Unit I - Reinforcing Number Concepts and Skills (20%)

Suggested Instruction and Assessment Time for this Unit: 10 classes(Assumes 55-60 minute classes)

Note: The skills developed in this unit are intended for further use and application where appropriate throughout the entire course.

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

A1 identify numbers as belonging to the various subsets of real numbers and recognize situations in which each of these subsets can be applied.

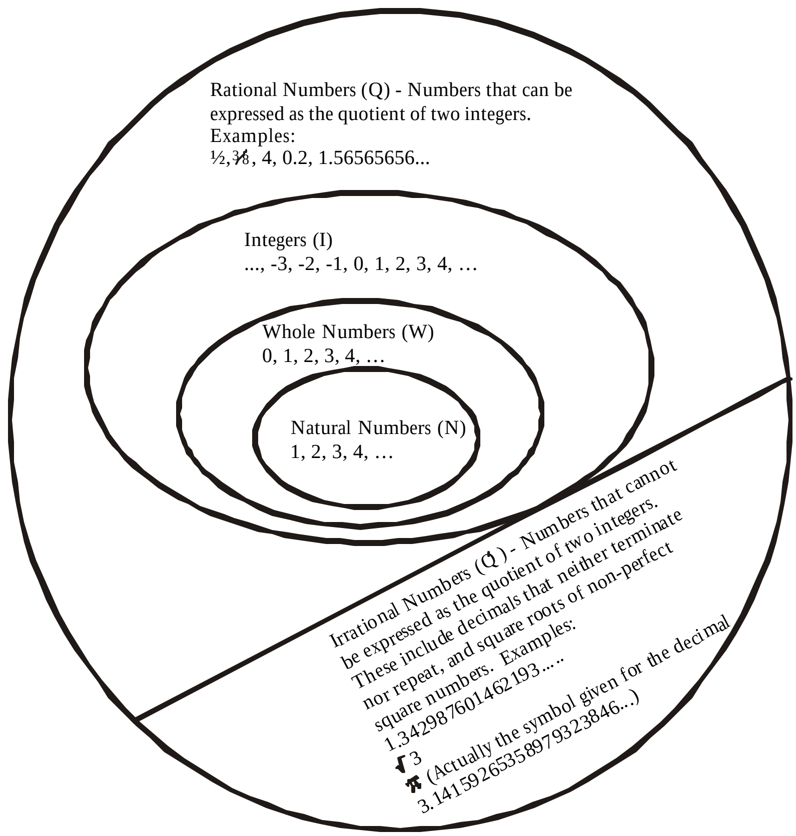
A2 write either a sentence or an inequality to describe domain and range for various problem situations.

Elaboration - Instructional Strategies/Suggestions

A1/A2

Students would have studied these subsets and their relationships in grade 9. This should be given just a quick review with specific reference to the following Venn Diagram:

The Real Number System



Students extend their grade 9 experiences by identifying situations in which each number system can be applied, and particularly to make the connection with domain and range. Domain and range are part of Mathematics 1204, 2204/2205 and 3204/3205. It is through review and sustained practice that students can solidify these concepts.

Worthwhile Tasks for Instruction and/or Assessment

A1.1 Classify each number according to ALL the subset(s) of the Real Numbers to which it belongs:

a) 5 b) -3 c) 2/5 d) 0 e) 2,000 f) g) 3.14

h) i) 2.155155155155155... j) 2.1474747474747474....

k) l) m) -9/6 n) -4.317495

o) 0.165165516555... p) -6.132133134... q) 72.04120647...

r) s) t) u)

v) w) x) y) 3π

z) -42,765

A1.2 Can a number be both rational and irrational ? Explain your answer.

A1.3 Which set of numbers contains all other sets ?

A1.4 Which set of numbers does not contain any other sets ?

A1.5 What other set(s) are contained by :
- a) Rational Numbers
- b) Whole Numbers
- c) Irrational Numbers
- d) Natural Numbers
- e) Integers

A1.6 Identify the set(s) of numbers that would be used in each situation below. There may be more than one possibility in each situation:

a) Natasha earns 5¢ for each container she returns for recycling. The number of containers is described by the ______________ numbers.

b) In a previous course, you learned the Pythagorean Relation which said that, for any right triangle, the square of the hypotenuse is equal to the sum of the squares of the sides; that is
$(\text{hypotenuse})^2 = (1^{st}\text{ leg})^2 + (2^{nd}\text{ leg})^2$
Meghan has a piece of cloth in the shape of a right triangle for a craft she is making. The 1^{st} leg was measured to be 5cm and the 2^{nd} leg was 4cm. The measure of the hypotenuse must be a(n) ________number.

c) The temperature of a glass of water must be a(n) ______________ number.

Resources

Mathematics 10 (Revised Edition), p.21-24

Additional work will be necessary to cover questions such as A1.6 related to the situations in which the various number systems arise.

Additional work will be necessary to cover outcome A2. However, *Mathematical Modeling: Book 1*, chapter 7 on Linear Programming will provide ideas on problems requiring restrictions such as problems A2.3 and A2.6.

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

A1 identify numbers as belonging to the various subsets of real numbers and recognize situations in which each of these subsets can be applied.

A2 write either a sentence or an inequality to describe domain and range for various problem situations.

