Outcomes

Students will be expected to

8N1 Continued

Achievement Indicators:

8N1.4 *Determine a square root of a given perfect square and record it symbolically.*

8N1.5 *Determine the square of a given number.*

Elaborations—Strategies for Learning and Teaching

The methods previously discussed are effective ways to identify perfect squares and to determine their square roots. It should be noted that none of these methods require the use of a calculator. Calculator usage can be discussed, but the focus should be on non-technological techniques referenced in the outcome.

Ultimately, using the various techniques, students should be able to make statements such as:

$$\sqrt{49} = 7 \quad \text{or} \quad 7^2 = 49$$

$$\sqrt{100} = 10 \quad \text{or} \quad 10^2 = 100$$

$$\sqrt{144} = 12 \quad \text{or} \quad 12^2 = 144$$

A discussion of inverse operations is appropriate here. Consider why squaring a number is the reverse of finding the square root of a number. Students should be able to identify squaring a number and taking a square root as inverse operations. Ask them to consider other inverse operations (addition and subtraction, multiplication and division, etc.). Relating this to non-mathematical situations may help students gain a better understanding of inverse operations. For example, when someone calls you on the phone, he or she looks up your number in a phone book (finding the phone number corresponding to the name). When caller ID shows who is calling, it has performed the inverse operation (finding the name corresponding to the number).

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Mental Math

- Here is a number pattern based on the squares of numbers. Can you complete this pattern? (8N1.5)

$$2^2 = 1 \times 3 + 1$$
$$3^2 = 2 \times 4 + 1$$
$$4^2 = 3 \times 5 + 1$$
$$5^2 = 4 \times 6 + 1$$
$$6^2 =$$
$$7^2 =$$
$$8^2 =$$
$$9^2 =$$
$$10^2 =$$

Problem Solving

- Ruth wants a large picture window put in the living room of her new house. The window is to be square with an area of 49 square feet. How long should each side of the window be? (8N1.4)

- The side length of a square is 11 cm. What is the area of the square? (8N1.5)

- A miniature portrait of Danny Williams is square and has an area of 196 square centimeters. How long is each side of the portrait? (8N1.4)

Resources/Notes

Math Makes Sense 8

Lesson 1.2: Squares and Square Roots

Lesson 1.3: Measuring Line Segments

ProGuide: pp. 9-14, 15-19, Master 1.16

CD-ROM: Master 1.25, 1.26

SB: pp. 11-13, 15-16, 17-21

Practice and HW Book: pp. 7-8, 9-10

Strand: Number

Outcomes

Students will be expected to

8N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

[C, CN, ME, R, T]

Achievement Indicators:

8N2.1 *Estimate, or approximate, a square root of a given number that is not a perfect square using the roots of perfect squares as benchmarks.*

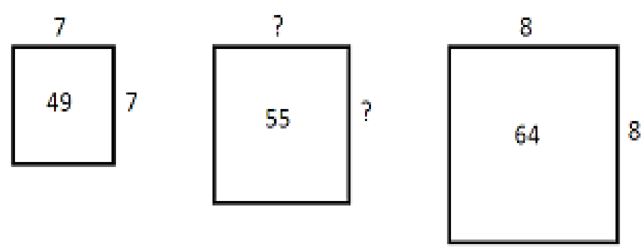
8N2.2 *Identify a whole number whose square root lies between two given numbers.*

Elaborations—Strategies for Learning and Teaching

As the discussion of perfect squares develops, students should notice that square numbers get farther apart the more the numbers increase. That is, there are many whole numbers that are not perfect squares. Develop the notion that numbers have approximate square roots that are decimal approximations located between two whole number square roots.
For example, the square root of 12 is between 3 and 4, because 12 is between 3^2 and 4^2. It is important to emphasize the difference between an exact square root and the decimal approximation.

