

Unit 5
Trigonometry
(20%)

Trigonometry

Outcomes

SCO: In this course students will be expected to

D4 solve problems using the sine, cosine and tangent ratios

B4 use the calculator correctly and efficiently

C28 solve simple trigonometric equations

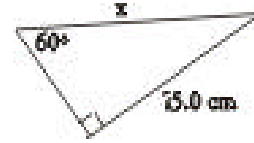
B3 demonstrate an understanding of the relationship between arithmetic operations and operations on equations and inequalities

Elaboration—Instructional Strategies/Suggestions

Trigonometry is the study of the relationships of the measures of the sides and angles of triangles. All problems that deal with geometric situations where length or angle measure is required can be modeled with diagrams or constructions of the geometric figures.

D4 In a previous course, students developed the trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ and applied them to problems involving right triangles. Students may need to review the use of trigonometric ratios to solve problems involving right triangles.

B4/C28/B3 Students will use their calculators correctly and efficiently for various procedures while working with trigonometric relationships. For example, when finding a missing side in a situation modeled by a right triangle, some students may set up an equation like the following;



$$\sin(60^\circ) = \frac{\text{length of side opposite } 60^\circ \text{ angle}}{\text{length of hypotenuse}} = \frac{15.0}{x}$$

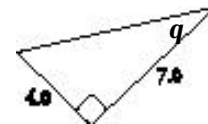
then multiply both sides by x : $x \sin(60^\circ) = 15.0$

then divide by $\sin(60^\circ)$, they would have the expression: $x = \frac{15.0}{\sin(60^\circ)}$
 $x = 17.3 \text{ cm}$

Students will also use trigonometric equations, like the one to the right, to find missing angles in right triangles.

$$\tan \theta = \frac{4.0}{7.0}$$

Most students would first change the ratio to a decimal, $\tan \theta = 0.57$.



Then students would find the angle measure θ by using “ \tan^{-1} .”

$$\begin{aligned} \theta &= \tan^{-1}(0.57) \\ \theta &= 29.68^\circ \\ \theta &= 30^\circ \end{aligned}$$

Some students may simply enter $\tan^{-1}\left(\frac{4.0}{7.0}\right)$ into their calculator and solve for theta.

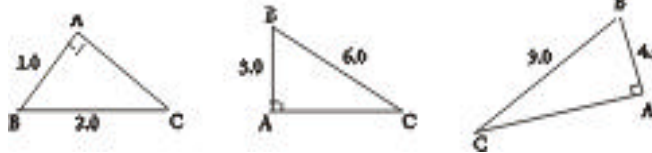
Since this chapter deals with measurement, students should be careful to properly use precision, accuracy and significant digits. Teachers may wish to discuss these at this time.

Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

Performance

D4/B4/C28/B1



1) Find the measure of angle C for each of the following:

a) Complete this table:

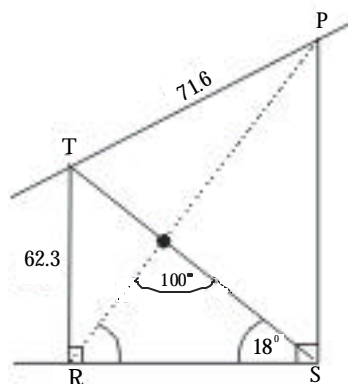
Length of hypotenuse	Length of shortest side	$m \angle C$

- b) Look for a relationship that seems to be true from examining the pattern in the table.
- c) Make a conjecture.
- d) Test your conjecture in a situation that you make up.

2) Oliver makes a shelter in the shape of an isosceles triangle. Use the given measures to find the height of the shelter.



3) In the town where Simon lives, some roads were constructed as in the diagram. Simon needs to know how much longer the road from S to P is than the road from R to T.



4) Richard leans the top of his 7.3 m ladder against the sill of a window that is 6.5 m above the ground. At what angle to the ground will his ladder be?

