Unit 1
The Algebra of 3-Space

(25 %)

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions

C19 solve problems involving systems of equation

C12 interpret geometrically the relationships between equations in systems

Elaboration – Instructional Strategies/Suggestions

C8 When given a situation that involves one or more relationships, students should be expected to translate among the representations of the relationships, e.g., between and among graphs, tables, written descriptions, and equations. For example, given three different taxi companies:

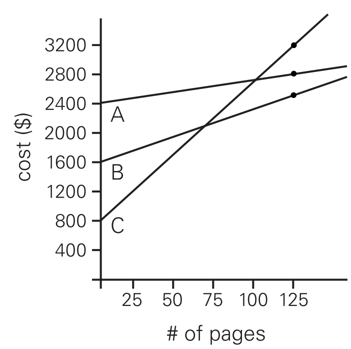
taxi A: charges $2.50 for pick up, then 50¢ per km
taxi B: charges $2.75 for pick up, then 45¢ per km
taxi C: charges $3.00 for pick up, then 40¢ per km

Students should be able to create a table of values relating the total taxi charge and the number of kilometres travelled. The table might help students construct a graph of the situation. Students should discuss what conditions would make this data discrete vs continuous.

C19 Students can then analyse and interpret the graph to determine common values between the total taxi charges. These common values are in fact the intersection points. Students should determine these by reading the graph. Graphing technology may also be used. Students might discuss why each taxi company has different pick up charges and how this is visible on the graph. They might talk about the kinds of numbers and the values that should be in the domain and range.

C12 Students can discuss what conditions would have to change so that all three lines are parallel. In contrast to that, have students create a situation that is different, so that a fourth line contains the intersection point for the graphs of taxi A and taxi B.

C8/C12 Students might be asked to describe the graphs, tables, or written descriptions as equations, and discuss the intersection point(s) of the graphs of the equations. Even though the points represent discrete points, lines are drawn for convenience of analysis. Students should consider how they might change an equation so that it causes a different intersection point. For example, given three cost equations and their graphs, students might be asked to interpret the meaning of each of the intersection points. They may also be asked to change equation *B* so that it shares the same intersection point as the point where *A* and *C* intersect.



Students might be given equations and graphs that represent a situation and be asked to describe the situation by looking at the values given in the equation. For example, given $c = 800 + 19.2p$, $c = 1600 + 20.8p$, and $c = 2400 + 22.4p$, where c is the cost of producing a book, and p is the number of pages, students should describe the three cost plans represented by the equations and their graphs. Ask students what questions can be answered from the graph that would be difficult to answer from the equation only.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C8/C12/C19

Performance

1. Mackin Industries manufactures toy wooden furniture. They have two plants; one in New Brunswick and one in Newfoundland and Labrador. At the moment they are focused on manufacturing dining room suites for doll houses. Each plant has certain fixed costs that must be paid regardless of how many dining room suites are sold. The New Brunswick plant has fixed costs of $3 000 and the Newfoundland and Labrador plant has fixed costs of $7 000. The Newfoundland and Labrador plant is newer and can manufacture goods more economically producing a profit of $250 on each dining suite manufactured. The New Brunswick plant produces a profit of only $150 on each dining suite manufactured. Which plant is more profitable in the long run?
   a) Use a graph to represent the profit for both plants.
   b) Is the data discrete or continuous? Explain.
   c) Determine and interpret the intersection point of the two lines from (a) through the points.
   d) For what levels of production is i) the New Brunswick plant more profitable? ii) the Newfoundland and Labrador plant more profitable?
   e) Interpret the vertical intercepts in terms of the problem.
   f) When you graphed the profit for both plants, what values did you use on the horizontal and vertical axes?
   g) When is it that each plant begins to have more income than expenses? Explain.
   h) Describe from your graphs how it is clear that each plant has a different profit rate.
   i) Make up a situation for a new plant in Nova Scotia whose aim is to cover its fixed expenses by selling only 25 dining sets.
   j) Change the Nova Scotia fixed expenses so that the graph for the Nova Scotia plant shares the same intersection point as the other two graphs.
   k) State the equation that represents each situation at each plant.
   l) Show how to solve algebraically to verify your answer in (c) above, and then (i) above.

Suggested Resources

The Algebra of 3-Space

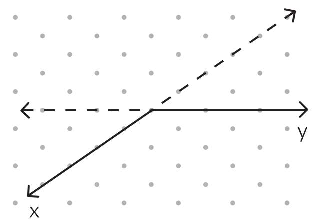
Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

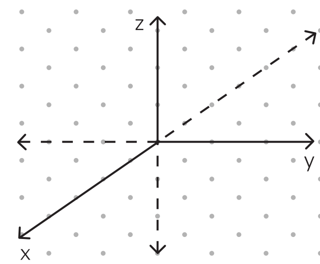
E1 demonstrate an understanding of the position of axes in 3-space

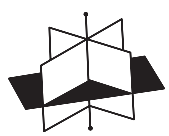
E2 locate and identify points and planes in 3-space

Elaboration – Instructional Strategies/Suggestions



E1/E2 When representing points in 3-space students need to agree on the positioning and use of the axes, and how that corresponds to the numbers in an ordered triple. The x-axis has positive values that get larger as they "come toward" the reader; the y-axis goes left and right with numbers getting larger from left to right. Students can model this by simply laying a piece of graph paper flat on their desk and drawing perpendicular lines with the x-axis being the one running towards them. The intersection is the origin. Ask them to place their pencil vertically at the origin. This would represent the vertical axis, or z-axis. Students can now locate points in four of the eight octants by building "cube towers" with height "z" and placing them on the graph paper using the x- and y-values given in the table, or by the ordered triple (*x*, *y*, *z*). For example, given the point (1, 2, 3), students might represent this by placing a tower 3 units high at the x, y-coordinate (1, 2). The top of the tower would represent the point (1, 2, 3). The other four octants would be under the students' desk as the vertical axis passes through the table top. The axes and octants can be modeled very nicely using pipe cleaners, or lin king cubes and rods. It might be a good idea for each group of students to have a model of the 3-space coordinate system to help with visualization.





E2 It is understood that if *A*, *B*, and *C* are non-collinear points, then *A*, *B*, and *C* determine a plane. Students should learn notation that would save a lot of unnecessary writing. Instead of writing "the set of all points (x, y) such that *x* and *y* satisfy some condition," students can write $\{(x, y) | (\text{some condition})\}$. For example, the x-axis in 3-space can be written as $\{(x,0,0) | x \in \mathrm{R}\}$. This reads "the set of all triples of real numbers with 2nd, 3rd coordinates 0." The x, y plane in 3-space can be written {(x, g, 0/x, y e r}, the x-z plane can be written {(x, z, 0/x, z e r} and the y-z plane can be written { y, z, 0) / y, z, e r }. These planes intersect in the point (0, 0, 0).

