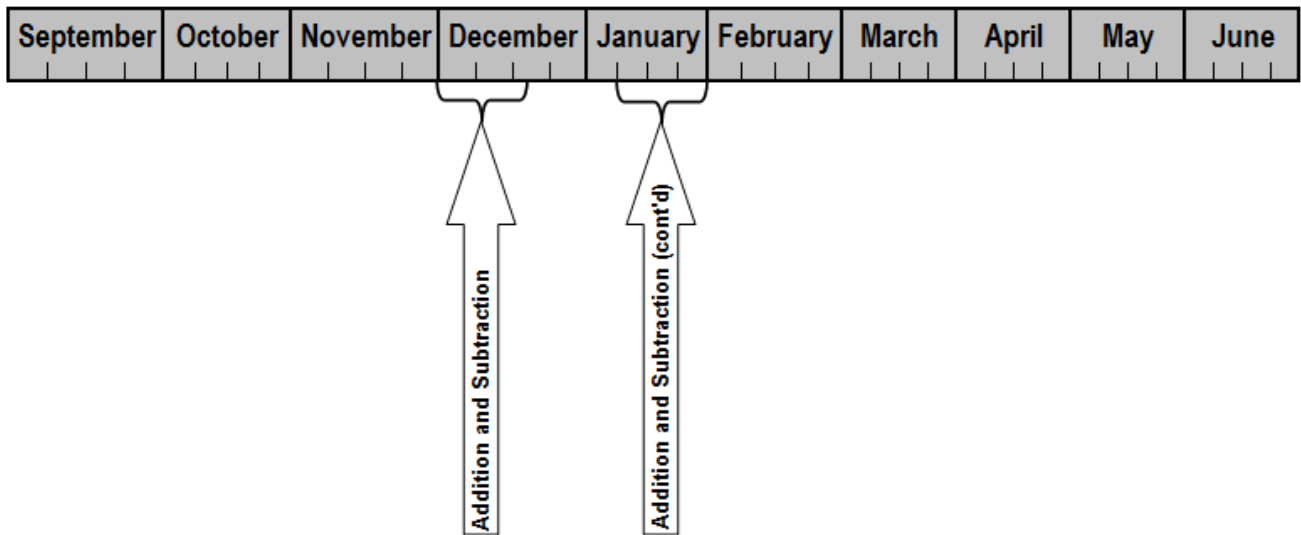


Addition and Subtraction

Suggested Time: $5\frac{1}{2}$ Weeks



This is the first explicit focus on addition and subtraction but as with other outcomes, it is ongoing throughout the year.

Unit Overview

Focus and Context

Prior to Grade 3 students explored addition and subtraction situations with 1 and 2 digit numbers with and without re-grouping. In Grade 3 the focus will be on combining and separating numbers to 1000. Students will develop a deeper understanding of situations involving addition and subtraction by creating, using and refining personal strategies. It is important that students be given many opportunities to share their thinking with classmates so that a bank of strategies for problem solving situations is explored. Through exploration of their personal strategies students should come to use the most effective strategies that work for them to solve problems.

“Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand computational methods that are over practiced without understanding are often forgotten or remembered incorrectly. On the other hand understanding without fluency can inhibit the problem solving process.” (Thornton 1990 and Hiebert 1999; Kamii, Lewis, and Livingston 1993; Hiebert and Lindquist 1990 in *Principles for School Mathematics (2000)* p. 35.

Math Connects

Students work with numbers naturally connects with all other mathematics strands Presenting students with problems that connect addition and subtraction with investigations of Statistics and Probability, Patterns and Relations, and Shape and Space further consolidate the integral world of mathematics. It is also important for students to see the connection between Mathematics and the real world. When students see this connection they tend to be more engaged in the problem solving process. Context for problems may arise through student initiated activities, teacher/student created stories and real world situations. Conceptual understanding of addition and subtraction will form the basis needed for later work in multiplication and division.

Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Number	3N6 Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: <ul style="list-style-type: none"> • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles. 	[C, CN, ME, PS, R, V]
Number	3N7 Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: <ul style="list-style-type: none"> • taking the subtrahend to the nearest multiple of ten and then compensating • think addition • using doubles. 	[C, CN, ME, PS, R, V]
Number	3N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.	[C, ME, PS, R]
Number	3N9 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by: <ul style="list-style-type: none"> • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in context that involve addition and subtraction of numbers. 	[C, CN, ME, PS, R, V]
Number	3N10 Apply mental mathematics strategies, such as: <ul style="list-style-type: none"> • using Doubles • making 10 • using Addition to Subtract • using the Commutative Property • using the Property of Zero to recall basic addition facts to 18 and related subtraction facts.	[C, CN, ME, PS, R, V]
Patterns and Relations (Variables and Equations)	3PR3 Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.	[C, CN, PS, R, V]

Strand: Number

Outcomes

Students will be expected to

3N10 Apply mental mathematics strategies, such as:

- using Doubles
- making 10
- using Addition to Subtract
- using the Commutative Property
- using the Property of Zero

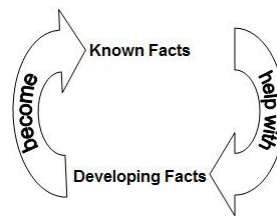
to recall basic addition facts to 18 and related subtraction facts.

[C, CN, ME, PS, R, V]

Elaborations—Strategies for Learning and Teaching

“Memorizing basic facts, perhaps with the use of flash cards, is very different from internalizing number combinations. Memorized knowledge is knowledge that can be forgotten. Internalized knowledge can’t be forgotten because it is a part of the way we see the world. Children who memorize addition and subtraction facts often forget what they have learned. On the other hand, children who have internalized a concept or relationship can’t forget it; they know it has to be that way because of a whole network of relationships and interrelationships that they have discovered and constructed in their minds.” (*Developing Number Concepts, Book 2: Addition and Subtraction* by Kathy Richardson, Page 43)

Grade 3 students will already have had experiences with mental math strategies. Now the focus will be on using the strategies to efficiently recall the facts. Efficient strategies are ones that can be done mentally and quickly. Some students will automatically develop strategies, while others will need direct teaching and practice. Strategy practice must directly relate to one or more number relationships. These strategies should be explicitly taught through demonstrations, think-a-louds, and modelling. It is important to note that the most useful strategy for a student is the one that they understand and are most confident to use. It is personal and they are able to connect it to concepts they already know.



In Grade 3, students use their increasing mathematical vocabulary along with everyday language. Students should be encouraged to use mathematical vocabulary in discussions and in their writing. The use of correct mathematical language is modelled repeatedly and consistently by teachers throughout the mathematics curriculum. It is important to note that a student’s knowledge about mathematical ideas and the use of mathematical language are connected.

“The purpose of the language in mathematics is communicating about mathematical ideas and it is necessary first to acquire knowledge about the ideas that the mathematical language describes.” (Marilyn Barns - Instructor Magazine April 2006)

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Ask students to complete the following problem:
According to the Commutative Property of Addition, which of the following means the same as $2 + 3 = 5$. Use pictures, numbers or words to explain how you know.
a) $3 + 2 = 5$ b) $5 - 2 = 3$ c) $2 + 3 + 2 = 7$ d) $5 - 3 = 2$
(3N10.1)

Performance

- Using centimetre grid paper, ask students to represent the following problem to show how it can be solved. Ms. Bursey divided her class into two teams to practice addition problems. She asked Team A to answer $7 + 2 =$. She asked Team B to answer $2 + 7 =$. What answers did the teams get? Ask students to write an addition number sentence to show their model. Ask students to compare the answers of the two addition sentences.
(3N10.1)

Resources/Notes

Math Makes Sense 3

Launch: Plants in Our National Parks

TG pp. 2 - 3

Lesson 1: Strategies for Addition Facts

3N10

TG pp. 4 - 7

Additional Reading:

Richardson, Kathy *Developing Number Concepts, Book 2: Addition and Subtraction*

Strand: Number

Outcomes

Students will be expected to

3N10 Continued

Achievement Indicator:

3N10.1 Explain or demonstrate the mental mathematics strategy that could be used to determine a basic fact, such as:

- *using doubles; e.g., for $6 + 8$, think $7 + 7$*
- *using doubles plus one, plus two; e.g., for $6 + 7$, think $6 + 6 + 1$*
- *using doubles subtract one, subtract two; e.g., for $6 + 7$, think $7 + 7 - 1$*
- *making 10; e.g., for $6 + 8$, think $6 + 4 + 4$ or $8 + 2 + 4$*
- *using addition to subtract; e.g., for $13 - 7$, think $7 + ? = 13$.*
- *using commutative property; e.g., for $3 + 9$, think $9 + 3$*
- *provide a rule for determining answers when adding and subtracting zero. When you add or subtract 0 to or from a number, the answer is the number you started with.*

Elaborations—Strategies for Learning and Teaching

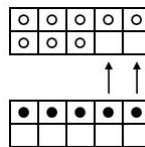
Students need opportunities to discuss and share the strategies they are using to determine the facts. Tasks like ‘Quiz-Quiz-Trade’ (explained below) can be used as an active way for students to apply a strategy.

