Introduction

Background

The mathematics curriculum for Atlantic Canada has been written in an effort to align the outcomes for student learning in mathematics with the recommendations of *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). This document identifies the primary goal for all students as the attainment of mathematical power—the ability to make mathematical connections, to reason logically, to communicate and apply mathematics effectively in problem situations. Since the late 1980s, several influential publications have affirmed this goal. In addition to *Curriculum and Evaluation Standards for School Mathematics*, these include two publications from the Mathematical Sciences Education Board—*Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989) and *Reshaping School Mathematics* (1990). As well, the National Council of Teachers of Mathematics (NCTM) published the companion standards documents *Professional Standards for Teaching Mathematics* in 1991 and *Assessment Standards for School Mathematics* in 1995. In April 2000, the NCTM published its newest document, *Principles and Standards for School Mathematics*, a revision, rewriting, and restatement of the 1989 *Curriculum and Evaluation Standards for School Mathematics*.

Foundation for the Atlantic Canada Mathematics Curriculum (1996) firmly establishes *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) as a guiding beacon for pursuing this vision, a vision that fosters the development of mathematically literate students. Curriculum design has been motivated by a desire to ensure that students benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience. More and more, students are being challenged to become problem solvers, to understand mathematical concepts by becoming active learners in highly interactive learning experiences. Computers and calculators are becoming common classroom tools, and innovations in assessment of student learning (which include portfolios and open-ended questions) are being used in classrooms.

Mathematics curriculum development in this region has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. The development process has brought together teachers and Department of Education officials to co-operatively plan and execute the development of curricula in mathematics, science, language arts, and some other subject areas. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the essential graduation learnings (EGLs), also developed regionally. The essential graduation learnings, and the contribution of the mathematics curriculum to their achievement, are presented in the Outcomes section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.

Foundation for the Atlantic Canada Mathematics Curriculum provides an overview of the philosophy and goals of the public school mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a framework of outcomes—general curriculum outcomes (GCOs), which relate to subject strands, and key-stage curriculum outcomes (KSCOs), which identify what students are expected to learn and be able to do by the end of grades 3, 6, 9, and 12.

Each course guide builds on the structure introduced in the foundation document by relating specific curriculum outcomes (SCOs) to each KSCO and providing suggestions for learning experiences, instruction, assessment, and resources.

Rationale

The purposes of high school mathematics are embedded in a context that is broad and consistent with accelerating changes in today's society—a society that is increasingly dominated by technology and quantitative methods. Predictions are that high school graduates in the future will change careers at least four or five times. If we are to develop a curriculum for students who need to be flexible with respect to the workplace and capable of lifelong learning, high school mathematics must emphasize a dynamic form of literacy, and high school mathematics instruction must maximize the opportunity for students to achieve outcomes dealing with a broad range of topics. Experiences must be provided that encourage and enable students to gain confidence in their mathematical ability, solve mathematical problems, reason and communicate mathematically, and understand the value of mathematics.

Expectations of employers and post-secondary institutions reflect the need for all students to understand the complexities and technologies of communication, to ask questions, to assimilate unfamiliar information, and to work co-operatively. These needs are best addressed by developing a curriculum that reflects the following beliefs:

€ **Knowing mathematics is "doing mathematics."**

Mathematics is more than just a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. Instructional settings and student activities should be developed and grow out of problem situations. This view of learning is summarized in *Everybody Counts* (Mathematical Sciences Education Board, 1989).

"In reality, no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: 'examine, represent, transform, solve, apply, prove, communicate'. This happens most readily when they are in groups, engage in discussion, make presentations, and in other ways take charge of their own learning."

€ **Mathematics has broad content encompassing many fields.**

Some aspects of doing mathematics have changed in the last decade. For example, quantitative techniques have permeated almost all intellectual disciplines, and this phenomenon has changed the fundamental mathematical ideas needed. Although traditional topics remain very important components of the curriculum, there is a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations and connections, mathematical modelling, and problem solving.

The integration of ideas from algebra and geometry is particularly strong, with graphical representation playing an important connecting role. Frequent references to graphing utilities indicate the value of computers with appropriate graphing software and/or a graphing calculator availability for students. Topics from statistics and probability are now elevated to a more central position for all students.

Arithmetic computation is not a direct object of study in the high school mathematics curriculum; however, conceptual and procedural understandings of number, numeration, and operations and the ability to make estimations and approximations to judge the reasonableness of results are strengthened in the context of applications and problem solving. Emphasis is placed on the role of technology and appropriate concepts and skills related to its use.

€ **Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself.**

New technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them. Because technology is changing mathematics and its uses, students should learn to use graphing calculators and computers as tools for processing information and performing calculations to investigate and solve problems.

The visualization approach offered through the use of graphing utilities such as the graphing calculator affords more students greater access to more mathematics. With the wide availability of technology comes additional decision making regarding what skills need to be developed mentally. Some aspects of further development in mathematics are facilitated when students reach an automatic response level with respect to certain basic skills.

Meeting the Needs of All Learners

An important emphasis in this curriculum is the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness as they begin this course and as they progress, but they must also remain aware of the importance of avoiding gender and cultural biases in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must be recognized as teachers make instructional decisions. While this guide for Mathematics 3207presents specific curriculum outcomes for the course, it must be acknowledged that all students may not progress at the same pace and may not be equally positioned with respect to attaining a given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for helping students to ultimately achieve the key-stage and general curriculum outcomes.

Curriculum Outcomes

Curriculum Outcomes Framework

The mathematics curriculum is based on a framework of outcomes—statements articulating what students are expected to know, be able to do, and value as a result of their learning experiences in mathematics. This framework comprises statements of the essential graduation learnings, general curriculum outcomes, key-stage curriculum outcomes, and specific curriculum outcomes. *Foundation for the Atlantic Canada Mathematics Curriculum* articulates general curriculum outcomes and key-stage curriculum outcomes. Curriculum guides provide specific curriculum outcomes for each course, together with elaborations and suggestions for related instructional and assessment strategies and tasks.