The Algebra of 3-Space

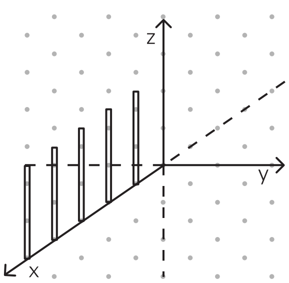
Worthwhile Tasks for Instruction and/or Assessment

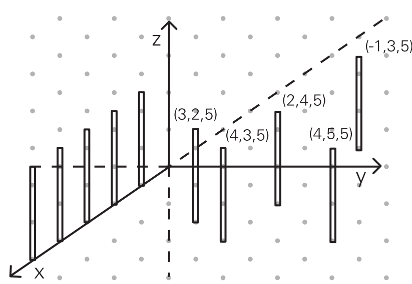
E1/E2

Activity

1. This activity will help students become more comfortable with the visualization of points in 3-space and with graphing those points/lines/planes on 2-dimensional paper. The diagrams given are there for the teacher's benefit.
   a) Position a piece of centimetre graph paper on a flat surface and draw an x - and y-axis anywhere on the grid. Position the paper so that the positive x-axis is 'coming toward' you. Note: It helps to angle these axes at approximately 135 to give the illusion of depth. The positive y-axis is to the right of the origin.
   b) Place your pencil vertically at the origin. This would represent the z-axis. The x-y plane is called the horizontal plane.
   c) Using linking cubes, or straws and a geoboard, build towers or place straws to represent the following points:
   i) (1, 0, 5) ii) (2, 0, 5) iii) (3, 0, 5) iv) (5, 0, 5)
   d) Describe how your towers or straws represent these points. [Ans: The points are represented by the tops of the towers or straws.]
   e) Describe two planes to which all of these points belong. [Ans: The vertical x-z plane, and the horizontal $z = 5$ plane.]
   f) Determine five other points that would also be on the $z = 5$ plane. State their coordinates and build and position towers to represent them. Describe how to represent the plane.

$\{(3,4,z) | -2 \le z \le 5 | z \in R\}$





   g) Using a pencil and isometric dot paper, represent all that you have done above.

Performance

2. a) Graph the line that passes through (3, 4, 0) and is parallel to the z-axis. Represent this line using set notation. [Ans: $\{(3,4,z) | z \in R\}$]
   b) Graph the same line, but make it a segment going from $z = -2$ to $z = 5$. Represent this segment using set notation. [Ans:]
   c) State the coordinate for the 'top' of this segment. [Ans: (3, 4, 5)].
   d) State 3 more coordinates on the plane $z = 5$.
3. Graph the points (3, 0, 0), (0, 2, 0) and (0, 0, 5). Join these three points and describe what the triangle represents. [Ans: a portion of a plane] What would you call the 3 points? [Ans: x-, y, and z-intercepts]

Suggested Resources

Win plot (free software downloadable from the web)

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions

C13 demonstrate an understanding that an equation in three variables describes a plane

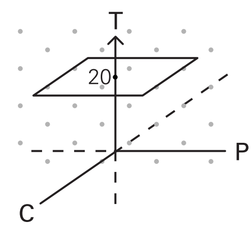
E2 locate and identify points and planes in 3-space

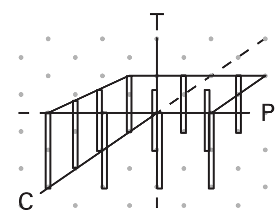
Elaboration – Instructional Strategies/Suggestions

C8 Students should model real-world situations to help them understand, represent, and visualize the planes in 3-space that represent the situations.

> The math club at Alma Mater High School produces a newsletter each month of the school year. Their total costs "T" are dependent upon the number of pages using colour "c", and the number of pages with pictures, "p". The first newsletter was produced with no colour and no pictures. The club will be charged $20 regardless of the number of pages. The club was then told that the total cost of the next edition would increase (if colour was used) at the rate of $5.00 per page. They were also told they could use pictures at no additional cost. Ask students to determine ten ordered triples (c, p, T) based on different numbers of pages, and use cubes to build towers to model the situation. (Some teachers may want to use straws and geoboards as the model. Students can cut straws to appropriate lengths and stick them onto the pegs of the geoboard to represent the coordinates.)

C13/E2 Some students should model the 1st edition of the newsletter, others the 2nd edition. Students should be asked to write the equations for the planes they have represented. Students working with the 1st edition should determine the equation $T = 20$ to represent their horizontal plane. With the 2nd edition, students should lay a piece of paper across the tops of the towers to 'see' the plane. They should realize that this plane is sloped away from them, and represents all the total costs, depending on the number of pages and unlimited use of pictures. The equation must be $T = 20 + 5c$. Students should then be told that for future editions there is an additional cost factor of $10 for every page on which a picture is placed. They should use towers to model this plane and represent it with an equation. Ask students to consider whether this plane represents the solution. They should realize that the solution is represented by a subset of points on that plane.





The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C8/C13/E2

Performance

1. The total cost "*T*" of producing a newsletter each month is dependent upon the number of pages with colour "*c*"' and whether there are pages with pictures "*p*" or not. There is an up-front charge of $50 for set-up costs (unlimited copies), an extra fee of $7.00 per page if colour is used on the page, and $3.00 if a picture appears on a page.
   a) Create a model for this situation, using cubes or straws.
   b) Place a piece of paper on the tops of the towers to represent the total cost plane. State the equation for the plane.
   c) Describe in your own words where and if the plane will intersect the $c-p$ plane.

Refer to the problem defined on page 50.

2. Suppose that 250 copies are produced of the first edition of the newsletter. The first edition has no colour or pictures
   a) Describe how this situation can be modeled and represented with a plane.
   b) State the equation for the plane.
   c) What would be the cost of 250 copies?
   d) Would there be an extra charge for more copies? Explain.

3. The second edition of the newsletter uses colour and will include 3 pages with pictures. 125 copies will be produced.
   a) Describe how this situation can be modeled and represented with a plane.
   b) State the equation for the plane.
   c) Describe in your own words where the plane will intersect the $c-p$ plane, if at all. If the intersection is a line, what is the equation for that line?

Suggested Resources

The Algebra of 3-Space

Outcomes

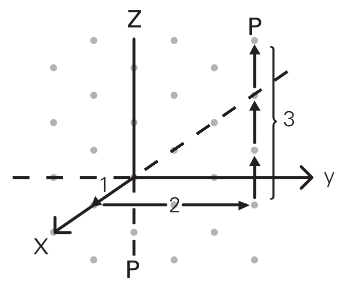
SCO: By the end of Mathematics 2204/2205, students will be expected to

E2 locate and identify points and planes in 3-space

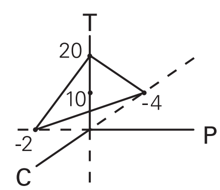
C13 demonstrate an understanding that an equation in three variables describes a plane

Elaboration – Instructional Strategies/Suggestions

E2 Students should explore the plotting of points in 3-space. For example, with an ordered triple like (1, 2, 3), students would begin with their pencil at the origin and move along the positive x-axis one unit (move the pencil toward them), then to the right two units, then up three units. Once several points have been plotted that should lie on a plane, students may still find it very difficult to 'see' the plane. Consequently, modeling these points is important for visualization.