Quiz-Quiz-Trade - Provide index cards with addition and subtraction facts pertaining to a strategy. E.g., doubles strategy

$$\begin{array}{l}
 1 + 1 = \quad 2 - 1 = \\
 2 + 2 = \quad 4 - 2 = \\
 9 + 9 = \quad 18 - 9 = \text{ etc.}
 \end{array}$$

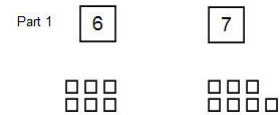
Give each student a card and ask them to find a partner. Next, students ask their partners to solve the fact on their card. They switch cards and repeat, then look for a new partner. Variation: Separate the students into addition facts and subtraction facts. Ask students to find their fact partner. E.g., $6 + 6$ will partner with $12 - 6$.

Making Ten – Provide students with a double ten frame and 2 sided counters. Give students a fact (e.g., $8 + 5$). Students will represent the number 8 on one ten frame and the number 5 on the other ten frame. Students will move counters from the ten frame with 5 to complete the ten frame representing 8.

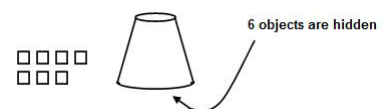


Students then verbalize what they did. E.g., “I took 2 from the 5 and put it with the 8 to make 10. Then, I added the 3 left over from the 5 and that was 13 so $8 + 5 = 13$ ”.

Using Addition to Subtract - Provide objects for counting, tub/container, number cards 0 to 9, recording sheet. Pick 2 number cards out of the bag (e.g., 6 and 7), take the number of objects for each card and find the total. Record your number sentence: $6 + 7 = 13$



Hide one of the groups of objects that match one of the number cards (e.g. 6) under the container. Record the subtraction sentence $13 - ? =$ the number of cubes left on the table. This activity can also be modelled using an overhead projector.



(continued)

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Present students with 'Numeral Wands' and call out a variety of addition/ subtraction facts including 0 facts. Note students who are having difficulty with the zero facts. (3N10.1)
- Fact Flash - Say or display, a variety of facts, one at a time, and ask students to record the sums/differences and reveal their answer. (3N10.2)

Journal

- Imagine that you are helping someone, younger than you, that is just learning to add and subtract. How would you explain addition and subtraction to him/her? Write down what you would say and do to tell someone how to complete the number sentences below:

$$4 + 5 = \underline{\quad} \qquad 9 - 5 = \underline{\quad} \qquad (3N10.1)$$

Student – Teacher Dialogue

- Ask students: Do you find it easy to add/subtract 0 to a number? If yes, why? If no, why not? (3N10.1)

Resources/Notes

Math Makes Sense 3

Lesson 1 (Cont'd): Strategies for Addition Facts

3N10

TG pp. 4 - 7

Strand: Number

Outcomes

Students will be expected to

3N10 Continued

Achievement Indicator:

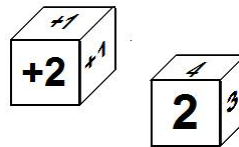
3N10.1 Continued

Elaborations—Strategies for Learning and Teaching

When discussing the concept of ‘adding zero to’ and ‘subtracting zero from’ a number, the property of zero should be emphasized. Using the part-part-whole concept with the use of manipulatives, it may be helpful to show two parts with one part being empty. Simple, real-life story problems would be good tools to illustrate the effect of adding or subtracting zero from a number. Sometimes students may think that when you add a number the sum must change and when subtracting a number, the difference must be less.

Whole	
Part	Part

Double Dice plus 1 or 2 – Prepare two cubes, one with numerals 1 – 9 and one with +1 and +2 stickers on it. Instruct the student to roll the number cube and double it. Next the student rolls the labelled cube and performs the operation. Variation: This can also be done with subtraction. (-1, -2)



General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Property of Zero – Using a set of 2 number cubes (one labelled 0, 2, 4, 6, 8, 10 and one labelled 0, 1, 3, 5, 7, 9), counters and the game board below, students play a game to reinforce that zero, when added to or subtracted from a number, has no effect on the answer. Players take turns rolling the number cubes, and adding or subtracting the numbers. If the answer is on the board the player gets to cover the number with a counter. Play continues until one player gets all 4 of their counters on the board.

8	14	3	0
5	11	7	19
0	3	9	2
16	11	17	16

(3N10.1)

Student-Teacher Dialogue

- Chant - Show cards representing a variety of missing addend number sentences for students to chant, or record on their whiteboard, the missing addend. E.g., $6 + \underline{\quad} = 13$. Ask students to explain how they figured out the missing addend. Possible responses might include: “I used addition”, “I counted up” or “I used doubles plus one.”

(3N10.1)

Resources/Notes

*Math Makes Sense 3***Lesson 1 (Cont'd):** Strategies for Addition Facts**3N10**

TG pp. 4 - 7

Strand: Number

Outcomes

Students will be expected to

3N10 Continued

Achievement Indicators:

3N10.2 Recall doubles to 18 and related subtraction facts.

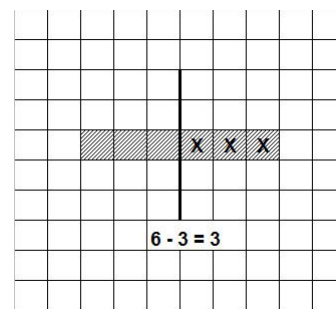
3N10.3 Recall compatible number pairs for 5 and 10.

Elaborations—Strategies for Learning and Teaching

Van de Walle (2008) suggests using “think-addition”, (using addition to subtract), as a powerful strategy for developing fluency with subtraction facts. An example of the “think-addition” strategy is when solving $12 - 5$, think “five and what makes 12?” Model the “think addition strategy” by talking about what you are thinking so that students can see the strategy in use and hear what the strategy sounds like.

Doubles in Subtraction – In ‘Doubles Equations’, one number is added to the same number. (E.g., “ $3 + 3$ ” or “ $4 + 4$ ”) Students can often recall these addition facts quickly. These equations can then be used in subtraction. E.g., if a student knows that “ $7 + 7 = 14$ ”, he/she can use this doubles fact to know the answer to “ $14 - 7$ ”.

Symmetrical Subtraction – Prepare a set of cards containing equations related to doubling and grid paper with a line as shown below. Ask the student to draw a card, e.g. $3+3 =$, from the doubles deck and colors squares going horizontally. Extending immediately to the right, the student colors the same number of squares. Using a bold color, the student traces the line of symmetry between the two sets of squares. Finally, she crosses out the squares on one side of the symmetry line and writes the matching subtraction equation below the picture. Each student should create as many sets of doubles as time allows.



Ten Frames are good for developing the part whole relationship for 5 and 10. It is important for students to be able to easily recall the number combinations for 5 and for ten. These understandings are very important in addition and subtraction fact work. Work with 5 and 10 lays the foundation for addition / subtraction of larger numbers. Frequent opportunities for students to practice number bonds to 5 and 10 during math warm-ups or morning routines are helpful.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Observe students as they are flashing number pairs for 5 and 10. Are students able to recall number pairs mentally or are they using manipulatives? (3N10.3)

Resources/Notes

Math Makes Sense 3

Lesson 1 (Cont'd): Strategies for Addition Facts

3N10

TG pp. 4 - 7

Strand: Number

Outcomes

Students will be expected to

3N10 Continued**Achievement Indicator:**

3N10.3 Continued

Elaborations—Strategies for Learning and Teaching

Chants can be fun ways to practice some strategies during morning/daily routines. Try this one for Make Ten strategy:

Say: 9

Students respond : 1

(Repeat for all combinations of 10)

Variations: Say: 9

Students clap, stomp or tap the number needed to make 10.

SNAP Ten - Deal out number cards, face down into 2 stacks. Player 1 lays the top card from his/her stack face up on the table. Player 2 lays the top card from his/her stack face up on the table. If that card makes a sum of 10 with the other card that is already on the table, player 2 should place it next to the other card and call SNAP. He/she has captured the two cards and gets to keep them. If the card does not make a SNAP, it remains face up in the center of the table. As play continues, the new card can be matched with any card that is already on the table that makes the sum of 10. Any player recognizing a match may call SNAP and collect the cards. Play continues until there are no matching cards remaining. The player with the most sets of cards is the winner. Variation: Game can be adapted to work with number pairs to 5.

While students are participating in tasks, encourage them to articulate their mathematical thinking by asking question such as:

- What strategy did you use?
- How did you figure it out?