Elaboration - Instructional Strategies/Suggestions

Students must recognize that each subset of the real numbers exists because there is a need for it. For example, while the Natural Numbers are sufficient for counting quantities such as people, they are not sufficient for expressing the situation in which there are no people, hence the need for whole numbers which contains the element 0. While the integers are sufficient for situations such as temperatures expressed to the nearest degree, they are not sufficient when more precise measurements for a part of a degree are required. Hence the need for rational numbers.

The need for irrational numbers is more difficult to convey. It is a good idea to examine the various types of decimals that can occur i.e. terminating, repeating, and non-repeating/non-terminating.

- Ask students to find an irrational number that lies between two given rational numbers in decimal form, e.g. Find an irrational number between 3.57 and $3.5\overline{8}$. By comparing answers and through class discussion, students should get a sense that there are an infinite number of irrational numbers between any two given rational numbers. (This would require use of a calculator.)

A brief history associated with the discovery of the number π would be useful. Even in ancient times, people knew that for any circle the ratio $\frac{circumference}{diameter}$ was a value slightly greater than 3. Around 1700 B.C. the Egyptians used the fraction $\frac{256}{81}$ to approximate this ratio, while the Greeks of about 220 B.C. used $\frac{22}{7}$ and used their letter π to represent it. The Chinese, Hindu, and Europeans also developed their own approximations, but in 1761 Johann Lambert proved there is no rational number equal to π.

Students should know that the square root of a non-perfect square number always gives an irrational number (e.g. $\sqrt{10} = 3.1622776601\ldots$). They should understand that while decimal approximations for such square roots are "good enough" in most everyday applications, in mathematics we try to express such answers in their *exact* forms using a radical. Maintaining exact form and thus avoiding multiple roundings when performing calculations gives more exact results.

- Through triangulation, a ground radar operator knows the horizontal distance to an incoming missile is $\sqrt{199}$ km. If the missile is at a height of 5.1 km, calculate the actual distance from the operator to the missile using

  i) the approximation $\sqrt{199} \approx 14.11$

  ii) the exact value

  Explain why it is important for the operator to use the exact value in his calculations if he intends to shoot down the missile.

Worthwhile Tasks for Instruction and/or Assessment

A2.1 Lenora has to design a switch that will automatically turn on a cooling fan if the temperature of a room reaches 25˚C.
Would the possible temperatures of the room be represented by the real numbers, the rational numbers, the irrational numbers, the integers, the whole numbers, or the natural numbers?

a) What inequality would describe the restriction on the temperatures when the fan is turned off?
b) What inequality would describe the restriction on the temperatures when the fan is turned on?

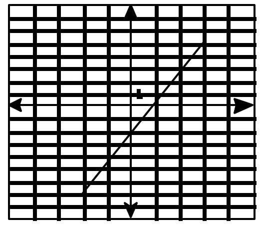
A2.2 Derek writes articles for the *Downhomer* magazine. He gets paid for each article he writes.

a) What is the smallest number of articles he can write?
b) What set of numbers can be used to describe the number of articles he can write?
c) The magazine will only accept a maximum of 36 articles per year from Derek. Write a sentence to describe the possible numbers of articles Derek is allowed to submit each year.
d) Use your answers in B) and C) to write an inequality describing the possible numbers of articles Derek can submit each year.

A2.3 Bonnie works at a sewing shop where she cuts the material for making dresses. She is paid $10.00 for each dress she cuts, but if she only cuts the material for part of a dress she gets paid based on the portion cut, e.g., if she cuts out 40% of a dress, then she gets paid 40% of $10.00 = $4.00

a) Is it possible for the number of dresses cut to be a natural number? a whole number? an integer? a rational number? an irrational number? a real number?
b) Bonnie is only allowed to work 5 hours per day and it takes her 40 minutes to cut one dress. If x is the number of dresses cut, write both a sentence and an inequality to describe the possible number of dresses Bonnie can cut each day.
c) If y is the amount of money she can earn, write a sentence and an inequality showing the amount of money Bonnie can earn each day.

A2.4 Examine the following graph.



a) What is the smallest x-coordinate on the graph?
b) What is the largest x-coordinate?
c) Can the x-coordinates be any real number between these smallest and highest values, or do they have to be integers? Why?
d) Write one inequality which summarizes your answers in a), b) and c).
e) Repeat a), b), c) and d) for the y-coordinates.

Resources

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

A1 identify numbers as belonging to the various subsets of real numbers and recognize situations in which each of these subsets can be applied.

A2 write either a sentence or an inequality to describe domain and range for various problem situations.

Elaboration - Instructional Strategies/Suggestions

Students should be able to recognize the need for numerical restrictions in a problem situation as well as identify specific domain and range. Start with a fairly simple problem and build toward formal terminology. Opportunities should also be provided for students to write domain and range in words as well as in symbols. Emphasize with students that they should first identify the set of numbers that applies and then think about the numerical restrictions needed. Inequalities should be written using the following as a format guide: $\{x \mid x \leq 3, x \in R\}$.

Bill sells cars at ***Rolls-Can-Barely Auto Sales.***

a) What is the smallest number of cars he can sell each week?
b) Would Bill sell only part of a car or would he sell only complete cars? Would the number of cars be integers or real numbers?
c) If x is the number of cars sold, combine your answers in a) and b) to write an inequality expressing the restriction on the number of cars sold.
d) Bill's boss feels that Bill could be selling more cars, so he introduces a performance quota in which he states that Bill should be selling at least 4 cars per week. Revise your inequality in c) to reflect this new situation.
e) Bill knows that based on his experience he cannot expect to sell any more than 25 cars even in his best week. Revise your inequality in d) to reflect this situation.

