Unit 5
Probability
(10 - 15 %)

Probability

Outcomes

SCO: In this course, students will be expected to

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes

G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

Elaboration—Instructional Strategies/Suggestions

G2 Every day students experience a variety of situations. Some involve making decisions based on their previous knowledge of similar situations.

- Should they do their math homework tonight or during their spare period before math class tomorrow?
- Should they challenge a friend to a game of racquetball or blockers?
- Should they buy a ticket on a car raffle?
- Should they take their umbrella today?

Before making the decision, what they must consider is, "What is the chance of this decision working out in my favour?"

In probability, the goal is to assign numbers between 0 and 1 inclusive to events that interest us, but for which we do not know the outcome.

In their previous studies (grades 7–9) students have created and solved problems using probabilities, including the use of tree and area diagrams and simulations. They have compared theoretical and experimental probabilities of both single and complementary events and dependent and independent events. They have examined how to calculate complementary events as well as two independent events, A and B. They have determined how to calculate the probability of A and B as $P(A) \times P(B)$.

Sometimes the task of listing and counting all the outcomes in a given situation is unrealistic, since the sample space may contain hundreds or thousands of outcomes.

G2/G3 The fundamental counting principle enables students to find the number of outcomes without listing and counting each one. For independent events, if the number of ways of choosing event A is $n(A)$ and the number of ways of choosing event B is $n(B)$, then $n(A \text{ and } B) = n(A) \times n(B)$, and

$$n(A \text{ or } B) = n(A) + n(B).$$

The first is the multiplication principle, the second, the addition principle.

Sometimes events are not independent. For example, suppose a box contains three red marbles and two blue marbles, all the same size. A marble is drawn at random. The probability that it is red is $\frac{3}{5}$. If the marble is then replaced, the probability of picking a red marble again is $\frac{3}{5}$. However, if it is not replaced, then when another marble is picked the probability of its being red is now $\frac{2}{4}$. The second selection of a marble is dependent on the first selection not being returned to the box.

continued ...

Probability

Worthwhile Tasks for Instruction and/or Assessment

Activity (G2/G3)

1) Two students are playing “grab” with a deck of special “grab” cards. One student has a triangular-shaped deck with 16 ones, 12 twos, 8 threes, and 4 fours. The other has a rectangular shaped deck with 10 each of ones, twos, threes, and fours. The decks are well shuffled and each student plays the top card simultaneously. A “grab” is made when two cards match (a double).
   a) There are 40 cards in each deck. What is the total number of pairs of cards that could be played?
   b) How many of these are “double ones,” that is, a one from the triangular deck and a one from the rectangular deck?
   c) How many are i) double twos? ii) double threes? iii) double fours?
   d) For equally likely outcomes, the probability of an event is “the number of outcomes that correspond to the event” divided by what?
   e) So, the probability of a double one is “what” divided by “the total number of pairs”?
   f) Use this principle and your answers to (c) to find the probability of i) a double one ii) a double two iii) any double.
   g) A circular deck has 10 ones, 20 twos, 10 threes, and no fours. Calculate the probability of a grab if a triangular deck is played against a circular deck.

Performance

2) Telephone numbers are often used as random number generators. Assume that a computer randomly generates the last digit of a telephone number. What is the probability that the number is
   a) an 8 or 9?
   b) odd or under 4?
   c) odd or greater than 2?

3) A airplane holds 176 passengers: 35 seats are reserved for business class, including 15 aisle seats; 40 of the remaining seats are aisle seats. If a passenger arrives late and is randomly assigned a seat, find the probability of that person getting an aisle seat or one in the business section.

4) Use the given table, which represents the number of people who died from accidents by age group to find the following: [in each case assume that one person is selected at random from this group]

Age	Number
0-4	[illegible]
5-14	4,229
15-24	19,975
25-44	27,201
45-64	14,738
65-74	8,459
75 and over	19,200

   a) the probability of selecting someone under 5 or over 74
   b) the probability of selecting someone between 16 and 64
   c) the probability of selecting someone under 45 or between 25 and 74

Suggested Resources

Flewelling, Gary et at., *Mathematics 10 A Search for Meaning*. Toronto: Gage 1987.

Probability

Outcomes

SCO: In this course, students will be expected to

G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

Elaboration—Instructional Strategies/Suggestions

continued ...

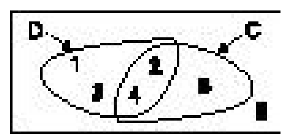
G3 How is the fundamental counting principle related to probability? Consider the marble situation described at the bottom of the previous page. The probability of selecting red is $P(c)=\frac{3}{5}$, while the probability of selecting blue is $P(b)=\frac{2}{4}$ (without replacement). The probability of selecting a red and a blue without replacement would be $P(r \text{ and } b)=\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}$.

Now, let us consider another situation:

Consider the experiment of a single toss of a standard die. There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6. Define certain events as follows:

A: observe a 2
B: observe a 6
C: observe an even number
D: observe a number less than 5.

$P(A)=\frac{1}{6}$ (observe a 2), $P(B)=\frac{1}{6}$ (observe a 6). What about $P(A \text{ or } B)$ (observe a 2 or 6)? This can be shown two ways: $\frac{n(A)+n(B)}{\text{total number of ways}}=\frac{1+1}{6}=\frac{2}{6}$ or $P(A \text{ or } B)=P(A)+P(B)=\frac{1}{6}+\frac{1}{6}=\frac{2}{6}$. Will this be true for any two events? The events "observe a 2", and "observe a 6" are called mutually exclusive events, or disjoint, because one can observe only a 2 or a 6, not both at the same time. On the other hand, events C and D above have at two elements in common and therefore are not mutually exclusive.



Consider the events C and D. The event (C or D) includes all the outcomes in C or D or both.

That is, P(C or D) = P(observe an even number or a number less than five)

= P (observe 2, 4, 6, or observe 1, 2, 3, 4)

Every outcome except 5 is included in (C or D). Thus there are exactly five favourable outcomes. Thus $P(C \text{ or } D)=\frac{5}{6}$

But $P(C)+P(D)=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$, which cannot be possible since it exceeds 1.

The outcomes 2 and 4 are contained in both C and D and must be removed. There is an overlap.

$$P(C \text{ or } D)=P(C)+P(D)-P(C \text{ and } D)=\frac{3}{6}+\frac{4}{6}-\frac{2}{6}=\frac{5}{6}$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G3)

1) Discuss whether the following pairs of events are mutually exclusive and whether they are independent.
   a) The weather is fine; I walk to work.
   b) I cut a deck of cards and have a Queen; you cut a 5.
   c) I cut the deck and have a red card; you cut a card with an odd number.
   d) I select a voter who registered Liberal; you select a voter who is registered Tory.
   e) I found a value for x to be greater than -2; you found x to have a value greater than 3.
   f) I selected two cards from the deck, the first was a face-card, the second was red.