(continued)

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Three in a row - Provide students with a blank 3 by 3 grid and a deck of cards containing numbers 0 – 9. Ask students to create their own game board by choosing 9 numbers from 0 to 18 to write into their blank 3 x 3 grid. They will place one of each of the nine numbers in each square. They may not write a number more than once. Place the deck of cards between the two players. Each partner draws a card and places it face up on the table. If possible, the partners will use both cards to form an addition or subtraction problem that will give them either a sum or difference on their card. If the sum and difference can be formed from the two cards, students may mark an X on the numbers on their ‘Three in a Row’ Game Board. If the number is not on the board, then the student will not mark a space on the game board. The winner is the student who gets 3 in a row first, vertically, diagonally or horizontally. (3N10.3)

Student-Teacher Dialogue

- Five Frame Flash/ Ten Frame Flash - Quickly show a 10-frame card and ask students to communicate how many more are needed to make 10. Students should show their answers to check accuracy. (3N10.3)

Portfolio

- Create a foldable on 11” x 17” paper. Fold the paper in half, lengthwise and then 3 times the other way. Cut on the fold line on the front piece of paper to form ‘doors’. Ask students to write a strategy on each door. Ask students to write facts that would relate to the strategy under each ‘door’. The last door would be used by students to explain one of the strategies. Ask students to explain one of their strategies to the class. (3N10.1)

Resources/Notes

Math Makes Sense 3

Lesson 1 (Cont’d): Strategies for Addition Facts

3N10

TG pp. 4 - 7

Strand: Number

Outcomes

Students will be expected to

3N10 Continued

Achievement Indicators:

3N10.1 Continued

3N10.4 Recall basic addition facts to 18 and related subtraction facts to solve problems.

Elaborations—Strategies for Learning and Teaching

“Fluency might be manifested in using a combination of mental strategies and jottings on paper or using an algorithm with paper and pencil, particularly when the numbers are large, to produce accurate results quickly. Regardless of the particular method used, students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate, and general” (NCTM, Principles and Standards, 2000, p 32).

If You Didn't Know - Pose the following task to the class: If you did not know the answer to $8 + 5$ (or any fact that you want the students to think about), what are some really good strategies you can use to get the answer? Explain that “really good” means that you don't have to count and you can do it in your head. Encourage students to come up with more than one strategy. Use a think-pair-share approach in which students discuss their ideas with a partner before they share them with the class. (Van de Walle, *Teaching Student-Centered Mathematics Grades K-3* p. 104)

What's the Same about the Zero Facts? - Display several zero facts, some with the 0 as the first addend, some with the 0 as the second addend. Ask students how these facts are alike. Is there a difference? Some students may need counters to visually represent the facts.

Provide pairs of students with snap cubes of two colors. Ask students to work together to create ‘fact families’. Each partner chooses a color and takes a number of cubes (you may designate a number range, for example, between 4 and 9). Students join their sets of cubes together and write a number sentence to reflect the ‘cube train’ (e.g. $4 + 9 = 13$). Students then turn the cube train around ($9 + 4 = 13$). Next, partners write the number they have altogether (13). One partner temporarily removes her/his cubes, and write the new number sentence showing subtraction ($13 - 4 = 9$). The other partner removes his/her cubes and writes the corresponding number sentence ($13 - 9 = 4$).

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to work in pairs to sort related fact cards according to the strategy they would use to solve them. Give students opportunity to justify their sorting. (3N10.1)
- Domino Group Work - Present each group of four students with, dominoes and one index card. The first person writes down an addition fact that goes with the domino and passes the card to the right. The next person writes another addition fact and passes it on. Repeat for two subtraction facts. When the group has completed their fact families they choose another domino and start over. Observe whether students are recognizing that doubles have only 2 facts. (3N10.4)
- Strategy Match (Part A) - Ask the students to work with a partner or in groups of 4. Give the students cards with a variety of facts to 18.

$5 + 6$	$18 - 9$	$7 + 7$	$15 - 7$	$6 + 3$
---------	----------	---------	----------	---------

Ask the students to look at the facts and explain the possible strategies that could be used to solve that fact. (3N10.1)

- Strategy Match (Part B) - Post the following headings: Near Doubles, Doubles, Make Ten, Property of Zero and Think Addition. Ask the students to place a given fact card under one of the headings and justify their placement. This activity should be repeated regularly as part of a Math Routine. (3N10.1)

Journal

- Ask students to explain the _____ strategy. Create problems that could be solved using this strategy.
- Tell students that you do not have to learn to subtract if you know how to add. Ask them if they agree or disagree? Why or why not? (3N10.1)

Resources/Notes

*Math Makes Sense 3***Lesson 2:** Relating Addition and Subtraction

3N10

TG pp. 8 - 10

Additional Activity:

Fastest Facts

TG: p. vi and 61

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

3PR3 Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.

[C, CN, PS, R, V]

Elaborations—Strategies for Learning and Teaching

An equation is a mathematical sentence with an equal sign. The amount on one side of the equals sign has the same value as the amount on the other side. For some students the equal sign poses a difficulty. (Keep in mind when using examples that students are working with facts to 18). Although they are comfortable with $4 + 5 = \square$, they interpret the equal sign to mean “find the answer”. Therefore when students see the sentence $\square - 4 = 5$, they may not be sure what to do as they think the answer is already there. Similarly, students may solve $4 + \square = 5$ by adding 4 and 5 to “get the answer”. The notion of an equation as an expression of balance is not apparent to them. It is important for students to recognize that the equal sign is viewed as a way to say that the same number has two different names, one on either side of the equal sign. The equal sign is “a symbol of equivalence and balance”. Small (2008) p. 586

The term ‘equation’ can be added to word walls and/or dictionaries and should be pointed out often.

The focus of this outcome is to ask students to develop strategies to help them solve equations when there is a symbol representing an unknown number, for basic addition facts to 18 and related subtraction facts. E.g.,

$$9 + \Delta = 16$$

$$16 - \Delta = 9$$

It is also very important to read and interpret equations in a meaningful way. In reading $9 + \Delta = 16$ you may say, “What do I need to add to 9 to get 16 ? or “If 16 is made up of two parts, and one part is 9, how many are in the other part?”

The book, *Equal Shmequal* by Virginia Kroll, would be useful in teaching this concept. Before reading the book, ask students to brainstorm the meaning of ‘equal’. Encourage symbols or examples as they come up. Read the story aloud. Model, using counters on a balance scale, each animal – the bee = 1, mouse = 2, etc. Demonstrate a balance of the animals, like a teeter totter. Ask students to explore the concept (preferably on their own balances or working in pairs), and continue to link the animals to the story, challenging them, for example, to balance a bear and two rabbits. Use language such as balance, equal, equality, sum, etc., as you demonstrate writing number sentences to match the balances.

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Resources/Notes

Math Makes Sense 3

Lesson 3: Addition and Subtraction Equations

PR3

TG pp. 11 – 14

Children’s Literature (not provided):

Kroll, Virginia. *Equal Shmequal*
ISBN: 1-57091-891-0

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

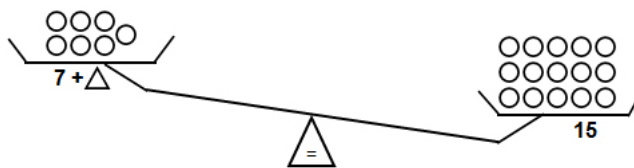
3PR3 Continued

Achievement Indicators:

3PR3.1 Explain the purpose of the symbol in a given addition or subtraction equation with one unknown.

Elaborations—Strategies for Learning and Teaching

Using a balance scale, counters (or other stacking manipulatives) and a recording sheet, ask students to place counters on the balance scale to represent the equation $7 + \Delta = 15$ by placing 7 counters in the left pan and 15 counters in the right pan.



Ask students to predict how many more counters are needed in the left pan to balance the scale. Record their predictions on a recording sheet (as shown below). Students add counters to the left pan to see if their predictions are correct and to determine the missing addend. Next, they complete the recording sheet. Ask them to repeat this task using other equations with one unknown number. Through this investigation and discussion, students should see that the symbol Δ representing the unknown number must be a number that will balance the equation.

equation with unknown #'s	prediction of unknown #'s	actual unknown #
$7 + \triangle = 15$		$7 + 8 = 15$

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies*Performance*

- Using a balance scale, ask students to demonstrate how to find the unknown numbers of the equations given ($11 = \Delta + 5$ or $15 = 18 - \Delta$). Ask questions like, how does the scale help you find the unknown numbers in the following equations:

$$11 = \Delta + 5$$

$$15 = 18 - \Delta$$

$$\Delta + 4 = 12$$

$$16 - \Delta = 9$$

(3PR3.1)

Resources/Notes*Math Makes Sense 3***Lesson 3 (Cont'd):** Addition and Subtraction Equations**PR3**

TG pp. 11 – 14

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

3PR3 Continued

Achievement Indicators:

3PR3.2 Create an addition or subtraction equation with one unknown to represent a given combining or separating action.