One effective model for approximating square roots is the number line method. Alternatively, consider the following approach. To estimate $\sqrt{55}$ to one decimal place, students should recognize that 55 lies between the perfect squares 49 and 64. Therefore, the square root of 55 must be between 7 and 8. Since 55 is about a third of the way between 49 and 64, we can estimate that the square root of 55 is about a third of the way between 7 and 8. Therefore, a good approximation of $\sqrt{55}$ is 7.3.

Van de Walle (2006 p.150) presents this method in a visual manner.



A discussion similar to the one above will lead to approximating $\sqrt{55}$ at 7.3.

Students could also be asked to identify a whole number that has a square root between 7 and 8.

$$7 < \sqrt{x} < 8$$
$$7^2 < x < 8^2$$
$$49 < x < 64$$

Any whole number between 49 and 64 has a square root between 7 and 8. There is more than a single correct answer.

Using patterns and estimation, students should also recognize that the square root of 3200 is between 50 and 60, but closer to 60.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Interview/Journal

- If a whole number has an approximate square root of 5.66, is the whole number closer to 25 or 36? How do you know? (8N2.1)

- In your own words, explain how you would estimate the square root of 75. (8N2.1)

- Jim measures each side of his mother's vegetable garden to be 3.2 m. Explain how Jim could reasonably estimate the area of the vegetable garden. (8N2.2)

Group Discussion

- Place pairs of numbers on the board. Ask students to use the strategy and identify a whole number whose square root lies between two given numbers. Have students write their answers on a card, or paper, and hold them up as a group. Discuss why all of the answers are not the same. (8N2.2)

Resources/Notes

Math Makes Sense 8

Lesson 1.4: Estimating Square Roots

ProGuide: pp. 20-25

CD-ROM: Master 1.27

SB: pp. 22-27

Practice and HW Book: pp. 11-12

Strand: Number

Outcomes

Students will be expected to

8N2 Continued

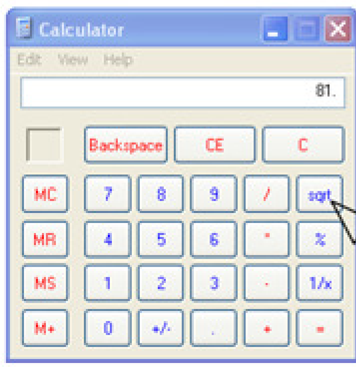
Achievement Indicators:

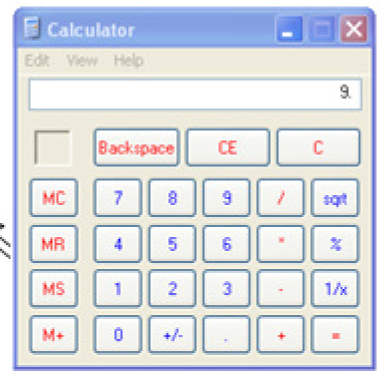
8N2.3 *Approximate a square root of a given number that is not a perfect square using technology, e.g. calculator, computer.*

8N2.4 *Explain why a square root of a number shown on a calculator may be an approximation.*

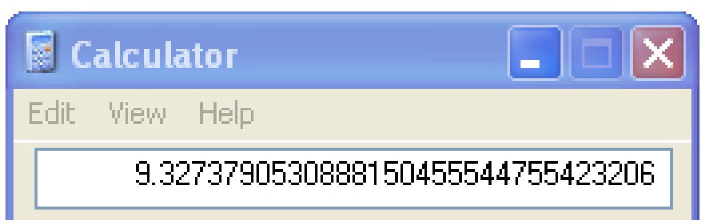
Elaborations—Strategies for Learning and Teaching

Calculators provide an efficient means of approximating square roots. It also provides a good opportunity to emphasize the difference between exact and approximate values. Students should conclude that when the square root is a terminating decimal, the original number was a perfect square.