E2/C13 A second way to sketch planes on graph paper is to use the three intercepts. First, have students find *x*, when *y* = 0 and *z* = 0. The point (*x*, 0, 0) is the x-intercept. Similarly, (0, y, 0) is the y-intercept and (0, 0, z) is the z-intercept. Students can visualize the plane that passes through these intercepts by joining the three intercepts and shading the interior of the triangle formed to represent the plane. For example, in the 3rd edition of the newsletter for the math club, the total cost is determined using the equation $T = 20 + 5c + 10p$.



Using pipe cleaners for the axes and wires for the lines joining intercepts, this plane could be projected into the (+,+,+) octant for practical purposes.

A third method is the "trace" method. The line where the plane cuts the plane $T = 0$ is the $c - p$ trace. The line where the plane cuts the plane $c = 0$ is the $T - p$ plane, and so on. Students should see that the traces are the lines that join the intercepts and that their equations can be found by setting one variable equal to zero and expressing the equation with the remaining two variables.

All three of the above methods should be examined using models, then translated onto grid paper.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

E2/C13

Performance

1. The total cost of producing a newsletter is given by the equation $T = 5 + .3c + .5p$ where "T" is the total cost, "c" is the number of colour pages, and "p" is the number of pages with pictures.
   a) Determine at least 5 ordered triples that belong to the set of points described by the given equation.
   b) Plot these points on a 3-space grid using isometric dot paper or a geoboard.
   c) Using the coordinates determined in (a) and plotted in (b) describe the plane that represents the given equation. Draw the plane on the dot paper.

2. a) Revisit #1 above. This time find the three intercepts, graph them and use these to draw the plane that represents the given equation.
   b) Find equations to represent the 3 traces for the equation given in #1 above.
   c) What is the connection between the traces and the intercepts?
   d) Model the traces using pipe cleaners for the 3-space axes, and fine wire for the traces.
   e) Use the model to produce the plane that represents the given equation, in the (positive, positive, positive) octant.

3. In 2-space, y = 3x describes a line with slope 3 that passes through the origin. Using set notation the line is described as $\{(x, y) | y = 3x\}$. In 3-space this same line is described as $\{(x, y, 0) | y = 3x\}$. Its slope is 3 and it passes through the origin (0, 0, 0).
   a) Where in 3-space is $\{(x, y, 3) | y = 3x\}$? What is its slope and y-intercept?
   b) Where in 3-space is $\{(x, y, -2) | y = 3x\}$? What is its slope and y-intercept?
   c) Where in 3-space is $\{(x, y, z) | y = 3x + 1\}$? Describe its appearance.

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

C13 demonstrate an understanding that an equation in three variables describes a plane

Elaboration – Instructional Strategies/Suggestions

C13 Students should be able to give an equation for a plane by:

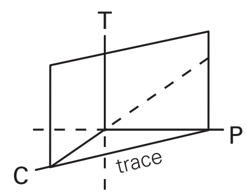
1) describing the relationship among the variables in words, then translating that description into symbols
2) being given two intercepts

For example, in the first case, given the newsletter situation, students could plot many points (model with cubes or straws) and describe the plane as the total cost of the newsletter, $20, plus additional amounts that depend on the number of pages using colour (at $5.00/page), and the number of pages with pictures (at $10.00/page). They would translate this into symbols: $T = 20 + 5c + 10p$.

In the second case, having the students join two given intercepts produces a trace through which an infinite number of planes can pass. If the intercepts are (2, 0, 0) and (0, 1.5, 0), the trace is a line whose equation can be found by first getting the

slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{-1.5}{2}$ or $\frac{-3}{4}$, en the

y-intercept: 1.5.



The resulting equation is , or $3x + 4y = 6$.

On the trace, the z-coordinates are 0. The vertical plane that contains the trace $3x + 4y = 6$ is made up of all the lines parallel to the trace, but with z-coordinates that are non-zero. Thus, the vertical plane whose intercepts are (2, 0, 0) and (0, 1.5, 0) can be represented with the equation $3c + 4p = 6$. When any coefficient for z other than zero is added to the equation, such as $3c + 4p - 5z = 6$, the resulting plane contains the trace $3x + 4y = 6$, however the plane is not vertical but oblique (it will also then have a z-intercept).

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C13

Performance

1. State the equation for the plane that represents each of the following situations.
   a) A music store sells CDs and cassettes. It makes a profit of $5.50 on each CD sold and $3.50 on each cassette sold. Its daily expenses are $250.00. What is the total profit per day?
   b) One litre of a 15% acid solution contains 150ml of acid and 850ml of water.
   c) There is a sale on socks and ties at George's favourite store. Socks sell for $5.00 a pair and ties for $10 each. How much might George spend?
   d) The cost of buying computer chips varies, depending on the chip purchased. One type "*x*" sells for $7.50, the other "*y*" $12.50. Determine an expression for the total cost of buying some "*x*" chips and some "*y*" chips.

2. Given the following information, determine the equation for the plane:
   a) (3, 0, 0) and (0, 5, 0)
   b) (0, 2.5, 0) and (0, 0, –7.5)
   c) (–5, 0, 0) and (0, 0, –3.5)

Suggested Resources

The Algebra of 3-Space

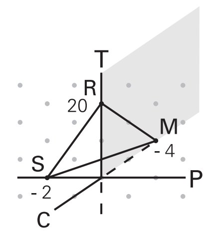
Outcomes

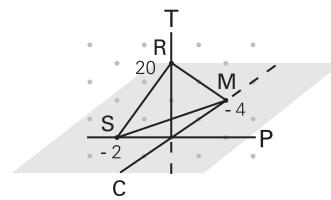
SCO: By the end of Mathematics 2204/2205, students will be expected to

C12 interpret geometrically the relationships between equations in systems

Elaboration – Instructional Strategies/Suggestions

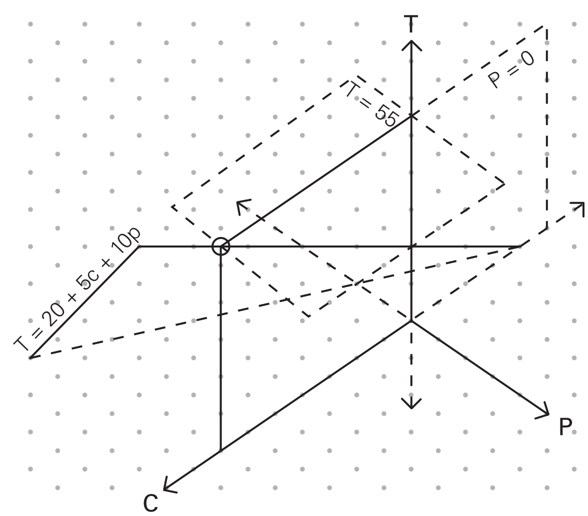
C12 A trace is a line that results from the intersection of a plane that represents a situation, with the plane determined by two axes. For example, students should use shading to see that the plane *RSM* intersects the *T* – *c* plane in the line *RM*. Also, they should visualize the line *SM* as the intersection of the *RSM* plane and the horizontal *c* – *p* plane. The intersection of the two planes determines a common set of points that determine a straight line. For example, the costs for the 3rd edition of the newsletter can be represented as a plane and as the equation $T = 20 + 5c + 10p$.