2) If 366 different possible birthdays are each written on a different slip of paper and put in a hat and mixed,
   a) find the probability of making one selection and getting a birthday in April or October
   b) find the probability of making one selection that is the first day of a month or a July date

3) A store owner has three student part-time employees who work independently of each other. The store cannot open if all three are absent at the same time.
   a) If each of them averages an absenteeism rate of 5%, find the probability that the store cannot open on a particular day.
   b) If the absenteeism rates are 2.5%, 3%, and 6% respectively for three different employees, find the probability that the store cannot open on a particular day.
   c) Should the owner be concerned about opening in either situation a) or b)? Explain.

4) There are 6 defective bolts in a bin of 80 bolts. The entire bin is approved for shipping if no defects show up when 3 are randomly selected.
   a) Find the probability of approval if the selected bolts are replaced, are not replaced.
   b) Compare the results. Which procedure is more likely to reveal a defective bolt? Which procedure do you think is better? Explain.

5) Mary randomly selects a card from an ordinary deck of 52 playing cards. What is the probability that Mary will select either an ace or a diamond? Below is Fred's solution. Explain what Fred is thinking. Will his attempt lead to a correct answer? Explain.

$$P(\text{ace or diamond}) = \frac{4+13}{52} = \frac{17}{52}$$

Journal

6) Consider the table of experimental results.

	Seldane	Placebo	Control	Total
Drowsiness	70	54	[illegible]	237
No drowsiness	711	[illegible]	[illegible]	[illegible]
	781	665	[illegible]	2072

   Comment on the following solution attempts.

   a) If one of the 2072 subjects is randomly selected, the probability of getting someone who took Seldane or a placebo is

$$\frac{781}{2072} + \frac{665}{2072} = \frac{1446}{4144} = 0.3489$$

   b) If one of the 2072 subjects is randomly selected, the probability of getting someone who took Seldane or experienced drowsiness can be found by:

$$\frac{781}{2072} + \frac{237}{2072} = \frac{1018}{2072} = 0.491$$

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

G4 apply area and tree diagrams to interpret and determine probabilities

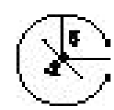
Elaboration—Instructional Strategies/Suggestions

G4 Students have studied area and tree diagrams since grade 7 and have applied them to help establish the sample space, or the total number of possible outcomes in a situation. In this course their experiences with these diagrams will be extended to probability tree diagrams and area diagrams that will help students visualize and calculate the probabilities given certain situations.

Consider the following situation. Students at Yore High School have two choices for where to eat lunch, in the cafeteria or elsewhere outside the school. Mildred, the manager of the cafeteria, needs to be able to predict how many students can be expected to eat in the cafeteria over the long run. Mildred asks the math class to conduct a survey. The results show that if a student eats in the cafeteria on a given day, the probability that he or she will eat there the next day is 72%. If a student does not eat in the cafeteria on a given day, the probability that he or she will eat in the cafeteria the next day is 38%. On Monday, 80% of the students ate in the cafeteria. What can Mildred expect for Tuesday?

A good way to organize all these statistics is with a probability tree diagram:

Geometric or area models will be useful to some students as these models provide a pictorial representation of the analysis which provides the students with a visual insight into the concept of probability. Consider the following situation. One of the events at your school's spring fair is a game of chance involving points. For each turn, a player spins and gets the points indicated in the area in that the spinner lands. Each player should add the numbers obtained by spinning twice. What are all the possible sums? What are the probabilities for obtaining each of these sums?



Students will notice that the spinner suggests that –2 will happen three-quarters of the time, while 5 will occur one quarter of the time. Using a grid of 16 squares to represent the probability of 1, they would draw a vertical line (as in fig. 1) to represent the probabilities for the first spin (1/4 and 3/4). They would then separate the grid horizontally (as in fig. 2) to represent the probabilities of getting a –2 or 5 on the second spin. They would then analyse the grid to find the probabilities of obtaining the sums –4, 3, and 10.

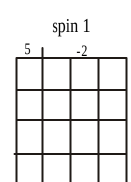


figure 1

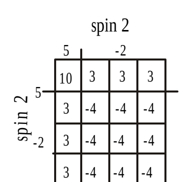


figure 2

$P(-4)=\frac{9}{16}$, $P(3)=\left(\frac{3}{16}\right)\times 2$, and $P(10)=\frac{1}{16}$.

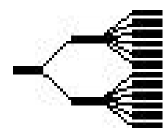
Now, using these results the students can be asked to create a situation where a player must accomplish something in order to win the game.

Probability

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper (G4)

1) This incomplete tree diagram lists all the outcomes of tossing a coin and then rolling a die.



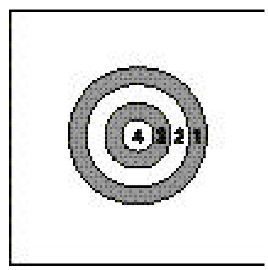
a) Copy and complete the diagram.
b) How many pairs of outcomes are there in this multiple event?
c) What is the probability of tossing a head on the coin and then rolling a six on the die?
d) What is the probability of tossing a head on the coin and then rolling an even number?
e) What is the probability of *not* tossing a head on the coin and then rolling an even number of the die?

2) In a restaurant there are four kinds of soup, 12 entrees, six desserts, and three drinks. How many different four-course meals can a patron choose from? If 4 of the 12 entrees are chicken and two of the desserts involve cherries, what is the probability that someone will order wonton soup, a chicken dinner, a cherry dessert, and milk?

3) Licence plates for cars often have three letters of the alphabet, then three digits from 0 to 9. How many possible different licence plates can be produced? What is the probability of having the plate "CAR 000"?

Performance (G5)

4) The dart board at the right consists of four concentric circles whose centre is the centre of the square board. The side length of the square is 36 cm. The circles have radii 2 cm, 4 cm, 6 cm, and 8 cm respectively. A dart hitting the bull's eye or one of the shaded rings scores the indicated number of points. A hit anywhere else on the board scores 0 points. Assume that a dart thrown at random hits the board. Determine the probability of scoring:
i) 4 points ii) 3 points iii) 2 points iv) 1 point v) 0 points



5) The following problem illustrates the usefulness of geometric probability. A tape recording is made of a meeting between a senator and her aide. Their conversation starts at the 21st minute on a 60-minute tape and lasts 8 minutes. While playing back the tape the aide accidentally erases 15 minutes of the tape.
a) What is the probability that the entire conversation was erased?
b) What is the probability that some part of the conversation was erased?
c) Suppose the exact portion of the conversation on the tape is not known, except that it began sometime after the 21st minute. What is the probability that the entire conversation was erased?

6) Consider finding the area of the region bounded by the ellipse $4x^2 + y^2 = 4$. Enclose the ellipse in a rectangle whose sides pass through the x- and y-intercepts, and then consider the rectangular region to be a dart board. Suppose several darts thrown at random hit the rectangular region.
a) Explain how probability can be used to approximate the area of the region bounded by the ellipse.
b) Explain how probability can be used to approximate the area of the region bounded by the equation $y = -x^2 + 4$.