Square roots can be approximated with calculators to any requested number of decimal places using rounding strategies.



$$\sqrt{87} \approx 9.3$$
$$\sqrt{87} \approx 9.33$$
$$\sqrt{87} \approx 9.327$$

Note: $\approx$ *means "approximately equal to"*

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

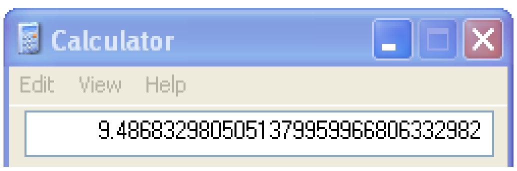
- Use a calculator to approximate these square roots, and identify which of the numbers under the radical signs are perfect squares.

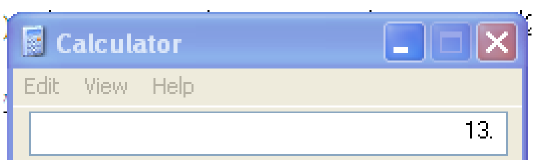
  $i.\ \sqrt{1600}$ $ii.\ \sqrt{1681}$ $iii.\ \sqrt{1212}$ $iv.\ \sqrt{1000}$ $v.\ \sqrt{2468}$

  (8N2.3)

Journal/Interview

- Kevin used his calculator to find the square roots of 90 and 169. Respectively his answers were:





  Are these exact answers? Explain your reasoning. (8N2.4)

- Emily wanted to find the area of a rectangle with a length of 9 *cm*. She knew that the width of the rectangle was the same as the lengths of the sides of an adjacent square. The area of the square was $38\,cm^2$. To find the side lengths of the square, she used her calculator as follows: $\sqrt{38} = 6.2$. Therefore the area of the rectangle is $9\,cm \times 6.2\,cm = 55.8\,cm^2$. Andrew solved the same problem as follows: $\sqrt{38} \times 9 = 55.5\,cm^2$. Account for the difference in results.

  (8N2.3, 8N2.4)

Resources/Notes

Math Makes Sense 8

Lesson 1.4: Estimating Square Roots

Technology: Investigating Square Roots with a Calculator

ProGuide: pp. 24-27

CD-ROM: Master 1.27

SB: pp.29

Practice and HW Book: pp. 11-12

Strand: Shape and Space

Outcomes

Students will be expected to

8SS1 Develop and apply the Pythagorean theorem to solve problems.

[CN, PS, R, T, V]

Achievement Indicator:

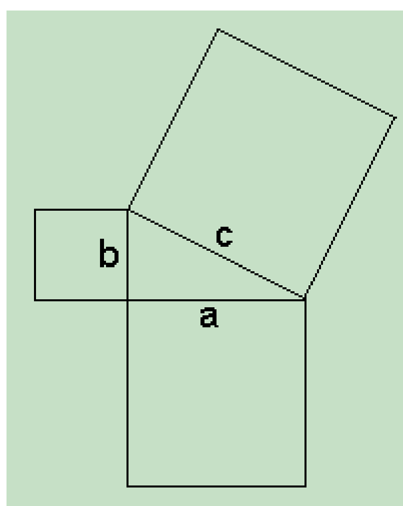
8SS1.1 *Model and explain the Pythagorean theorem concretely, pictorially or using technology.*

Elaborations—Strategies for Learning and Teaching

Pythagoras was born in the late 6th century BC on the island of Samos. He was a Greek philosopher and religious leader who was responsible for important developments in mathematics, astronomy and the theory of music. Pythagoras is famous also for the fact that he allegedly paid his first student. Frustrated that no one would listen to his learning, he decided to "buy" a student.

It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule in construction. In Egypt, Pythagoras studied with the engineers, known as "rope-stretchers", who built the pyramids. They had a rope with 12 evenly spaced knots. If the rope was pegged to the ground in the dimensions 3-4-5, a right triangle would result. This enabled them to lay the foundations of their buildings accurately. Pythagoras generalized this relationship and is credited with its first geometrical demonstration.