Teachers and administrators are expected to refer to the curriculum outcomes framework to design learning environments and experiences that reflect the needs and interests of the students.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression*, *citizenship*, *communication*, *personal development, problem solving*, *technological competence, and spiritual and moral development.*

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading, and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Graduates will demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.

See *Foundation for the Atlantic Canada Mathematics Curriculum*, pages 4–6.

General Curriculum Outcomes

General curriculum outcomes are statements that identify what students are expected to know and be able to do upon completion of study in mathematics. General curriculum outcomes contribute to the attainment of the essential graduation learnings and are connected to key-stage curriculum outcomes. The seven general curriculum outcomes for mathematics are organized in terms of four content strands: number concepts/number and relationship operations; patterns and relations; shape and space; and data management and probability.

Number Concepts/Number and Relationship Operations

- Students will demonstrate number sense and apply number theory concepts.
- Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Patterns and Relations

- Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Shape and Space

- Students will demonstrate an understanding of and apply concepts and skills associated with measurement.
- Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Data Management and Probability

- Students will solve problems involving the collection, display, and analysis of data.
- Students will represent and solve problems involving uncertainty.

Key-Stage Curriculum Outcomes

Key-stage curriculum outcomes (KSCOs) are statements that identify what students are expected to know and be able to do by the end of grades 3, 6, 9, and 12 as a result of their cumulative learning experiences in mathematics. This curriculum guide lists key-stage curriculum outcomes for the end of grade 12 (see p. 17). Specific curriculum outcomes for Mathematics 3207 are referenced to key-stage curriculum outcomes on these same pages.

Specific Curriculum Outcomes

Specific curriculum outcomes are statements which contribute to the achievement of the key-stage curriculum outcomes identifying what students are expected to know and be able to do at a particular grade level.

In the table that follows, the specific curriculum outcomes for Mathematics 2204/2205 and Mathematics 3204/3205 are listed.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers, and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, and number system characteristics (e.g., density).

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205

By the end of Mathematics 2204/2205, students will be expected to

A1 demonstrate an understanding of irrational numbers in applications

A3 demonstrate an understanding of the application of random numbers to statistical sampling

A4 demonstrate an understanding of the conditions under which matrices have identities and inverses

A5 demonstrate an understanding of the properties of matrices and apply them

By the end of Mathematics 3204/3205, students will be expected to

A3 demonstrate an understanding of the role of irrational numbers in applications

A4 demonstrate an understanding of the nature of the roots of quadratic equations

A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations

A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations

A7 describe and interpret domains and ranges using set notation

A9 represent non-real roots of quadratic equations as complex numbers

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations, and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies, and mental mathematics.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205

By the end of Mathematics 2204/2205, students will be expected to

B1 demonstrate an understanding of the relationship between operations on fractions and rational algebraic expressions

B2 demonstrate an understanding of the relationship between operations on algebraic and matrix equations

B4 use the calculator correctly and efficiently

B5 analyse and apply the graphs of the sine and cosine functions

B6 derive and analyse the Law of Sines, Law of Cosines, and the formula

B11 develop and apply the procedure to obtain the inverse of a matrix

B12(Adv) derive and apply the procedure to obtain the inverse of a matrix

B13 solve systems of equations using inverse matrices

B14(Adv) determine the equation of a plane given three points on the plane

B15 solve systems of "*m*" equations in "*n*" variables

By the end of Mathemathics 3204/3205, students will be expected to

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

B2 demonstrate an understanding of the recursive nature of exponential growth

B4 calculate average rates of change

B8 determine probabilities using permutations and combinations

B10 derive and apply the quadratic formula

B11(Adv) analyse the quadratic formula to connect its components to the graphs of quadratic functions

B12 apply real number exponents in expressions and equations

B13 demonstrate an understanding of the properties of logarithms and apply them

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205

By the end of Mathematics 2204/2205, students will be expected to

C1 model situation with sinusoidal functions

C2 create and analyse scatter plots of periodic data

C3 determine the equations of sinusoidal functions

C4(Z) determine the equations of sinusoidal functions expressed in radians

C5 determine quadratics functions using systems of equations

C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions

C9 analyse tables and graphs of various sine and cosine functions to find patters, identify characteristics, and determine equations

C10(Adv) analyse tables and graphs of various sine and cosine functions to find patterns, identify characteristics, and determine equations

C12 interpret geometrically the relationships between equations in systems

C13 demonstrate an understanding that an equation in three variables describes a plane

C14 demonstrate an understanding of the relationships between equivalent systems of equations

C15 demonstrate an understanding of sine and cosine ratios and functions for non-acute angles

By the end of Mathematics 3204/3205, students will be expected to

C1 model real-world phenomena using quadratic functions

C2 model real-world phenomena using exponential functions

C3 sketch graphs from descriptions, tables, and collected data

C4 demonstrate an understanding of patterns that are arithmetic, power and geometric and relate them to corresponding functions

C8 describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships

C9 translate between different forms of quadratic equations

C10(Adv) determine the equation of a quadratic function using finite differences

C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships

C15 relate the nature of the roots of quadratic equations and the x-intercepts of the graphs of the corresponding functions

C16 demonstrate an understanding that slope depicts rate of change

C17 demonstrate an understanding of the concept rate of change in a variety of situations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205

By the end of Mathematics 2204/2205, students will be expected to

C16(Adv) demonstrate an understanding of sine and cosine ratios and functions for non-acute angles expressed in radians

C17(Adv) solve problems by determining the equation for the curve of best fit using sinusoidal regression

C18 interpolate and extrapolate to solve problems

C19 solve problems involving systems of equations

C21 describe how various changes in the parameters of sinusoidal equations affect their graphs

C22(Adv) describe how various changes in the parameters of sinusoidal equations, expressed in radians, affect their graphs

C23 identify periodic relations and describe their characteristics

C24 derive and apply the reciprocal and Pythagorean Identities

C25 prove trigonometric identities

C27 apply function notation to trigonometric equations

C28 analyse and solve trigonometric equations with and without technology

C29(Adv) analyse and solve trigonometric equations with and without technology, expressing solutions in radians

C30 demonstrate an understanding of the relationship between solving algebraic and trigonometric equations