Suggested Resources

Probability

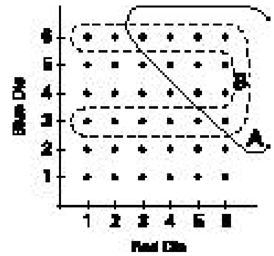
Outcomes

SCO: In this course, students will be expected to

G5(Adv) determine conditional probabilities

Elaboration—Instructional Strategies/Suggestions

G5(Adv) Ask the students a question such as What is the probability that event A occurs if it is known that event B has occurred? You should, through specific examples and some discussion, be able to get the class to arrive at a definition for conditional probability. For example:



If two dice, one red and one blue, are thrown and it is known that the blue die shows a number divisible by three, ask students what the probability is that the total on both dice is greater than 8? The condition that the number on the first die be divisible by three changes the sample space under consideration.

In particular, the new sample space contains only the 12 points shown inside the dashed closed curve at the right. In light of the fact that all 36 points in the original sample space were assumed to be equally likely, students should agree that it seems reasonable to say that all 12 points in this sample space are equally likely. For how many of these points would the total be greater than 8? Given the condition that the number on the blue die is divisible by three, students should calculate the probability of having a total greater than 8 is equal to $\frac{5}{12}$.

For any two events A and B, the symbol " $P(A|B)$ " is used to designate the probability that event A occurs given that event B has occurred. This is called a conditional probability because the condition is given that event B has occurred.

To evaluate $P(A|B)$ reconsider the above problem. Let the original sample space be the set of 36 possible outcomes shown in the diagram, let A be the set of points for which the total number of spots showing is greater than 8, and let B be the set of points for which the number of spots showing on the first die is divisible by three. Then $A \cap B$, pronounced '**A** intersect B' consists of the 5 points indicated in the diagram by the triangular shape. In this case, to determine the conditional probability P(A*B), divide the number of points in $A \cap B$ by the number of points in B. Of course, if the points of the original sample space were not equally likely, the result could not be obtained by simply counting points. Therefore, the probability of event A given that event B has occurred is defined as the probability of $A \cap B$ divided by the probability of B.

The probability that event A occurs if it is known that event B has already occurred is known as "conditional probability." It is symbolized as $P(A|B)$, and calculated using

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{5}{12}$$

If the first of three tosses of a fair coin is heads, find the probability of getting exactly two heads in three tosses.

Solution: Let A be the event "getting exactly two heads."

Let *E* be the event "getting a head on the first throw."

The event $(A \cap E) = \{HHT, HTH\}$

$$\text{so}, P(A \cap E) = \frac{2}{8} = \frac{1}{4}, P(E) = \frac{1}{2}$$

$$\therefore P(A|E) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G5(Adv))

1) What is the probability of getting two fives when two dice are thrown and it is known that at least one landed with a five up?
2) Assuming the probability of being born male is 0.5. In a family of three children it is known that at least one child is male. What is the probability that all three children are male?
3) A weather report indicates an 80% probability of rain on Monday, 60% on Tuesday, and 20% on Wednesday. What is the probability that it will rain on at least one of the three days?
4) In the MAKE-A-NUMBER game, you draw a Condition Card. Then you draw two Number Cards from a stack of only five cards and place them side-by-side to make a two-digit number. If the two-digit number fits your Condition Card, you score one point.

1. Condition

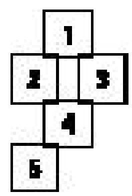
The number is divisible by 3.

Probability: ________

2. Condition

The sum of the digits of the number is 5.

Probability: ________



3. Condition

The number is greater thatn 40.

Probability: ________

4. Condition

The number is a prime number.

Probability: ________

5. Condition

The tens digit of the number is greater than the ones.

Probability: ________

6. Condition

The units digit of the number is divisible by the tens digit

Probability: ________

Determine the probability of scoring with these Condition Cards.

Suggested Resources

Shulte, Albert P., ed. *Teaching Statistics and Probability. 1981 Yearbook*. Reston, VA: NCTM, 1981.

Probability

Outcomes

SCO: In this course, students will be expected to

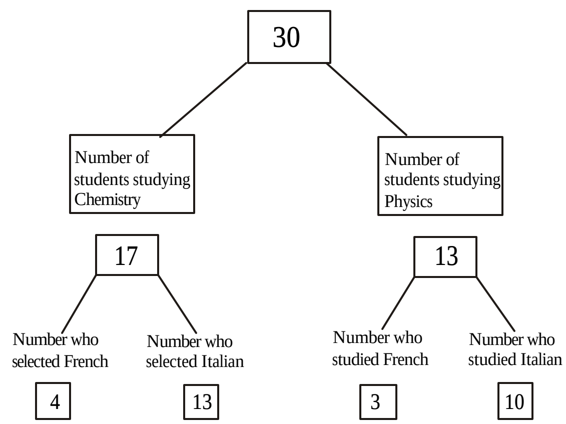
G5(Adv) determine conditional probabilities

Elaboration—Instructional Strategies/Suggestions

G5(Adv) Tree diagrams are often used to organize all the possible combined outcomes of a multiple event. Each student in a class of 30 students studies French or Italian and one Science, Physics or Chemistry. Their choices are shown in the table.

	Chemistry	Physics
French	4	3
Italian	13	10

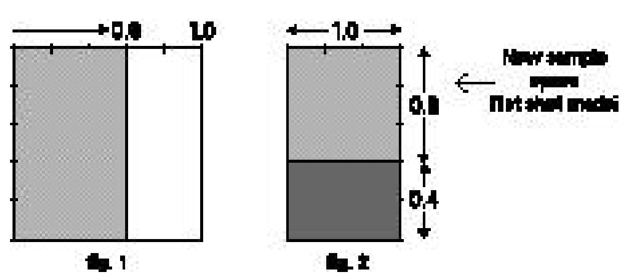
If a student is selected at random from the class, what is the probability that the student studies French given that the student studies chemistry.



Let F represent event "student studied French". Let C represent event "student studied Chemistry".

$$P\left(F/C\right) = \frac{P\left(F \cap C\right)}{P\left(c\right)} = \frac{4}{17}$$

An area diagram example: Suppose that Tom is a 60% free throw shooter in basketball. At the end of a game he was fouled and his team is losing by two points. He will shoot "one-and-one." What is the probability that he misses the second shot? To solve this problem, students could use an area model like that on the right. The probability of making the first shot is shown in fig. 1, then if he makes the first shot, he gets the second shot. Fig. 2 shows the probability of missing the second shot.

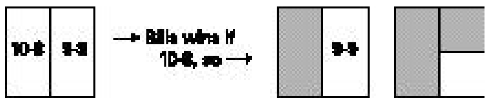


Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G5(Adv))

1) Two gamblers play a game for a stake that goes to the first player to gain 10 points. If the game is stopped when the score is 9 to 8, in favour of Bill, what is the probability that Bill will win when the game is resumed? Use an area model to help. (It is assumed that both players have equal chances of winning each point.)
   If the score is 9–8 then the next score will be ...



   If the game goes to 9–9, either one might win.
   a) What can you conclude from this?
   b) What would be the solution to the problem if Bill was winning 9–7 when the game is stopped?

