Unit III - Solving Polynomial Equations
(25%)

Suggested Instruction and Assessment Time for this Unit: 12 classes(Assumes 55-60 minute classes)

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B10 recognize and use the language of polynomials

C1 solve linear equations

Elaboration - Instructional Strategies/Suggestions

B10 Students should know the following terminology associated with polynomials: degree, constant term, leading coefficient, zeros, roots, x-intercepts, y-intercepts, constant polynomial, linear polynomial, quadratic polynomial, cubic polynomial, quartic polynomial.

They should also understand the x-coordinate of a y-intercept is always 0, and the y-coordinate of an x-intercept is always 0. Following from this, they should be able to explain why 0 is substituted for x determines the y-intercept and why 0 is substituted for y determines the x-intercepts.

C1 Students are expected to recognize the relationship between x-intercepts, roots, and zeros. While these are the same for any given polynomial, zeros are generally connected with the function, roots are generally connected with the equation, and x-intercepts are connected with the graphical form of the function. Students should be able to readily find the intercepts of linear equations.

Linear equations are solved by isolating the variable on one side of the equation and the constants on the other side. Teachers should review linear equations of the following types:

$ax + b = c$

$ax + b = cx + d$

$a(bx+c) = d$

$a(bx + c) = d(ex + f)$

Note that all cases above should include exercises in which one or more of a, b, c, d, e, and/or f is a fraction. Some answers should be rational numbers, with the requirement that they be expressed in exact form (e.g. 2/3, not 0.67).

Students should be able to check an answer by substituting in the original equation and evaluating each side of the equation independently to see if they “balance”.

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B10.1 Given the polynomial $4x^3 - 5x^2 - x^4 + 1$, identify:

a) the type of polynomial (constant, linear, quadratic, etc.)
b) the leading coefficient
c) the constant term
d) the linear term

C1.1 Find the root of each equation:

a) x - 5 = 10 b) 2x - 3 = 0 c) $y - \frac{1}{4} = \frac{1}{2}$

d) 3x + 5 = 2x - 2 e) 2(x - 1) = 10

B10.2 Find the zero of each linear function:

a) f(x) = 2x - 4 b) y = 5x - 3

B10.3 Explain why the x-coordinate of a y-intercept for any function is always 0.

B10.4 Explain why 0 must first be substituted for y in any polynomial function in order to find the x-intercept(s).

B10.5 Find the y-intercept and the x-intercept of each linear function:
a) y = 2x - 1 b) 3x + 2y = 6

B10.6 Find the *coordinates* of the y-intercept of $y = -2x^3 + 3x^2 - 4x + 5$

Pencil and Paper

C1.1 Solve: a) $\frac{2}{3}x - \frac{1}{2} = \frac{4}{5}x - \frac{9}{2}$ b) $\frac{-3}{4}(x+3) = \frac{2}{3}(x-2) - 8$

c) -2(5 + x) - (3 - 5x) = -½[4 - (-2 - 3x) + x]

Resources

Advanced Mathematics, Brown, Richard and Robbins, David. p.47-50

Mathematics 10 (Revised Edition), p.124-127 See also Extra Practice 10, question #1 in the Teacher's Resource of this text.

Math in Context 9, p.539-540 (sec. 10.2-10.4) B10 & B11

Access (alg) GA #1-240, GB 1-144, GC 1-192, GE 1-212, GF 1-200, GG 1-80

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

B11 identify, add, subtract, and multiply complex numbers

B12 verify complex numbers as solutions to polynomial equations.

Elaboration - Instructional Strategies/Suggestions

B11/B12

These outcomes are related and should be completed together. Students should recognize that $\sqrt{-1} = i$ (the imaginary unit) and that the square root of any negative number involves the use of the imaginary number e.g. $\sqrt{-4} = 2i$ and $\sqrt{-8} = 2i\sqrt{2}$ Likewise, they should also be able to distinguish between pure imaginary numbers of the form b*i* and complex numbers of the form a + b*i*. In addition, *they should recognize that any number, including real numbers and pure imaginary numbers, can be written in complex form* ex. 4 = 4 + 0*i* and -3*i* = 0 - 3*i*.