By the end of Mathematics 3204/3205, students will be expected to

C18 demonstrate an understanding that the slope of a line tangent to a curve at a point is the instantaneous rate of change of the curve at the point of tangency

C19 demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function

C20(Adv) represent circles using parametric equations

C22 solve quadratic equations

C23 solve problems involving quadratic equations

C24 solve exponential and logarithmic equations

C25 solve problems involving exponential and logarithmic equations

C27 approximate and interpret slopes of tangents to curves at various points on the curves, with and without technology

C28 solve problems involving instantaneous rate of change

C29 analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships

C30 describe and apply rates of change by analyzing graphs, equations, and descriptions of linear and quadratic functions

C31 analyse and describe the characteristics of quadratic functions

C32 demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: The concept of patterns and relationships have extensive application including number patterns, those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205

By the end of Mathematics 2204/2205, students will be expected to

By the end of Mathematics 3204/3205, students will be expected to

C33 analyse and describe the characteristics of exponential and logarithmic functions

C34 demonstrate an understanding of how the parameter changes affect the graphs of exponential functions

C35(Adv) write exponential functions in transformational form, and as mapping rules to visualize and sketch graphs

C36 demonstrate an understanding of the relationship between angle rotation, and the coordinates of a rotating point

C37(Adv) describe and apply parameter changes within parametric equations of circles

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) and/or procedures (e.g., proportions) to determine measurements indirectly.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205.

By the end of Mathematics 2204/2205, students will be expected to

D1 derive, analyse, and apply angle and arc-length relationships

D2 demonstrate an understanding of the connection between degree and radian measure and apply them

D3 apply sine and cosine ratios and functions to situations involving non-acute angles

D5 apply the Law of Sines, the Law of Cosines, and the formula "area of a triangle $ABC = \frac{1}{2}bc\sin A$" to solve problems

By the end of Mathematics 3204/3205, students will be expected to

D1 develop and apply formulas for distance and midpoint

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as (i) the direction, orientation, and perspectives of objects in space, (ii) the relative shapes and sizes of figures and objects, and (iii) how a change in shape relates to a change in size. Geometric concepts, properties, and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205.

By the end of Mathematics 2204/2205, students will be expected to

E1 demonstrate an understanding of the position of axes in 3-space

E2 locate and identify points and planes in 3-space

By the end of Mathematics 3204/3205, students will be expected to

E3 write the equations of circles and ellipses in transfornmational form, and as mapping rules to visualize and sketch graphs

E4 apply properties of circles

E5 apply inductive reasoning to make conjectures in geometric situations

E7 investigate and make and prove conjectures associated with chord properties of circles

E8 investigate and make and prove conjectures associated with angle relationships in circles

E9 investigate and make and prove conjectures associated with tangent properties of circles

E11 write proofs using various axiomatic systems and assess the validity of deductive arguments

E12 demonstrate an understanding of the concept of converse

E13 analyse and translate between symbolic, graphical, and written representations of circles and ellipses

E14 translate between different forms of equations of circles and ellipses

E15 solve problems involving the equations and characteristics of circles and ellipses

E16 demonstrate the transformational relationship between the circle and the ellipse

GCO F: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display, and analysis of data involves (i) attention to sampling procedures and issues, (ii) recording and organizing collected data, (iii) choosing and creating appropriate data displays, (iv) analysing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean), and (v) formulating and evaluating statistical arguments.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205.

By the end of Mathematics 2204/2205, students will be expected to

F1 draw inferences about a population from a sample

F2 identify bias in data collection, interpretation and presentation

F4 demonstrate an understanding of how the size of a sample affects the variation in sample results

F6(Adv) explore periodic data to determine the equations of sinusoidal curves using regression analysis

F7 draw inferences from graphs, tables, and reports

F8 apply characteristics of normal distributions

F9 construct, interpret, and apply 90% box plots

F10 interpret and apply histograms and probability bar graphs

F11 determine, interpret and apply confidence intervals

F14 formulate hypotheses and null hypotheses

F15 design and conduct experiments/surveys to explore sampling variability

F16 demonstrate an understanding that the type of experiment/survey affects the organization and communication of results

F18 test hypotheses and interpret the results

F19(Adv) apply and interpret the chi-square $\left(c^2\right)$ statistic

F20(Adv) collect data about two populations and analyse it using the chi-square statistic

By the end of Mathematics 3204/3205, students will be expected to

F1 analyse, determine and apply scatter plots and determine the equations for the curves of best fit, using

GCO G: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails (i) determining probabilities by conducting experiments and/or making theoretical calculations, (ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment, and (iii) analysing problem situations to decide how best to determine probabilities.

The following are the SCOs for Mathematics 2204/2205 and Mathematics 3204/3205.

By the end of Mathematics 2204/2205, students will be expected to

G1 construct and apply 90% box plots and normal probability distributions, and determine confidence intervals

G2(Adv) connect probability with the chi-square (c^2) statistic to interpret its meaning

G3 graph sample distributions and interpret them using 90% box plots, probability bar graphs, and the language of probability

By the end of Mathematics 3204/3205, students will be expected to

G1 develop and apply simulations to solve problems

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes

G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

G4 apply area diagrams and tree diagrams to interpret and determine probabilities of dependent and independent events

G5(Adv) determine conditional probabilities

G7 distinguish between situations that involve combinations and permutations

G8 develop and apply formulas to evaluate permutations and combinations

G9 demonstrate an understanding of binomial expansion and its connection to combinations

G10 connect Pascal's Triangle with combinatorial coefficients

G11(Adv) connect binomial expansions, combinations, and the probability of binomial trials

G12(Adv) demonstrate an understanding of and solve problems using random variables and binomial distributions

Key-Stage Curriculum Outcomes

Key-stage curriculum outcomes (KSCOs) are statements that identify what students are expected to know and be able to do by the end of grades 3, 6, 9, and 12 as a result of their cumulative learning experiences in mathematics. This curriculum guide lists key-stage curriculum outcomes for the end of grade 12 (see p. 18). Specific curriculum outcomes for Mathematics 3207 are referenced to key-stage curriculum outcomes on these same pages.