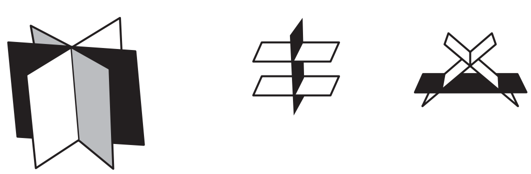




Suppose that the math club used no pictures, then $p = 0$ would represent the T - c plane. Where that plane intersects the plane $T = 20 + 5c + 10p$ would describe all the values for *T* that might result, depending on the number of pages using colour. Since the total number of pages in the newsletter might range from 1 to some large number, there would be many solutions to the problem. Students should examine a situation where the two planes do not intersect and examine the equations to see how the corresponding coefficients in the two equations are related.

If students know that the total cost for the newsletter was \$55.00, they can represent this as the constant plane $T = 55$. When this is added to the graphs just studied, there would be three planes intersecting at one point: the point (7, 0, 55). Students should interpret this point as a newsletter with 7 pages with colour, no pictures and with a total cost of \$55.00. Students should also realize that three planes can intersect, creating only one line (like the pages in a book), two different lines, or 3 different lines as shown below. They should examine the corresponding coefficients in the three equations to see how this happens.





The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C12

Activity

1. The total cost in dollars of producing one high school yearbook is given by the equation $T = 1250 + 2c + 1.5p$, where "T" is the total cost, "c" the number of pages with colour, and "p" the number of pages with pictures.
   a) Draw the graph of the plane that represents the total cost in dollars.
   b) Draw the graph of the plane $T = 1250$. What does this plane represent? [Ans: various numbers of pages where there is no extra charge for colour or pictures.]
   c) Look at the intersection of these two planes. How many points are on the intersection? Describe what they represent. (They represent points that form a line. These points are various values for "c" and "p" that result in yearbooks that will result in a total cost of $1250. The only meaningful point in this set of points for this context would be (0, 0, 1250), the rest must include either a negative number of pages with colour or pictures).
   d) Draw the graph of the $T - c$ plane. Describe what this plane represents. [Ans: The total cost for a book with any number of pages that have colour and no pages with pictures.]
   e) Describe the intersection of the $T - c$ plane and the plane of the original equation $T = 1250 + 2c + 1.5p$. [Ans: The $T - c$ plane represents $p = 0$, a book with colour pages, but no pictures. The intersection then would be the various total costs resulting as the number of colour pages vary.]
   f) Is it possible that all three planes discussed above would all intersect at a point? What is the meaning of that point? [Ans: Should intersect at (0, 0, 1250)—the number of pages of colour and the number of pages with pictures that result in the cost of the book being $1250.]
   g) If the initial set up costs were reduced to $1000 but the other costs don't change, describe the relationship between this plane and the plane representing $T = 1250 + 2c + 1.5p$. [Ans: parallel, students should examine the corresponding coefficients in the two equations to see why they planes are parallel.]
   h) The company offers a sale: the cost per picture-page is reduced by 50 . How does this affect the plane $T = 1250 + 2c + 1.5p$?

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B15 solve systems of "*m*" equations in "*n*" variables with and without technology

Elaboration – Instructional Strategies/Suggestions

B15 In Mathematics 1204, students have solved systems of 2 equations with 2 variables, using graphs, technology, and the substitution method. In this course, students will solve 2 equations with 2 variables, using graphs, substitution, elimination, and the inverse matrix method (B13, p. 62). They will also solve three or more equations with three or more variables, using technology (B13, p. 62). In the process of moving from solving 2×2 systems to systems, students should experience solving 3 equations with only 2 variables and 2 equations with 3 variables. The need for solving systems like these has arisen from examining the graphs of intersecting planes. For example, the elimination method could be introduced (see C14, p. 60) to solve the following system of 3 equations in 2 variables:

1) $x + y = 5$
2) $x - y = 1$
3) $3x - 5y = -1$

Students will learn that they can add equation 1) and 2) to get

or subtract equation 2) from 1) to get

They should realize that substituting both of these values into equation 3),

$$3(3) - 5(2) = -1$$
$$9 - 10 = -1$$
$$-1 = -1$$

satisfies equation 3) and that (3, 2) is the solution to the given system and that the system of equations is consistent. They should know that if the 3rd equation is not satisfied, then the system has no solution and the system of equations is inconsistent (no intersection) (see C12). The elimination method may also be the quickest way to solve a system of 2 equations with 3 variables. For example:

1) $x - y + z = 8$
2) $3x - 2y + 5z = 4$

Multiplying equation 1) by –2, then adding equation 2) gives:

$-2x + 2y - 2x = -16$
$3x - 2y + 5z = 4$
$x + 3z = -12$
$x = -12 - 3z$

Similarly, multiplying equation 1) by –3 and adding equation 2) leads to $y = -20 - 2z$. Thus, the solution to the system is $(x, y, z) = (-12 - 3z, -20 - 2z, z)$. The "$z$" is called a parameter and leads to the understanding that there are an infinite number of solutions (intersection is a line) (see C12, p. 56).

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B15

Pencil/Paper

1. Solve the following systems, and determine whether they are consistent or inconsistent.

$$\begin{cases} x - 2y - 3z = 0 \\ 2x + y = 0 \\ x + z = 0 \end{cases}$$

2. Show that the line of intersection of the planes $x + 2y - 4z + 8 = 0$ $3x - y - z + 6 = 0$ lies in the plane $5x - 11y + 13z - 14 = 0$.

3. Determine which pairs of planes are parallel. For each pair that is not parallel, find the intersection:
   a) $x + y - 3z = 4$ and $x + 2y - z = 1$
   b) $5x - 2y + 2z + 1 = 0$ and $5x - 2y + 2z - 3 = 0$
   c) $x + 3y - z - 4 = 0$ and $2x + 6y - 2z - 8 = 0$
   d) $x - 3y - z + 3 = 0$ and $2x + 4y - z - 5 = 0$

C14 demonstrate an understanding of the relationships between equivalent systems of equations

$$\begin{cases} x - y - 3z = 3 \\ 3x - y - z = 5 \end{cases}$$

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

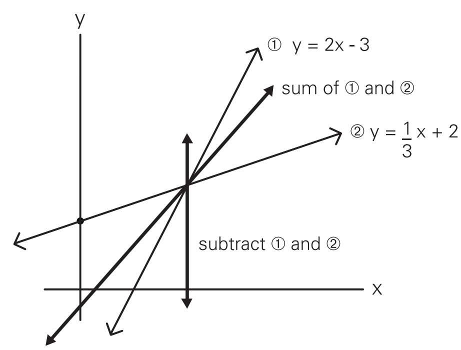
C19 solve problems involving systems of equations

C14 demonstrate an understanding of the relationship between equivalent systems of equations

Elaboration – Instructional Strategies/Suggestions

C14 Before students use the elimination method to solve systems of equations, the method must be developed with the students. The development that follows is in two dimensions. Students must understand the relationships between equivalent systems of equations before they can use the method with meaning. To develop this understanding, have students graph two equations that represent related situations so that the intersection of the graphs can be interpreted with meaning. Have students add the equations together and graph the resulting equation. Have them subtract the 2 equations and graph the resulting equation. Have students conjecture about the effect on the intersection point of the system when a new system is developed from the old by adding and/or subtracting (new graphs are produced, but the intersection point remains the same. The new equations produce an intersection equivalent to the original system.

