Introduction

Introduction

Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers". This is supported by Van de Walle (1997, 2), who states, "Children's learning of mathematics is based on constructivist theory which asserts that children must actively construct ideas rather than passively absorb them. NCTM publications also advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), which brought together teachers and Department of Education officials for the planning and development of curricula in mathematics, science, and language arts in both official languages.

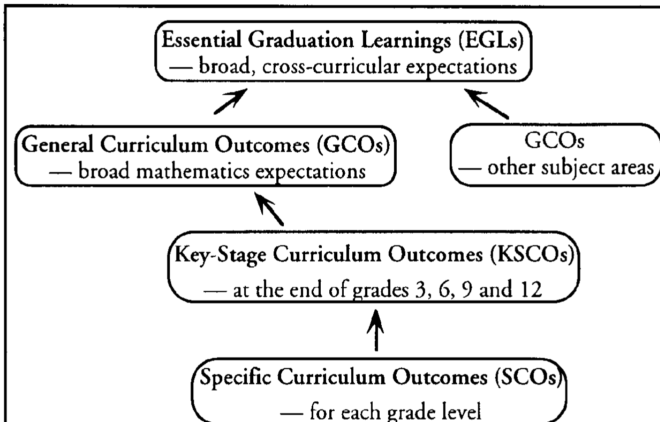


Figure 1: Outcome Framework

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). The mathematics curriculum is designed to support the six Essential Graduation Learnings. While the curriculum contributes to students' achievement of each of these, the communication and problem solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)

Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. It is supplemented by curriculum guides that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO.

The Atlantic Canada Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; and iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Program Organization

This primary curriculum guide presents specific curriculum outcomes for grades 1, 2, and 3. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.

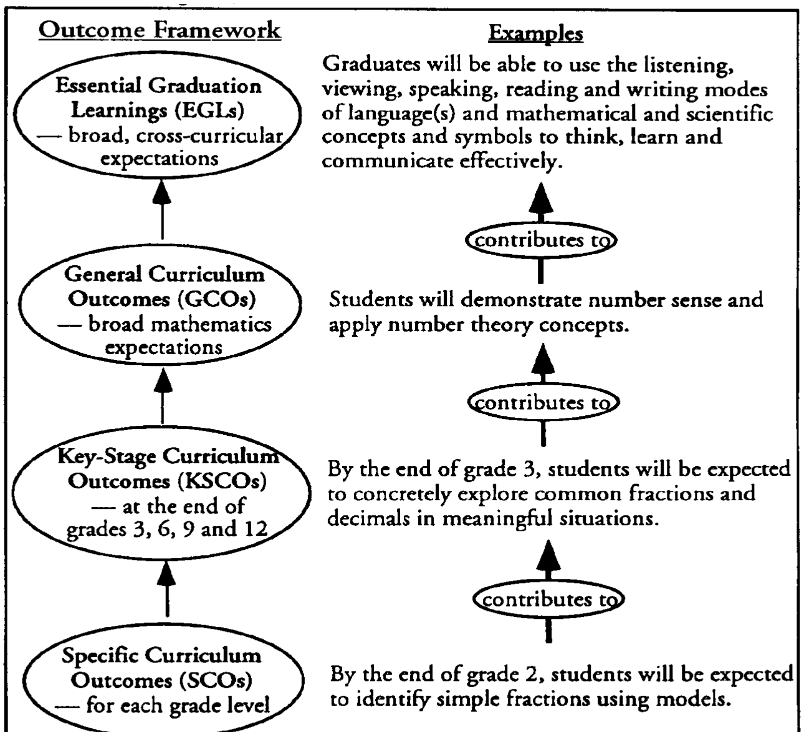


Figure 2: Examples of Outcomes

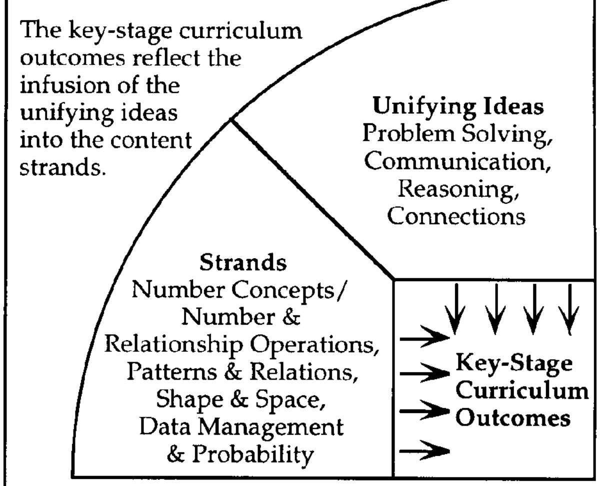
It is important to emphasize that the presentation of the specific curriculum outcomes follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists for the structuring of the program. As well, some outcomes like those pertaining to patterns and data management may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province like an election, an exhibition, or a fair.

Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)

Figure 3 — Integrating Strands and Unifying Ideas



These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

Problem Solving

Van de Walle (1997, 48) describes a problem as being "any task which involves wrestling with a new idea." Wrestling with a new idea implies that students are actively involved in construction of their own knowledge. Setting up an environment where activities are built around problems and exploration of ideas involves a rethinking of how mathematical activities are presented to students. Constance Kamii (1982, 31) illustrates this point when talking about numeracy development in young students. Preparing for snack in kindergarten often involves the distribution of cups for juice. Asking one student from each table to bring enough cups to his/her table is a very different request from telling one student from each table to bring four cups to his/her table. In the first case, students have to decide what *enough* means in this situation. They have to wrestle with the idea of *enough* and figure out a way to follow through on the teacher's instructions. In the second case, students do not have to wrestle with the idea of *enough* because the teacher has told them the number of cups needed.

The NCTM *Curriculum and Evaluation Standards* (1989, 23) emphasizes that "problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity." Schroeder and Lester (1989, 32) distinguish between teaching about problem solving, teaching for problem solving and teaching via problem solving. For example, if the situation is to decide if you have enough of something, you are teaching for problem solving. If the discussion focusses on how to act a situation out to solve the problem, you are teaching about problem solving. If you want to teach that you need four trips to take cups to a table, if there are sixteen students and you carry four cups at a time, and students figure this out on their own , then you are teaching via problem solving. The *Interactions Teacher's Resource Binder Kindergarten* (1994) suggests many problem-solving situations which can be explored with students.

The communication that takes place when students share solutions to problems often gives teachers opportunities to talk about, model and assess strategies that students can use to solve a problem. To help students develop a variety of strategies that they can use to solve routine and non-routine problems, it is important to provide large-group, small-group and individual problem-solving sessions. As students develop their own problem-solving approaches, they can apply them in new situations. Learning about, for and via problem solving is interrelated as teachers help students clarify their conceptual understanding of mathematical ideas.

Communication

The students involved in figuring out *enough* in the situation described previously may use different strategies to solve the problem. They may apply their experiences of setting the table at home and place the cups at each table by matching one cup to each person/chair. This acting out of the problem helps students arrive at a possible solution. From prior experiences of using the tables in the classroom, students may have discovered that four chairs fit comfortably at each table. These students apply the strategy of prior knowledge and counting to confirm four cups are needed. Sharing solutions to problems gives students opportunities to clarify their thinking as they interact with others. Discussion may lead to other problems which may be explored. For example,

- at one table there was one cup not used. As students talked about how this could have happened, they realized there could be a number of reasonable possibilities:

  - the student who gave out the cups matched one cup to each chair. Later the class realized there was one student sick that day so there was an extra cup at the table.

  - the student who gave out the cups counted the children who were sitting at the table and gave out four cups. That day apple juice was being served, and one student did not like apple juice. Although there were four students at the table, only three cups were needed.

Mathematics was traditionally viewed as a silent activity. However, students should be expected to communicate their thinking/reasoning in mathematics in the same way they do in other curriculum areas. Many students need to talk a situation through in order to understand it, and this should be encouraged. As students talk to one another, each gets a broader perspective on a problem. Sometimes, in talking to another student, a child is required to confront a misconception. In addition, talking provides the teacher another opportunity to gain insight into what students do and do not understand. Opportunities for shared writing, talking, dramatization of a problem situation, discussion of mathematical ideas in stories, the labelling of models, drawing, painting, use of manipulative materials and writing can help students develop and clarify mathematical concepts and understandings.