3PR3.3 Provide an alternative symbol for the unknown in a given addition or subtraction equation.

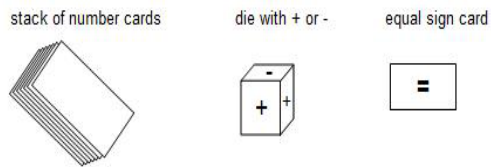
Elaborations—Strategies for Learning and Teaching

Prepare a deck of number cards and an ‘operations’ dice (you may use a regular dice and cover the numbers with stickers containing the operations). Have a student choose 2 cards from the deck and roll the die to find the operation. E.g. 8, 3, operation -.

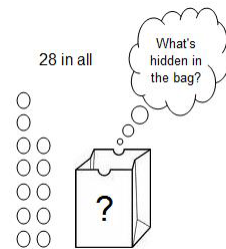
Ask the student to place one of the numbers first, then the operation card and finally the second number after the equal sign. E.g., $8 - ? = 3$

Ask the student to record the equation on a recording sheet using a symbol to represent the unknown number.

Ask the student to determine the missing number and explain how he/she arrived at the answer.



Present students with counters, blocks, link-its, etc. Working in pairs, have Student A take a handful of objects and count to find the total. Student B should record the total. Next, Student A takes some of the objects and puts them in a paper bag and asks, “What’s Hidden?”. Student B creates an addition or subtraction equation to find the missing part. Then they dump the objects and check the solution. They change rolls and repeat the process.



Explain to students that a symbol is not a complex picture that it is a simple representation.

Students should be exposed to using varying symbols to represent the unknown. For example, a square, circle or triangle can be used.

$6 + \Delta = 18$ $6 + \square = 18$

 General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Paper and Pencil

- ‘Number of the Day’ Equations - Ask students to create addition and subtraction equations, with unknowns and with the ‘Number of the Day’ on one side of the equation. E.g., The ‘Number of the Day’ is 16. Possible equations with an unknown could include:

$$16 = 8 + \Delta$$

$$\square + 6 = 16$$

$$18 - \square = 16$$

(3PR3.2, 3PR3.3)

- Ask students to create their own addition and subtraction equations with an unknown number. Encourage them to create different symbols to represent the unknown numbers. Play music and ask students to walk around the room. When the music stops, students give their equation to a classmate standing near them. They then take the equation card to their desks to find the unknown and explain to the student, who created the problem, how they arrived at the answer. (3PR3.2, 3PR3.3)

Resources/Notes

Math Makes Sense 3

Lesson 3 (Cont'd): Addition and Subtraction Equations

PR3

TG pp. 11 – 14

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

3PR3 Continued

Achievement Indicators:

3PR3.4 Solve a given addition or subtraction equation with one unknown that represents combining or separating actions, using manipulatives.

3PR3.5 Solve a given addition or subtraction equation with one unknown, using a variety of strategies, including guess and test.

Elaborations—Strategies for Learning and Teaching

Present students with varying problems like:

Ms. Best needs 18 pieces of construction paper for art class. She has 7 pieces, how many more pieces of construction paper does she need?

Students use manipulatives to solve the problem. Observe to see if students start with 18 and separate 7 from the group to find the unknown or if they start with 7 and add up to 18.

To solve addition or subtraction equations with one unknown, students need to explore different strategies. One strategy is with the use of manipulatives outlined in 3PR3.4.

Other examples of strategies may include, but are not limited to, the following:

Guess and Test strategy - This strategy is based on trying different numbers. The key is to think after each try and change or revise guess when necessary. E.g., $7 + \Delta = 16$

(Think $7 + 7 = 14$, that is too low.

Think $7 + 8 = 15$, that is too low but close to 16.

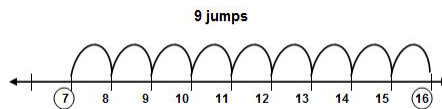
Think $7 + 9 = 16$. So the missing number is 9).

Mental Math strategy - E.g., $7 + \Delta = 16$

(Think doubles. I know $7 + 7 = 14$.

14 is only 2 away from 16 so the missing number must be 9).

Number Line strategy - Create a number line with the start point being 7. Then count up to 16, keeping track by using the number line. E.g., $7 + \Delta = 16$



 General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Performance

- Present students with an equation where there is an unknown and ask them to model with manipulatives how to find the missing number. (3PR3.4)

Portfolio

- Present students with equations, involving addition and subtraction, where there is one unknown number on either side of the equal sign.

E.g., $15 - \Delta = 9$

$$\Delta + 8 = 13$$

$$17 = \text{pentagon} + 11,$$

$$7 = \text{pentagon} - 4$$

Ask students to solve the equations and then choose one and explain their strategy. (3PR3.5)

Resources/Notes

Math Makes Sense 3

Lesson 3 (Cont'd): Addition and Subtraction Equations

PR3

TG pp. 11 – 14

Strand: Patterns and Relations (Variables and Equations)

Outcomes

Students will be expected to

3PR3 Continued

Achievement Indicators:

3PR3.6 Solve a given addition or subtraction equation when the unknown is on the left or the right side of the equation.

3PR3.7 Explain why the unknown in a given addition or subtraction equation has only one value.

Elaborations—Strategies for Learning and Teaching

It is important that students read and solve equations when the unknown number is on either the left of the equals sign or the right of the equal sign.

Example of unknown on the left: $12 + \Delta = 18$

Example of unknown on the right: $18 = \Delta + 12$

Present students with an equation such as:

$$17 = 8 + \Delta$$

Demonstrate, using manipulatives, how to find the unknown number. Begin with 17 counters. Secretly place 8 under a cup. Ask students to tell you how many you put under the cup by viewing what is left. Ask other guiding questions like:

Could the number be anything else?

After demonstrating this process to students, ask students to find missing numbers in various equations using manipulatives.

After experimenting with solving equations with unknowns using concrete materials present students with a task similar to the following. Tell students that there are 18 counters. Show them 5 and ask them what the missing part must be.

Counters in My Pocket - Say: "I have 15 counters. Five are in my hand." Ask: "How many are in my pocket? How do you know?"

 General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Performance

- Present student with two numbers and ask them to create equations where one of the numbers are unknown. E.g., 14, 6

Possible equations: $14 - \square = 6$, $6 + \Delta = 14$, $14 = 6 + \Delta$, etc.
(3PR3.6)

Paper and Pencil

- Present students with equations where one part is unknown. Ask students to record the equation including the missing part. (3PR3.6)

Journal

- Ask students to respond to the following:
 - (i) Sean says if he makes 16 cupcakes and only puts icing on 7, there will be 9 without icing. Do you agree or disagree?
(3PR3.7)
 - (ii) Sara saw $14 = 6 + \Delta$
She said that the Δ represents 10. Is she correct? Explain using pictures, numbers and words. (3PR3.6)

Resources/Notes

Math Makes Sense 3

Lesson 3 (Cont'd): Addition and Subtraction Equations

PR3

TG pp. 11 – 14

Strand: Number

Outcomes

Students will be expected to

3N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.

[C, ME, PS, R]

Achievement Indicators:

*3N8.1 Estimate the solution for a given problem involving the sum of two 2-digit numerals;
e.g., to estimate the sum of 43 + 56, use 40 + 50 (the sum is close to 90).*

Elaborations—Strategies for Learning and Teaching

Estimation is a mental “process of producing an answer that is sufficiently close to allow decisions to be made” (Reys 1986, p. 22).

“Students should be encouraged to explain their thinking, frequently, as they estimate. As with exact computation, sharing estimation strategies allows students access to others’ thinking and provides many opportunities for rich class discussions.” (*Principles and Standards for School Mathematics*, 2000, p. 156).

When students estimate first and then calculate, they refine their estimation strategies. When estimating, the context will determine if an exact answer or an estimate is appropriate and whether a high estimate or a low estimate is more appropriate. In discussing estimating sums and differences, give students the following context:

Karen is taking piano lessons and her piano teacher asked her approximately how much time she practiced on Saturday and Sunday. Karen knew she practised 43 minutes on Saturday and 56 minutes on Sunday. To find an estimate for $43 + 56$, Karen may use one of the strategies below:

Front-end Strategy - The front-end strategy is a method of estimating computations by keeping the first digit in each of the numbers and changing all the other digits to zeros. This strategy can be used to estimate sums and differences. Note that the front-end strategy always gives an underestimate for sums. Think: $43 \rightarrow 40$ and $56 \rightarrow 50$. $40 + 50 = 90$. Karen could say she practiced about 90 minutes.