2) As archers, Rita hits the target $\frac{2}{5}$ of the time and David $\frac{1}{3}$, of the time. They are going to have a contest with David shooting first. They alternate shots until one wins by hitting the target. Who is favoured? What is each contestant's probability of winning?
3) A certain restaurant offers select-your-own desserts. That is, a person may select one item from each of the categories listed:
   a) Using a tree diagram, list all possible desserts that can be ordered.

Ice Cream	Sauce	Extras
vanilla	chocolate	cherries
strawberry	caramel	peanuts
chocolate mint		

   b) Would you expect the choices of a dessert to be equally likely for most customers?
   c) If the probability of selecting chocolate ice cream is 40%, and vanilla is 10%, chocolate sauce is 70%, and cherries 20%, describe the dessert with the highest probability of being selected.

4) A certain model of automobile can be ordered with one of three engine sizes, with or without air conditioning, and with automatic or manual transmission.
   a) Show, by means of a tree diagram, all the possible ways this model car can be ordered.
   b) Suppose you want the car with the smallest engine, air conditioning, and manual transmission. A General American agency tells you there is only one of the cars on hand. What is the probability that it has the features you want, if you assume the outcomes to be equally likely?

5) Jennifer dresses in a skirt and a blouse by choosing one item from each category.

Skirts	Blouses
tan plaid gray	white pink 1 pink 2
stripe 1 stripe 2	red
stripe 3	

   a) Show, by means of a tree diagram, all the outfits she can make if one has three striped skirts and two pink blouses and only one of everything else.
   b) What is the probability of her wearing something striped and white knowing that she already has a striped skirt on?

Suggested Resources

Newan, Claire et al. *Exploring Probability. Quantitative Literacy Series.* White Plains, NY: Dale Seymour Publications, 1987.

Probability

Outcomes

SCO: In this course, students will be expected to

G1 develop and apply simulations to solve problems

Elaboration—Instructional Strategies/Suggestions

G1 Simulation is a procedure developed for answering questions about real problems by running experiments that closely resemble the real situation.

Suppose the students want to find the probability that a three-child family contains exactly one girl. If students cannot compute the theoretical answer and do not have the time to locate three-child families for observation, the best plan might be to simulate the outcomes for three-child families. One way to accomplish this is to toss coins to represent the three births. A head could represent the birth of a girl. Then, observing exactly one head in a toss of three coins would be similar, in terms of probability, to observing exactly one girl in a three-child family. Students could easily toss the three coins many times to estimate the probability of seeing exactly one head. The result gives them an estimate of the probability of seeing exactly one girl in a three-child family. This is a simple problem to simulate, but the idea is very useful in complex problems for which theoretical probabilities may be nearly impossible to obtain.

Students need experience thinking through complete simulation processes. When choosing a simple device to model the key components in the problem they have to be careful to choose a model that generates outcomes with probabilities to match those of the real situation. Students could use devices such as coins, dice, spinners, objects in a bag, and random numbers.

Students need to understand that the experimental probability approaches the theoretical probability as the number of trials increases. They should also realize that knowing the probability of an event gives them no predicting power as to what the outcome of the next trial will be. However, after enough trials, they should be able to predict with some confidence what the overall results will be.

When conducting simulations students should follow a certain process such as the one outlined: (see next page for an actual class activity).

Step 1: State the problem clearly.

Step 2: Define the key components.

Step 3: State the underlying assumptions.

Step 4: Select a model to generate the outcomes for a key component.

Step 5: Define and conduct a trial.

Step 6: Record the observation of interest.

Step 7: Repeat steps 5 and 6 until 50 trials are reached.

Step 8: Summarize the information and draw conclusions.

Probability

Worthwhile Tasks for Instruction and/or Assessment

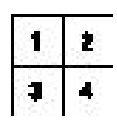
Pencil and Paper (G1)

1) Consider the following problem:
   Marie has not studied for her history exam. She knows none of the answers on the seven-question true-and-false section of the test. She decides to guess at all seven. Estimate the probability that Marie will guess the correct answers to four or more of the seven questions.
   Ask students to complete the following:
   a) What are you being asked to do?
   b) To perform a simulation, what assumptions should you make?
   c) Describe the model you would choose to perform the simulation.
   d) Pretend that you are watching the simulation. Describe what you observe for the entire simulation.
   e) What conclusion do you think would be made?

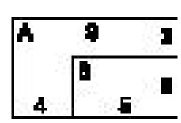
2) Suppose a stick or a piece of raw spaghetti has been broken at two random points. What is the probability that the three pieces will form a triangle? (The pieces must touch end to end.)
   a) Describe the process that might be used to estimate the answer using experimental probability.
   b) Instead, Robert is going to use a simulation. He assumes the spaghetti is 100 units long, and he is going to generate two random numbers between 0 and 100 using each as a side of a triangle. How would Robert find the third side? How would Robert check to see if the numbers represent the lengths of the side of a triangle?
   c) Perform this simulation to find the answer.

Performance (G1)

3) Dale, a parachutist, jumps from an airplane and lands in a field. What are the chances that Dale will land in a particular numbered plot? Make a field grid using a normal sheet of graph paper divided into four equal areas.



   a) Model the situation by tossing a thumbtack onto the grid from a metre or more away. (If the tack bounces off the sheet—do not count it as a toss.)
      In your response consider several questions:
      Is there an equal chance to land in each plot?
      How many times did Dale land in plot 1?
      Discuss the experimental probability results versus the theoretical probability results for the given field.
   b) Conduct the experiment again, but use a field divided into plots A and B to find the probability that Dale will land in Plot A.
   c) Perform a simulation to answer the same problem as in a). Compare the results of the simulation with that of the theoretical. Comment.



4) Perform simulations to solve the following problems:
   a) What is the probability that all five children in a family will be girls?
   b) A couple leaves for work anytime between 7:00 and 8:00 am. Their newspaper arrives any time between 6:30 and 7:30 am. What is the probability that they get the paper before they leave for work?

Suggested Resources

Zawojewski, Judith. *Dealing With Data and Chance*. *Curriculum and Evaluation Standards for School Mathematics Addenda Series. Grades 5–8*. Reston, VA: NCTM, 1992.

Probability

Outcomes

SCO: In this course, students will be expected to

G7 distinguish between situations that involve permutations and combinations

Elaboration—Instructional Strategies/Suggestions

G7 Before describing different situations in terms of permutations and combinations, students need to have an opportunity to solve simple counting problems (see elaboration for G2, p. 142). They may wish to organize their work into systematic lists and/or tree diagrams. As the number of choices increases, they will see the need for a way to count more efficiently. For example:

a) How many different routes can you take from Sydney to Halifax through Antigonish?
b) How many routes are there from Antigonish to either Halifax or Sydney?

Following this, the class might be split into two groups—one will do Problem A, the other Problem B. Students should present their solution to the class.

Problem A: Suppose there were three people, Adam, Marie, and Brian, standing in line at a banking machine. In how many different ways could they order themselves?

Problem B: The executive of the student council has five members. In how many ways can a committee of three people be formed?

Solutions might look like:

Problem A: using a systematic list: A M B, A B M, M B A, M A B, B A M, B M A.