Specific Curriculum Outcomes

Specific curriculum outcomes are statements identifying what students are expected to know and be able to do at a particular grade level, and which contribute to the achievement of the key-stage curriculum outcomes.

In the tables that follow, the Mathematics 3207 specific curriculum outcomes are related to the Key-Stage outcomes.

SCOs and KSCOs for Mathematics 3207

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers, and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, and number system characteristics (e.g., density).

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry–grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO i: demonstrate an understanding of number meanings with respect to the real numbers	**A1** demonstrate an understanding of recursive formulas
KSCO ii: order real numbers, represent them in multiple ways (including scientific notation), and apply appropriate representations to solve problems	**A2** determine, describe, and apply the value for "*e*" **A3** represent arithmetic and geometric sequences as ordered pairs and discrete graphs. **A4** represent a series in expanded form and using sigma notation
KSCO iii: demonstrate an understanding of the real number system and its subsystems by applying a variety of number theory concepts in relevant situations	**A5** demonstrate an understanding for the use of, and need for, radian measure in the domain of trigonometric functions
KSCO iv: some post-secondary intending students will be expected to explain and apply relationships among real and complex numbers	**A6** explain the connections between real and complex numbers **A7** translate between polar and rectangular representations

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry–grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO i: explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism **KSCO ii:** derive, analyse, and apply computational procedures (algorithms) in situations involving all representations of real numbers **KSCO iii:** derive, analyse, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations	**B1** describe the relationships between arithmetic operations and operations on rational algebraic expressions and equations **B2** develop, analyse, and apply algorithms to generate terms in a sequence **B3** develop, analyse, and apply algorithms to determine the sum of a series **B4** apply convergent and divergent geometric series **B5** evaluate and apply limits **B6** determine and apply the derivative of a function **B7** derive and apply the power rule **B8** derive and apply the general rotational matrix $\begin{pmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{pmatrix}$
KSCO iv: apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving real numbers	
KSCO v: some post-secondary intending students will be expected to apply operations on complex numbers to solve problems	**B9** apply operations on complex numbers both in rectangular and polar form **B10** develop and apply DeMoivre's Theorem for powers

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry–grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO i: model real-world problems using functions, equations, inequalities, and discrete structures	**C1** model problem situations using discrete structures such as sequences and recursive formulas
KSCO ii: represent functional relationships in multiple ways (e.g., written descriptions, tables, equations, and graphs) and describe connections among these representations	**C2** model problem situations with combinations and compositions of functions **C3** model real-world phenomena using polynomial functions and rational functions **C4** model situations with periodic curves **C5** use tables and graphs as tools to interpret expressions
KSCO iii: interpret algebraic equations and inequalities geometrically and geometric relationships algebraically	**C6** demonstrate an understanding for asymptotic behaviour **C7** demonstrate an understanding for slope functions and their connection to differentiation **C8** explore and describe the connections between quadratic equations and their inverses **C9** examine, interpret, and apply the relationship between trigonometric functions and their inverses
KSCO iv: solve problems involving relationships, using graphing technology as well as paper-and-pencil techniques	**C10** analyse and solve polynomial, rational, irrational, absolute value, and trigonometric equations **C11** analyse and solve polynomial, rational, and absolute value inequalities **C12** demonstrate an understanding for the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of the tangent line and their applications **C13** extend the understanding of exponential growth and decay through multiple contexts **C14** analyse relations, functions, and their graphs **C15** determine the equations of polynomial and rational functions

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry–grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO v: analyse and explain the behaviours, transformations, and general properties of types of equations and relations	**C16** analyse the effect of parameter changes on the graphs of functions and express the changes using transformations
KSCO vi: perform operations on and between functions	**C17** explore and analyse the graphs of the reciprocal trigonometric functions **C18** demonstrate an understanding for recursive formulas and how recursive formulas relate to a variety of sequences **C19** investigate and interpret combinations and composition, of functions **C20** factor polynomial expressions **C21** perform various transformations using multiplication of matrices **C22** explore and verify trigonometric identities
KSCO vii: some post-secondary intending students will be expected to describe and explore the concept of continuity of a function	**C23** explore and describe the connections between continuity, limits, and functions
KSCO viii: some post-secondary intending students will be expected to investigate limiting processes by examining infinite sequences and series	**C24** demonstrate an understanding of divergence and convergence **C25** demonstrate an intuitive understanding of the concept of limit **C26** investigate and apply the concept of infinity by examining sequences and series
KSCO ix: some post-secondary-intending students will be expected to make connections among trigonometric functions, polar coordinates, complex numbers, and series	**C27** represent complex numbers in a variety of ways **C28** construct and examine graphs in the complex and polar planes

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements using appropriate units and using formulas (e.g., surface area, Pythagorean Theorem) and/or procedures (e.g., proportions) to determine measurements indirectly.

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO i: measure quantities indirectly, using techniques of algebra, geometry and trigonometry	**D1** describe and apply the connection between arc length and radian measure
KSCO ii: determine measurements in a wide variety of problem situations and determine specified degrees of precision, accuracy, and error of measurements	
KSCO iii: apply measurement formulas and procedures in a wide variety of contexts	
KSCO iv: demonstrate an understanding of the meaning of area under a curve	**D2** demonstrate an understanding of how to approximate the area under a curve using limits

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as (i) the direction, orientation, and perspectives of objects in space, (ii) the relative shapes and sizes of figures and objects, and (iii) how a change in shape relates to a change in size. Geometric concepts, properties, and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by transversal intersecting parallel lines.