The Pythagorean relationship states that $c^2 = a^2 + b^2$ where *a*, *b*, and *c* represent the sides of a right triangle. The longest side, or the hypotenuse, is *c* and the shorter sides, or legs, are *a* and *b*. An area interpretation states that if a square is made on each side of a right triangle then the sum of the areas of the two smaller squares will equal the area of the square on the longest side.



$$a^2 + b^2 = c^2$$

Continued

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

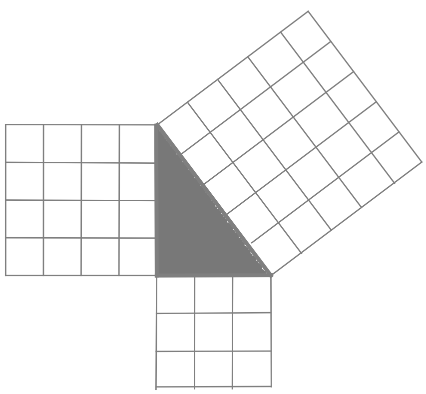
Investigation

- Refer to the NL government website for *Developing Pythagorean Activity Worksheet*.

  www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/

  (8SS1.1)

- A simple way to illustrate the Pythagorean relationship is to give groups of students a variety of right triangles which have whole number sides, such as the $3cm-4cm-5cm$ triangle, the $6cm-8cm-10cm$ triangle, or the $5cm-12cm-13cm$ triangle (or ask students to draw such triangles). Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each triangle. Place the squares on the sides of the triangle as shown. Find the area of each square. Ask students what they notice. (8SS1.1)



Resources/Notes

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Technology: Verifying the Pythagorean Theorem

ProGuide: pp. 29-36

SB: pp. 31-38

Strand: Shape and Space

Outcomes

Students will be expected to

8SS1 Continued

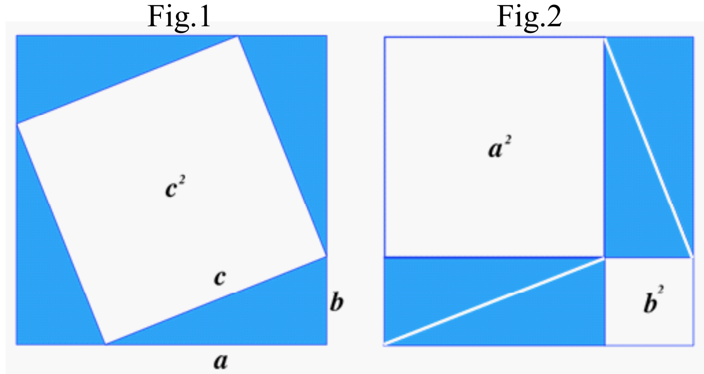
Achievement Indicator:

8SS1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

One of many proofs of the Pythagorean Theorem is given here.

Fig.1 Fig.2



Consider the white space captured by the four congruent right triangles in Figure 1. The area of that white space is c^2. Also notice that the diagram in Figure 1 is the same size as the diagram in Figure 2. Therefore, they have the same overall area. Next, notice that if the four congruent right triangles from Figure 1 are rearranged as in Figure 2, the inner white space is rearranged into two separate square regions with areas of a^2 and b^2. However, because the 4 triangles are identical they will cover the same area regardless of how they are arranged, which means the trapped white space must also be the same in each diagram. From this we can conclude that $a^2 + b^2 = c^2$.

The Pythagorean relationship can be further explored using Tangrams. Have students place one of the small triangles in the centre of the paper and trace around it. Label the hypotenuse *c* and the legs *a* and *b*. Use Tangram pieces to form a perfect square along each side of the triangle. Trace around the squares. They should determine that two small triangles were used on sides *a* and *b*, and four small triangles were needed to make the square on side *c*. Discuss how the perfect squares of *a* and *b* combined to make a perfect square on side *c*. Repeat this using the medium triangle, and then again using the large triangle. Have students compare the three drawings. Discuss the relationship of the areas of the squares along each leg of the right triangle to the area of the squares along the hypotenuse. They should conclude that the sum of the areas of the squares on the legs is equal to the square of the hypotenuse.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Technology

- An animated proof of this theorem can be found at *http://www.usna.edu/MathDept/mdm/pyth.htm*l. SmartBoard users can locate an animated proof at *www.smarttech.com*. (8SS1.1)