Problem B: using a systematic list : if Adam, Marie, and Brian along with Dennis and Elaine were on the executive, then to select committees of three, starting with Adam, Marie and Brian, the five permutations in the answer to A above would result in the same five people being the committee, so they represent one combination.

The essential difference between these two situations needs to be discussed and emphasized. Eventually, Problem A should be described as a permutation (order is important), Problem B as a combination (order not important).

Probability

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper (G7)

1) For each of the following, decide whether permutations or combinations are involved.
   a) The number of committees of two that can be formed from a group of 12 people.
   b) The number of possible lineups for a baseball team that can be formed from 12 people without regard to position (a baseball team consists of nine players, as follows: pitcher; catcher; first, second, and third basemen; shortstop; right, centre, and left fielders).
   c) The number of five-letter licence plates that can be formed from 12 different letters.
   d) The number of six subsets that can be formed from 12 different letters.
   e) The number of five-man basketball teams that can be formed from 10 players.
   f) The number of ordered triples that can be formed from 10 different numbers.
   g) The number of ordered triples that can be formed from the numbers

      1, 1, 1, 3, 3, 5, 5, 5, 5, and 4.
2) The manager of a baseball team needs to decide the batting order for the season opener. In how many ways can the first four batters be arranged on the batting roster? Is this a permutation or combination question? Explain.
3) As a promotion, a record store placed 12 tapes in one basket and 10 compact discs in another. Pierre was the one millionth customer and was allowed to select 4 tapes and 4 compact discs. To find how many selections that can Pierre make, does one use permutations or combinations? Explain.
4) Three identical red balls (R) and two identical white balls (W) are placed in a box. How many ways are there of selecting the balls in the following order?

   RWRRW
5) a) Find the total number of arrangements of the letters of the word "SILK."
   b) Find the total number of arrangements of the letters of the word "SILL."
   c) How are your answers in a) and b) alike? How are they different?

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations

G8 develop and apply formulas to evaluate permutations and combinations

Elaboration—Instructional Strategies/Suggestions

A6 As students refine their methods of counting, moving from tree and area diagrams and listing through the fundamental counting principles, they should learn to recognize and use $n!$ (n factorial) to represent the number of ways to arrange n distinct objects. For example, the product rule can be used to find the number of possible arrangements for three people standing in a line. There are three people to choose from for the front of the line. For each of these choices, there are two people to choose from for the second position in the line. For each of these choices, there is one person to choose from the end of the line. Therefore, there are $3 \times 2 \times 1$ or six possible arrangements.

In another example, at a music festival, eight trumpet players competed in the Baroque class. After the judging, they were awarded first, second, third... down to eigth place. In how many ways could their placements be awarded?

If all the trumpet players were given a position first, second, third, ... , eighth, then the total number of possible standings could be calculated by using reasoning like: There are eight people eligible for first, which leaves seven eligible for second, six people for third ... leading to a calculation $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. This product can be written in a compact form as 8! and is read "eight factorial."

In general, $n! = n(n-1)(n-2)\ldots(3)(2)(1)$, where $n \in w$ and $0! = 1$.

A6/G8 If there are only three prizes to be given to the 8 trumpeters, how many ways could placement be awarded?

Students should reason that eight people could come first, only seven could come second, and six could come third $\Rightarrow 8 \times 7 \times 6 \Rightarrow 336$. This could be worded "How many permutations are there of eight distinct objects taken three at a time?"

The symbol commonly used to represent this is ${}_8P_3$, or ${}_nP_r$ for the number of "*n*" objects taken "*r*" at a time. Students should notice that

$${}_8P_3 = 8 \times 7 \times 6$$

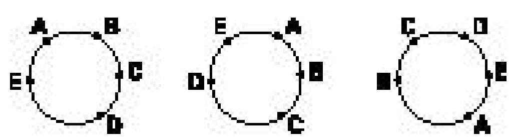
$$also,\ {}_8P_3 = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$so,\ {}_8P_3 = \frac{8!}{5!}$$

$${}_8P_3 = \frac{8!}{(8-3)!}$$

$$so,\ {}_nP_r = \frac{n!}{(n-r)!}$$

Students should note that when five people are to be arranged in a straight line there would be 5! or 120 ways to do this. However, if the same five people were to be arranged around a table in the order, say A, B, C, D, and E, their relative position to each other would not be distinguishable.



Thus, the total number of arrangements would be:

$$\frac{{}_5P_5}{5} = \frac{5!}{5} = 4! = 24$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper (A6)

1) The town of Karsville, which has 32 505 automobiles, is designing its own licence plates for residents to place on the front of their automobiles.
   a) Ask students to use counting principles to determine the best of the following three options and explain their choice:
      i) a licence made from using four single-digit numerals from 1 to 9
      ii) a licence made of three single-digit numerals from 1 to 9 and one letter from the alphabet
      iii) a licence made from three single-digit numerals from 1 to 9 and two letters from the alphabet.
   b) Ask students to select the best combination of single-digits from 1 to 9 and letters from the alphabet to suit the purposes of this town and to defend their selection.

2) In a box there are three black marbles and two white marbles.
Without looking in the box, choose two of the five marbles. How many ways are there to select two marbles that are the same colour? Each a different colour?

Pencil and Paper (A6/G8)

a) Indicate which of the following are true (T) and which are false (F).

   i) $\frac{5!}{4!} = 5 \times 4$

   ii) $10 \times 9 \times 8 = \frac{10!}{7!}$

   iii) ${}_8P_2 = 56$

   iv) ${}_{100}P_4 = 100 \times 100 \times 99 \times 98 \times 97$

b) Create a story where each true expression above would be used in the solution.

7) There are five points, no three of which are collinear, on a plane.
   a) How many segments can be formed using these five points as endpoints?
   b) If consecutive points are joined, a convex polygon is formed. How many diagonals does this polygon have?

8) A local pizza restaurant has a special on its four-ingredient 20 cm pizza. If there are 15 ingredients from which to choose, how many different "specials" are possible?

9) Explain why the following theorem would be true:

   A circular arrangement of 'n' items can be calculated using: $\frac{{}_nP_n}{n} = (n-1)!$

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations

G8 develop and apply formulas to evaluate permutations and combinations

G7 distinguish between situations that involve permutations and combinations

Elaboration—Instructional Strategies/Suggestions

A6/G8 Refer back to the problem where there are five members on the executive of the student council. If these five were elected from a list of 10 candidates for executive position, such as president, vice president, secretary, the number of ways 10 people can be slotted into five positions would be found using permutations ${}_{10}P_5 = \frac{10!}{(10-5)!} = 30\ 240$.

A6/G8/G7 From these five people a committee of three is struck. If the five people are represented by A, B, C, D, and E, then clearly a committee with A, B, and C is the same as a committee with C, A, and B. So, the order of the selection is not important and the arrangement is called a combination. Therefore, since ABC, ACB, BAC, BCA, CAB, and CBA are all considered the same committee, they represent only one committee of three selected from the five people. The number of permutations of A, B, and C is 3!. Thus, the number of committees from the original list of 10 candidates

$$= \frac{\text{number of ways the executive was chosen}}{3!}$$

$$= \frac{30240}{3!}$$

$$= 5040$$

$$\text{That is } {}_{10}C_3 = \frac{{}_{10}P_5}{3!} = 5040$$

and the number of committees from the five member executive selected would be

${}_5C_3 = \frac{{}_5P_3}{3!} = 10$.