- ❐ Have students graph $y = 2x - 3$ and and find the intersection point [Ans: 3, 3]. Then have students add the two equations and graph, then subtract the two equations and graph. Compare the two systems (use dotted lines, for one system and solid lines for the other to help distinguish between them).



Students should then investigate systems whose equations have been altered through multiplication by a constant and/or addition and subtraction. For example, beginning with the system have students multiply the first equation by 3 and add it to the second to form a new 3rd equation. Then have them subtract the 2nd equation from three times the first to form a 4th equation. Graphing equations 3 and 4 will produce another system with the same intersection point (equivalent system).

C19 Students will need to practise the elimination method. It is important to practise for a purpose, and in context as much as possible. Students should also practise the other methods previously studied and work with systems that are a mixture of ‘m’ equations in ‘n’ variables. They should discuss and defend their choice of methods. It is not intended that an assessment item be focussed on any one method.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C14

Performance

1. Given $3y - 2x + 5 = 0$ and $4y + 3x - 7 = 0$.
   a) explain, without graphing, how to determine another linear equation that shares the same intersection point as the two equations above. What is the equation?
   b) state a different equation that also passes through the same intersection point. Explain how this was determined.

2. Given $3x - 5y = 2$.
   a) sketch the graph.
   b) multiply each term in the equation by 3.
   c) explain in words why the graph of the equation (b) will be the same as the graph in (a).

3. Given this system $2x - \frac{1}{2}y + 5 = 0$ and $3x - 4y - 1 = 0$.
   a) describe your first steps in solving this system, using elimination, and explain why.
   b) your first steps in solving the system $3x - y = -2$ and $5y + x - 7 = 0$ using substitution and explain why.
   c) what advantage you think there is in using the elimination method versus the substitution method.

$$A = \begin{cases} x - 3y = 0 \\ 2x + 4y = 0 \\ x + 5y = 8 \end{cases}$$

4. Three cab companies charge different rates. After how much time will two cabs have the same cost? Will there be a time when all three cost the same?

   Yellow Cab: cost = \$1.50 + 5¢/second
   White Cab: cost = \$2.75 + 4.5¢/second
   Blue Cab: cost = \$2.13 + 5.25¢/second

5. System A has a solution. Find it. System B does not. Why not?

$$B = \begin{cases} x - 3y = 0 \\ 2x + 4y = 10 \\ x + 5y = 4 \end{cases}$$

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B11 develop and apply the procedure to obtain the inverse of a matrix

B12(Adv) derive and apply the procedure to obtain the inverse of a matrix

B13 solve systems of equations using inverse matrices

B2 demonstrate an understanding of the relationship between operations on algebraic and matrix equations

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

Elaboration – Instructional Strategies/Suggestions

B11/B12(Adv)/B13 From earlier grades students have used matrices to represent network graphs and inventory lists. In this course, students will use matrices to represent systems of equations. This will enable students to solve large systems with less manipulation, and with technology. It will also enable students to explore the properties of matrices.

Students will develop (Adv-level students will derive) and apply procedures for solving systems of linear equations (2 by 2 and 3 by 3), using matrices (inverse matrix method). Students should learn to use these procedures, using appropriate technology.

One of the more common uses of matrices is to solve a system of equations (see B13, page 70 for the solving). In general, every linear system can be represented by a matrix equation of the form $AX = B$, where A is the matrix made from the numerical coefficients of the linear equations, B is the column matrix made from the constant terms, and X is the column matrix whose elements are the variables.

B2/A4 As is the case with linear equations, students strive to isolate the term with the variable, then isolate the variable. For example, when solving $2x + 5 = 9$; students first isolate the variable term by subtracting 5 from both sides giving $2x = 4$, then they divide both sides by two to isolate the variable itself.

Since division is not an operation with matrices, another look at the procedure above indicates that the division by 2 could be described as multiplication by the inverse of the coefficient of x. With matrices, to isolate the variable matrix, multiplication by the inverse [A^{-1}] will take place. The matrix equation $AX = B$ has the solution $X = A^{-1}B$ if A^{-1} exists (e.g., A is a square matrix and its determinant is not zero). See the bottom of p. 66 for the definition of determinant.

continued ...

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B13

Pencil/Paper

1. Represent the following systems using matrices

$$A=\begin{cases}3x+5y=7\\2x+6y=10\end{cases}\qquad B=\begin{cases}-3x-7y+11=0\\-2x+y=0\end{cases}$$

$$C=\begin{cases}2x-3y+5z=11\\\frac{1}{2}x-\frac{2}{5}y-z=\frac{12}{13}\\5x-7y-3z=\frac{2}{3}\end{cases}\qquad D=\begin{cases}5x-y=2z\\2y-3z=1\\2z-3x=5y\end{cases}$$

2. Given the following matrices, write the system they represent using algebraic terms.

a) $\begin{bmatrix}-5 & 2 & 0\\1 & -1 & 1\\2 & -3 & 5\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}8\\11\\2\end{bmatrix}$ b) $\begin{bmatrix}3 & 5\\-2 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$

3. Barb and Helen are solving equations.

Barb:

(1) $3x-7y=5$

(2) $-7y=-3x+5$

(3) $y=\frac{3}{7}x-\frac{5}{7}$

Helen:

$\begin{cases}2x+7y=5 & ①\\3z+2x=-3 & ②\end{cases}$

$7y-3z=8$ ③

$7y=3z+8$ ④

$\frac{1}{7}(7y)=\frac{1}{7}(3z)+\frac{1}{7}(8)$ ⑤

$y=\frac{3}{7}z+\frac{8}{7}$

a) What did Barb do to get from line ② to line ③?
b) How did Helen get ③ ?
c) Describe what Helen is doing to get from ④ to ⑤ ? How is this similar to what Barb did?

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B13 solve systems of equations using inverse matrices

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

A5 demonstrate an understanding of properties of matrices and apply them

B11 develop and apply the procedure to obtain the inverse of a matrix

B12(Adv) derive and apply the procedure to obtain the inverse of a matrix

Elaboration – Instructional Strategies/Suggestions

B13/A4/A5 To go further with the example on the previous page necessitates finding the multiplicative inverse of the matrix *A* which has the notation A^{-1}. The inverse of a square matrix *A* is a matrix *B* when the product of *A* and *B* is the identity

matrix $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or in the case of 3 by 3 matrices: .