Nadine babysits for her neighbours during the summer to earn some pocket money. She is paid \$6.00 per hour, but she is also paid for working part of an hour. For example, if she works for 0.4 hours, then she earns 0.4 × \$6.00 = \$2.40. She never works more than 40 hours in a week.

a) Write an inequality to represent the possible number of hours worked in a week. Use *x* to represent the number of hours. Include the set of numbers you should use.
b) What is the smallest amount of money Nadine can earn each week? What is the largest amount? Explain how you calculated each amount.
c) If *y* is the amount of money Nadine earns in a week, write an inequality showing the possible amounts she can earn. Include the set of numbers used.
d) Suppose Nadine had to work a minimum of 8 hours per week and a maximum of 40 hours per week. What is the new domain and range for this problem?

Worthwhile Tasks for Instruction and/or Assessment

A2.5 John pays a flat rate of $19.95 per month for internet service regardless of how many hours he uses.

a) If H is the number of hours, write the restriction showing how many hours John can possibly have his computer online in one month. What set of numbers did you use and why?
b) If B is the amount John is charged each month for internet service, write the restriction showing how much John will be charged each month. What set of numbers did you use and why?
c) Which inequality in a) and b) is the domain? the range? Explain.
d) Construct a table of values using H and B and plot at least eight of the ordered pairs on a grid.
e) How does your graph show the domain and range?

A2.6 Heather cuts material to make couches for Spinney Manufacturing. She is paid $13.50 for each complete couch bundle she cuts. She is also required to complete at least 10 bundles per week.

a) If she gets nothing for partly completed bundles, write an inequality for the number of couch bundles Heather must cut each week. What set of numbers did you use and why?
b) Heather and a group of other workers complained to the company that they should get paid for partial bundles; for example, if they complete ¼ of a couch bundle then they should get ¼ of $13.50, or $3.375. The company eventually agreed to this scheme. Revise your inequality statement in a) to reflect this new situation.
c) If *E* is the money earned by Heather, write an inequality to show the possible amounts of money Heather can earn for one week. Explain how you found your answer.
d) Heather can only work up to 5 hours per day, 5 days per week. It takes her 30 minutes to cut a couch bundle.
   i) Revise your inequality in b) to show the possible numbers of couch bundles she can cut.
   ii) Revise your inequality in c) to show the possible amounts of money Heather can earn for one week.
e) If *T* is the amount of time Heather works each week, write an inequality to show the possible amounts of time Heather can work. Explain how you found your answer. (Don't forget the answer must be related to the minimum number of couches to be cut!)

Resources

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

A3 apply fraction concepts both mentally and with pencil and paper algorithms including:

- lowest common denominator (LCD)
- Equivalent fractions
- Reducing to simplest form
- Improper vs mixed fraction
- Factors
- Prime factorization to find LCD's
- Ordering fractions by relative size

Elaboration/Instructional Strategies

A3 This outcome is mainly a review of skills from Grades 6-9; however, there should be special emphasis on performing calculations MENTALLY when reasonable. The focus should be on developing mental strategies for performing such calculations. For example, students should be able to perform the following calculations mentally:

- Reduce 16/24 to lowest terms
- Find the LCD of these fractions: -7/8 and 5/12
- Convert to a mixed fraction: 25/3
- Convert $-\left(3\frac{2}{5}\right)$ to an improper fraction.
- Put these fractions in order of relative size:
  - a) 3/5 and 6/13 (students can see that 3/5 is a little more than 1/2 and 6/13 is a little less)
  - b) $\frac{-2}{7}$ and $\frac{-3}{-8}$ ($\frac{-3}{-8}$ becomes $\frac{3}{8}$ and a positive is always less than a negative. The other fraction is still negative)
  - c) $\frac{3}{7}$ and $\frac{1}{4}$ (students can use either the common denominator approach, or a common *numerator* approach to convert $\frac{1}{4}$ to $\frac{3}{12}$ and realize that this has to be smaller than $\frac{3}{7}$. As an aid, students can visualize a pie divided into 7 equal pieces as opposed to 12 and think about 3 out of 12 pieces vs. 3 out of 7).

Students should be able to use prime factorization to find the LCD of two fractions in cases where the LCD is not obvious e.g. find the LCD of 7/24 and -15/26. Since $24 = 2^3 \times 3$ and $26 = 2 \times 13$, then the LCD must be $2^3 \times 3 \times 13 = 312$.
Students may find the following strategy helpful:

Write the prime factorization of each denominator. The LCD must contain any common factors (in this case there is only one common factor, which is 2) and any factors not common (the remaining two 2's and the 3 from the factorization of 24, and the remaining 13 from the factorization of 26), so the LCD = 2 (the common one) ×2×2×3×13 = 312.

$$24 = \boxed{2} \times 2 \times 2 \times 3$$
$$26 = \boxed{2} \times 13$$

Since this is essentially the same as finding the LCM (Least Common Multiple) of two numbers, further information and background work on this skill may be found in *Mathematics 10 (Revised Edition)*, sec 1-1.