A combination of "*n*" objects taken "*r*" at a time is any subset of size "*r*" taken from the "*n*" objects. The number is denoted by $\binom{n}{r}$ (read "*n*" choose "*r*"), or ${}_nC_r$.

The number $\binom{n}{r}$ can be evaluated by investigating the connection between permutations and combinations.

Since ${}_nP_r = \frac{n!}{(n-r)}$

Thus, in general, $\binom{n}{r} = \frac{{}_nP_r}{r!}$

$\therefore {}_nC_r = \frac{n!}{r!(n-r)!}$

For example: A committee of size 4 and a committee of size 3 are to be assigned from a group of 10 people. How many ways can this be done if no person is assigned to both committees? Solution: First committee ${}_{10}C_4 = \binom{10}{4} = 210$ ways, and there are 6 people left for the second committee. Second committee ${}_6C_3 = \binom{6}{3} = 20$ ways. Therefore the two committees can be assigned $210 \times 20 = 4200$ ways. Note: If the smaller committee was selected first then

$\binom{10}{3}\binom{7}{4} = 120 \times 35 = 4200$ ways.

Probability

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper (A6/G8/G7)

1) a) Which of the following will produce the number of greatest magnitude? (Use estimation first.) Which will produce the smallest ?

i) $6!$ iv) $3! \cdot 4$ vii) $\frac{9!}{7!}$

ii) $11!$ v) $\frac{9!}{2!}$ viii) $\frac{100!}{2!}$

iii) $\frac{15!}{12!}$ vi) $\frac{9!}{2!}$ ix) $4! - 3!$ x) $\frac{7!}{6!}$

b) Pick three of the above expressions and create a problem in which these symbols would be used in the solution.

Pencil and Paper (A6)

2) Write each as a ratio of factorials.

a) 7 ´ 6 ´ 5 d) 30 ´ 29 ´ 29 ´ 12 ´ 11 ´ 10 ´ 9

b) 19 ´ 9 ´ 8 ´ 7 ´ 5 ´ 19 e) 20 ´ 19 ´ 18 ´ 17

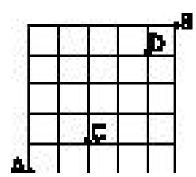
c) 10 ´ 9 ´ 8 ´ 7 ´ 6 f) $\frac{50 \times 49 \times 48 \times 47 \times 46}{5!}$

Performance (G8/G7)

3) A government committee of size 9 is to be selected from five liberals, four reformers, and four new democrats. How many ways can this be done if each of the three parties must be equally represented?

4) Explain in words why you think a combination lock is called a combination lock instead of a permutation lock.

5) A fly goes from A to B in the grid by travelling only to the right or upwards. How many possible routes are there? How many routes are there that go through C, but not through D?



6) Linda, Gino, and Sam each draw 3 cards from a deck of 52 playing cards and do not replace them.

a) If Linda goes first, in how many ways can she pick 3 cards?

b) In how many ways can Gino draw his cards after Linda has drawn hers?

c) Finally, in how many ways can Sam draw her cards?

7) A quarterback on a football team has seven different plays to use in a game. In order to confuse the defence of the other team, the quarterback does not want to repeat the same sequence of plays too often. How many different sequences of three plays has she to choose from if no play is repeated?

8) Mr. Burble teaches 182 students mathematics at Harry High. He tells his students that they must do these six problems, but that they can do them in any order. Is it possible for each of his students to do them in a different order? Explain.

Suggested Resources

Probability

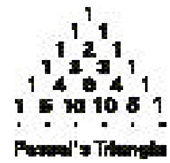
Outcomes

SCO: In this course, students will be expected to

G10 connect Pascal's Triangle with combinatorial coefficients

G9 demonstrate an understanding of binomial expansion and its connection to combinations

Elaboration—Instructional Strategies/Suggestions



G10 Students should be asked to take binomials like $(x + y)$ and find simplified expressions for $(x+y)^2$, $(x+y)^3$, $(x+y)^4$, etc., and look for patterns in their coefficients. They should be able to find a connection between the expansion power and that same row in the Pascal's Triangle with respect to the coefficient values. $((x+y)^0) = 1$ is the top row, row 0).

G9 The counting techniques, discussed in G3, p. 142, can be useful in the multiplying of polynomials. Looking at the product of $(a + b)(c + d) = ac + bc + ad + bd$, students should notice that each term in the expansion has one factor from $(a + b)$ and one factor from $(c + d)$. e.g., ac has two factors a and c. The a is from $(a + b)$ and the c is from $(c + d)$. Thus the number of terms in the expansion is four since there are two choices from $(a + b)$ and two choices from $(c + d)$. Students should also notice that since there are two factors $(a + b)$, and $(c + d)$ there are two factors in each term of the expansion.

$$(a+b)(c+d) \text{ ° } ac + bc + ad + bd \text{ ° } 4 \text{ (each term has two factors)}$$

two factors
in each term

The product of one binomial and itself follows the same pattern.
$(x+y)^2$ ° $(x+y)(x+y)$ ° $xx + xy + yx + yy$, but the multiplication would be completed by collecting the like terms and using exponents: $x2 + 2xy + y2$. Students should consider $(x+y)^5 = (x+y)(x+y)\ldots(x+y)$ ° $xxxxx + xxxyy + \ldots + yyyyy$. Each term is made up of five factors and using exponents will look like $x^a\,y^b$ where

$$xxxxx \rightarrow x^5\,y \rightarrow 5 + 0 = 5$$
$$a + b = 5, \text{ e.g., } xxxyy \rightarrow x^4\,y \rightarrow x^{4\,y^1} \rightarrow 4 + 1 = 5$$
$$xxxyy \rightarrow x^3\,y^2 \rightarrow 3 + 2 = 5$$

G10/G9 In collecting the like terms, how many terms will be made up of the two factors $x^2\,y^3$? To answer this students should count the number of ways to make $x^2\,y^3$, e.g., the two factors of x must come from two of the five factors in each term of $(x+y)^5$. This can be done $\binom{5}{2}$ or ${}_5C_2 = 10$ ways. The three factors of y must come from the remaining three factors in each term of $(x+y)^5$ and this can be done in only one way. So the coefficient of x^2y^3 $({}_5C_3)$ will be 10. Students should note that these coefficients are values in the fifth row of Pascals' Triangle.