Round each number to the nearest multiple of 10. E.g., $43 + 56 = \underline{\quad}$
Think: 43 can be rounded to 40 and 56 can be rounded to 60 so $40 + 60 = 100$. Karen could say she practiced about 100 minutes.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Tell students that Matthew has 95¢. He wants to buy a pack of gum that cost 50¢ and a bottle of water that cost 35¢. He estimates that he does not have enough money to buy both. Is he correct? Use pictures, numbers and words to explain. (3N8.1)

Journal

- Ask students to respond to the following:
 - Ryan estimated that $35 + 46$ would be about 70. What strategy might he have used for his estimate?
 - Julia needs 24 popsicle sticks for her art project. She has 15 collected. She estimates that she will need about 10 more to make 24. Is her estimate reasonable? Use pictures, numbers and words to explain. (3N8.1)

Performance

- Estimating Sums - Students play in pairs. Students will take turns choosing two numbers from the game board and circling them. Next they add the two numbers using an estimation strategy. Students record points according to the chart below and keep playing until all the numbers on the board are used up.

23	29	68	19	74
5	11	18	21	49
60	13	33	27	15
12	36	35	55	8
15	39	17	46	44

Total Between	0 and 20	100 and 150	20 and 40	40 and 60	60 and 80	80 and 100
Points	1	2	3	2	1	0

The player with the highest score is the winner. After giving the students several opportunities to play this estimating game, ask students: How did estimating help you get more points? Explain your estimation strategy.

Resources/Notes

*Math Makes Sense 3***Lesson 4:** Estimating Sums

3N8

TG pp. 15 - 17

Children's Literature (provided):

Goldstone, Bruce.

*Greater Estimations***Additional Reading** (provided):Small, Marian (2008) *Making Math Meaningful to Canadian Students*, K-8 p.160-161

Strand: Number

Outcomes

Students will be expected to

3N9 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by:

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in context that involve addition and subtraction of numbers.

[C, CN, ME, PS, R, V]

Elaborations—Strategies for Learning and Teaching

Research has shown that students will create different strategies for adding and subtracting. A classroom climate that fosters communication and sharing of personal strategies will allow for many methods to be explored. Students will choose strategies that make sense to them.

Some examples of personal strategies for addition and subtraction are provided. These strategies can be used for 3 digit addition and subtraction as well.

Personal Strategies for Addition

Add Tens, Add Ones, Then Combine	Move Some to Make Tens
36+58 30 and 50 is 80 6 and 8 is 14 80 and 14 is 94	36 +58 Take 2 from the 36 and put it with 58 to make 60. Now you have 34 and 60 more is 94.
Add on Tens, Then Add Ones	Use a Friendly Number and Compensate
36 +58 36 and 50 make 86 Then I added on the other 8. 86 and 4 is 90 and 4 more is 94	36 + 58 36 and 60 is 96. That's 2 extra so it's 94.

Personal Strategies for Subtraction

Subtract Tens from the Tens, Then Subtract Ones	Subtract Tens, Then Ones
83-36 80 subtract 30 is 50. Subtract 6 more is 44. Now add in the 3 ones left from the 83. It's 47	83-36 83 subtract 30 is 53 Then subtract 6: 3 makes 50 and 3 more is 47
Subtract Extra Tens, Then Add Back	Add to the Whole then Compensate
83-36 83 subtract 40 is 43. That is 4 too many. 43 and 4 is 47.	83-36 Give 3 to 83 to make 86. 86 subtract 36 is 50 now give back the 3. It is 47.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Resources/Notes

Math Makes Sense 3

Lesson 5: Adding 2-Digit Numbers

3N9

TG pp. 18 - 21

Additional Activity:

First to 10

TG: p. vi and 62

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicator:

3N9.1 Model the addition of two or more given numbers, using concrete or visual representations, and record the process symbolically.

Elaborations—Strategies for Learning and Teaching

Visual representations may include, but are not limited to, hundreds charts, number lines, place value mats and base ten materials.

What’s in the Basket? - Provide a basket, Base ten materials (rods and small cubes) and a recording sheet.

Numbers	Base 10 Representation	Addition Problem
45		$40 + 30 = 70$
36		$5 + 6 = 11$
		$70 + 11 = 81$
45		
36		
		$\begin{array}{r} 1 \\ 45 \\ + 36 \\ \hline 81 \end{array}$

Students work in pairs. Player A chooses a handful of base ten rods and small cubes to represent a 2 digit number. Both players record the number on their recording sheet. Player A puts his base ten materials into the basket. Player B repeats the process. Both players write an addition problem to represent the joining of the base ten materials that were selected. After both partners figure out the total, they count the value of the base ten materials in the basket and check to confirm their answer.

Give students a deck of number cards. Ask students to choose 2 or more cards from the deck. Write the addition equation and then find the sum using a hundreds chart or number line. Observe the students as they are solving the equation. Ask students to explain their solution. Which number are they starting with? What strategies are they using for adding on the hundreds chart?

E.g., $29 + 36 =$

Example of student explanation may be:
 “I started with 36 because it’s the largest number. I moved down 3 rows on the hundreds chart which is 30, which is 1 more than 29 so then I moved back one space. So $29 + 36 = 65$ ”

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Present students with story problems such as Eric has 27 hockey cards, Shania has 42 hockey cards and Jenna has 29 hockey cards. If the children combined their collections, how many hockey cards would they have all together?

Ask students to model the addition problem with base-ten blocks and record in their math journal. (3N9.1)

Resources/Notes

Math Makes Sense 3

Lesson 5 (Cont'd): Adding 2-Digit Numbers

3N9

TG pp. 18 - 21

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicators:

3N9.2 Create an addition or subtraction story problem for a given solution.

3N9.3 Determine the sum of two given numbers, using a personal strategy; e.g., for $326 + 48$, record $300 + 60 + 14$.

Elaborations—Strategies for Learning and Teaching

When tasks involving computation are rooted in problems, students see the purpose in using computation. Take advantage of problems that arise daily to create story problems. E.g., giving back change from a recess order, ordering books for a book order, etc. The ‘Number of the Day’ can be given as a solution and ask students to create an addition or subtraction story for the solution.

Quick Draw Addition - Prepare a bag of 2-digit numeral cards and a recording sheet. For this task, students work in pairs.

Numbers Drawn	Show how you found the sum
1.	
2.	
3.	
4.	
5.	
Choose one of your problems and explain your strategy	

Ask students to choose two numeral cards. They add the numbers together to find the sum, using any strategy they want. After 5 draws students choose any addition problem and explain their strategy.

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Portfolio

- Present students with a given solution and ask them to create addition or subtraction story problems. Students can illustrate their problems with a visual and present to the class. (3N9.3)

Journal

- Ask students to respond to the following:
How would you find the sum of 322 and 86? Can you use a different strategy? (3N9.3)

Paper and Pencil

- Exit cards - Give student 1-, 2-, or 3-digit numbers (as appropriate for the time of the year) and an 'exit card'. E.g., 27 and 45. Before the class ends, students are asked to create a story problem using the given numbers and then solve it using pictures, numbers and words. Students pass in their 'exit cards' as they leave the class. This type of assessment can be repeated often throughout the year. (3N9.2)

Resources/Notes

Math Makes Sense 3

Lesson 5 (Cont'd): Adding 2-Digit Numbers

3N9

TG pp. 18 - 21

Strand: Number

Outcomes

Students will be expected to

3N6 Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:

- adding from left to right
- taking one addend to the nearest multiple of ten and then compensating
- using doubles.

[C, CN, ME, PS, R, V]

Achievement Indicators:

3N6.1 Add two given 2-digit numerals, using a mental mathematics strategy, and explain or illustrate the strategy.

3N6.2 Explain how to use the “adding from left to right” strategy; e.g., to determine the sum of $23 + 46$, think $20 + 40$ and $3 + 6$.

3N6.3 Explain how to use the “taking one addend to the nearest multiple of ten and then compensating” strategy; e.g., to determine the sum of $28 + 47$, think $30 + 47 - 2$ or $50 + 28 - 3$.

Elaborations—Strategies for Learning and Teaching

Students invent many strategies over time, but will eventually settle on two or three that are most efficient for them. Record students’ thinking on the board for all students to see as this will help other students try the strategies as well. Hearing others explain their reasoning helps students develop mathematical language as well as written communication about their mental math strategies.

The two parts that make up the whole are the addends. For example, in $23 + 46 = 69$, the ‘23’ and ‘46’ are the addends. It is not necessary to expect students to use these terms. However, it is good for you to model this language as it gives students a name for these particular numbers if they wish to.