Worthwhile Tasks for Instruction and/or Assessment

Teachers should use assessment tools that allow for evaluation of mental math as well as pencil and paper algorithms. Some ways to do this are:

- Timed quizzes with no calculator, scrap paper or space on the paper to do rough work
- Observation
- Oral questioning

Timed quizzes could include items such as those below:

A3.1 Reduce 16/24 to lowest terms

A3.2 Find the LCD of these fractions: -7/8 and 5/12

A3.3 Convert to a mixed fraction: 20/7

A3.4 Convert to an improper fraction: $-5\frac{1}{4}$

A3.5 Put these fractions in order of relative size:

a) 4/7, 6/13 and 4/3

b) $\frac{-3}{-5}$ and $\frac{5}{-7}$

c) $\frac{3}{7}$ and $\frac{1}{4}$

Oral questioning could include some of the same types of items but students could also be asked to explain processes e.g. How would you decide which of these two fractions, 7/15 and 14/27, is greater without finding an LCD?

Resources

http://www.mathmax.com (Click on Basic Math 8/e, then Chapter 2, Extra Practice. You can also find plenty of other Extra practice for this and other outcomes on this site. There are extra practice sheets along with answers, reviews, explanations, and quizzes. You will need *Adobe Acrobat Reader*™ to view these files).

Math in Context 9, p.519-520 (sec. 5.2 and 5.3)

Further practice may be necessary, particularly for the mental math component

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B1 add, subtract, multiply and divide fractions (both mentally and using pencil and paper), and work with patterns and graphs based on operations with fractions.

Elaboration/Instructional Strategies

B1 Students should be able to perform all of these operations mentally in cases where it is reasonable to do so. For example, students might need pencil and paper to add 50/27 + 10/33, but they should be able to subtract
4 - ¾ mentally.

Emphasize mental "tricks" to allow students to perform calculations efficiently e.g.

- To simplify 4 - ¾ mentally, first realize 4 is the same as and

  from is , so I am left with
- To simplify $\frac{5}{6} \div \frac{1}{6}$ mentally, students can think, how many $\frac{1}{6}$ are there in $\frac{5}{6}$?

Where possible, and for simple cases only at this stage, links should be made to related algebraic skills e.g.

- Simplifying $\frac{2}{3} \times \frac{3}{4}$ and require the same types of skills.

Students should also work with patterns and graphs based on these operations. See sample questions in the *Worthwhile Tasks* section that follows.

There are two algorithms for division of fractions which can be considered. The most commonly used algorithm is known as the invert-and-multiply algorithm. This is the algorithm which students are commonly taught in the intermediate grades. It simply involves inverting the divisor and multiplying by it. For example, $\frac{4}{3} \div \frac{1}{2} = \frac{4}{3} \times \frac{2}{1} = \frac{8}{3}$ *or* $2\frac{2}{3}$

Another algorithm to consider is the common denominator algorithm. Using this algorithm involves finding a common denominator and then dividing the numerators. For example:

$$\frac{4}{3} \div \frac{1}{2} = \frac{8}{6} \div \frac{3}{6} = 8 \div 3 = 2\frac{2}{3}$$

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B1.1 a) Add: b) Subtract:

c) Subtract: d) Subtract:

e) Subtract: f) Simplify:

g) Simplify i) ii)

iii)

h) Simplify:

d) How many one-sevenths are in 8/7? How many one-fourteenths are in 8/7? How many one-twenty eighths are in 8/7?

j) If you have half a pie, divide it into three equal parts and then eat one of these pieces, what fraction (in reduced form) of the original full pie remains?

k) What is ½ of 2? of ¼? of ¾?

l) What is ¾ of ½ ?

m) What is $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times\frac{7}{8}\times\frac{8}{9}\times\frac{9}{10}\times 1000$?

Pencil and Paper

B1.2 Start with the fraction 3/4 (step 1). Find 2/5 of this fraction (step 2), then 2/5 of the result (step 3), and so on. Plot these points on a graph using the step numbers as the x-coordinates and the results of the calculations as the y-coordinates.

a) Describe the shape of the graph.

b) Will the graph ever intersect or go below the x-axis? Explain.

B1.3 The *Sir Robert Bond Bridge* over the Exploits River is $4\frac{1}{8}m$ above the normal water level. During a spring flood, the water began rising at a rate of $\frac{4}{5}m/h$. Assuming the water continued to rise at this rate, how long would it take to reach the bridge?

Resources

http://www.mathmax.com Click on *Prealgebra 3/e*, then Chapter 3 or Chapter 4, Extra Practice.

http://www.mathmax.com/develmath/chapter/chep/SEC2_2_EP12.html

You will need *Adobe Acrobat Reader*™ to view the above files.

Mathematics 10 (Revised Edition), p.16 #'s 1-4

Math in Context 9, p.22-23 (these are more low level questions) p.520-522 (sec. 5.6 - 5.13)

Mathpower Nine, p.136-147

Further work may be necessary for questions like G) iii) involving rational expressions.

Access (Alg) Section AG, # 1-140
Sec AI , # 1-160

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B1 add, subtract, multiply and divide fractions (both mentally and using pencil and paper), and work with patterns and graphs based on operations with fractions.

Elaboration - Instructional Strategies/Suggestions

B1 It is very important to revisit estimation skills with fractions so that students will be able to judge the reasonableness of the results they acquire through pencil and paper algorithms. For example, in a problem such as, $3\frac{3}{4} \times 5\frac{1}{5}$ students should s able to mentally round these numbers so that a simpler problem, 4×5, is created. They should then be able to compare their exact answer with the estimate to determine whether the answer achieved using the standard algorithm appears reasonable.