Students should examine the pattern changes in the signs between terms when $(x-y)^5$ is expanded. Because the second term in the expression $(x-y)^5$ could be considered negative $(-y)$, then the terms in the expansion that have odd numbers of y-factors will be negative. When exponents or coefficients are included in the binomial to be expanded $(x^2 + 3y)^3$ students should be aware that for every x-factor, there is now an x^2-factor, and for every y-factor there is now a $3y$-factor, e.g., when x is replaced with x^2 and y with $3y$ the expansion becomes:

$$(x^2 - 3y)^3 = \binom{3}{3}(x^2)^3 - \binom{3}{2}(x^2)^2\,3y + \binom{3}{1}(x^2)(3y)^2 - \binom{3}{0}(3y)^3$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil (G10/G9)

1) What is the coefficient of *the* x^4y^2 term in each of the following?
   a) $(x + y)^6$
   b) $(x - 2y)^6$
   c) $(2x + y)^6$
   d) $(3x - 2y)^6$

2) When examining the terms from left to right, find the specified term in each expansion.
   a) 10 th in $(x - y)^{12}$
   b) 20 th in $(2x - 1)^{19}$
   c) 8 th in $(a + b)^{10}$
   d) 2 nd in $(x^3 - 5)^7$
   e) 3 rd in $(1 - 2x)^9$
   f) 15 th in $(1 + a^2)^{24}$

3) a) Find the sum of the elements in each row, for the first six rows of Pascal's Triangle.
   b) Find the number of subsets in a 0–, 1–, 2–, 3–, 4–, and 5– element set.
   c) How are parts (a) and (b) related?
   d) How many elements are there in an n-element set?

4) Find a decimal approximation for 1.02^{10} by writing it as $(1 + 0.02)^{10}$ and calculating the first five terms of the resulting binomial series.

Journal (G10/G9)

5) Betty Lou missed math class today. Helen phoned her at night to tell her about how combinations are helpful when expanding binomials. Write a paragraph or two about what Helen would have told her.
6) When expanding $(a^2 - 2b)^5$, Wally gets confused about the exponents in his answer. Write a paragraph to Wally to help him remember how to record the exponents on this expansion.

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

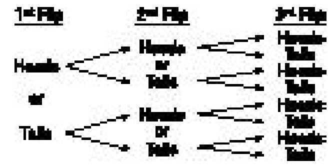
G11(Adv) connect binomial expansions, combinations, and the probability of binomial trials

G1 develop and apply simulations to solve problems

B8 determine probabilities using permutations and combinations

Elaboration—Instructional Strategies/Suggestions

G11(Adv) Many experiments consist of more than two parts, and if these parts are independent of one another, students can use the concept of a product model or tree diagram to help them with their counting and probability calculations. For example, when flipping a fair coin three times, a tree diagram determines that the sample space has eight outcomes. The probability of any one of them being selected is 1/8. P (of any one of eight outcomes) = $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.



G1/G11(Adv)/B8 The tossing of three coins discussed above could be used as a simulation model. Say the students want to solve the problem "in a family with three children what is the probability that the first two children are girls and the third is a boy."

Experiments that consist of repeated trials of a simple experiment (e.g., tossing a coin) using a model with only two possible outcomes (heads or tails) are called binomial trials. Suppose students needed to find the probability of getting exactly three heads in 10 tosses of a fair coin. Since each trial has two choices there would have to be $2^{10} = 1024$ branches on a tree diagram. The answer would be the sum of all the probabilities of the branches that contain three heads and seven tails. The number of ways 3 heads and 7 tails could be arranged is the "ten choose three ($_{10}C_3$)" or $\binom{10}{3}$, and so the probability of this happening would be $\frac{\text{\# of success}}{\text{total number of outcomes}} = \frac{_{10}C_3}{2^{10}}$.

Another way to consider this is that the probability of any one of the 2^{10} branches being selected would be the product of $\frac{1}{2}$ for every H and $\frac{1}{2}$ for every T or $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$. Since every branch with three heads and seven tails has the same probability the answer is the number of these branches times the probability for each branch. The number of branches will be the number of ways of choosing the three heads out of ten tosses $\binom{10}{3}$. Hence, $\binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$. Students should compare $\frac{\binom{10}{3}}{2^{10}}$ and $\binom{10}{3}\frac{1^3}{2}\frac{1^7}{2}$ to see how one is the same as the other.

In general, in binomial trials there are two outcomes for each of n trials. One of the two outcomes is a "success," the other a "failure." These are labelled p and q respectfully and $q = 1 - p$. The number of successes in n trials is labelled s. Thus the probability of getting s successes and $n - s$ failures in n binomial trials is $\binom{n}{s}p^s q^{n-s}$. For example, suppose students conduct an experiment of flipping a coin. The coin is bent, so the probability of heads (success) is 0.3. If they flip the coin five times, what is the probability of three tails and two heads? Students should now be able to answer this with $P(3T, 2H) = \binom{5}{2}(0.3)^2(0.7)^5$. Students should recognize this as a term in the binomial series that comes from expanding $(0.7 + 0.3)^5$. Students might want to use the 'randBin' feature on their calculators, or other software technology to conduct experiments or simulations where random samples are needed from populations that include only two possible outcomes (e.g., yeses, and nos).

Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G11(Adv)/G1)

1) Find the probability of getting 10 heads in 15 throws of a bent coin if the probability of heads on the bent coin is $\frac{2}{3}$.

2) Find the probability of getting exactly two ones in six rolls of a fair die.

3) If n = 4 and $p = \frac{1}{2}$, for what value of s will $\binom{n}{s}p^s q^{n-s}$ be largest? Answer the same question for n = 4 and $p = \frac{1}{3}$ and for n = 5 and $p = \frac{2}{3}$.

4) If Jamie is serving he wins a tennis game against Sam with probability $\frac{4}{5}$, but if he is receiving he wins with probability $\frac{2}{5}$. Jamie and Sam agree to play five games, and Jamie bets that he can win two in a row. If Jamie wins the toss, should he elect to serve or receive? Draw two tree diagrams and verify your answer.

5) A teacher made up a fair 10-item true and false test. Kira missed a few days just before the test and thought if she answered the questions randomly selecting *T*s and *F*s, she might do allright. When she was done, she had 4 *T*s and 6 *F*s. What is the probability that Kira's 4 *T*s and 6 *F*s are correct? Show how to find the answer two ways.

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

G12(Adv) demonstrate an understanding of and solve problems using random variables and binomial distributions

B8 determine probabilities using permutations and combinations

Elaboration—Instructional Strategies/Suggestions

G12(Adv)/B8 Once students have developed the pattern described on the previous page $\binom{n}{s}p^s q^{n-s}$, they can use it to calculate the probabilities of other related events:

For example:

Let x = number of times the bent coin is "heads" in five flips.
Let $P(x)$ = probability that it is "heads" x times.
Therefore, with the probability of heads being 0.30 ...

no head: $P(0) = \binom{5}{0}(0.3^0)(0.7^5) = 0.16807$

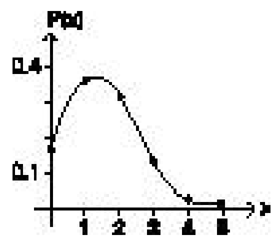
one head: $P(1) = \binom{5}{1}(0.3^1)(0.7^4) = 0.36015$

$\vdots \qquad \vdots \qquad \vdots$

five heads: $P(5) = \binom{5}{5}(0.3^5)(0.7^0) = 0.00243$

As a check on the answers, students should realize that x is certain to take on one of the values 0 through 5. So P (0 or 1 or 2 or 3 or 4 or 5) must equal 1 or 100%.