Students should be able to recognize quickly that adding or subtracting two complex numbers follows a process similar to adding or subtracting polynomial expressions. That is, the like parts are combined. Similarly, when multiplying two complex numbers such as (3 + 2*i*)(4 - 2*i*) the process is the same as multiplication of two binomial expressions. In addition, students should realize that the *simplified* result is another complex number, not a trinomial as when two linear polynomials are multiplied (the exception is when complex conjugates are multiplied, in which case the result is a real number).

Students will also be expected to simplify expressions such as $(i)^{13}$ and $(4i)^{4}$.

Point out to students that when multiplying the square roots of two negative numbers, these numbers must first be converted to imaginary numbers before multiplication can be performed e.g.

Correct method: $\sqrt{-4} \times \sqrt{-9} = 2i \times 3i = 6i^2 = -6$

Incorrect method: $\sqrt{-4} \times \sqrt{-9} = \sqrt{-4 \times -9} = \sqrt{36} = 6$

Students will not be asked to find complex roots of polynomial equations except in simple cases when the square root property for solving quadratic equations can be applied e.g.

$$x^2 + 9 = 0$$
$$x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm 3i$$

Some students enrolled in this course may have only completed Math 1204 and are concurrently enrolled in Mathematics 2204 or 2205. Hence they may have had no experience with the quadratic formula which would allow them to generate complex solutions. They will, however, be given complex number solutions to polynomial equations and asked to verify that they are solutions or asked to evaluate polynomial functions for complex values of the independent variable. If students are currently doing Mathematics 3204 and are familiar with the quadratic formula, teachers should then extend the study to include quadratic equations which are not factorable and require the use of the

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

B11.1 a) Evaluate: $\sqrt{-81}$ b) Simplify: $(i)^{10}$ c) Simplify: $(-2i)^6$

d) Multiply: $\sqrt{-5} \times \sqrt{-20}$ e) Multiply: $(5 - 2i)(5 + 2i)$

f) Multiply: $\left(1+\sqrt{-3}\right)\left(1-\sqrt{-3}\right)$ g) Simplify: $(5 + 2i) + (-9 - 5i)$

h) Simplify: $-(3 - 2i) - (5 - 5i)$

i) Express as a mixed radical in simplest form: $\sqrt{-72}$

j) How many terms are in the simplified form of:

$(50 + 101i)(251 + 72i)$?

k) Evaluate: $\sqrt{\frac{-9}{4}}$

B12.1 a) What are the roots of $4x^2 + 16 = 0$?

b) Which of these equations must have complex roots?

i) $2x^2 - 12 = 0$ ii) $x^2 + 3 = 0$ iii) $-x^2 = -16$

iv) $-3x^2 + 15 = 0$

Pencil and Paper

B11.2 a) Simplify: $\left(7-i\sqrt{3}\right)\left(4+2i\sqrt{3}\right)$ b) Simplify: (13 + 4i)(12 - 9i)

B12.2 a) Verify that $1 + 2i$ and $1 - 2i$ are roots of $x^2 - 2x + 5 = 0$.

b) Verify that $2 - 3i$, $2 + 3i$, and 2 are zeros of the function $f(x) = x^2 - 6x^2 + 21x - 26$.

c) Verify that $5 - 4i$, $5 + 4i$, $1 - 3i$, $1 + 3i$ are roots of the polynomial equation $x^4 - 12x^3 + 70x^2 - 172x + 369 = 0$.

d) From your work in parts a), b) and c), what conjecture can you make about complex roots of polynomial equations?

e) Test your conjecture in part d). In each of the following cases, a complex root is given for the equation. Find another possible complex root and verify that it is also a root:

i) $x^2 - 2x + 16 = 0$, one root is $1 + 4i$.

ii) $3x^2 - 24x + 60 = 0$, one root is $4 - 2i$.

iii) $x^3 + 5x^2 + 11x + 15 = 0$, one root is $-1 - 2i$.

