Unit II - Exponents
(15-20%)

Suggested Instruction and Assessment Time for this Unit:
7-10 classes(Assumes 55-60 minute classes)

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B7 apply the Laws of Exponents for both integer and rational exponents.

B8 relate rational exponents to radicals and use this relationship to simplify expressions

Elaboration - Instructional Strategies/Suggestions

B7 A quick review of the Laws of Exponents should be sufficient. These laws were studied at the grade 9 level. It is recommended that the Laws of Exponents be practiced with integer exponents first and then with rational exponents.

Students should be reminded that $b^0 = 1$, $b \neq 0$ and that $b^{-n} = \frac{1}{b^n}$ (they should realize that this means they will need to take the reciprocal of the base and change the sign of the exponent. Hence, it will be easier to rewrite a problem like $\left(\frac{2}{5}\right)^{-3}$ as $\left(\frac{5}{2}\right)^{3}$ instead of $\left(\frac{1}{2/5}\right)^{3}$).

Teachers may wish to discuss an example such as the following sequence to help lead students to an understanding of negative exponents using a pattern for a specific case.

..., 16, 8, 4, 2, 1, ½, ¼, **c**, ... = ..., $2^4, 2^3, 2^2, 2^1, 2^?, 2^?, 2^?$,...

Students should observe the following pattern: each successive term in the original sequence is divided by 2 to get the next term, the corresponding powers of 2 in the second sequence decrease by 1. Have them replace the question marks using this pattern. They should connect the above with $1/4 = 1/2^2 = 2^{-2}$

B8 Rational exponents are new for most students. Depending on their mathematical knowledge students may also need an introduction to radicals other than square roots to become familiar with the terminology:

$\sqrt[q]{b}$ where q is the index and b is the radicand. Students may need to see a couple of specific examples e.g. $\sqrt[4]{16} = 2$ because 2×2×2×2 = 16. It should also be pointed out that for square roots, the index is understood to be 2 by default and it normally is not written.

One way to approach the concept of rational exponents is to start with a few specific examples such as:

Since $\sqrt{5} \times \sqrt{5} = \sqrt{5^2} = 5$ and $5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5$, then we can deduce that $\sqrt{5} = 5^{\frac{1}{2}}$

Similarly, since $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = \sqrt[3]{5^3} = 5$ and $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5$, then $\sqrt[3]{5} = 5^{\frac{1}{3}}$.

For rational exponents with numerators other than 1, students may need to see an example such as $5^{\frac{2}{3}} = \left(5^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{5}\right)^2$ or $\sqrt[3]{5^2}$ before moving on to the more

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B7.1 Simplify: a) $(2^3)^2$ b) $(-2x^3y^2)^4$

B7.2 Calculate: a) 4^{-2} b) $\left(\frac{4}{5}\right)^3$ c) $\left(\frac{-3}{2}\right)^{-2}$

B8.1 Rewrite using positive rational exponents:

a) $\left(\sqrt[3]{9}\right)^2$ b) $\left(\sqrt{2a}\right)^5$

B8.2 Calculate: a) $\sqrt[3]{125}$ b) $(-8)^{\frac{2}{3}}$ c) $\left(\frac{4}{9}\right)^{\frac{-1}{2}}$

d) $3^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 3^{\frac{1}{6}}$ e) $\left(5^{\frac{1}{2}}\right)^3\times 5^{\frac{1}{2}}$ f) $\frac{3^{5.5}}{3^{7.5}}$

B8.3 Simplify:

a) $x^{\frac{2}{3}}\left(x^{\frac{1}{3}}-x^{\frac{-1}{3}}\right)$ b) $(x+y)^{\frac{-1}{2}}(x+y)^{\frac{3}{2}}$ c) $y^{\frac{4}{5}}\left(y^{\frac{6}{5}}+y^{\frac{-4}{5}}\right)$

Paper and Pencil

B7.3 Simplify:

a) $\left(5na^{-2}\right)^3\cdot\left(n^3a\right)^{-2}$ b) $\left(a^{-1}-b^{-1}\right)^{-1}$ c) $\left(a^{-2}+b^{-2}\right)^{-1}$

d) $\frac{x^5+x^{-2}}{x^{-3}}$

B8.4 Simplify: $\left(x^{\frac{3}{2}}+x^{\frac{-1}{2}}\right)^2$

B8.5 Simplify using positive rational exponents where necessary.

a) $\sqrt[5]{32x^4}$ b) $\left(\frac{\sqrt[3]{27}x^{-3}}{y^4}\right)^{\frac{1}{2}}$

B8.6 Simplify and write with positive rational exponents:

a) $\sqrt{x}\cdot\sqrt[3]{x}\cdot\sqrt[6]{x}$ b) $2x^{\frac{1}{3}}\left(x^{\frac{2}{3}}y^{\frac{-1}{3}}\right)^{-5}$ c) $\frac{3^{\frac{-1}{3}}-3^{\frac{2}{3}}}{3^{\frac{5}{3}}}$ d) $\frac{x^{\frac{-4}{3}}-x^{\frac{2}{3}}}{x^{\frac{-1}{3}}-x^{\frac{-4}{3}}}$

B8.7 Rewrite using powers of the same base and then simplify:

a) $\frac{27\cdot 3^5}{9^5}$ b) $6^{-100}\times 36^{59}\div\left(\sqrt[4]{216}\right)^{24}$

Resources

Advanced Mathematics, Brown, Richard and Robbins, David. Sections 5.1 and 5.2, p.147-151 (up to #24) and p.152-155 (up to #20)

Mathematics 10 (Revised Edition) p.48-60 and 67-71. In addition, p.18-20 provide a good review of radicals other than square roots.