B11 In order to generalize the inverse of a matrix, students should find particular inverses and look for a pattern. They might begin by looking at the situation and trying to determine the values for *a*, *b*, *c*, and *d* that would satisfy the equation. Or, they might try several examples with a determinant of 1, observe the pattern, then examine matrices with a determinant of 2, 3, etc., to redefine the pattern. (Note: the inverses should be quickly found using technology.) For example, using technology, students should be able to see that

the inverses of is . When the determinant of A is one, then

students should see the pattern—reverse the order of the numbers on the major diagonal:

becomes ; record the opposite numbers for the numbers on the minor

diagonal .

B12(Adv) Advanced level students should derive the inverse of a matrix. When a matrix is multiplied by the inverse, the result should be the identity matrix *I* since with real numbers, one is the result when a number is multiplied by its multiplicative inverse

.

To find the inverse of a matrix must exist such that

$AB = BA = I$. yields the following

system of equations:

solving each pair produces $a = 3$, $b = -1$, $c = -5$, and $d = 2$ thus is the inverse of *A* and is written A^{-1}.

continued...

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

A4/A5

Activity

1. Purpose: to find the identity of a matrix. Finding the inverse of a matrix necessitates knowing the identity matrix. As in the real number system, a matrix multiplied by the identity results in the original matrix. That is, if A is a matrix, and I is the identity matrix, then $AI = A$. Students need to find I:

   a) Start with any matrix A, try and then

   using matrix multiplication, set up two systems to solve

   c

   for a, b, c, d.

   b) Replace $I = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with the values for a, b, c, and d.

   c) Does $AI = A$? Does $IA = A$?

   d) Create several other matrices like A and redo step C above.

   e) Explain why a matrix must be a square matrix in order to have an identity matrix.

   [illegible] $b + 2d = 2$, $3b + 4d = 4$

   f) Given and , find AB. Find BA. What conclusion do you think you can make? Try some different matrices for A and B, then make a conjecture.

B11

2. Purpose: Develop mental math procedures to obtain the inverse of a matrix.

   a) Use a graphing calculator

   i) enter matrix

   ii) look at matrix A and matrix A^{-1} on the screen. Are the same digits in A^{-1} as in A? Describe what is different about them. Record these matrices on paper.

   iii) enter matrix

   iv) look at matrix B and matrix B^{-1} on the screen. Answer ii) above for matrix B^{-1} and B.

   v) enter matrix and repeat ii) and iii) above for matrix C and C^{-1}.

   b) Describe the pattern that seems to exist that would help you obtain the inverse of a matrix. Use the terms major diagonal (upper left to lower right) and minor diagonal.

continued ...

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B11 develop and apply the procedure to obtain the inverse of a matrix

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

B12(Adv) derive and apply the procedure to obtain the inverse of a matrix

Elaboration – Instructional Strategies/Suggestions

B11/A4 Students should check this using the MATRX functions in their graphing calculators. Teachers should provide students with several examples to try whose determinant is 1 to help them observe the pattern.

B12(Adv) Students should be asked to generalize this derivation of the inverse of a matrix by solving, in the same way as above, this: Let the inverse of *M* be

$$M^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

(Teachers might have different groups each obtain the value for one of *a*, *b*, *c*, or *d*).

$$\begin{array}{lll} B\times(1) & aA+bC=1 & (1) \\ A\times(2) & aB+bD=0 & (2) \\ (4)-(3) & aAB+bBC=B & (3) \\ & \underline{aAB+bAD=0} & (4) \\ & bAD-bBC=-B & \\ & b(AD-BC)=-B & \\ & b=\dfrac{-B}{AD-BC} & (5) \end{array}
\qquad
\begin{array}{lll} B\times(1) & cA+dC=0 & (1) \\ A\times(2) & cB+dD=1 & (2) \\ (4)-(3) & cAB+dBC=0 & (3) \\ & \underline{cAB+dAD=A} & (4) \\ & dAD-dBC=A & \\ & d(AD-BC)=A & \\ & d=\dfrac{A}{AD-BC} & (5) \end{array}$$

Similarly, by substituting the values for *b* and *d* into the given equations (1), students will be able to determine values for *c* and *d*.

From this students would conclude the inverse of the matrix is:

They should also notice and discuss that the difference in the product of the diagonals (the determinant) plays an important part, and that if it were zero the inverse could not be found (singular matrix).

... continued

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B11

2.

c) If A and A^{-1} are inverses, what will happen if you multiply the two matrices? Does the order in which you multiply them matter? Try this for the 3 sets of matrices and their inverses from (a) on page 65.

d) Repeat steps a), b), and c) above for the following matrices.

$E = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$ $\quad F = \begin{pmatrix} 1 & -1 \\ 4 & -2 \end{pmatrix}$

(If you get decimals in your answers, you may want to express the matrix with fractions to help you see the pattern.)

e) Re-express D^{-1}, E^{-1}, and F^{-1} with any fractions factored out.

f) For A, B, C multiply the two numbers on the major diagonal, then subtract the product of the minor diagonal. This number is called the determinant. Evaluate the determinant for D, E, and F.

g) How is the determinant important when determining the inverse of a matrix?

h) Find the inverse of this matrix mentally. Then check using your calculator.

B12(Adv)/A4

Performance

3. Given a matrix and its inverse , work with A and A^{-1} to obtain values for a, b, c, d so that .

B11/B12(Adv)/A4

Pencil/Paper

4. Given the following systems, show how you could use matrices and inverse matrices to solve the system. You don't need to solve at this point.

a)

b) $\begin{cases} 3x + 2y - z = 1 \\ 5y = -3x + 4z - 5 \\ x = 3y - 5z + 3 \end{cases}$

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B11 develop and apply the procedure to obtain the inverse of a matrix

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

Elaboration – Instructional Strategies/Suggestions

B11 As students explore patterns with determinants other than one, they should be able to include this determinant value in their pattern to obtain an inverse. For example, they should now explore such matrices and their inverses using technology.

Give students matrix $A = \begin{pmatrix} 5 & 4 \\ 2 & 2 \end{pmatrix}$. Ask them to find the inverse using technology

and express it with fractions, if possible . The pattern they had

developed (p. 64) should have given .

They should notice that technology gave them numbers that were each of those obtained from the pattern. They should try some others that will lead them to the same discovery. To answer why , ask them to subtract the product on the minor diagonal of A from the product on the major diagonal (10 – 8 = 2). Tell them this is called the determinant and that it affects the pattern. Have them describe how.

A4 Students should now try other matrices with different determinants to see how consistent the pattern is. When using technology, they will get an error statement if they try to find the inverse of a matrix whose determinant is zero. Discuss why.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B11/A4

Mental Math

1. Find the determinant, if possible, given the following matrices.

a) b) c) d)

2. State the inverse matrix, if possible, for each of the above matrices.

A4

Journal

3. Explain the conditions under which a matrix would have no inverse.
4. Explain the conditions under which a identity matrix cannot be produced.
5. Explain why the determinant is important when finding the inverse of a matrix.

Performance

6. Illustrate that and are inverses of one another.

Adv-level

7. Given and find:

a) b) A^{-1} c) B^{-1} d) $(A + B)^{-1}$

Adv-level

8.

a) Find det M and state why M must have an inverse.
b) Evaluate M^2 and compare it to M^{-1}.