B12.3 Given $f(x) = x^3 + x^2 - 2x + 1$, find $f(3 - 2i)$.

Resources

Using Advanced Algebra, p.182-185 p.190-195

Advanced Mathematics, Brown, Richard and Robbins, David. p.50, #14-18

Access (trig) Sec CA 1-76, CB 1-96, CC 1-156, CF 1-36, DC (only use those questions of the form $x^2 + a = 0$)

Further work may be necessary to help students achieve outcome B12

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

C2 factor polynomial expressions and use factoring to solve polynomial equations in one variable of degree 2 or higher

Elaboration - Instructional Strategies/Suggestions

C2 Students should at this point easily understand that a linear equation has only one root. They may find it useful to "build" higher degree equations by first multiplying some linear factors together. As they consider each factor, students should also consider the root of that factor. Therefore, when the multiplication process is complete, students should be able to look back at the factors they started with and recognize that they are connected to the roots of the polynomial that has been created. e.g.,

Given factors:	Associated Linear equations	Roots
x + 1	x + 1 = 0	x = -1
x - 3	x - 3 = 0	x = 3
2x + 1	2x + 1 = 0	x = -1/2

Find the product of these three factors and check to see if these roots are also roots of the polynomial equation produced. (Hopefully, this will also lead students to the conclusion that the degree of the equation determines the number of roots (i.e. a linear equation has one root, a quadratic has two, a cubic has three, and so on) provided that when a double root occurs, it is counted as two roots, a triple root counts as three roots, etc.

Important Note: Level II students doing this course would not have seen the quadratic formula. Therefore, they would not be expected to solve quadratic equations requiring its use. Students would be expected to find irrational or complex roots of polynomial equations, in special cases such as when the square root property can be applied as in outcomes B11/B12. However, if all students are doing Mathematics 3204 and have therefore worked with the quadratic formula, problems requiring the quadratic formula can be included.

Students should have experience with solving quadratic equations by factoring from Mathematics 1204 (see (i) below), and they would have studied factoring of the various types of quadratic polynomials in Grade 9. The difference here is that students will now move into the special cases for solving higher degree equations (see ii), iii), iv) and v) below).

i) Solve quadratic equations involving missing constant terms (i.e. common factors will be used), trinomials (both $x^2 + bx + c = 0$ and $ax^2 + bx + c = 0$), and differences of two perfect squares.

ii) Solve quartic equations which have a quadratic form (e.g. $x^4 - 10x^2 + 9 = 0$). It is expected that these types will be done at the same time as the quadratic equations in part i).

iii) Solve a polynomial with no constant term (i.e. a GCF must be removed first to begin solving).

iv) Solve cubic equations by grouping.

v) Solve polynomial equations using a combination of methods from cases i), ii), iii), and iv).

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

C2.1 What are the roots of $(x + 3)(3x - 1)(x + 2)^2 = 0$?

C2.2 What root is common to both of these equations? Explain why this root is common.
$3x^2 + 9x = 0$ and $4x^3 + 20x^2 + 16x = 0$

C2.3 What are the roots of $(x^2 - 1)(x^2 + 1) = 0$?

C2.4 Which cubic polynomial would have zeros 2i and 2?

a) $P(x) = (x^2 - 2i)(x - 2)$

b) $P(x) = (x^2 + 2i)(x + 2)$

c) $P(x) = (x^2 + 4)(x - 2)$

d) $P(x) = (x^2 -4)(x - 2)$

Pencil and Paper

C2.5 Find all roots, real or imaginary:
a) $x^4 - 16 = 0$.

b) $x^3 = -16x$.