KSCOs	SCOs
By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to	*By the end of Mathematics 3207, students will be expected to*
KSCO i: extend spatial sense in a variety of mathematical contexts	
KSCO ii: interpret and classify geometric figures, translate between synthetic (Euclidean) and coordinate representations, and apply geometric properties and relationships	**E1** model real-world phenomena with a variety of functions/relations
KSCO iii: analyse and apply Euclidean transformations, including representing and applying translations as vectors	
KSCO iv: represent problem situations with geometric models (including the use of trigonometric ratios and co-ordinate geometry) and apply properties of figures	
KSCO v: make and test conjectures about, and deduce properties of and relationships between, two- and three-dimensional figures in multiple contexts	
KSCO vi: demonstrate an understanding of the operation of axiomatic systems and the connections among reasoning, justification and proof	**E2** develop and evaluate mathematical arguments and proofs **E3** prove using the principle of mathematical induction
KSCO vii: some post-secondary-intending students will be expected to represent and apply vectors in three dimensions algebraically and geometrically	
KSCO viii: some post-secondary-intending students will be expected to explore and apply, using multiple representations, circles, ellipses and parabolas and, in three-dimensional, spheres and ellipsoids	

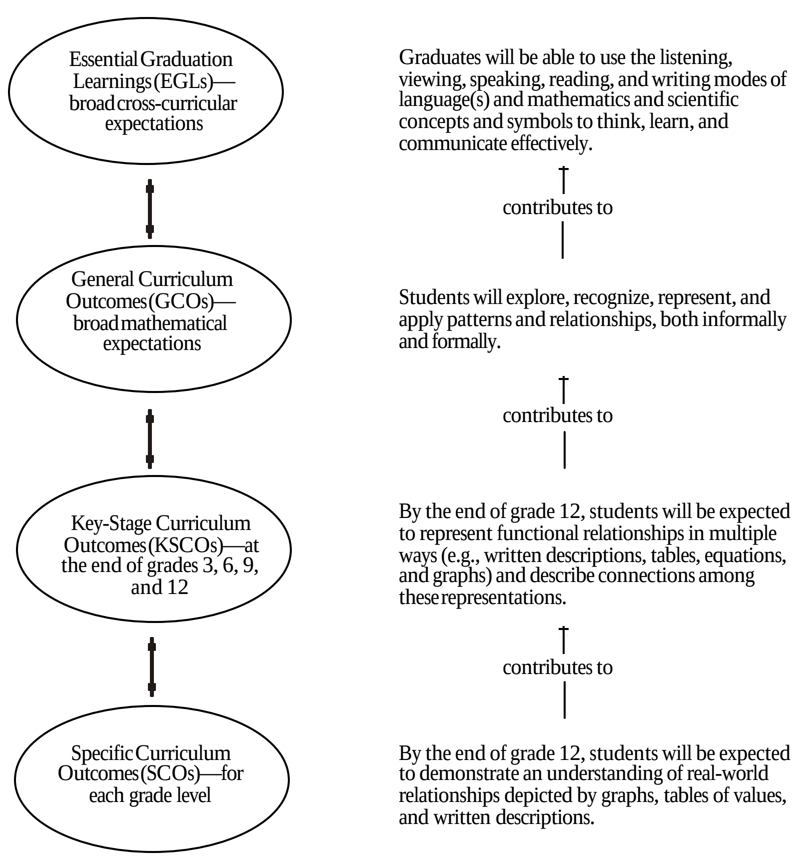
Program Design and Components

Program Organization

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication and problem-solving EGLs relating particularly well to the curriculum's unifying ideas. (See the Outcomes section of *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document identifies curriculum outcomes at key stages of the student's school experience. The specific curriculum outcomes for Mathematics 3207 represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and ultimately, the essential graduation learnings.

It is important to emphasize that **the presentation of the specific curriculum outcomes** in the guide follows a **teaching sequence.** Student and teacher resources have been developed to complement the curriculum document. At times, a different sequence will need to be followed depending on whether pre-requisite courses are being done prior to or concurrently with this course.

Outcomes Framework **Examples**



It is recognized that students' understandings of concepts will vary in terms of depth and breadth. *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) recommends that the study of mathematics for every student revolve "around a core curriculum differentiated by the depth and breadth of the treatment of the topics and by the nature of the applications" (p. 9). While it is expected that all students will work toward achievement of the same outcomes, it is recognized that students will demonstrate different levels of performance.

<table>
<tr><th>Program Level</th><th>Course 1</th><th>Course 2</th><th>Course 3</th><th>Course 4</th></tr>
<tr><td>Advanced</td><td rowspan="2">Mathematics 1204</td><td>Mathematics 2205</td><td>Mathematics 3205</td><td>Mathematics 3207</td></tr>
<tr><td>Academic</td><td>Mathematics 2204</td><td>Mathematics 3204</td><td>Mathematics 3103</td></tr>
<tr><td>Practical</td><td>Mathematics 1206</td><td>Mathematics 2206</td><td>Mathematics 3206</td><td></td></tr>
</table>

Students who are about to begin Level III will select an appropriate course depending on

- their success in previous mathematics courses
- their interests
- their academic/career goals in and beyond high school

The high school mathematics curriculum has been moulded into four courses, each designed for **110 hours** minimum of instruction. All students, at all program levels, will work toward achievemnet of the same **key-stage curriculum outcomes**. While the key-stage curriculum outcomes are intended as targets for all students, all students will not be expected to achieve them at a single level of performance. As well, there will be an additional small percentage of students who will see their outcomes significantly altered in individual educational program.

Most of the specific curriculum outcomes for each grade level are the same, but not all. It is expected that some students cab move from one program level to another.

The students in the practical level courses will be expected to meet the same key-stage curriculum outcomes and most of the same specific (course) outcomes as those in academic and advanced levels. As well, the instructional environment and philosophy should be the same at all levels. The significant difference between practical and academic courses is with respect to the level of performance expected in regard to each outcome.

In general, the practical course should be characterized by a greater focus on concrete activities, models, and applications, with less emphasis given to formalism, symbolism, computational or symbol-manipulating facility, and mathematical structure. Academic and advanced level courses will involve greater attention to abstraction and more sophisticated generalizations, while the practical level course would see less time spent on complex exercises and connections with advanced mathematical ideas.

Typically, students who choose practical courses are those who lack confidence and may have experienced considerable difficulty in mathematics throughout their schooling. In addition, their literacy skills may not be on a par with students of the same age. They may need more time on task with new concepts in order to understand them and may need connections presented in more explicit ways. They often exhibit lower self-esteem (in relation to mathematics) and require a pace that accommodates the revisiting and reinforcing of concepts, skills, and knowledge. These students need equal or perhaps greater access than their peers to technology.