Suggested Resources

Brueningsen, Chris et al, *Real-World Math with the CBLTM System*, Texas Instruments, 1994

Meiring, Steven P., “*A Core Curriculum: Making Mathematics Count for Everyone*”, Addenda Series 9–12, Reston, VA, NCTM, 1992

“The Geometer’s Sketchpad,” Software, Key Curriculum Press, Everyville, CA

Trigonometry

Outcomes

SCO: In this course students will be expected to

D5 apply the Law of Sines, the Law of Cosines, and the formula “area of a triangle $ABC = \frac{1}{2} bc \sin A$ ” to solve problems

B4 use the calculator correctly and efficiently

Elaboration—Instructional Strategies/Suggestions

D5/B4 Students will interact with the teacher to develop a procedure for obtaining measurements of triangles that are not right-angled (oblique triangles). They should develop, through teacher-led discussion or a directed activity, the Law of Cosines and the Law of Sines, as well as a new formula for finding the area of any triangle.

To develop the Law of Sines students might begin like this (or they might use a geometry software package to explore in the same way):

Questions 1–3 refer to the acute $\triangle ABC$ on the right.

1) Measure each side to the nearest tenth of a centimetre.

a) c b) a c) b

2) Measure each angle to the nearest degree.

a) $\angle A$ b) $\angle B$ c) $\angle C$

3) Calculate each of the following to 1 decimal place.

a) $\frac{a}{\sin A}$ b) $\frac{b}{\sin B}$ c) $\frac{c}{\sin C}$

4) Repeat questions 1–3 for the obtuse $\triangle ABC$.

5) Draw an acute triangle, $\triangle ABC$, of your own and repeat questions 1–3.

6) Draw an obtuse triangle, $\triangle ABC$, of your own and repeat questions 1–3.

7) Based on the results of question 3, what can you conclude about the

relationship between $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$

Students should conclude that these ratios are equal. Any one proportional statement is the formula called the Law of Sines.

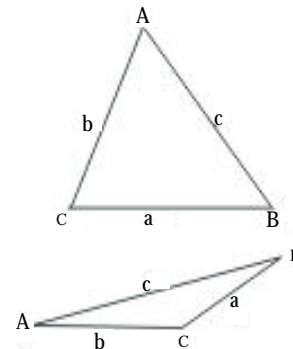
$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \text{or} \quad \frac{b}{\sin B} = \frac{c}{\sin C}$$

Students should know that some students may use the formula in this form.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Have students think about when they would use the Law of Sines. They should conclude that it will be used either when they know two angles and any side and are looking for another side, or, they are given two sides and the non-included angle and need to find a side.

Students should also recognize the convention that the lower case letter indicating a side length is placed opposite the angle named with the corresponding upper case letter.



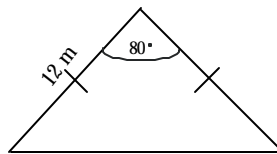
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

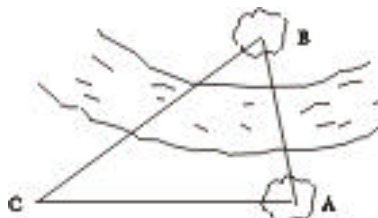
Performance

D5/B4

- 1) A flower garden is having pressure treated wood placed around its border. Find the perimeter of the garden.

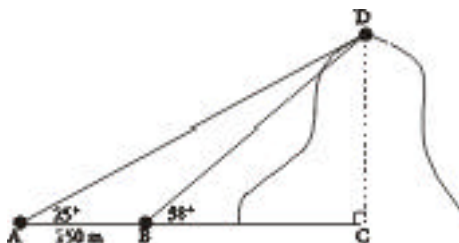


- 2) Surveyor Sally had to determine the distance between two large trees situated on opposite sides of a river. She placed a stake at C, 100.0 m from point A. Sally then determines the angle measures at points C and A to be 45° and 80° respectively.



- a) Ask students to help her find the distance between the trees.
b) Ask students to create a different problem using the above diagram.

- 3) Surveyors cannot get to the inside centre of a mountain easily. Therefore, a mountain's height must be measured in a more indirect way. Find the height of the mountain.