Reasoning

Many students help set the table at home and have had to make decisions about whether there were enough cups on the table. But is *enough* always the same? What would happen if guests were invited to dinner? Young students can explore this problem through their own drawings of what their tables would look like at home if an aunt and uncle were there. As students share drawings, they have the opportunity to justify their thinking. It is during this sharing that teachers can assess students' developing understanding of number. For example,

- I drew the table and put a chair for my mom, dad, sister and me. Then I put a chair for my aunt and a chair for my uncle. I put a cup on the table beside each chair. I know I have enough cups because there is one by each chair.
- I know there are four people in my family. I drew those four cups. Then I drew a chair for my aunt and a chair for my uncle. I gave them each a cup. Now there are enough cups.
- There are only three people in my family. My aunt and uncle will make two more people. I used the blocks to count how many cups I needed. I drew five cups on the table.

In the above example, students' concrete experiences of setting a table at home helped them represent their understanding in the form of pictures. Pictures supported their ability to solve the problem and communicate their ideas. In time, their thinking, supported by concrete materials and pictures, will move to an abstract level of understanding. Primary students need frequent experiences with concrete materials to help them think about, explore with, talk about and reason with mathematical ideas. Talking and reasoning help students develop appropriate language to describe mathematical ideas and relationships.

Mathematical Connections

To develop students' conceptual understanding in each of the mathematics strands, it is important for students to continually link their prior knowledge to new experiences. The more ways that students are given to test an idea, the better able they are to develop an understanding of mathematical concepts. The NCTM *Curriculum and Evaluation Standards* (1989, 32) states that it is important for teachers to plan activities which will help students make connections:

- between the real world and other curriculum areas
- within and among mathematical ideas
- between conceptual knowledge and symbolism

Connections Between the Real World and Other Curriculum Areas

The informal experiences that students have had with mathematical ideas can be built on and expanded during the primary years. Many hands-on experiences at the learning centres provide real-world situations in which to use mathematics. At the store, students develop numeracy skills as they count money, and at the sand and water table, they develop insights about capacity and mass. In the block corner, students develop spatial awareness as they explore space and shape. Other curriculum areas promote the development of mathematical ideas. Patterns and relationships can be explored through language arts, science, art, music, and movement activities. Spatial awareness is further developed through the physical education program. The environment itself offers opportunities for students to make connections to mathematical ideas being explored in the classroom. Patterns in nature, on wrapping paper and wallpaper, as well as time sequencing, are a few examples of real-life connections to the mathematics strand Patterns and Relations. Use of real-world experiences both inside and outside the classroom makes mathematics learning relevant for students. The use of literature helps students build connections between their background knowledge and mathematical ideas presented in stories. Stories can build a sound base for problem solving as students relate their language and experiences to the language of mathematics. *Read Any Good Math Lately?* (Whitin and Wilde, 1992) gives suggestions for using stories and poems to develop mathematical ideas.

Connections Within and Among Mathematical Ideas

The interrelatedness of the mathematics strands makes it possible for students to see connections within and among mathematical ideas. Often, as an activity is developed to focus on one strand, other strands can be included. For example, students examining a set of shapes might also count the number of sides of each shape and order them according to size.

Connections Between Conceptual Knowledge and Symbolism

At the primary level, mathematical concepts are often linked with concrete and pictorial experiences. The focus is on the development of a conceptual understanding of mathematical ideas that will help students move to an abstract level of understanding.

Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

Developing Mathematical Understanding

The content of the mathematics curriculum is arranged under four strands which are built upon from one grade level to the next. Each strand - Number Concepts/Number and Relationship Operations, Patterns and Relations, Shape and Space and Data Management and Probability - can be thought of as the major areas of content to be explored in mathematics. At all ages, conceptual understanding of curriculum areas will depend on prior experiences with the topics. "Mathematical concepts that children are in the process of constructing are not the well-formed ideas conceived by adults" (Van de Walle 1997, 32).

Prior to beginning kindergarten, students act and talk mathematically - sharing cookies, arranging pebbles by size, comparing their height against that of a sibling and pleading for "just two more books."