By way of a brief illustration, students at all levels should develop an understanding of exponential relationships. Students taking practical level courses have as much need as others to understand the nature of exponential relationships, given the place of these relationships in universal, everyday issues such as provincial and national debt and world population dynamics. The nature of exponential relationships can be developed through concrete, hands-on experiments and data analysis that do not require a lot of formalism or symbol manipulation. The more formal and symbolic operations on exponential relationships will be much more prevalent in the academic and advanced courses.

Students who have career aspirations which involve the study of mathematics at post-secondary should choose the advanced level of mathematics including Mathematics 3207. Other students who choose the academic level but still feel there is some chance that mathematics will be a component of their post-secondary program should at the very least include Mathematics 3103 in their high school program.

Content Organization

The NCTM *Curriculum and Evaluation Standards for School Mathematics* establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. *Foundation for the Atlantic Canada Mathematics Curriculum* further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum (see pages 7–11). Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas.

These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured to provide opportunities for students to communicate mathematically, that teacher encouragement and questioning should enable students to explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines, and/or the world around them.

The mathematical content identified in the strands should promote the development of depth as well as breadth. For this depth to be developed, a number of common connections must be visible to unify the core content. The unifying connections are as follows:

Mathematical Modeling

As students study the strands of mathematics, they need to see its value in making predictions in the real world. Some basic mathematical structures used in modeling include graphs, equations, tables, and algorithms. Furthermore, students need to understand the limitations of modeling real situations, which are most often very complex.

In some situations the modeling appears to be straightforward e.g., Vectors can be used to model an aeroplane in a wind current; exponential functions can be used to model population growth; quadratics can be used to model trajectory paths; and trigonometric functions can be used to model wave motion. At other times the model may require transformations in the data. Regression analysis will allow us to better understand data from some real situations. In examining population growth, logarithms plotted against time may produce a better fit of the data to make predictions. Probability simulation may be used to model processes involving gambling, insurance, and genetics.

Relations and Functions

The high school mathematics program emphasizes is the study of relationships between two quantities. Across all strands of the mathematics program, students need to see the various ways in which one quantity can vary in relation to another. This study will precede the basic notion of function, how input and output are related, and how functions may be described in various ways such as verbally, graphically, algebraically, and numerically in tables. A formula such as the one for the area of a circle, $A(r) = pr^2$, does not in itself provide a meaning of the relationship. Students need to see how a change in the radius "*r*" results in a corresponding change in the area "*A*(*r*)". This can be described verbally or graphically.

The function concept is basic to the development of mathematical thinking. The type of mathematical reasoning that has students understand that the value of one variable may depend on the value of another pervades all strands of mathematics. In trigonometry, the functions are periodic; in discrete mathematics the functions are not continuous, but progress in steps; in exponential functions the functions are used to model growth and decay. In geometry we can examine the function relationships that exist between the image and its object for a given transformation, and in probability we may also view the probability of an event as a function of the number of choices available.

Communicating Mathematics

Communicating in mathematics helps students to develop insight into the nature of mathematics. Much of mathematics involves solving problems where students are required to develop, interpret, and analyse algorithms. When students are given a problem, they should be given opportunities to share the various ways they solved the problem so that they can compare the effectiveness, the efficiency, and the relative appropriateness of the methods used. It is through this type of communication that students deepen their understanding and extend their ability to reason. Technology continues to advance, resulting in a change in the type of problems that we can solve. It is important for students to be able to communicate by using technology to solve problems.

Mathematical arguments help students address questions such as “How do I know if I’m correct?” “Is this always true?” “Is there any solution to satisfy these conditions?” When students are asked to justify a result, they must be able to see how things fit together in a natural way. Mathematical justification communicates a student’s understanding and allows the student to express ideas in many different ways, including discussions of what is and is not accepted. Students may, for example, be asked to clarify when it is appropriate to use the exponential function to model a situation.

A formal proof can illustrate the power of the axiomatic structure in mathematics. Being able to move from examples to a deductive proof convinces us of the truth of conjectures.

Mathematical discourse should be part of every lesson, since it promotes both reasoning and understanding in mathematics.

Multiple Representations

Real understanding in mathematics is present when students are able to use and choose representations to clarify and communicate. Students who are in control of their learning may choose or find the representation they find most useful. For example, a student who has studied the quadratic function demonstrates mathematical power when he or she is able to move between the graph and the equation to find solutions to the quadratic equation or inequality and understands the implications of these solutions.

An understanding of the multiple ways of representing an idea or solving a problem as well as the recognition of the equivalence of the various representations results in deeper understandings of mathematical structure and process. For example, if students examine the same geometry concepts from an Euclidean, analytic, and transformational approach, they will develop a much stronger intuitive understanding of these concepts.

Learning and Teaching Mathematics

What students learn is fundamentally connected to how they learn it. The view of learning mathematics as an integrated set of intellectual tools for making sense of mathematical situations has created a need for new forms of classroom organization, communication patterns, and instructional strategies. The teacher is no longer the sole dispenser of knowledge but is rather a facilitator and educational conductor whose major roles include

- creating a classroom environment to support the teaching and learning of mathematics
- setting goals and selecting or creating mathematical tasks to help the students reach these goals
- stimulating and managing classroom discourse so that the students are clearer about what is being taught
- analysing student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions

Good mathematics teaching and learning take place in a range of situations. Instructional settings and strategies should create a climate that reflects the constructive, active view of the learning process. This means that learning does not occur by passive absorption and imitation but rather as students actively assimilate new information and construct their own meanings.

Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate. What students learn about particular concepts and procedures and their own mathematical thinking depends upon the ways in which they engage in mathematical activity in their classrooms. Their dispositions toward mathematics are also shaped by such experiences. Consequently, the goal of developing students' mathematical power requires careful attention to pedagogy as well as to the curriculum.

Mathematics instruction should vary and should include opportunities for group and individual assignments, discussion between teacher and students and among students, appropriate project work, practice with mathematical methods, and exposition by the teacher.