Journal

- 4) In class, Marlene said that she could find AB in question 2(above) using the Law of Sines. Is she correct or not? Explain.
- 5) Billy is using $\frac{a}{\sin A} = \frac{b}{\sin B}$ to solve a problem. Billy's dad said that in his day he would have used $\frac{\sin A}{a} = \frac{\sin B}{b}$. Is Billy or his dad correct? Explain.

Suggested Resources

Meiring, Steven P., "A Core Curriculum Making Mathematics Count for Everyone", Addenda Series 9–12, NCTM, 1992

"Geometers' Sketchpad"
Key Curriculum Press

Trigonometry

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B3 demonstrate an understanding of the relationship between arithmetic operations and operations on equations and inequalities

Elaboration—Instructional Strategies/Suggestions

D5/B4/C28/B3 Teachers should help students see how the Law of Cosines works and its similarity and relationship to the Pythagorean Theorem.

The Law of Cosines stated symbolically is $c^2 = a^2 + b^2 - 2ab \cos C$

Applied to the given diagram, the Law of Cosines would state:

$$AB^2 = BC^2 + AC^2 - 2(BC)(AC) \cos C$$

$$AB^2 = 12^2 + 10^2 - 2(12)(10) \cos 72^\circ$$

$$AB^2 = 144 + 100 - 240(0.3090)$$

$$AB = 13.0$$



Usually this formula is used to find a missing side, given the other two sides and the angle opposite the missing side. The formula can be restated as $b^2 = a^2 + c^2 - 2ac \cos B$, or $a^2 = b^2 + c^2 - 2bc \cos A$, depending on which side the students want to find.

Sometimes students will use the formula to find an angle measure when all three side lengths are given. They don't need to arrange the formula first. For example, given $a = 12.0$ cm, $b = 9.5$ cm, and $c = 7.2$ cm, students could find any of the angle measures. If they wanted $\angle A$:

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$12.0^2 = 9.5^2 + 7.2^2 - 2(9.5)(7.2) \cos A$$

$$144 = 90.25 + 51.84 - 136.8 \cos A$$

Next, they need to isolate the variable term.

$$144 - 90.25 - 51.84 = -136.8 \cos A$$

then isolate the variable:

$$\begin{aligned} \frac{1.91}{-136.8} &= \frac{-136.8 \cos A}{-136.8} \\ -0.01396 &= \cos A \\ \angle A &= \cos^{-1}(-0.01396) \\ \angle A &= 91 \end{aligned}$$

Teachers could also discuss the re-arranged version of the formula

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Students should notice the similarity of the last two steps in the above calculation to the steps used when solving right triangle trigonometric equations. They would conclude that the angle A measures about 91° .

Students need to think about using the Law of Cosines when they have a situation where there is no right angle, and they need a third side length, given the other two, or an angle measure given all the side lengths.

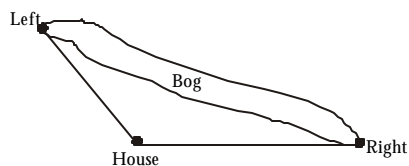
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

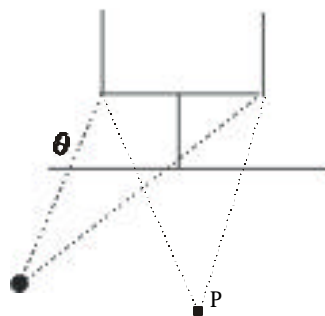
Performance

D5/B4/C28/B3

- 1) A farmer wants to find the length of the back of his property which is all bog land. He knows that his left and right boundary lines connect near his house at an angle of 147° . The left boundary length is 90 m and the right is 110 m. Suggest a means of finding this length.
- 2) Terry is building an A - frame cabin in the woods. The length of each of two rafters is 8.50 m. If the angle of the top of the frame is to be 46° , calculate the proposed width of the cabin at the base.



- 3) a) A football player is attempting a field goal. His position on the field is such that the ball is 7.5 m from the left upright of the goal post and 10.0m from the right up- right of the goal post. The goal posts are 4.3 m apart. Find the angle marked q .
- b) If the ball is moved to the middle of the field, position P, then the ball is equidistant to both uprights, approximately 8.5 m each. Find the angle corresponding to q from this position.