C2.6 What is the complete factorization of the polynomial $18x^3 - 9x^2 - 2x + 1$?

a) $(2x - 1)(3x - 1)^2$
b) $(2x - 1)(3x + 1)(3x - 1)$
c) $(2x - 1)(9x^2 -1)$
d) $(2x + 1)(3x + 1)(3x - 1)$

Resources

Mathematics 12 (National Edition), p.64-67

Advanced Mathematics, Brown, Richard and Robbins, David. p.54-55

Mathematics 12: Principles & Process p.147, 157-159

Access (alg), sec JA 1-100, JB 1-212, JC 1-136, JD 1-80

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

C2 factor polynomial expressions and use factoring to solve polynomial equations in one variable of degree 2 or higher.

Elaboration - Instructional Strategies/Suggestions

Students should be aware why certain methods are inappropriate because they cause the loss of a root from an equation:

ex. Solve x(2x - 1) = 3(2x - 1)

Correct Methods

x(2x - 1) - 3(2x - 1) = 0

(2x - 1)(x - 3) = 0

x = ½, x = 3

OR

$2x^2 - x = 6x - 3$

$2x^2 - 7x + 3 = 0$

(2x - 1)(x - 3) = 0

x = ½, x = 3

Incorrect Method

$$\frac{x(2x-1)}{(2x-1)} = 3$$

x = 3

"Cancelling" the common factor reduces the degree of the original equation, and therefore the number of roots obtained.

Worthwhile Tasks for Instruction and/or Assessment

C2.7 Solve:

a) $2x^3 - x^2 + 2x - 1 = 0$

b) $10w^3 + 5x = 6w^2 + 3$

c) $2x^4 - 10x^2 + 8 = 0$

d) $2x^4 + 7x^2 = 15$

e) $x(x - 1) = 2(x - 1)$

f) $4x(2x -3) = 2(2x - 3)$

Resources

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

C3 solve polynomial equations of degree 3 or 4 using the Rational Roots Theorem to identify a factor and synthetic division to obtain and solve a depressed equation.

Elaboration - Instructional Strategies/Suggestions

Note: Long division may be used in place of Synthetic Division.

Teachers will have to be careful when assigning questions for this outcome as students must EITHER be able to factor all depressed equations in order to obtain rational or integer roots OR the depressed equations must be of the form $ax^2 + c = 0$ so that the square root property can be applied.

One common mistake while solving polynomial equations of the form $ax^n + k = 0$, where $n>2$, is that students try to apply procedures similar to the square root property for quadratic equations of the form $ax^2 + c = 0$ e.g. When solving $x^3 - 8 = 0$, they might proceed as follows:

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

Using the Rational Roots Theorem to identify 2 as a root and synthetic division to obtain the quadratic depressed equation will show that there must be two other roots (which will actually be complex!).

Many students do not realize that synthetic division can only be applied when using *linear divisors of the form x - a*.

Note: Teachers may wish to do background work on long division of polynomials before moving into synthetic division. This can be found in the text *Mathematics 10 (Revised Edition)*, p.117-121.

It is important for students to note that when dividing two polynomials, the ***remainder must be 0 before they can determine if the divisor is a factor of the polynomial.*** Students will often find a quotient and assume it can be used to form the depressed equation without checking this.

Students should be made aware of the Remainder Theorem which says that if "x - a" is a linear divisor of a polynomial function p(x), then p(a) is the remainder. Students may think this is not very useful since they can obtain depressed equations and check remainders at the same time by using synthetic division, but the Remainder Theorem is quicker for many equations such as

$x^3 - 1 = 0$, or $x^{100} + 1 = 0$

Students should also be able to check their division answers using the relationship:

Dividend = (Divisor)(Quotient) + Remainder. They may want to first

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

C3.1 a) What are the possible rational roots of $x^3 - 2x - 21 = 0$?

(b) Does $x + 2$ divide evenly into $2x^3 + x^2 + 3x + 7$? Why or why not?

c) Explain what is meant by a depressed equation.

d) How many roots (real and imaginary) must a polynomial of degree 100 have? Why?