Students should be given problems such as:

$5 - (\frac{1}{2} + \frac{1}{2} \times \frac{1}{3})^2 \div \frac{1}{9}$

This will allow them to apply their fraction skills and practice the order of operations.

Worthwhile Tasks for Instruction and/or Assessment

B1.4 As a big time investor, I invested $1 in the stock market. At the end of the first year I had $\frac{5}{4}$ of what I started with. At the end of the second year I had $\frac{5}{4}$ of what I had in the previous year, and so on.

a) Without using a calculator, construct a table of values showing the amount of money I had at the end of each of the first 8 years. Express all amounts as mixed fractions.
b) Plot the points, connect them with a smooth curve and describe the shape of the graph.
c) Explain why the y-intercept is located at (0,1).
d) What is the domain of this graph?
e) Use your graph to estimate how much money I would have after 4.5 years.
f) My friend Sadie deposited a different amount of money two years earlier than me, but by the time I invested my money she had exactly the same amount as I did. If her money grew consistently at exactly the same rate as mine, how much money did she start with?

B1.5 Beth had $\$1\frac{4}{5}$ on Day 0. Each successive day her Dad gave her $\$\frac{2}{5}$.

a) Construct a table of values showing how much money Beth would have on each of the first ten days and draw the graph.
b) Describe the shape of the graph. Why is it different than the shapes of the graphs you drew in B1.2 and B1.4?
c) Does it make sense to connect the points on the graph? Why or why not?
d) Why don't you need any negative numbers on the x-axis to draw this graph? Give a verbal description of the types of numbers you would use for the x-coordinates you used in this problem.
e) What is the domain of this graph?

Resources

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B2 simplify complex fractions containing rational numbers.

Elaboration - Instructional Strategies/Suggestions

B2 This outcome is intended to introduce students to simplifying complex fractions. Students are not expected to simplify complex fractions containing variables since this will be done in unit 4. However, some simple fraction problems involving variables may be included at this point. Students may use one of the following methods to simplify complex fractions:

Example: Simplify $\dfrac{\frac{-3}{2}-\frac{5}{7}}{\frac{1}{14}+\frac{-3}{28}}$

Method 1

$$\frac{\frac{-42}{28}-\frac{20}{28}}{\frac{2}{28}+\frac{-3}{28}}$$
$$=\frac{\frac{-62}{28}}{\frac{-1}{28}}$$
$$=\frac{-62}{28}\div\frac{-1}{28}$$
$$=\frac{-62}{28}\times\frac{28}{-1}$$
$$=62$$

When students are comfortable with the concept and procedure, as a short cut the denominators in the first step could be eliminated to simply get $\dfrac{-42-20}{2+-3}=\dfrac{-62}{-1}=62$

Method 2 Find the LCD of all fractions within the complex fraction and multiply it by both numerator and denominator of the complex fraction:

$$\frac{28\left(\frac{-3}{2}-\frac{5}{7}\right)}{28\left(\frac{1}{14}+\frac{-3}{28}\right)}$$
$$=\frac{28\left(\frac{-3}{2}\right)-28\left(\frac{5}{7}\right)}{28\left(\frac{1}{14}\right)+28\left(\frac{-3}{28}\right)}$$
$$=\frac{-42-20}{2+-3}$$
$$=62$$

Note: Students do not always see why both numerator and denominator must be multiplied by the same quantity or why the LCD is chosen. Point out that multiplying any number by 1, returns that same number.

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B2.1 Simplify each of the following complex fractions:

a) $\dfrac{\frac{2}{3}}{\frac{5}{3}}$ b) $\dfrac{\frac{-4}{5}}{\frac{3}{5}}$ c) $\dfrac{\frac{5}{4}}{\frac{5}{7}}$ d) $\dfrac{\frac{-7}{15}}{\frac{1}{3}}$ e) $\dfrac{1-\frac{1}{4}}{1+\frac{1}{4}}$

Pencil and Paper

B2.2 Simplify:

a) $\dfrac{1+\frac{4}{9}}{1-\frac{2}{3}}$ b) $\dfrac{\frac{8}{27}-8}{\frac{1}{3}+1}$ c) $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}{\frac{1}{-8}}$ d) $\dfrac{\frac{-3}{5}-\frac{4}{7}}{1-\frac{2}{14}}$

e) $\dfrac{1-\left(\frac{2}{3}\right)^2}{1-\left(\frac{1}{5}\right)^2}$ f) $\dfrac{-\frac{1}{2}+\left(\frac{-1}{2}\right)^3}{\left(\frac{3}{2}\right)^2-1}$

B2.3 The formula $s=\dfrac{d}{t}$ is used to find the average speed, *s*, of an object which has moved a distance *d* in time *t*. Find the average speed of a car which has moved $\dfrac{3}{5}km$ in $\dfrac{3}{400}hours$.

B2.4 If a = 4/5 and b = -2/3, evaluate:

a) $\dfrac{5a^2-9b}{25a+4b}$ b) $\dfrac{-10a^3-15b^2}{3a-2b}$ c) $\dfrac{a^2-b^2}{b^2-a^2}$

Answers

B2.1 a) 2/5 b) -4/3 c) 7/4 d) -7/5 e) 3/5

B2.2 a) 13/3 b) -52/9 c) - 8 d) -41/30 e) -125/234

Resources

Using Advanced Algebra, p.133-134
(The number of exercises is somewhat limited)

Mathematics 11 (National Edition), p.15, #9

Further work will be necessary

Access (Alg) Section AG, # 1-140

Access (Alg) Sec AI , # 1-160

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B3 add, subtract, multiply and divide decimal numbers mentally.