Instructional settings should include varied learning environments that encourage the development of specific co-operative behaviours. Students should be expected to work together to help each other, and at the same time they can be expected to complete individual projects. Students develop strategies and skills in asking questions, listening, showing and explaining to others how to do things, finding out what others think, and determining what way to complete a project.

Summary of Changes in Instructional Practices

Moving away from exclusively relying on:

- teacher and text as only sources of knowledge
- rote memorization of complex facts and procedures
- extended periods of individual practising of routine tasks
- instruction based only on teacher exposition
- paper and pencil manipulative skill work
- the relegation of testing to an adjunct role with the sole purpose of assigning grades

toward practices that include

- the active involvement of students in constructing and applying mathematical ideas
- problem solving as a means as well as a goal of instruction
- effective questioning techniques that promote student interaction
- the use of a variety of instructional formats (small groups, explorations, peer instruction, whole class, project work)
- use of computers and calculators as tools for learning and doing mathematics when appropriate
- student communication of mathematical ideas orally and in writing
- the establishment and application of the interrelatedness of mathematical topics
- the systematic maintenance of student learnings and embedding review in the context of new topics and problem situations
- assessment of learning as an integral part of instruction

Integrating Technology

The integration of computers, graphics calculators, video technology, and other technologies into the mathematics classroom allows students to

- explore individual or groups of related computations or functions quickly or easily
- create and explore numeric and geometric situations for the purpose of developing conjectures

€ perform simulations of situations that would otherwise be impossible to examine
€ easily link different representations of the same information
€ model situations mathematically
€ observe the effects of simple changes in parameters or coefficients
€ analyse, organize, and display data

All of these situations enhance discovery learning and problem-solving potential. At the same time, teachers have the opportunity to use technology to communicate with fellow mathematics teachers, to share lessons with experts, and to expose their students to information that would otherwise be inaccessible.

Students will need to learn to make judgments as to when the use of technology is appropriate and when it is not. It is very important that mental math and computational estimation skills be fostered throughout the senior high program. From time to time teachers should ask students to perform basic computation tasks without the aid of technology to ensure that such basic skills are maintained. This can be done in topics such as Trigonometry by focusing on exact values as well as the decimal approximations produced by calculators. There are certain basic facts that students should acquire in this course which should reach an automatic response level. For example, all students should know that Cos $45^{o} = \frac{\sqrt{2}}{2}$ and that 45^{o} in radian measure is $\frac{p}{4}$.

Learning Resources

This curriculum document represents the central resource for the teacher of mathematics with respect to *Mathematics 3207* of the high school mathematics program. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the curriculum outcomes have been met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent to which they support the curriculum goals. Schools, school districts, and Departments of Education should work together in making professional resources available to teachers as they seek to broaden their instructional and mathematical skills. As well, manipulative materials and appropriate access to technological resources need to be at hand.

It is highly recommended that teachers familiarize themselves not only with *Foundation for the Atlantic Canada Mathematics Curriculum*, but also with *Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics, Assessment Standards for School Mathematics*, and *Principles and Standards for School Mathematics*

(NCTM, April 2000). Because of the extent of information contained in these documents, teachers are cautioned that assimilation of the ideas contained will require much reflection, discussion, and rereading. All high school mathematics teachers may wish to join the National Council of Teachers of Mathematics (NCTM) for professional growth. Membership can include a subscription to *The Mathematics Teacher*, a journal that contains a wealth of information and practical teaching suggestions. Institutional or individual membership can be obtained by telephoning 1-800-235-7566 (NCTM order office), or by contacting the NCTM representative on your mathematics teacher association's executive. The authorized resource for this course is Mathematical Modeling: Book 4 by Thompson-Nelson, 2002.

Assessing and Evaluating Student Learning

In recent years there have been calls for change in the practices used to assess and evaluate students' progress. Many factors have set the demands for change in motion, including the following:

- **new goals for mathematics education as outlined in *Curriculum and Evaluation Standards for School Mathematics***

  The *Curriculum Standards* provide educators with specific information about what students should be able to do in mathematics. These goals go far beyond learning a list of mathematical facts; instead, they emphasize such competencies as creative and critical thinking, problem solving, working collaboratively, and the ability to manage one's own learning. Students are expected to be able to communicate mathematically, to solve and create problems, to use concepts to solve real-world applications, to integrate mathematics across disciplines, and to connect strands of mathematics. For the most part, assessments used in the past have not addressed these goals. New approaches to assessment are needed if we are to teach and address the goals set out in *Curriculum and Evaluation Standards for School Mathematics.*

- **understanding the bonds linking teaching, learning, and assessment**

  Much of our understanding of learning has been based on a theory that viewed learning as the accumulation of discrete skills. Cognitive views of learning call for an active, constructive approach in which learners gain understanding by building their own knowledge and developing connections between the facts and concepts. Problem solving and reasoning become the emphases rather than the acquisition of isolated facts. Conventional testing, which includes multiple choice or having students answer questions to determine if they can recall the type of question and the procedure to be used, provides a window into one aspect of what a student has learned. Assessments that require students to solve problems, demonstrate skills, create products, and create portfolios of work reveal more about the student's understanding and reasoning of mathematics. If the goal is to have students develop reasoning and problem-solving competencies, then teaching must reflect such, and in turn, assessment must reflect what is valued in teaching and learning. Feedback from assessment directly affects learning. The development of problem-solving and higher order thinking skills will only become a realization if assessment practices are in alignment with these goals.

In planning assessment, it is important to decide whether technology will be permitted. Certain assessment items become trivial when technology is used. It is recommended that since technology is an integral part of certain aspects of the curriculum, it should be permitted when those aspects are assessed. However there may be times when assessment tools are created for use with others aspects of the curriculum where it is appropriate to decide that technology will not be permitted. It is important if students are permitted to use technology during classroom activities that this be mirrored in assessment. Likewise, when the goal is for students to demonstrate mental facility, then calculator use can interfere.

€ **limitations of the present methods used to determine student achievement**

Does the present method of assessment provide the student with information on how to improve performance? Assessment methods should provide accurate information about students' academic achievement and information to guide teachers in decision making to improve both learning and teaching.