Paper and Pencil

C3.2 The equation $x^3 + 3x^2 - x - 3 = 0$ has a root at $x = 1$. What are its other roots?

a) {-1, -3} c) {1, -3}
b) {-1, 3} d) {1, 3}

C3.3. What is the value of k if 2 is a zero of $f(x) = 2x^2 - kx - 4$?

a) -4 c) 0
b) -2 d) 2

C3.4 How many roots does the polynomial equation $8 - 4x^2 - 5x^4 = 0$ have?

C3.5 What is the remainder when $P(x) = 2x^3 + x^2 + x + 7$ is divided by $x + 2$?

C3.6 a) A polynomial equation has been partially solved using synthetic division as shown. What are the roots of the equation?

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

b) What is the dividend in a)?

C3.7 When a polynomial function p(x) is divided by $x - 2$, the quotient is $2x^2 - x + 3$, and the remainder is 5. Find p(x).

Resources

Mathematics 12: Principles & Process, p.149-150, 153, 157-159

Mathematics 12 (National Edition), p.72-78

Advanced Mathematics, Brown, Richard and Robbins, David. p.67-75

Access (alg), sec EI 1-132 (good review for either long division or synthetic division)

Access (trig), sec GA (all exercises), GB 1-124, *GH 5-9, 11, 21-32, 41-48, 55,56, 59, 61-64, 81-84, 107-112, 145-148, 153-156, 181-192 *this is a little disjointed to avoid depressed quadratics which cannot be factored*
Sec GI 1-108

Outcomes

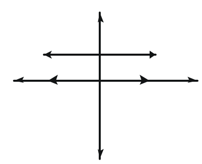
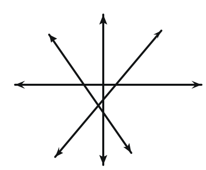
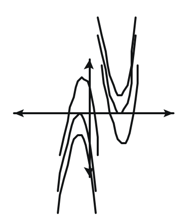
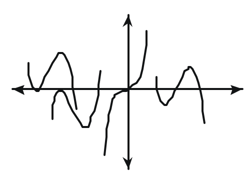
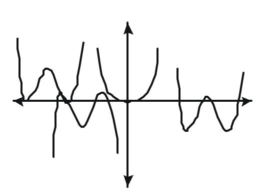
SCO: By the end of Mathematics 3103 students will be expected to:

C4 recognize basic graphs of polynomial functions to degree 4

C5 use graphs to obtain precise polynomial functions

Elaboration - Instructional Strategies/Suggestions

C4 Students should be able to see that the greatest number of x -intercepts of a polynomial graph is the same as the degree of the polynomial function; however, it is possible that the degree is greater than the number of x-intercepts (e.g. when there are points of tangency to the x-axis to indicate double roots or when there are complex roots that cannot be shown as x-intercepts). The more reliable method is to look at the number of "turning points", which is always one less than the degree of the function. Students could fill in a table similar to the following:

Type of Polynomial	Degree	Greatest Number of x-intercepts	# of Turning Points	Sketches
Constant	0		0 or infinitely many 0	
Linear	1	1	0	
Quadratic	2	2	1	
Cubic	3	3	2	
				

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

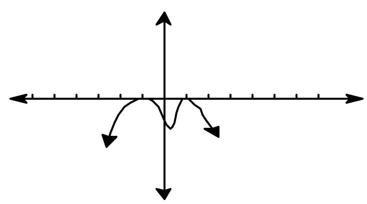
C4.1 What are the x-intercepts of the function $f(x) = -3(x - 1)(2x - 3)(x + 3)$?

C4.2 Which quadratic function has a graph with x-intercepts 5 and -1, and a y-intercept of 3?

$$y = \frac{-3}{5}(x-5)(x+1) \qquad y = \frac{5}{3}(x-5)(x+1)$$

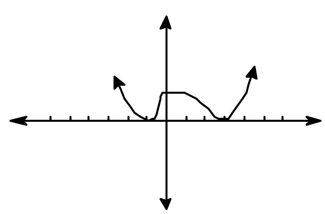
C4.3 What is the equation of the following graph?