A4 develop place value and rounding concepts for decimal numbers.

Elaboration - Instructional Strategies/Suggestions

B3 Students may need a few minutes to review pencil and paper algorithms. For mental calculations, focus on compatible numbers that use simple arithmetic facts. The following are examples of questions involving decimals which students should be able to do mentally.

- 7.2 × 100 means we will need to move the decimal in 7.2 two places right to get 720.
- 0.12 ×0.12. (Think 12×12 = 144 and put in 4 decimal places to get 0.0144.)
- Since 72 ÷ 36 = 2, then 7.2 ÷ 3.6 = 2 and 72 ÷ 3.6 = 20 and 0.72 ÷ 3.6 = 0.2.
- 0.8 + 0.9 + 0.6
- 0.93-0.96
- 700-0.07
- $\frac{0.00024}{8}$

To divide when the divisor is not a whole number,

a) move the decimal point (multiply by 10, 100, and so on) to make the divisor a whole number;
b) move the decimal point (multiply the same way) in the dividend the same number of places; and
c) place the decimal point directly above the new decimal point in the dividend and divide as though dividing whole numbers.

To divide by a power of ten, such as 10, 100, or 1000, and so on,

a) count the number of zeros in the divisor, and
b) move the decimal point that number of places to the left.

To divide by a tenth, hundredth, thousandth, and so on,

a) count the number of decimal places in the divisor, and
b) move the decimal point that number of places to the right.

Students should understand why these procedures work.

A4 Students should be able to use these concepts to help them compare and order decimal numbers. Students should also be able to check whether a given calculation is reasonable even when the numbers are not compatible e.g. Explain, without actually doing the exact calculation, why this calculation cannot be correct: 3.6 ÷ 0.91 = 5.1. A student should be able to estimate that this is about the same as 3.6 ÷ 0.9 which equals 4.

Worthwhile Tasks for Instruction and/or Assessment

B3.1 Calculate each of the following mentally:

- -1.5 × 0.3
- 1.8 ÷ -0.06
- 0.532 ÷ 0.0001
- 0.11 × 0.11
- -24 ÷ (-300)
- 600 ÷ 0.06

B3.2 How many digits are to the left of the decimal for each of the following:

a) 1.2 ÷ 0.2
b) 0.602 ÷ 100
c) 70.09 × 0.001
d) -9 ÷ 0.03

B3.3 Explain whether it is possible for this statement to be correct:

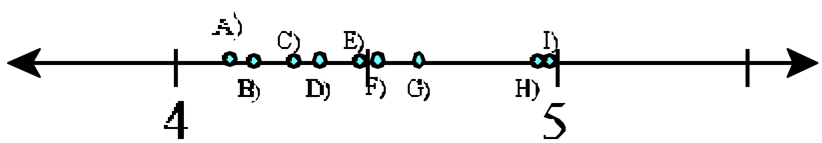
0.045 ÷ 0.09 = 5

A4.1 Round each number to the nearest integer, the nearest tenth, the nearest hundredth, and the nearest thousandth.

a) 5.79146
b) -2.65961
c) 4.12131415...
d) $7.\overline{15}$

A4.2 Match each number with the correct position on the given number line:

$4.\overline{89}$, 4.35, 4.9, 4.16, 4.375, 4.49, $4.6\overline{5}$, 4.515, 4.213154968913...



Resources

http://www.mathmax.com (Click on *Prealgebra 3/e*, then Chapter 5, then Chapter Review)

http://www.mathmax.com/basicmath/chapter/chep/ch4s2.pdf

http://www.mathmax.com/basicmath/chapter/chep/ch4s3n4.pdf

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

A5 convert numbers from:
- percentage to decimal
- decimal to precentage
- percentage to fraction
- fraction to percentage
- decimal to fraction
- fraction to decimal

Elaboration/Instructional Strategies

A5 This should be quick review for most students. When converting from fraction form to one of the other forms, students should be given opportunities to do so mentally using skills developed in outcome B2. For example, students should be able to convert 4/5 mentally by thinking of 4.0 ÷ 5 = 0.8 = 80% or 4/5 = 8/10 = 80%. While this skill should be practised until it becomes almost automatic, it is not intended that these skills be practised using a block of time but instead be used throughout the course. Students should also be able to convert repeating decimals to fractions. There are a number of ways to do this:

i) Method #1: = 0.454545454545...

Let x = 0.454545454545...
Multiply by a power of 10 large enough to bring out the first series of repeating digits. In this case, multiply by 100 to bring out the first repeating 45 (i.e. the first period):
100x = 45.4545454545...
x = 0.4545454545...
99x = 45 by subtracting the two lines. The repeating portion of each line is eliminated. Solving the resulting equation for x gives x = 45/99, which reduces to 5/11.