Adding left to right

Add the tens and add the ones and then combine them together

E.g., $46 + 12 =$

$40 + 10 = 50$

$6 + 2 = 8$

$50 + 8 = 58$

So $46 + 12 = 68$

Taking one addend to the nearest multiple of 10 and then compensating

E.g., $69 + 28 =$

69 is close to 70

$70 + 28 = 98$

69 + 28 is 1 less

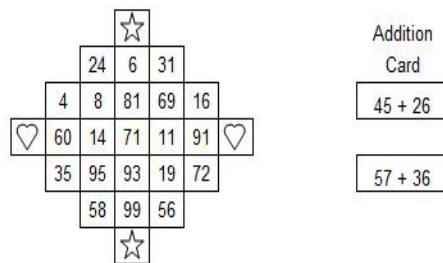
So $69 + 28 = 97$

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Stars and Hearts - Present students with a deck of 2-digit addition equations whose sums are on the game board illustrated below. Students shuffle the cards. Player 1 picks a card, solves the equation and explains the strategy to his partner. If the sum is on the game board he/she may cover the number with a counter. Player 2 then chooses a card from the deck and repeats the process. The winner is the first player to cover 3 numbers in a row on the board.



(3N6.1)

Presentation

- Show and Tell - Students pick a 2-digit number expression, spend time preparing a presentation on how they would mentally add the numbers and explain it to their group or to the class. Students may use visuals and or concrete materials to aid in their explanation.
E.g., $23 + 87$ (3N6.1, 3N6.5)

Resources/Notes

Math Makes Sense 3

Lesson 6: Using Mental Math to Add

3N6

TG pp. 22 - 23

Additional Reading (provided):

Van de Walle , John A. and Lovin, LouAnn (2006) *Teaching Student Centered Mathematics 3 - 5*, pp.100 - 112

Strand: Number

Outcomes

Students will be expected to

3N6 Continued

Achievement Indicators:

3N6.4 Explain how to use the “using doubles” strategy; e.g., to determine the sum of $24 + 26$, think $25 + 25$; to determine the sum of $25 + 26$, think $25 + 25 + 1$ or doubles plus 1.

3N6.5 Apply a mental mathematics strategy for adding two given 2-digit numerals.

Elaborations—Strategies for Learning and Teaching

Using Doubles

Use a doubles fact you know to help find the sum

E.g., $32 + 30 =$

$30 + 30 = 60$

$32 + 30$ is 2 more

So $32 + 30 = 62$

During Daily Warm-ups or Morning Routines, is an excellent time to apply and reinforce mental math strategies. E.g., Ask: If it is the 16th of the month, what will the date be in 2 weeks? Ask student to tell the class which strategy he/she used to arrive at an answer.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- **Pick and Add** - Students will work with a partner. The object of the game is to get to 100 first. Students will need a recording sheet each, and a deck of 2-digit number cards between them. Player 1 chooses a card from the deck and adds it to the starting point of zero. They record their equation and the new starting point. Player 2 chooses a card and records the equation, and his/her new starting point. Play continues with students taking turns and adding to their running total. The winner is the student who reaches 100 first.

Recording Sheet

Starting Point	Equation
0	$0 + 22 = 22$
22	$22 + 13 = 35$
35	

Students choose one equation and explain or illustrate the strategy they used. Then share their strategy with their partner.

(3N6.5)

Student-Teacher Dialogue

- In a conversation with a student ask:
 - What is the sum of $25+28$? Which strategy did you use?
 - What is the sum of $39+28$? Which strategy did you use?
 - What is the sum of $64+33$? Which strategy did you use?

(3N6.2, 3N6.3, 3N6.4)

Resources/Notes

Math Makes Sense 3

Lesson 6: Using Mental Math to Add

3N6

TG pp. 22 - 23

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Elaborations—Strategies for Learning and Teaching

In Grade 3, students continue to work on combining and separating larger numbers in a variety of ways as they solve 2- and 3-digit addition and subtraction problems. Allowing students to use personal strategies will add to their understanding of number and provide a concrete foundation for flexible methods of computation. Some students may choose to use base-ten materials on a place value mat, a hundred chart, etc. Provide a variety of materials for students to manipulate as they use strategies that is most meaningful to them.

E.g., $245 + 330$ can be viewed as $200 + 45 + 300 + 30$, then $200 + 300$ and $45 + 30$. Strategies invented by classmates should be discussed, shared and explored by others. This allows for exposure to a variety of strategies so that students can choose those that make sense to them. Personal strategies are generally faster than the traditional algorithm and makes sense to the person using them.

It is important to reinforce proper mathematics vocabulary. “The terms ‘regroup’, ‘trade’ and ‘exchange’ are used rather than the terms ‘carry’ or ‘borrow’. This is because carrying and borrowing have no real meaning with respect to the operation being performed, but the term ‘regroup’ suitably describes the action the student must take” (Small, 2008 p.170).

It is also important that the addition and subtraction of numbers be put into a context for students. Students enjoy learning when it makes sense to them. As much as possible, create stories to paint a picture for why it is necessary for them to perform the operation and arrive at an answer.

Achievement Indicator:

3N9.1 Continued

Having students use models is vital in understanding the relationship between the physical action of joining and or separating two groups and the symbolic representation. Students can use base-ten materials to concretely represent the joining and separating of groups.

Students use a spinner to find two 3 digit numbers. They create a number sentence and explain the strategy they used to solve the problem. Then students use base-ten materials to show their workings concretely and visually.

Numbers Spun	Number Sentence	Strategy Used	Use Base 10 Blocks to show your working

(continued)

General Outcome: Develop Number Sense

Suggested Assessment Strategies*Performance*

- Tell students that two schools are joining together to raise money to contribute to a children's hospital. One school raised \$121.00 and the other school raised \$193.00. Ask students to model the addition of the two numbers (i.e. 121 and 193) using base-ten materials. Ask students to record their work pictorially and symbolically to show how they solved the equation. Discuss with the students if this strategy worked well for them or if they have another strategy that they would prefer to use. This task can be repeated regularly throughout the year, beginning with 1-digit numbers and progressing through to 2-digit and 3-digit numbers.
(3N9.1)

Resources/Notes*Math Makes Sense 3***Lesson 7: Adding 3-Digit Numbers****3N9**

TG pp. 24 - 27

Game: Tic Tac Add**3N9**

TG p. 28

Additional Activity:

Tic-Tac-Toe Squares

TG: p. vi, 63 and 64

Additional Reading (provided):Van de Walle, John A. and Lovin, LouAnn (2006) *Teaching Student Centered Mathematics Grades K-3*, p. 158

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicators:

3N9.2 Continued

3N9.3 Continued

3N9.4 Refine personal strategies to increase their efficiency.

3N9.5 Solve a given problem involving the sum or difference of two given numbers.

Elaborations—Strategies for Learning and Teaching

It is important that students be involved in solving meaningful and worthwhile addition and subtraction tasks that connect to everyday life. Model the creation of stories in mathematics routines by using the date or number of days in school as a given solution. Students can use games, scores, money and other relevant experiences to help create their own stories for any number.

Sum it Up - The object of this task is to make the greatest sum. Provide students with two decks of number cards; deck A - 3 digit numbers, deck B - 2 Digit numbers. Students choose a card from each deck and find the sum using their personal strategy. Ask students to record their work. After completing this centre, ask students to identify their largest sum and place the number on a number line.

Through various experiences working individually and with small and whole group, students will have opportunities to discover their own personal strategies for computation. “The goal may be that each student has at least one or two methods that are reasonably efficient, mathematically correct, and useful with lots of different numbers. Expect different students to settle on different strategies.” (Van De Walle, *Teaching Student-Centered Mathematics Grades K-3*, p. 165,)

Whatever strategy students use, they need to be encouraged to understand and explain why it work.

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Portfolio

- Ask students to create and write an addition and /or subtraction story problem for a given solution. If the answer is 121, what could the problem be? Ask students to write the corresponding number sentence and then solve the problem using pictures numbers and words. This assessment lends itself well to being part of a mathematics routine an should be repeated throughout the year using a variety of 1-, 2- and 3- digit numerals.

(3N9.2)

Student-Teacher Dialogue

- Provide students with two numbers. Ask students to find the sum and explain the strategy they have used. Students may use base ten or other manipulatives to aid in their explanation. Observe students for correct use of math language and depth of understanding. (3N9.3)

Performance

- Players each draw two 2 and/or 3-digit numeral cards and adds them. The player with the largest sum collects all cards. In the event of a tie each player keeps one card, selects another and finds the new sum. The discarded card goes to the bottom of the deck. Play ends when there are no cards left for each person to select 2 cards. (Another version can be played using subtraction – the largest difference collects the cards).

Question students' thinking by asking what strategy they used to find the sum or difference. (N9.4, N9.5)

Resources/Notes

Math Makes Sense 3

Lesson 7 (Cont'd): Adding 3-Digit Numbers

3N9

TG pp. 24 - 27

Strand: Number

Outcomes

Students will be expected to

3N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.