Mathematics 10: Principles & Process p.21-23 and p.103-109

http://www.mathmax.com/ini_alg/chapter/bk5imipp/chapter10/bk5c10s2.html (This is an interactive math site)

Math in Context 9, p.24-29 (more low level questions) p.538 (sec. 9.10)

Using Advanced Algebra, p.68-71, p.176-181

Access (alg), BB 1-76, BC 1-100, BD 1-96, BE 1-64, BF 1-60, BG 1-196

Access (trig), BB 1-120, BG 1-80, BH 1-80, BI 1-140, BK 1-8

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B9 simplify expressions involving radicals

Elaboration - Instructional Strategies/Suggestions

B9 This work would have been practiced in relation to outcomes B7 and B8; however, it is also intended that students be able to multiply binomial radical expressions such as $(\sqrt{3}+1)(\sqrt{5}-2)$ and $\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)$. Students should know what a conjugate is and be able to multiply conjugates of simple irrational numbers mentally [e.g. $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=5-2=3$], whereas conjugates of more complex irrational numbers may require pencil and paper [e.g. $\left(2\sqrt{5}-3\sqrt{8}\right)\left(2\sqrt{5}+3\sqrt{8}\right)$]. Students should be able to simplify an expression such as $\left(3\sqrt{2}\right)^2$ mentally. These skills will then be carried over to rationalizing denominators when the denominator is either monomial or binomial in nature.

It is recommended that students work with more complex expressions involving radicals, making sure they properly apply the order of operations e.g. simplify: $(1-\sqrt{3})^2-(1-\sqrt{3}^2)+4\sqrt{3}(-2-3\sqrt{3})$

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B9.1 a) Simplify : $\left(\sqrt{6}-4\right)\left(\sqrt{6}+4\right)$

b) Rationalize the denominator: $\frac{2}{\sqrt{3}}$

c) Simplify: $\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)$

Paper and Pencil

B9.2 a) Multiply: $\left(5\sqrt{3}+3\sqrt{5}\right)\left(3\sqrt{3}+2\sqrt{5}\right)$

b) Multiply: $\left(\sqrt{x+h}-x\right)\left(\sqrt{x+h}+x\right)$

c) Rationalize the denominator of each of the following:

i) $\frac{2+\sqrt{2}}{2-\sqrt{2}}$ ii) $\frac{3\sqrt{2}}{2\sqrt{3}-5\sqrt{2}}$ iii) $\frac{(1+x)}{\left(1-\sqrt{x}\right)}$ iv) $\frac{x-1}{\sqrt{x+3}-2}$

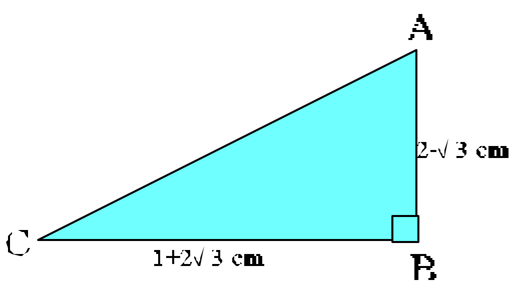
d) Simplify: $\left(5-\sqrt{2}\right)\left(5+\sqrt{2}\right)^2-3\sqrt{12}\left(\sqrt{3}+\sqrt{6}\right)$

B9.3 Simplify:

a) $\frac{5}{2\sqrt{3}}+\frac{4}{\sqrt{75}}$ b) $\frac{1}{\sqrt{3}-2}-\frac{2}{\sqrt{3}+2}$

B9.4 The sides of a square are $\sqrt{10}-2\sqrt{2}$ units long. Find:
a) the area in simplest radical form;
b) the length of a diagonal in simplest radical form.

B9.5 For the right triangle shown, find
a) the length of the hypotenuse in simplest form;



b) the ratio of the hypotenuse to side BC in simplest form;
c) the ratio of side AB to BC in simplest form.

Resources

Mathematics 10 (Revised Edition), p.37-45

Mathematics 10: Principles & Process, p.117-129

Mathematics 11 (National Edition), p.28 - 32

http://www.mathmax.com/introalg/chapter/bk3c9ep.html

Using Advanced Algebra, p.171-175
p.197 (Chapter Review)
p.199 (Chapter Test)

Access (alg) MF 1-28, MG 1-128, MD 1-68, MF 29-80, MH 1-88, MK 1-52, ML 1-76

Some extra practice may be necessary.