(a) $y = -3(x - 1)(x + 1)$
(b) $y = -3(x - 1)^2(x + 1)^2$
(c) $y = 3(x - 1)(x + 1)$
(d) $y = 3(x - 1)^2(x + 1)^2$



C4.4 What is the function for the polynomial graphed below?

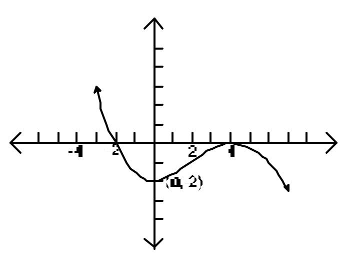
(a) $y = a(x - 1)^2(x + 3)^2$, $a<0$
(b) $y = a(x - 1)^2(x + 3)^2$, $a>0$
(c) $y = a(x + 1)^2(x - 3)^2$, $a<0$
(d) $y = a(x + 1)^2(x - 3)^2$, $a>0$



Paper and Pencil

C5.1 Find an equation of the cubic polynomial P(x) if P(-3) = P(-1) = P(2) = 0 and P(0) = 6.

C5.2 Find the function for the graph shown.



Resources

Mathematics 12 (National Edition), p.42-45

Advanced Mathematics, Brown, Richard and Robbins, David. p.62-67

Mathematics 12: Principles & Process, p.154-156

Outcomes

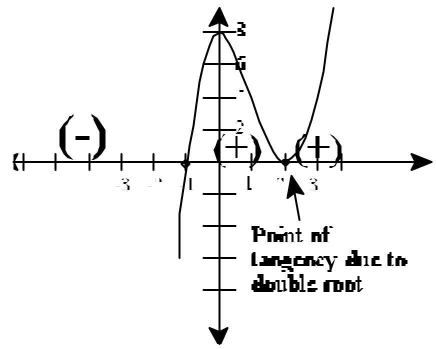
SCO: By the end of Mathematics 3103 students will be expected to:

C4 recognize basic graphs of polynomial functions to degree 4

C5 use graphs to obtain precise polynomial functions

Elaboration - Instructional Strategies/Suggestions

C4 Students should be able to sketch the graph of a polynomial function using the x-intercepts and one other point, recognizing that the graph automatically crosses the x-axis at the roots except at double roots (or roots that occur an even number of times) where points of tangency occur. Triple roots will cause a "flat" portion around that x-intercept. Constructing a sign graph first may help e.g.



Draw the graph of $y = 2(x+1)(x-2)^2$

Identify roots: x = -1

x = 2 (Double root)

Test point: (0, 8)

See sign graph on axes.

C5 Students should also be given graphs of polynomial functions (showing x-intercepts and one other point) or information about graphs and be expected to come up with the exact functions.

Ex. For a polynomial function p(x), p(1)= p(2)= p(-5) = 0, and p(4) = 6. Find p(x).

Solution: x = 1, 2, and -5 must be x-intercepts and therefore zeros of the function, so (x - 1), (x - 2) and (x + 5) must be factors. We can write the function as y = a(x - 1)(x - 2)(x + 5), where "a" is an unknown coefficient or "stretch factor" for the graph.

Since y = 6 when x = 4, we substitute these values for x and y, then solve to find a:

6 = a(4 - 1)(4 - 2)(4 + 5)

6 = a(54)

6/54 = a

1/9 = a

Writing the function using p(x) instead of y gives

p(x) = 1/9(x - 1)(x - 2)(x + 5)

Note: Many students will simply write p(x) = (x - 1)(x - 2)(x + 5) as the function without finding the stretch factor. Remind them that p(4) = 6 means that when x = 4, then y = 6. Have them do a brief mental calculation by substituting x = 4 into this function to show that it returns 54 as the result, then ask them, "By what number would you have to divide (or by what fraction would you have to multiply) in order to

Worthwhile Tasks for Instruction and/or Assessment

C4.5 Sketch the graphs of $f(x) = (x - 2)(x + 3)(x + 2)$ and $g(x) = -(x - 2)(x + 3)(x + 2)$. Show all x-intercepts, a test point, and a sign graph. Compare the two graphs and state any similarities and differences.