Resources/Notes

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Technology: Verifying the Pythagorean Theorem

ProGuide: pp. 29-36

SB: pp. 31-38

Strand: Shape and Space

Outcomes

Students will be expected to

8SS1 Continued

Achievement Indicator:

8SS1.2 *Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.*

Elaborations—Strategies for Learning and Teaching

Whenever a triangle has a right angle and two known side lengths, the Pythagorean theorem should come to mind for students. Students should be provided with experiences that involve finding the length of the hypotenuse. Such situations should cause little difficulty. They should also experience situations where the hypotenuse and one side are known and the other side is to be found. Some discussion will be required here. It is possible, but not necessary, to use formula rearrangement in this situation. To find a missing leg, students can rearrange the formula before substituting values.

$$(leg_1)^2 + (leg_2)^2 = (hyp)^2$$
$$(leg_1)^2 + (leg_2)^2 - (leg_2)^2 = (hyp)^2 - (leg_2)^2$$
$$(leg_1)^2 = (hyp)^2 - (leg_2)^2$$

Whether formula rearrangement is used first, or side lengths are substituted into the Pythagorean theorem immediately, the procedure reiterates the concept of preservation of equality that students were introduced to in grade 7.

It is important to present diagrams of right triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. They should also recognize that the hypotenuse is the longest side of the triangle. While the use of technology is permissible students should be encouraged to attempt to find an unknown side without the use of calculators. This will help develop mental skills and number sense.

There are many opportunities to use the Pythagorean relationship. Typical applications include finding the distance between two points on a coordinate plane when the points are not vertical or horizontal with respect to each other, finding how high a ladder will reach, and finding the length of the diagonal of a square or rectangle.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Problem Solving

- An airplane is flying at an elevation of $5000m$. The airport is 3 kilometers away from a point directly below the airplane on the ground. How far is the airplane from the airport? (8SS1.2)

- Ross has a rectangular garden in his backyard. He measures one side of the garden as $7m$ and the diagonal as $11m$. What is the length of the other side of his garden? (Hint: draw a diagram.) (8SS1.2)

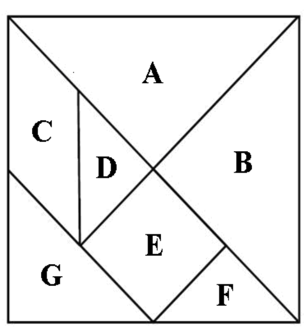
- The dimensions of a rectangular frame are $30cm$ by $50cm$. A carpenter wants to put a diagonal brace between two opposite corners of the frame. How long should the brace be? (8SS1.2)

- A triangular flower garden is created where two walkways intersect at right angles. The flower garden extends $2m$ along one walkway and $1.5m$ along the other.
  - (i) Steve wants to put a border around the whole garden. What length of border will be required?
  - (ii) Steve wishes to spray the area for pests. He needs to know the area of the garden to determine the size of pesticide container to purchase. What is the area of the garden?

(8SS1.2)

Enrichment

- Designate the side length of the square made of the seven Tangram pieces as 1 unit. Using the Pythagorean theorem, determine the lengths of all sides of each of the seven Tangram pieces.



(8SS1.2)

Resources/Notes

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Lesson 1.6: Exploring the Pythagorean Theorem

Lesson 1.7: Applying the Pythagorean Theorem

ProGuide: pp. 29-34, 44-49, Master 1.19, 1.21

CD-ROM: Master 1.28, 1.30

SB: pp. 31-36, 39-51

Practice and HW Book: pp. 13-14, 18-20

Strand: Shape and Space

Outcomes

Students will be expected to

Achievement Indicators:

8SS1.3 *Explain, using examples, that the Pythagorean theorem applies only to right triangles.*

8SS1.4 *Determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem.*

8SS1.5 *Solve a given problem that involves Pythagorean triples, e.g. 3, 4, 5 or 5, 12, 13.*

Elaborations—Strategies for Learning and Teaching

Students should be able to use the Pythagorean theorem to determine if three given side lengths are, or are not, the sides of a right triangle.