Journal

D5

- 4) Explain why the Law of Cosines might be a useful relationship to try to remember.

Suggested Resources

Brueningsen, Chris et al,
*Real-World Math with the
CBLTM System*, Texas
Instruments, 1994

Meiring, Steven P., "*A Core
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Elaboration—Instructional Strategies/Suggestions

Teachers may want to discuss with students that the Law of Cosines ‘looks’ like the Pythagorean Theorem with an adjustment factor to make up for the lack of a right angle.

$$c^2 = a^2 + b^2 - \text{adjustment factor}$$

They may want to have students examine several situations where the measure of $\angle C$ has various values, say $40^\circ, 50^\circ, 70^\circ, 85^\circ, 90^\circ, 100^\circ, 110^\circ, 120^\circ, \dots$

Students would evaluate $2ab \cos c$ and note that as

$m\angle C \rightarrow 90^\circ, 2ab \cos C \rightarrow 0$. As $m\angle C$ gets larger and larger beyond 90° , $2ab \cos c$ gets smaller and smaller.

Talk to students now about:

if $m\angle C = 90^\circ$, then $c = \sqrt{a^2 + b^2}$ (Pythagorean Theorem)

as $m\angle C = 90^\circ$, gets closer to 90° then $\cos C$ gets smaller

and therefore $2ab \cos C \rightarrow 0$.

as $m\angle C$ gets farther from 90° , $\cos C$ should be larger therefore

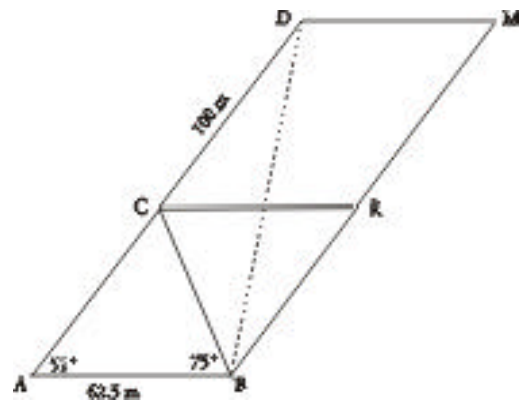
$2ab \cos C$ gets larger and larger and the impact of the adjustment

factor becomes greater.

Students should be prepared to combine both the Laws of Sines and Cosines in the same question when required. This is a challenging problem for Mathematics 2206 and some hints may be required.

For example, farmer Jones’ property (ABRC) is shaped like a parallelogram (see diagram). He is given more land (DMRC). He needs to know the distance from D to

B. He would first have to use the Law of Sines to get BC, then the Law of Cosines to BD.



Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

Activity

D5/B4

- 1) Given $\triangle ABC$ with $b = 4.0$ cm and $c = 3.0$ cm:
 - a) Ask students to determine “a” if $m\angle A = 90^\circ$.
 - b) Ask students to evaluate $2bc \cos A$, if $m\angle A = 90^\circ$.
 - c) Ask students to determine “a” again using $a^2 = b^2 + c^2 - 2bc \cos A$, given $m\angle A = 90^\circ$. Make a conjecture about this new formula.
 - d) Ask students to construct an accurate diagram of $\triangle ABC$ with $b = 4.0$ cm, $c = 3.0$ cm, and $m\angle A = 80^\circ$ and measure the length of a
 - e) Ask students to
 - i) predict if $a < 5.0$ cm, $a = 5.0$ cm, or $a \geq 5.0$ cm and explain their choice.
 - ii) measure with a ruler the length “a” and record it.
 - iii) calculate the length “a” using $a^2 = b^2 + c^2 - 2bc \cos 80^\circ$.
 - f) Ask students to repeat step (e) given $m\angle A = 60^\circ, 65^\circ, 85^\circ, 89^\circ$.
 - g) Ask students to describe the pattern and how it fits with their conjecture in (c).
 - h) Ask students to repeat step (e) given $m\angle A = 91^\circ, 95^\circ, 100^\circ, 120^\circ$
 - i) Ask students to describe the pattern and how it fits with their conjecture in (c).
 - j) Ask students to make a statement about how the formula $a^2 = b^2 + c^2 - 2bc \cos A$ can be used.
 - k) Glen conjectured that if he knew the side measures, he could determine the $m\angle A$ using this new formula. Do you agree or disagree? Explain.
 - l) Colin conjectured that if he knew the length of “b” and “a” and the $m\angle C$ he could determine the length C. Explain what Colin must be thinking.