The independent variable x is called a random variable since you cannot be sure what value x will have on any one run of the random experiments. The dependent variable *P(x)* is the probability that the value is x. So P is a function of a random variable. The graph of *P(x)* for the above situation is shown.



The function P shows how the total probability, 1.00000, is "distributed" among the possible values of x. This function of a random variable is often called a probability distribution. Since this particular distribution has probabilities that are terms of a binomial series, it is called binomial distribution. It is skewed left since the probability of heads is only 0.3.

Binomial distributions occur when students perform a random experiment repeatedly, and each time there are only two possible outcomes (e.g., heads or tails, boy or girl, win or lose, yes or no). Students have already learned that a normal distribution is the result of recorded measurement of the same phenomena repeated over and over and over again. Since the binomial distribution is the result of a very similar action or fact repeated over and over and over again, it would be expected that it too would approach a normal distribution if the given probability is 0.5. This can be simulated quickly using the 'randBin' feature of the graphing calculator or other software technology. For example: 'randBin' $(10,0.5,10) \rightarrow L_1$. Once they have found the probability distribution, they can use the properties of probability to calculate the probabilities of related events.

For example, if the bent coin is flipped five times, as above, then the probability of getting at least two heads is $P(x \geq 2) = P(2) + P(3) + P(4) + P(5)$.

$= 0.3087 + 0.1323 + 0.02835 + 0.00243$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G12(Adv))

1) Heredity Problem: If a dark-haired mother and a dark-haired father have a recessive gene for light hair, there is a probability of them having a light-haired baby. For this to happen, each must have a large *x* (dark hair) and a small *x* (light hair) gene. In order for the baby to be light-haired, it must have two small *x* genes
   a) What is their probability of having a dark-haired baby?
   b) If they have three babies, calculate $P(0)$, $P(1)$, $P(2)$, and $P(3)$, the probabilities of having exactly 0, 1, 2, and 3 dark-haired babies, respectively.
   c) Show that your answers to part b are reasonable by finding their sum.
   d) Plot the graph of the probability distribution, *P*.

2) Multiple Choice Test Problem: A short multiple choice test has four questions. Each question has five choices, exactly one of which is right. Willie Makitt has not studied for the test, so he guesses at random.
   a) What is his probability of guessing any one answer right? Wrong?
   b) Calculate his probabilities of guessing 0, 1, 2, 3, and 4 answers right.
   c) Perform a calculation that shows your answer to part b is reasonable.
   d) Plot the graph of the probability distribution in part b.
   e) Willie passes the test if he gets at least three answers right. What is his probability of passing?

4) What is the probability of getting exactly 50 heads when 100 coins are tossed?

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

G11(Adv) connect binomial expansions, combinations, and the probability of binomial trials

G12(Adv) demonstrate an understanding of and solve problems using random variables and binomial distributions

B8 determine probabilities using permutations and combinations

Elaboration—Instructional Strategies/Suggestions

G11(Adv)/G12(Adv)/B8 Using the ideas developed over the last two two-page spreads, students can investigate some of the claims typically made in television and newspaper advertising. For example, a television commercial states that 8 out of 10 cats prefer Purrfect Chow. The claim is based upon a particular test in which 8 out of 10 cats chose Purrfect when given a choice between it and another cat food. A complaint is made by a rival cat food manufacturer. They say that 8 out of 10 would not be unusual, if it is assumed that cats have no particular preference for *Purrfect*. Assuming that cats will choose equally between one food or another randomly, what is the probability of them choosing Purrfect, and what does this mean with respect to the claim made by the other manufacturer?

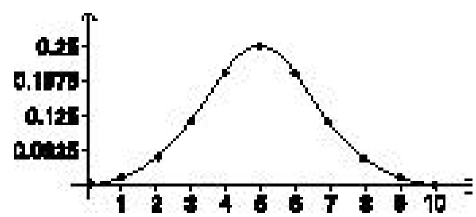
In their solution attempts, students could use the binomial model to calculate the probability that exactly 8 of 10 chose Purrfect:

$$\binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 = 45\left(\frac{1}{256}\right)\left(\frac{1}{4}\right) = \frac{45}{1024} \approx 0.0439$$

If R is the number choosing Purrfect, then the full probability distribution would be:

r	0	1	2	3	4	5	6	7	8	9	10
$\binom{10}{r}\frac{1}{2}\left(\frac{1}{2}\right)^{10-r}$	$\frac{1}{1024}$	$\frac{10}{1024}$	$\frac{45}{1024}$	$\frac{120}{1024}$	$\frac{210}{1024}$	$\frac{252}{1024}$	$\frac{210}{1024}$	$\frac{120}{1024}$	$\frac{45}{1024}$	$\frac{10}{1024}$	$\frac{1}{1024}$

The graph of this distribution looks quite normal, since the probability is 0.5 that cats will choose Purrfect over the other food choice.



From the graph the result “8 or more out of 10” is likely to occur in only about 5% of all samples of 10 cats. Based on previous study 5% is not very likely and suggests that the assumption made by the rival manufacturers is probably wrong. It appears likely that more than 50% of cats would indeed choose Purrfect.

The probability of 8 or more of the cats choosing Purrfect can be calculated using:

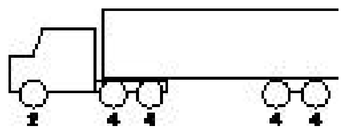
$$P(8 \text{ or more}) = \frac{45}{1024} + \frac{10}{1024} + \frac{1}{1024} = \frac{56}{1024} \approx 0.055.$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Performance (G11(Adv)/G12(Adv))

1) Eighteen-Wheeler Problem: Large tractor-trailer trucks usually have 18 tires. Suppose that the probability of any one tire blowing out on a cross-country trip is 0.03.



Ask students the following:
a) What is the probability that any one tire does not blow out?
b) What is the probability that
   i) none of the 18 tires blows out?
   ii) exactly one tire blows out?
   iii) exactly two tires blow out?
   iv) more than two tires blow out?
c) If the trucker wants to have a 95% probability of making the trip without a blowout, what must the reliability of each tire be? That is, what is the probability that any one tire will blow out?

2) Sally claims that she can predict which way a coin will land, either heads or tails. Tommy throws the coin eight times and Sally gets it right six times. Ask students to calculate, on the basis of a binomial model, the probability of
a) getting six coin tosses correct out of eight
b) getting six or more coin tosses correct out of eight
c) Ask students if they think the result supports her claim? Explain your answer.

6) A blind taste test is organized to see if people can tell the difference between two different brands of orange juice. They have 10 “tastes”. After each taste they have to say whether it is juice A or juice B. Ask students how often they would expect the participants to get it right before they were reasonably convinced that they could actually tell the difference.

7) A list of people eligible for jury duty contains about 40% women. A judge is responsible for selecting six jurors from this list.
a) If the judge’s selection is made at random, what is the probability that three of the six jurors will be women?
b) Prepare a probability distribution table and graph for the number of women among the six jurors.
c) The judge’s selection includes only one woman. Ask students if they think this is sufficient reason to suspect the judge of discrimination? Explain.

Suggested Resources