[C, ME, PS, R]

Achievement Indicator:

3N8.2 Estimate the solution for a given problem involving the difference of two 2-digit numerals; e.g., to estimate the difference of $56 - 23$, use $50 - 20$ (the difference is close to 30).

Elaborations—Strategies for Learning and Teaching

Estimating sums and differences is valuable because it helps predict an answer and check a calculation. When using estimation in a problem solving context, there are important things to keep in mind. What is best, an exact answer or an estimate? How important is it for the estimate to be close to the exact value? Is it better to have a low or high estimate?

The following are some strategies to explore:

Front-end Strategy – When estimating $77 - 24$

Write each number to the number of tens. 77 has 7 tens. 24 has 2 tens. Subtract the tens: 7 tens subtract 2 tens = 5 tens. The estimate is about 50.

Closest ten Strategy – When estimating $77 - 24$

Write each number as an approximation by rounding the number to the closest ten. For example 77 is 3 away from 80 so we round to 80. 24 is 4 away from 20. Subtract: $80 - 20 = 60$

Number of Tens Strategy – When estimating $77 - 24$

Using the number of tens to determine estimate. For example 24 has two tens. Subtract 2 tens: $77 - 20 = 57$.

Estimating Differences – Students play in pairs. One at a time, students choose two numbers from the game board and circle them.

99	24	42	22
35	54	4	34
3	32	62	43
40	12	44	55
27	52	91	16

Next the student estimates the difference between the two numbers. The student checks to see the range in which the estimate falls on the chart below and records his/her points. Keep playing until all the numbers are used up. The player with the highest score wins. Ask students: How did estimating help you get more points?

Total	0 to 10	11 to 29	30 to 49	50 to 69	70 to 89	90 or more
Points	1	2	3	2	1	0

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:
 - (i) There are 63 pencils left in the Grade 3 classroom supplies. There are 25 students and each child gets a new pencil. About how many pencils are left in the classroom supplies? Lisa estimated 40 pencils are left and Yolanda estimated 43 pencils are left. The class agrees with both estimates. Using pictures, numbers and words explain how this is possible.
 - (ii) Erin has 83 coloured beads to make necklaces for her friends. She uses 37 beads to make a necklace for Julia. About how many beads does Erin have left?
- (3N8.2)

Student-Teacher Dialogue

- Within the Range - Write 2 numbers on the board. E.g., 28 ↔ 38. Ask students to find combinations of numbers that, when added or subtracted, fall within the range of the given numbers.
 - E.g., 40 - 4 falls within the range of 28 and 38.This activity lends itself well to a mathematics routine and can be repeated using 1- and 2- digit numerals.
- (3N8.2)

Resources/Notes

*Math Makes Sense 3***Lesson 8: Estimating Differences****3N8**

TG pp. 29 - 31

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicator:

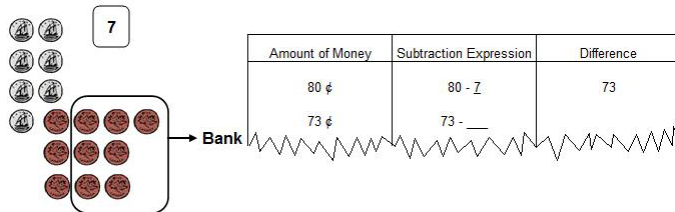
3N9.6 Model the subtraction of two given numbers, using concrete or visual representations, and record the process symbolically.

Elaborations—Strategies for Learning and Teaching

In subtraction, the minuend is the whole, the number on the top in the vertical form or the first number in the horizontal form. For example, in $12 - 10 = 2$, 12 is the minuend. It is not necessary to expect students to use these terms, however, it is good to expose them to the language.

Literature connection - *Shark Swimathon* by Stuart J. Murphy. Read the story together and ask the students to describe what is happening in each illustration. Talk about what Coach Blue writes on the sign at the end of each day. Ask “How many laps did the team swim at the end of the day?”, “How many more laps do they need to swim?” Discuss the strategy Coach Blue used to subtract. Encourage students to pose other strategies that can be used to subtract.

Money Be Gone - Provide students with 8 dimes, 50 pennies for the bank and a deck of number cards (1 through 15).



Each player starts with 8 dimes. Shuffle the deck of number cards and place face down. Taking turns, each player takes a card and subtracts that amount to give to the bank. If the player does not have the exact change, he/she must exchange a dime for 10 pennies and then subtract the amount on the card. The ‘winner’ is the player who gets rid of all of their money first. Place 40 dimes and 50 pennies for the ‘bank’. Each player starts with two 1 dollar coins. Taking turns, players roll two dice, create a 2-digit number from their roll and then subtract that amount to give to the bank.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Portfolio

- Present students with two multi digit numbers. Ask students to find the difference and model their thinking using one of the following: base-ten, hundreds chart, number line, money, etc. (3N9.6)

Paper and Pencil

- Spin the spinner twice and record the numbers. Write the subtraction problem. Use base-ten materials to represent the minuend concretely and pictorially. Subtract the other number from the base-ten materials, making all necessary trades and recording the changes on the recording sheet. E.g.,

Numbers	Show how you found the difference	Base 10 Representation
47	$47 - 29 =$	
29	$47 - 20 = 27$ $27 - 9 = 18$	

(3N9.6)

Journal

- Havy Jo’s best score on her video game yesterday was 43. Her score today is 95. How many points did Havy Jo earn today? Ask students to explain their thinking. (3N9.4, 3N9.5)

Resources/Notes

Math Makes Sense 3

Lesson 9: Subtracting 2-Digit Numbers

3N9

TG pp. 32 - 35

Additional Activity:

Let’s Go Shopping

TG: p. vi and 65

Children’s Literature (not provided):

Murphy, Stuart J.

Shark Swimathon

ISBN: 978-0064467353

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicators:

3N9.7 Determine the difference of two given numbers, using a personal strategy; e.g., for $127 - 38$, record $38 + 2 + 80 + 7$ or $127 - 20 - 10 - 8$.

3N7 Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:

- taking the subtrahend to the nearest multiple of ten and then compensating
- think addition
- using doubles.

[C, CN, ME, PS, R, V]

3N7.1 Subtract two given 2-digit numerals, using a mental mathematics strategy, and explain or model the strategy used.

Elaborations—Strategies for Learning and Teaching

Connect Three - Player 1 chooses 2 numbers from the list (shown below). Player 1 subtracts the 2 numbers. If the difference is on the grid, he/she may place a counter on that square. Player 2 repeats the process using a different colored counter. Once a number is covered it cannot be covered again. The winner is the person to get 3 counters in a row, horizontally, vertically or diagonally.

Observe students as they play the game. Question students about the strategies they are using to find the difference. It is important to note whether they are subtracting the smaller number from the larger number.

list

82	41	79	35	56	27	60
29	19	52	38			
33	41	15	55			
22	14	44	26			
47	23	25	21			

Through games and centres such as Subtraction Rounds, observe and question the mental math strategies that students are using to find the difference between two 2 digit numbers.

Subtraction Rounds - Choose a student to help model this game to the class. Shuffle and divide a stack of 2-digit number cards evenly between both players. Each player, in turn, flips the tops two cards from his/her own pile and calculates the difference between the numbers. He/She, records the number sentence, the difference and explains the strategy used. The differences are totalled after 5 rounds and the player with the lowest score wins.

	Player 1		Player 2	
	Subtraction Sentence	Difference	Subtraction Sentence	Difference
Round 1				
Round 2				
Round 3				
Round 4				
Round 5				
Total				

General Outcome: Develop Number Sense

Suggested Assessment Strategies*Performance*

- Present students with a subtraction problem. E.g.,
Cameron has 73 dinkies. He shares 47 of them with his brother, Jacob. How many does Cameron have now? Ask students to solve and explain their strategy. (3N9.7)
- Show And Tell - Students pick a 2 digit number expression, spend time preparing a presentation on how they would mentally subtract the numbers and explain it to their group or to the class. Students may use visuals and or concrete materials to aid in their explanation. (3N9.7)

Resources/Notes*Math Makes Sense 3***Lesson 9 (Cont'd):** Subtracting 2-Digit Numbers**3N9**

TG pp. 32 - 35

*Math Makes Sense 3***Lesson 10:** Mental Math to Subtract**3N7**

TG pp. 36 - 37

Strand: Number

Outcomes

Students will be expected to

3N7 Continued

Achievement Indicators:

3N7.2 Explain how to use the “taking the subtrahend to the nearest multiple of ten and then compensating” strategy; e.g., to determine the difference of $48 - 19$, think $48 - 20 + 1$.

3N7.3 Explain how to use the “think addition” strategy; e.g., to determine the difference of $62 - 45$, think $45 + 5$, then $50 + 12$ and then $5 + 12$.

3N7.4 Explain how to use the “using doubles” strategy; e.g., to determine the difference of $24 - 12$, think $12 + 12 = 24$.