C4.6 Sketch the graph of $y = x^3 - 9x + 2x^2 - 18$.

C4.7 Sketch the graph of a cubic polynomial function $p(x)$ such that $p(-2) = 0$ and $p(x)>0$ only when $x > 3$.

C4.8 Use a graphing calculator to draw the graph of $p(x) = 12x^3 - 20x^2 - 23x - 5$. Copy the graph on your notebook and estimate the zeros of the function to one decimal place using the graph. Check your answers by substituting in the function.

Resources

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

C6 solve equations involving radicals and determine extraneous roots

Elaboration - Instructional Strategies/Suggestions

This will involve equations with radicals in one or two places e.g.

$\sqrt{x+1}=2$

$\sqrt{x+1}-\sqrt{x}=2$

Students should understand that the opposite (or inverse) operation for the square root is squaring, therefore to "cancel" the square root in the equation the operation of squaring will have to be involved. They will have to always isolate a square root on one side of the equation and then square both sides

e.g. to solve $\sqrt{x+1}-\sqrt{x}=2$, it will first have to be rewritten as

$\sqrt{x+1}=2+\sqrt{x}$

Note that the more "complicated" square root was isolated in order to make subsequent steps a little easier to handle.

Both sides will now have to be squared.

$\left(\sqrt{x+1}\right)^2=\left(2+\sqrt{x}\right)^2$

A common mistake here is that students do not square the right hand side as a binomial.

$x+1=4+4\sqrt{x}+x$

Since there is still a square root left, this will have to be isolated:

$-3=4\sqrt{x}$

$\frac{-3}{4}=\sqrt{x}$

$\left(\frac{-3}{4}\right)^2=\left(\sqrt{x}\right)^2$

$\frac{9}{16}=x$

Check the answer by substituting in the ORIGINAL equation to see if it is extraneous.

Note: Depending on the nature of the students, teachers may also wish to discuss restrictions on the replacement set for the problem e.g. $\sqrt{x+1}$ means $x \geq -1$ and $\sqrt{x}$ means $x \geq 0$. Therefore, any possible solutions must fit the restrictions; however, even those that do must still also be checked using substitution.

Worthwhile Tasks for Instruction and/or Assessment

Mental Math

C6.1 What is the solution of

a) $\sqrt{x} = 3$? b) $\sqrt{x-2} = 4$

C6.2 Solve the equation for x:

a) -1 c) 1

b) 0 d) no solution

Pencil and Paper

C6.3 Solve each of the following equations:

a) b)

c) d)

e) f)

Resources

http://www.mathmax.com Intermediate Algebra, Chapter 6, Section 6.6 Extra Practice

Mathematics 11 (National Edition), p. 33 - 37

Mathematics 12 (National Edition), p.108-110

Mathematics 11: Principles & Process, p.130-134

Mathematics 12: Principles & Process, p.72-73

Using Advanced Algebra, p.318-320

Access (alg), MI 1-220

Outcomes

SCO: By the end of Mathematics 3103 students will be expected to:

C6 solve equations involving radicals and determine extraneous roots

Elaboration - Instructional Strategies/Suggestions

Solve: $\sqrt{x} = 2$. This problem has the restriction that $x \geq 0$

$\left(\sqrt{x}\right)^2 = (2)^2$

$x = 4$ A check by substituting in the original equation gives $\sqrt{4} = 2$, so $x = 4$ is the solution.

Solve: $\sqrt{x} = -2$. Restriction is still $x \geq 0$

$\left(\sqrt{x}\right)^2 = (-2)^2$

$x = 4$ A check by substituting in the original equation gives $\sqrt{4} = -2$, so $x = 4$ is an extraneous solution.

Worthwhile Tasks for Instruction and/or Assessment

Resources