Of course, this procedure would need to be varied for other cases such as where there are digits that are not part of the repeating pattern. Notice that in the second line of this solution we multiply by a power of 10 that brings out the non-repeating portion of the decimal:

1000x = 345.4545454545...
10x = 3.4545454545...
990x = 342
x = 342/990
x = 19/55

ii) Method #2: This method involves writing the decimal as an infinite geometric series (this is a series such as 3+6+12+24+... in which each term is obtained by a constant amount, in this case 2. The constant amount is called the constant ratio because it can be found by taking a term and dividing it by the previous term), finding the constant ratio between successive terms and using the formula for the sum of an infinite geometric series, S. , where t_1 is the first term and r is the constant ratio between successive terms:

= 0.45 + 0.0045 + 0.000045 + 0.00000045 + .., so t_1 = 0.45 and r = 0.01

Therefore

To convert , rewrite it as , convert 0.3 to 3/10 and to 1/22, and then add the results.

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

A5.1 Convert each of the following decimals to fractions in lowest terms:

a) 0.3 b) 1.8 c) -0.25 d) 0.005

A5.2 Convert each percentage to a decimal:

a) 89% b) 12.08% c) 0.1%
d) 121% e) 99.98%

A5.3 Convert each percentage to a fraction in lowest terms:

a) 35% b) 12.5%

Pencil and Paper

A5.4 Convert each fraction to a decimal:

a) 2/5 b) 7/8 c) 8/9 d) -2/3

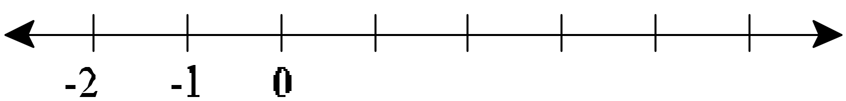
A5.5 Explain the procedure for converting:

a) fraction to a decimal
b) terminating decimal to a fraction
c) repeating decimal to a fraction
d) percentage to a decimal and vice versa
e) percentage to a fraction and vice versa

A5.6 Convert each repeating decimal to a fraction in reduced form:

a) $0.\overline{4}$ b) $0.2\overline{57}$ c) $3.\overline{17}$ d) $3.\overline{9}$

A5.7 Place all numbers in A5.1 - 5.4 and 5.6 on the following number line in the correct order from least to greatest:



Resources

Mathematics 10 (Revised Edition), p.16 #'s 5-6

http://www.mathmax.com Click on *Basic Math 8/e*, then Chapter 4, then Chapter Review or one of the Extra Practice selections. For work on Percentages, click on Chapter 6, Extra Practice, Sections 6.1-6.2

Math in Context 9, p.48
p.522 (sec. 5.14)

Mathpower 9, p.338-339

Access (Alg) Sec AK # 1-76 & sec AL # 1-100

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B4 solve simple equations involving percentages

B5 apply percentage increase and decrease in problem solving situations

Elaboration/Instructional Strategies

B4 Students should be able to work with percentages of the form a% of b = c, where students are given any two of a, b, or c, and asked to find the missing value. Work with students to develop number sense for percentage. The most fundamental thing to understand is that a percentage is a portion of some quantity. Students should also be able to use mental math skills to perform many calculations with percentages.

It will be important for students to realize that a percentage less than 100% of something will give a smaller quantity than the original quantity; however, a percentage more than 100% of that quantity will give a larger quantity. Teachers may need to review situations in which it makes sense to talk about more than 100% of something. For example,

- You have $2000 in your account now and $2200 next year. This means next year you will have 110% of what you have now (students should not confuse this with percentage *rate* of increase, which would be 10% in this case).

Students should also recognize that when a quantity decreases by a certain percentage then - 100% subtract the rate of change is the percent remaining. E.g. If an item priced at $55 is discounted by 20%, then the customer will have to pay 80% of $55.

B5 Students should be presented with situations involving repetitive calculations of percentage in order to see the effects in growth or decay situations. Students are to use mental math strategies when appropriate. They should plot some points on a graph to see the general curve obtained. They should also be able to calculate the rate of increase or decrease given a table of values or a graph and interpret these graphs to obtain other required information. For example,

- You start a job earning $300 per week in the first year with a guarantee of a 10% raise in each of three years and 5% per year for each year thereafter. Use a spreadsheet to determine your salary after 5 years, after 10 years, after 20 years. Graph the salary per week for the first week of each year and discuss the appearance of the graph.

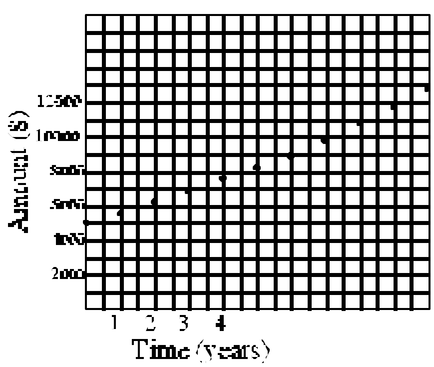
- You have a starting salary of $200 per week + 3% commission on all sales. Your pay check indicated that for the first week, you had a gross income of $297.50. What was your total sales for the week?

- You have a starting salary of $200 per week + 3% commission on all sales up to $5000. After $5000 your commission increases to 3.5%. What was the total sales in a week where your pay check indicated a gross income of $425.00?

Worthwhile Tasks for Instruction and/or Assessment

B4.1
a) What is 15% of $20?
b) Given 30% of b is 18, what is b?
c) Given a% of 12 is 48, what is a?
d) Sadie sees a new pair of shoes discounted by 35% off the original price. If the original price is $80, what percentage of the original price will she pay? If she has $60 to spend, will she have enough money to pay for the shoes including sales tax of 15%?
e) What is 20% of 30% of $50? By what one percentage could you multiply the $50 to get the same answer?