The converse of the Pythagorean theorem states that if the sides of a triangle have lengths *a*, *b* and *c* such that $a^2 + b^2 = c^2$, then the triangle is a right triangle. The emphasis should not be on this idea being called a converse to a theorem. Students have rarely encountered the situation where an idea can be supported logically in both directions. Some discussion will be necessary to make students comfortable with the notion "if A leads to B, then sometimes B leads to A." The goal here should be to reach the understanding that if the Pythagorean theorem works for a given triangle, then it is a right triangle.

A Pythagorean triple consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (*a*, *b*, *c*), and a well-known example is (3, 4, 5). If (*a*, *b*, *c*) is a Pythagorean triple, then so is (k*a*, k*b*, k*c*) for any positive integer k. For example, since (3, 4, 5) is a triple then so is (6, 8, 10) and so on.

Right triangles with non-integer sides do not form Pythagorean triples. For instance, the triangle with sides $a = b = 1$ and $c = \sqrt{2}$ is right, but $\left(1,1,\sqrt{2}\right)$ is not a Pythagorean triple because $\sqrt{2}$ is not a positive integer.

Some Pythagorean triples with $c < 100$ are indicated below.

(3, 4, 5)	(5, 12,13)	(7, 24,25)	(8, 15,17)
(9,40,41)	(11,60,61)	(12,35,37)	(13,84,85)
(16,63,65)	(20,21,29)	(28,45,53)	(33,56,65)

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Determine whether each triangle with sides of given lengths is a right triangle. (8SS1.4)

  (i) $9\,cm$, $12\,cm$, $15\,cm$

  (ii) $16\,mm$, $29\,mm$, $18\,mm$

  (iii) $9\,m$, $7\,m$, $13\,m$

Investigation

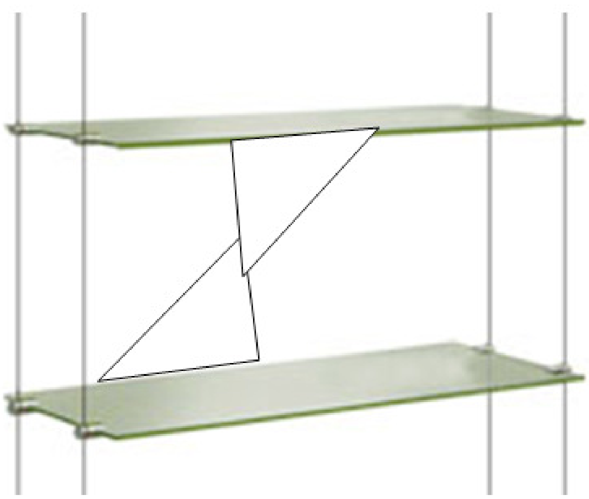
- Draw triangles other than right triangles. Measure the side lengths and check to see if the Pythagorean theorem works for these non-right triangles. (8SS1.3)

Journal

- Carpenters often use a 3-4-5 triangle to determine if corners are square $(90°)$. Explain, in your own words, why this works. (8SS1.5)

Enrichment

- How could the Pythagorean theorem be used to determine if these shelves are parallel? (8SS1.4)



- Refer to the NL government website for the *Finding Pythagorean Triples* Activity. The “Take It Further” activity on page 45 of the student book is related to this worksheet.

  www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/

  (8SS1.5)

Resources/Notes

Math Makes Sense 8

Lesson 1.6: Exploring the Pythagorean Theorem

ProGuide: pp. 37-43

CD-ROM: Master 1.29

SB: pp.39-45

Practice and HW Book: pp. 15-17

Lesson 1.6: Exploring the Pythagorean Theorem

ProGuide: pp. 37-43, Master 1.9

CD-ROM: Master 1.29

SB: pp. 41-42, 44-45

Practice and HW Book: pp. 15-17