Considerable informal learning about matching (developing one-to-one correspondence as cookies are shared), ordering (arranging pebbles by size), comparing (height), and classification (sorting toys) takes place in the preschool years. Smith (1997, 34-47) presents many home experiences students may have had with these early mathematical concepts, including the following:

Matching	Classification
- one sock for each foot - buttonhole for each button - party hat for each person - setting the table - toys (one driver for each car) - interaction during the sharing of books, songs, and poems	- sorting toys at home (colour, shape, size, and number) - clean-up (books on the bookshelf) - laundry (towels and socks) - interaction during the sharing of books, songs, and poems

Comparing	Ordering/Seriation
- earliest experiences are opposites - weather (cold/hot) - shoes (large/small) - toys (heavy/light) - cooking (more/less) - compare quantities (more fewer cookies) - interaction during the sharing of books, songs, and poems	- events in the day (breakfast, lunch, dinner) - drawings (family members in order of size) - nesting toys - cooking utensils (measuring cups/spoons) - early counting - interaction during the sharing of books, songs, and poems - counting backwards (reverse seriation) often presented in rhymes, songs, books

Early mathematical concepts such as matching, classification, comparing and ordering help students develop logical thinking processes which they can apply in problem-solving situations. Matching is the concept of one-to-one correspondence and is one of the earliest mathematical concepts to develop. It forms a basis for the use of our number system and is a prerequisite skill for the more difficult tasks of conservation. Smith (1997, 45) defines classification (sorting) as the ability to see the *sameness* that defines members of a group and comparing as the ability to see *differences*. Ordering involves sequencing and is another foundation of our number system. All of these concepts are interrelated and are developed through activities in each of the strands of the mathematics curriculum. For example:

Patterns and Relationships

- extending patterns involves matching, comparing, and ordering

Data Management and Probability

- creating and interpreting a picture graph involves matching, sorting, comparing, and ordering

Students' prior experiences and their understanding of mathematical ideas may be at very different levels. It is important that teachers interact with and observe students to ensure each child is "receiving challenging mathematical experiences" (Smith 1997, 7).

Adapting to the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student readiness as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Support Resources

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. These guides should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in supporting the specific curriculum outcomes. Manipulative materials and technological resources (e.g., software and videos) should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM) and the *Addenda Series* and *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* or *Elementary and Middle School Mathematics: Teaching Developmentally* (John van de Walle), *Developing Number Concepts Using Unifix Cubes* (Kathy Richardson), and *About Teaching Mathematics; A K-8 Resource* (Marilyn Burns).

Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students's lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

Assessing Student Learning

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessing and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment would include:

- the use of a wide variety of assessment strategies and tools
- aligning assessment strategies and tools with the curriculum and instructional techniques
- ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborates good assessment practices and it served as a guide for developing the student assessment component of the mathematics foundation document.

Program Assessment

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for grades 1, 2, and 3. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled “Suggestions for Teaching and Learning,” provides a clarification of the specific curriculum outcome(s), as well as suggestions for teaching strategies and activities which could be used to help students achieve the outcome(s). While the strategies and activities presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s). They will illustrate ways to work toward the achievement

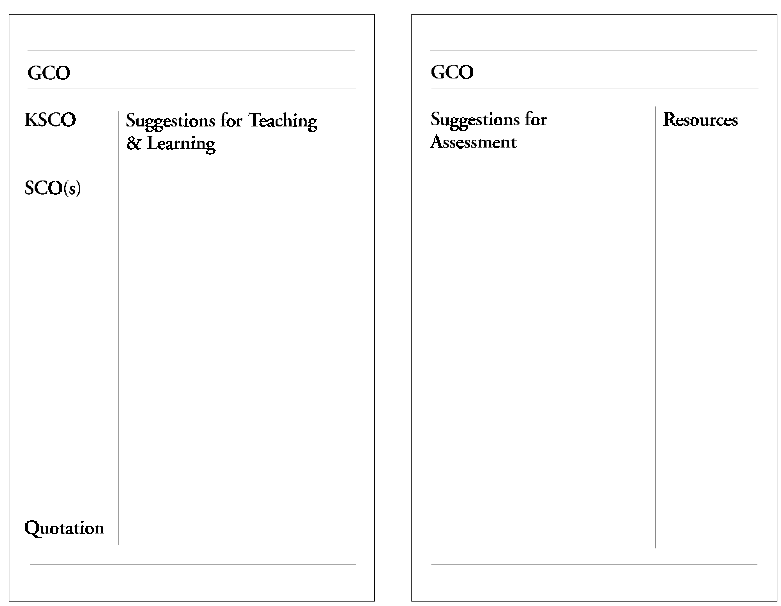


Figure 4: Layout of a 2-Page Spread

of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activites are introduced in this column of the layout by the symbol❑.

The third column of the two-page spread, entitled "Suggestions for Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings if used for assessment (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).