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

B13 solve systems of equations using inverse matrices

C19 solve problems involving systems of equations

Elaboration – Instructional Strategies/Suggestions

A4/B13/C19 Students should understand that arranging numbers into a matrix is a useful procedure and that subsequent operations can be performed. Investigations can show that there is a parallel between the identity (I) matrix and the number 1 in the real number system, although it isn't a true parallel because (I) only exists for square matrices. (See A5) Students will identify matrices, identify and apply inverse matrices, and use these and appropriate technology to solve problems involving systems of equations.

On page 62 the solution to a system was demonstrated using matrices, but not completely finished. Students had $AX = B$, or .

Multiplying both sides by the inverse: $A^{-1} AX = A^{-1} B$
Since the determinant 0, then $A^{-1} A = I$ and $IX = X$

Then all there is to do is $A^{-1} B$

They can conclude that the solution to the system is the point (4, –2).

Students will model real-world situations with two, three, or more equations, using two, three, or more variables. For example, bolts, nuts, and washers are produced using three different machines. The chart below shows the number of hours each machine spends on each of the three parts. Find how many of each can be produced in a day if machine 1 is used 6 hours per day, machine 2 for 7 hours per day, and machine 3 for 8 hours per day.

	Bolts	Nuts	Washers
machine 1	2	1	2
machine 2	1	1	4
machine 3	1	3	1

To solve this problem, students may first establish a system of three equations, using three variables.

$$2b + n + 2w = 6$$
$$b + n + 4w = 7$$
$$b + 3n + w = 8$$

They should then re-express the system as matrices and use technology and the inverse matrix method to find values for *b*, *n*, and *w*.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B13/A4

Performance

1. Solve each of the systems, using the inverse matrix method.

a) $\begin{cases} x-2y=0 \\ -2x+4y=3 \end{cases}$ b) $\begin{cases} 4x=11+3y \\ 2x=9+5y \end{cases}$ c) $\begin{cases} 3x-4y=-2 \\ 2x+y=5 \end{cases}$

2. Solve these systems:

a) $\begin{cases} x+2y+3z=3 \\ x+y+2z=1 \\ y-4z=2 \end{cases}$ b) $\begin{cases} \frac{1}{2}x-\frac{2}{3}y+5z=2 \\ \frac{2}{3}x+\frac{2}{5}y-3z=1 \\ \frac{1}{5}x-\frac{1}{2}y-\frac{4}{7}z=0 \end{cases}$

C19/B13

3. Determine the intersection of the planes P_1, P_2 and P_3, if possible. If not possible, explain why not.

a) $\begin{cases} P_1: x-2y+3z=9 \\ P_2: x+y-z=4 \\ P_3: 2x-4y+6z=5 \end{cases}$ b) $\begin{cases} P_1: x+2y+z=12 \\ P_2: 2x-y+z=5 \\ P_3: 3x+y-2z=1 \end{cases}$ c) $\begin{cases} 3a=5b+2c+4d+e=35 \\ 2a+4b-c \quad 3d+6e= \quad 15 \\ 4a-2b-3c+d+2e=18 \\ -5a+b+4c-d-3e=-18 \\ -2a+5b+6c-2d+e=-19 \end{cases}$

4. Three robots agreed to have a race. The sum of their speeds was 30 km per hour (kph). The robots had names: C3P–oh, R2D–too, and PI3–dotcom. PI3–dotcom's speed plus one third of C3P–oh's speed was 22 kph more than the R2D–too's speed. Four times the C3P–oh's speed plus three times the R2D–too's speed minus twice the PI–dotcom's speed was 12 kph. Find how fast (to the nearest kph) each robot ran.

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

A5 demonstrate an understanding of properties of matrices and apply them

Elaboration – Instructional Strategies/Suggestions

A5 Matrices provide convenient, logical ways to store information. Students will discover that may properties for real numbers apply to matrices. However commutativity under multiplication in general does not apply to matrices. Students should discover that the commutative property does not hold true for matrices under multiplication. They should also be aware that square matrices are the only matrices that can have an identity, and of the conditions under which square matrices have inverses. Students will be expected to discover answers to the following questions:

a) If you add matrices, is the order in which the matrices appear important? Why or why not?
b) If you multiply matrices, is the order in which the matrices appear important? Why or why not?
c) What conditions or characteristics must be considered for adding or multiplying matrices?

Conclusion:

Commutative Property for Addition

$$A + B = B + A$$

Information can be organized into a series of columns and rows. This information can be manipulated according to specific criteria. Example: Medal standings for countries during the Atlanta Olympics were reported to the public in a variety of fashions, including total number of medals, number of gold, silver, and bronze medals, and the point standings. The information for the end-of-day results were placed in a matrix on a computer and manipulated to convey these results. The following results are the medal accumulations for four countries:

	Day 1			Day 2			Day 3		
	Gold	Silver	Bronze	Gold	Silver	Bronze	Gold	Silver	Bronze
CAN	0	0	1	0	1	0	0	2	1
USA	2	0	0	2	1	2	3	1	1
ENG	0	1	1	1	0	0	0	2	1
RUS	1	2	1	2	0	2	2	3	1

How many gold, silver, and bronze medals did each of the four countries receive at the end of the second day? Would it make any difference if the medals on Day 1 and Day 2 were added in the opposite order?

Conclusion:

continued ...

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

A5

Pencil/Paper

1. The following table shows the numbers of different kinds of shirts sold by a store over 3 months. The store sells t-shirts, short-sleeve, crew-neck, and long-sleeve shirts.

	Jan S M L	Feb S M L	Mar S M L
T	10 15 25	5 10 20	15 5 20
SS	3 2 5	1 2 5	10 12 22
C	15 20 25	20 22 17	17 10 5
LS	12 15 20	15 15 25	1 3 2

   a) The owner of the store needs to total all the small t-shirts sold in the first 3 months of the year. He wonders if he can add January and February, then add March, or if he should add February and March, then add January. Do you think it will make a difference? Show that it does or doesn't.
   b) Test your conclusion from (a) by adding all the medium long sleeve shirts in the same way.
   c) Will the total of crew neck shirts (large) be different for Jan + Feb than for Feb + Jan? Show your work.

$

   d) In March the price for shirts forms a matrix like this:

T	12
SS	10
C	15
LS	15

   The prices are the same for all sizes. How much money did the owner take in during the month of March? Keep in mind that you still want a total for each size. Determine if it makes a difference how the two matrices are organized. Explain.

Journal

2. Use the results from #1 above to talk about the commutative and associative properties for addition, and the commutative property for multiplication. Give examples for each.

... continued

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

A5 demonstrate an understanding of properties of matrices and apply them

Elaboration – Instructional Strategies/Suggestions

A5 On their calculator have students enter the Olympic results by day (expressed as matrices). Perform the operations in the order appropriate to the statement

$$(A + B) + C = R_1$$

$$A + (B + C) = R_2$$

Are the results identical? (Is $R_1 = R_2$?) If yes, then one might conjecture that associative property is true for addition of matrices. Restriction: *A*, *B*, *C* must have the same dimensions.