B5.1
a) If the HST (Harmonized Sales Tax) is 15%, by what number would you multiply the price of an item to get the final price including tax?
b) Pat has a new car that she just bought for approximately $25,000. The dealer tells her that this particular model depreciates by 20% per year on average.
   i. What is the dollar amount of depreciation in the first year? in the second year? in the third year?
   i. Why is the dollar amount of depreciation in the second year more than the dollar amount of depreciation in the third year?
   ii. Estimate how long (to the nearest year) it will take the car to depreciate to less than $7000. Show all relevant calculations.
c) Brian bought a new computer in 1995 for $1500. In 1996 it was worth $1050, in 1997 it was worth $735, in $514.50. What was the annual rate of depreciation? If he wanted to sell it in 2001, what should be his asking price?
d) Leo has a fat bank account that he began 8 years ago with a small investment. The growth of his money is shown by the graph.
   i) What is the approximate rate of increase?
   ii) How long did it take for his money to double?
   iii) Why does the graph appear to be curved instead of linear?



Resources

http://www.mathmax.com (Click on *Prealgebra 3/e*, then Chapter 6)

Math in Context 9, p.243 -
p.531-534 (sec. 7.16-8.11)

Mathpower 9, p.340-349

Access (alg) Sec FB #1-80, Sec FC #1-72, Sec FD #1-72, Sec FF #1-68

Extra practice may be necessary.

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B6 apply the order of operations with rational numbers, simple irrational numbers, and algebraic expressions

Elaboration/Instructional Strategies

B6 This should begin with the order of operations with integers and then moving quickly to fractions and radicals. Although the order of operations is certainly emphasized in the grade 7 to 9 curriculum, students often forget to apply them - particularly with algebraic expressions and irrational numbers. Students often have trouble with simple expressions such as 7 - 2(x + 5). In this case they may mistakenly first do 7 - 2 and then multiply the result by the terms in the brackets, or they may remember to "multiply the -2 by the brackets" but forget to use the distributive property. Similar problems occur in trigonometry when using a formula such as the Law of Cosines where students might face an expression such as

$$c^2 = 6^2 + 7^2 - 2(6)(7)\cos 60^\circ$$

Students may have little difficulty simplifying a fraction such as $\frac{5-1}{5-3}$ but will simplify an algebraic fraction such as $\frac{x-1}{x-3}$ inappropriately by first "cancelling the x's" to be left with either $\frac{-1}{-3}$ or $\frac{1-1}{1-3}$. This outcome is intended to address such skill deficiencies.

The following questioning techniques may help:

1) What is 7(4)?
   What is 7(3 + 1)?
   Which of the following would give the same result as 7(3 + 1)? Explain your choice.
   7(3) + 1 or 7(3) + 7(1)

2) Ask students to explain why $\frac{2+5}{2+12} = \frac{1}{2}$ and not $\frac{6}{13}$ or $\frac{5}{12}$. Ask them to think about the order operations and why this problem is equivalent to $\frac{x+5}{x+12}$. They must understand that although the brackets were not originally written, they are implied by the fraction format. After considering a number of similar problems, students should then make a connection to algebraic fractions and see that an expression such as $\frac{x+5}{x+12}$ cannot be simplified simply by "*cancelling x's*".

Students may need a brief review of writing mixed radicals when given an entire radical by extracting the largest perfect square factor e.g. $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

Teachers may also wish to have students perform a number of similar calculations using the distributive property instead of simplifying the brackets first. Exercises should increase in level of difficulty to include fractions, decimals, positive exponents, and irrational numbers e.g. Simplify $(\sqrt{2})^2 - 10(\frac{1}{5} - \frac{7}{10})$

Students should then tackle simplifying related algebra problems such as

$2(x + 1)^2 - 3(2x^2 - 5)$
$(2x - 3y)^2 - 4(x^2 - y^2) + 12xy$

From the intermediate grades, students may remember the acronym BEDMAS (Brackets, Exponents, Division and Multiplication in the order they occur, Addition and Subtraction in the order they occur).

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B6.1 What is $\sqrt{300}$ as a mixed radical in simplest form?

B6.2 Which of these mixed radicals can be simplified further? Explain.

A) $3\sqrt{10}$ B) $-2\sqrt{20}$ C) $4\sqrt{26}$

B6.3 Simplify $(5\sqrt{3})^2$

B6.4 Simplify -3x - (-2x - 1)

B6.5 Simplify $\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

B6.6 Simplify: -3x + 2(6x - 3)

Pencil and Paper

B6.7 Simplify: $\left(\frac{2}{3}\right)^2 - \frac{1}{2}\left(4 - \frac{2}{3}\right)^2$

B6.8 Simplify 2[5x - 3(x + 1)]

B6.9 Simplify 3x - [(2x - 6) - x]

B6.10 Simplify 6 - [4(x - 3) - (3 - x)]

B6.11 $3(x - 2)(x + 4)^2 - (2 - x)(x^2 + 3x + 2)$

Resources

Mathematics 10 (Revised Edition), p.37-38

Mathematics 11 (National Edition), p.69 - 72

Mathematics 11: Principles & Process, p. 122-125

http://www.mathmax.com/interalg/chapter/ch_ep/chrs6.pdf

Access (Alg)
Sec MB # 1-84, Sec MC # 1-76, Sec CA # 121-148, Sec EJ # 1-140, Sec MF #1-28, Sec MG 1-20 & 29-64