What Is Assessment?

Assessment allows teachers to communicate to students what activities and learning outcomes they truly value. In order for teachers to assess students effectively in a mathematics curriculum that emphasizes applications and problem solving, they need to employ devices that recognize the reasoning involved in the process as well as in the product. *Assessment Standards for School Mathematics* (NCTM, 1995, p. 3) defines assessment as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes."

Assessment can be informal or formal. Informal assessment occurs while instruction is occurring. It is a mind-set, a daily activity that helps the teacher answer the question, is what is taught being learned? Its primary purpose is to collect information so that the teacher can make decisions to improve instructional strategies. For many teachers the strategy of making annotated comments about a student's work is part of the informal assessment. Assessment must do more than determine a score for the student. It should do more than portray a level of performance.

It should direct teachers' communication and actions. Assessment must anticipate subsequent action.

Formal assessment requires the organization of an assessment event. In the past, mathematics teachers may have restricted these events to quizzes, tests, or exams. As the outcomes for mathematics education broaden, it becomes more obvious that these assessment techniques become more limited. Some educators would argue that informal assessment provides better quality information because it is in a context that can be put to immediate use.

Why Should We Assess Students?

We should assess students in order to

- improve instruction by identifying successful instructional strategies
- identify and address specific sources of the students' misunderstandings
- inform the students about their strengths in skills, knowledge, and learning strategies
- inform parents of their child's progress so that they can provide more effective support
- certify the level of achievement for each outcome

If we assume that assessment is integral to instruction and that it will enable effective intervention in instruction, then it is essential that teachers develop a repertoire of assessment strategies.

Assessment Strategies

Some assessment strategies that teachers may employ include the following:

Documenting Classroom Behaviours

In the past teachers have generally made observations of students' persistence, systematic working, organization, accuracy, conjecturing, modelling, creativity, and ability to communicate ideas, but often failed to document them. Certainly the ability to manage the documentation played a major part. Recording information signals to the student those behaviours that are truly valued. Teachers should focus on recording only significant events which are those that represent a typical student's behaviour or a situation where the student demonstrates new understanding or a lack of understanding. Using a class list, teachers can expect to record comments on approximately four students per class. The use of an annotated class list allows the teacher to recognize where students are having difficulties and identify students who may be spectators in the classroom. For summative purposes, grades should reflect the degree to which students achieve the curriculum outcomes.

Using a Portfolio or Student Journal

Having students assemble responses to various types of tasks on a regular basis is part of an effective assessment scheme. Responding to open-ended questions allows students to explore the bounds and the structure of mathematical categories. As an example, students are given a triangle in which they know two sides, or an angle and a side and they are asked to find out everything they know about the triangle. This is preferable to asking students to find the side of a triangle in a trigonometry question because it less prescriptive and allows students to explore the problem in many different ways and gives them the opportunity to use many different procedures and skills. Students should be monitoring their own learning by being asked to reflect and write about questions such as

- What is the most interesting thing you learned in mathematics class this week?
- What do you find difficult to understand?
- How could the teacher improve mathematics instruction?
- Can you identify how the mathematics we are now studying is connected to the real world?

In the portfolio or in a journal teachers can observe the development of the students' understanding and progress as a problem solver. Students should be doing problems that require varying lengths of time and represent both individual and group effort. What is most important is that teachers discuss with their peers what items are to be part of a meaningful portfolio and that students also have some input into the assembling of a portfolio.

Projects and Investigative Reports

Students will have opportunities to do projects at various times throughout the year. For example, they may conduct a survey and do a statistical report, they may do a project by reporting on the contribution of a mathematician, or the project may involve building a three dimensional shape. Students should also be given investigations in which they learn new mathematical concepts on their own. Excellent materials can be obtained from the National Council of Teachers of Mathematics including the *Student Math Notes.* (These news bulletins can be downloaded from the Internet.)

Written Tests, Quizzes, and Exams

Written tests have been accused of being limited to assessing a student's ability to recall and replicate mathematical facts and procedures. Some educators would argue that asking students to solve contrived applications usually within time limits provides us with little knowledge of the students' understanding of mathematics. However, a test that is properly developed can be the most valid and reliable method of collecting information about the degree to which students have achieved the curriculum outcomes.

How might we improve the use of written tests?

- Our challenge is to improve the nature of the questions being asked, so that we are gaining information about the students' understanding and comprehension as well as their procedural knowledge.
- Tests must be designed so that questions being asked reflect the expectations of the outcomes being addressed.

Teachers must also reflect on the quality of the test being given to students. Are they being asked to evaluate, analyse, and synthesize information, or are they simply being asked to recall isolated facts from memory? Teachers should develop a table of specifications when designing their tests.

In assessing students we have a professional obligation to ensure that the assessment reflects those skills and behaviours that we truly value. The bottom line is that good assessment is equivalent to good instruction and therefore promotes student achievement.

Course Organization

Course Design

This section of the Mathematics 3207 guide presents mathematics curriculum outcomes that students are expected to achieve as part of their studies. However, teachers are encouraged to examine the curriculum outcome that precede and follow this course.

Mathematics 3207 is organized into five units: Sequences and Series, Developing a Function Toolkit I, Developing a Function Toolkit II, Trigonometry, and Complex Numbers. The presentation of the specific curriculum outcomes in each unit reflects a suggested teaching sequence.

The Two-Page Spread

The following pages detail curriculum outcomes. Each two-page spread is dedicated to a small number of specific curriculum outcomes. As much as possible, connections are made through references to other pages of related outcomes or topics.

At the top of each page the overarching unit topic is presented, with the appropriate specific curriculum outcome(s) (SCOs) displayed in the left-hand column. The second column presents the elaboration which includes instructional strategies and suggestions, as well as some examples that might be used to illustrate achievement of outcomes. The third column includes worthwhile tasks for instruction and/or assessment purposes. While the strategies, suggestions, and examples are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communication, reasoning, and connections.

The final column is entitled Suggested Resources and will, with your additions, over time become a collection of useful references to resources that are particularly valuable with respect to achieving the outcome(s) given.