Associativity for multiplication can be demonstrated using the same test as above for associativity under addition. The assumption must be that each of the matrices can be multiplied according to the definition of matrix multiplication.

For example, if *A* is a 3×2, *B* is a , and *C* is a , then becomes $A_{3\times2}\left(B_{2\times4}C_{4\times3}\right)$ multiplication with respect to the dimensions, resulting in a 3×3 matrix as would

Students might conjecture that multiplication of matrices is associate as long as the dimensions allow for multiplication to occur as in the above example.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

A5

Performance

1. Make up three matrices and call them *A*, *B*, and *C*. Show that . Does this mean that associativity under multiplication holds? Explain.

A5

Activity

2. a) Plot the points $A(2, 3)$, $B(5, -2)$, and $C(7, 6)$ and join them to form .
   b) Make a 2x3 matrix called M of the coordinates for *A*, *B*, and *C*.
   c) Multiply the matrix *M* by the matrix $T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
   d) Where must you place matrix *T* so that you can multiply? Explain.
   e) Record the answer matrix as coordinates, label them , plot them, and join them. Describe the transformation obserred.
   f) Describe in your own words what you have done from a) – e).
   g) Now multiply the answer matrix by . Show how you do this.
   h) Record this 2nd answer matrix as coordinates, label them , and join them. Describe the transformation observed.
   i) Using matrix *M*, *T*, and *P*, see if you can move to position with one set of multiplications. Determine all the possible orders of the matrices for this to work. For example, can they be in the order *T M P*? Describe the order in which you could multiply the three matrices once they are positioned.
   (j) Conjecture about the effect of multiplying a collection of ordered pairs by T, by P, by the product of T and P in general. Test your conjecture with at least one more figure.

$(A \cdot B)C$

$P = \begin{pmatrix} & \\ 0 & -1 \end{pmatrix}$

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

C5 determine quadratic functions using systems of equations

Elaboration – Instructional Strategies/Suggestions

C5 Students will use 3 equations with 3 variables to find the equation of a quadratic relation. For example, Wally, the human cannon ball, is fired from his cannon at a target 200m distant on the horizon and 8m above the ground. The mouth of the cannon is 3m above the ground. Assuming that his path is parabolic and that he just barely misses a tower 9.9m high, 20 metres from the target, what is his maximum height during flight?

To find the equation of the path students might begin by overlaying the Cartesian coordinate system and determining 3 key points: (1, 3) the mouth of the canon; (180, 9.9) the top of the tower; and (200, 8) the location of the target. To get the quadratic equation that passes through the three points (1, 3), (180, 9.9), and (200, 8), students will use the model $ax^2 + bx + c = y$ and set up a system of equations to determine values for *a*, *b*, and *c*.

- from (1, 3): $a(1)^2 + b(1) + c = 3$
- from (180, 9.9): $a(180)^2 + b(180) + c = 9.9$
- from (200, 8): $a(200)^2 + b(200) + c = 8$

Students will find . With these coefficients students can write the equation $-0.000671x^2 + 0.16x + 2.84 = y$.

To solve the problem, students need to determine the maximum height obtained by Wally. They can do this by graphing the equation, and tracing the graph, or using other calculation features with technology.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

C5

Performance

1. A man had his car repaired four times at the same service centre. From his bills, he recorded the following information.
   a) The man used a quadratic model to represent the cost of service in terms of the number of hours of work required. Is the model reasonable? Give reasons for your answer.
   b) Find an equation for the situation.
   c) Based on the equation, what might a 5 hour job cost?
   d) If the bill came to $50, how much time might you expect it would have taken?

Hours of work	Total
1	$44
3	$68
6	$104
$4\frac{1}{2}$	$86

2. A rocket attains a maximum height of 250 m when it is fired at a target that is 200 m distant on the horizon, 180 m from this target and on the direct line of fire is an 85 m tall building. Will the rocket reach the target, and if so, by how much will it clear the building?
3. A golf ball is thrown horizontally a distance of 80 m but only goes 10m off the ground. The ball is thrown from a height of 1m above the ground. If the flight of the ball is parabolic, what equation will describe the path of the ball?
4. Water is discharged through a tap near the bottom of a water tank. The depths of water remaining in the tank with respect to the time of flow in the first 100s have been recorded in the table shown.
   a) Would you use a quadratic model to represent the situation? Give your reasons.
   b) Find an equation for the depth of water remaining in terms of the time of flow.
   c) When will all the water in the tank be emptied?

Time (s)	Depth (cm)
0	100
20	81
40	64
60	49
80	36
100	25

Suggested Resources

The Algebra of 3-Space

Outcomes

SCO: By the end of Mathematics 2204/2205, students will be expected to

B14(Adv) determine the equation of a plane given three points on the plane

Elaboration – Instructional Strategies/Suggestions

B14(Adv) In three-dimensional space, the equation of a plane is given by $Ax + By + Cz + D = 0$ as long as A, B, and C are not simultaneously zero, and

.

To find the equation of the plane that contains the three points

E = (10, 1, 1)
F = (2, –2, 2) and
G = (12, 7, 3)

students should first divide the model equation $Ax + By + Cz + D = 0$ by A, so that the coefficient of x is

This may be written as: $x + Py + Qz + R = 0$

Students can now plug in the given values for x, y, and z and solve the resulting system for P, Q, and R:

Rearrange:

$$P + Q + R = -10$$
$$-2P + 2Q + R = -2$$
$$7P + 3Q + R = -12$$

Students would now enter the coefficients into matrix M and N.

matrix matrix

Then, using the inverse matrix method Frac ENTER, they would get

and substitute this back into the equation, getting , the

equation of the plane which contains the given points E, F, and G.

The Algebra of 3-Space

Worthwhile Tasks for Instruction and/or Assessment

B14(Adv)

Pencil/Paper

1. Find the values for A, B, C and D that would give the equation for each of the following planes:
   a) the $y – z$ plane
   b) the $x – z$ plane
   c) the plane parallel to the $x – y$ plane, 4 units above it
   d) a plane parallel to the $x – z$ plane, 2 units to the left of it
2. Find the plane that contains each of the following sets of points
   a) (2, 4, 7), (1, 0, 5), and (–2, –3, 8)
   b) (1, –2, 3), (4, 12, 0) and (1, 3, –2)

Performance

3. Find the equation of the plane that passes through the origin and the line of intersection of the planes defined by $3x + 4y – 7z – 2 = 0$ and $2x + 3y – 4 = 0$.
4. Ted is building a tree house for his son, Junior. There are 3 branches in the tree on which Ted thinks he could build the main platform. However, he doesn't think that it will be level and he needs to know the measurement of supports he could use to make the platform level. He superimposes a 3-dimension grid over a sketch of the tree branches and determines the coordinates for the 3 points of contact. He needs the equation of the plane that contains these coordinates: (0, 0, 0), (4, 3, 7), and (–2, –5, 8). Find the equation of the plane.

Suggested Resources