Journal

D5

- 2) How can you help yourself remember the formula for the Law of Cosines? (**Hint:** it is the Pythagorean Theorem, plus or minus some adjustment factor. How can I remember the adjustment factor?)

Suggested Resources

Meiring, Steven P., “A Core Curriculum Making Mathematics Count for Everyone”, Addenda Series 9-12, NCTM, 1992

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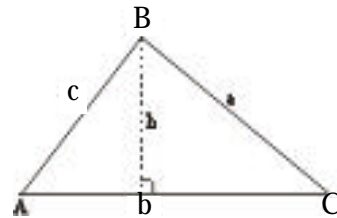
Elaboration—Instructional Strategies/Suggestions

To determine the area formula, “area of a triangle” $= \frac{1}{2} bc \sin A$, students should be asked to explain how they would find the area of triangle ABC.

Students would write $A = \frac{1}{2} \text{base} \times \text{height}$.

In this triangle, the base is c, so

$$A = \frac{1}{2} \text{base} \times \text{height}$$



The teacher would then ask students to replace the “h” with an expression using $\sin A$.

Students would write $\sin A = \frac{h}{c}$

or, $h = c \sin A$

when this is substituted into the formula $A = \frac{1}{2} bh$ we get

$$A = \frac{1}{2} b (c \sin A)$$

or without brackets $A = \frac{1}{2} b c \sin A$

Students should apply this formula in various problem-solving situations involving area. To use this formula to find area, students should realize that they need any two sides and the included angle measure of any triangular shape. When the area of a triangular shape is given, students can use this formula to find any of the missing three measures, if the other two are given. For example, if the area of a triangular region on a stage was to be carpeted with 37 square metres of carpet, and two adjacent sides measures of the carpet were given as 12.0 m and 6.7 m, then the angle between these sides could be found:

$$\text{Area} = \frac{1}{2} b c \sin A$$

$$37 = \frac{1}{2} (12.0)(6.7) \sin A$$

$$\frac{37}{40.2} = \sin A$$

$$\angle A = 67^\circ$$

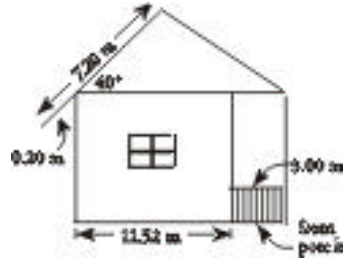
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

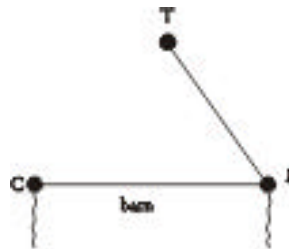
Performance

D5

- 1) Hilary wants to paint the triangular gable ends of her log cabin. She knows that a can of paint will cover 39 m^2 . She expects to have to paint two coats. If a can costs \$29.95, how much money will she have to spend?



- 2) Cousin Barney is building a new corral on the side of his barn for his new lamb, Huey. He measures the barn length to be 15.25 m. There is already a fence from one end of the barn to a tree (T) with a length 21.62 m. Barney has just spread seed that covers 120.50 m^2 inside the triangular region C - B - T. How long will the fence be that goes from C to T?



Journal

- 3) Ask students to explain why “ $c \sin A$ ” from the formula:

area of a triangle = $\frac{1}{2}bc \sin A$ is the same as “ h ” in the formula area of a

$$\text{triangle} = \frac{bh}{2}.$$

Suggested Resources

Meiring, Steven P., “A Core Curriculum Making Mathematics Count for Everyone”, Addenda Series 9–12, NCTM, 1992