3N7.5 Apply a mental mathematics strategy for subtracting two given 2-digit numerals.

Elaborations—Strategies for Learning and Teaching

Math Strategies:

Taking the subtrahend to the nearest multiple of ten and then compensating. E.g.,

$$69 - 28 =$$

28 is close to 30

$$69 - 30 = 39$$

39 + 2 more

$$\text{So } 39 + 2 = 41$$

Think addition

E.g., To determine the difference between 62 and 45, think: 5 more than 45 will get me to 50, 10 is 60... I've added 15 so far and 2 more is 62, so my difference is 17.

Using Doubles

Use a doubles fact you know, to help find the difference. E.g.,

$$62 - 30 =$$

$$30 + 30 = 60$$

$$60 - 30 = 30$$

32 is 2 more than 30

$$\text{So } 62 - 30 = 32$$

Since not all students invent strategies, it is important that strategies invented by classmates need to be discussed, shared and explored by others. This allows for exposure to a variety of strategies for students to choose ones that make sense to them. A good place to reinforce mental math strategies would be during a morning routine or in math warm-ups.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Loop Game - A loop game is a fun way for students to practice mental math strategies. Loop games also provide opportunities to pause and question students' thinking when they mentally compute. It is not necessary to question every student. Target specific students. This could be an on-going assessment, done many times throughout the year. It can be embedded in a mathematics routine or warm-up.

By pausing throughout, to share strategies, students hear various ways to compute, mentally. To play, simply put questions like the following, on cards and give each student a card. Any student can begin by reading their card to the group. The student who has the corresponding difference reads their card. The game continues until the game loops back to Student One.

Student One: I am 10, What is 40-10?

Student Two responds: I am 30, What is 22-14?

I am 10, What is 40-10?

I am 30, What is 22-14?

I am 8, What is 41-12?

I am 29, What is 36-18?

I am 18, What is 26-21?

I am 5, What is 67-56?

I am 11, What is 42-14?

I am 28, What is 86-73?

I am 13, What is 60-33?

I am 27, What is 40-20?

I am 20, What is 93-84?

I am 9, What is 78-66?

I am 12, What is 50-33?

I am 17, What is 37-18?

I am 19, What is 50-24?

I am 26, What is 72-36?

I am 36, What is 43-37?

I am 6, What is 87-62?

I am 25, What is 22-18?

I am 4, What is 47-24?

I am 23, What is 99-98?

I am 1, What is 42-18?

I am 24, What is 83-76?

I am 7, What is 52-37?

I am 15, What is 61-39?

I am 22, What is 29-15?

I am 14, What is 31-28?

I am 3, What is 60-39?

I am 21, What is 82-66?

I am 16, What is 90-59?

I am 31, What is 53-18?

I am 35, What is 41-39?

I am 2, What is 44-11?

I am 33, What is 52-18?

I am 34, What is 51-11?

I am 40, What is 73-63?

Resources/Notes

Math Makes Sense 3

Lesson 10 (Cont'd): Mental Math to Subtract

3N7

TG pp. 36 - 37

(3N7.5)

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicators:

3N9.6 Continued

3N9.2 Continued

3N9.7 Continued

Elaborations—Strategies for Learning and Teaching

To consolidate understanding of ‘regrouping’, students need continuous experiences modelling with concrete materials such as base-ten materials. Students need to make the connection between the operation and what it physically looks like. “The literature has been clear, as has conventional practice, that you move students from the concrete to the symbolic. Teachers know that students learn through all of their senses, so the use of concrete materials, or manipulatives, makes sense from the perspective alone. However, what makes the use of manipulatives even more critical in mathematics is that most mathematical ideas are abstractions, not tangibles.” (Small, 2008. *Making Math Meaningful to Canadian Students K-8*, p. 639)

To practice representing with concrete materials and visuals, ask students to choose two number cards (1-, 2- or 3-digit numbers). Create a story problem and number sentence. Ask them to model how to solve the problem with base-ten materials. Students can represent their model with pictures.

Numbers Chosen	Number Expression	Base Ten Representation
136, 294	$294 - 136 =$	

Zig Zag Subtraction - Player 1 chooses two numbers from the list and finds the difference. If the difference is on the game board player one covers the number. Player two repeats process. Play continues until a player can put three counters in a row (across, down, diagonally). Question students thinking about strategies they use to find the difference.

- 136 305 523 708 664 419 251

572	185	169	245
387	289	413	272
141	457	283	403
359	528	168	218

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Present students with two numbers. E.g., 266 and 39 ask them to demonstrate with base 10 how to subtract 39 from 266. Ask students to explain their models. (3N9.6)

Portfolio

- Present students with a two or three digit number. Ask them to create a subtraction story for the given number where the number is the solution. Write the number sentence for the story. Solve the problem using concrete or visual representation. Ask students to record their representation. (3N9.6, 3N9.2)

Performance

- Subtraction Connect Four - Player one chooses a number from Group A and one from Group B. They work out the difference between the two numbers. If the answer appears on the grid, player one places the counter on the number. If the number is not there or is already covered, player one misses their turn. Player two repeats the process. The winner is the first player to have four counters in a row (in any direction). This game can be used as a centre where the teacher may observe and question students thinking about strategies they use to find the differences. Observe to see if students are making reasonable choices from Group A and Group B to connect four.

**Subtraction
Connect 4!!**

Group A			Group B		
116	198	54	82	117	127
197	85	152	145	207	51

91	71	146	63	85
42	32	134	7	29
55	28	1	3	9
11	70	80	122	116
10	135	53	101	81
52	125	130	73	37

(3N9.7)

Resources/Notes

Math Makes Sense 3

Lesson 11: Subtracting
3-Digit Numbers
3N9

TG pp. 38 - 41

Strand: Number

Outcomes

Students will be expected to

3N9 Continued

Achievement Indicators:

3N9.4 Continued

3N9.5 Continued

Problem Solving Strategies:

Working Backward

Elaborations—Strategies for Learning and Teaching

When students are involved in creating and solving problems they are more engaged. Problems, in context, help students understand the purpose of using the operations and help them make mathematical connections to the real world. Put numbers into a context as much as possible so that students are more interested and motivated to find an answer.

Students have had experience solving addition and subtraction using personal strategies. As students begin to take more risks with personal strategies, encourage them to make connections between known and new strategies, as well as between their personal strategies and the strategies of their classmates. Therefore plenty of opportunities need to be provided for students to share their thinking and their strategies with peers.

Tasks such as ‘Problem of the Day’ provide students with opportunities to think about what the problem is asking, what operation they need to use and what strategies they will use to solve the problem. Also, students need to create their own problems involving addition and subtraction and these problems can be added to the problem bank for ‘Problem of the Day’.

Strategy Focus - Working Backwards -This strategy involves starting with the end result and reversing the steps to determine the information about the original situation, in order to figure out the answer to the problem. Students need to be given a variety of opportunities to work through authentic problems in a variety of situations.

“The context of the problems can vary from familiar experiences involving students’ lives or the school day to applications involving the sciences or the world of work.” *Principles and Standards for School Mathematics*, NCTM (2000), p. 52

E.g., Ryan wants to find the weight of his dog. He steps on the scale holding his pet dog. The scale reading is 43 kg. Alone, Ryan weighs 35 kg. How much does his dog weigh?

To solve this problem using the working backwards strategy; start with the total weight of Ryan and his dog (41kg). Next use your knowledge of Ryan’s weight (35kg) and subtract it from the total weight. By finding the difference you will find the weight of the dog.

 General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Present students with a problem such as:
Mr. Lush is taking the primary and elementary students skating. There are 213 primary students and 198 elementary students. How many students will be going skating?
Observe to see if the correct operation is being used and ask students to explain their strategy. (3N9.5)

Paper and Pencil

- Ask students to create their own addition and subtraction story problems using 1-, 2- or 3-digit numbers. Students can share their problems for others to solve. (This task can be used in mathematics routines and should be repeated throughout the year). (3N9.2, 3N9.5)

Journal

- Present students with problems such as:
Travis baked blueberry muffins over the weekend. Each day during the week he took four muffins to school to share with his friends. On Saturday when he counted there were 18 left. How many had he baked?
Mrs. Piercey bought five flags of different Canadian Provinces, to use in a Social Studies class activity. She added them to the flags she already had in the classroom. She borrowed two more flags. In the end ten flags were used in the activity. How many flags were there in the classroom already?

Observe students to see if they are using the 'Working Backwards' strategy and / or if they applied any other previously learned strategy.

Resources/Notes

Math Makes Sense 3

Lesson 12: Solving Addition and Subtraction Problems

3N9

TG pp. 42-45

Math Makes Sense 3

Lesson 13: Strategies Toolkit

3N9

TG pp. 46-47

