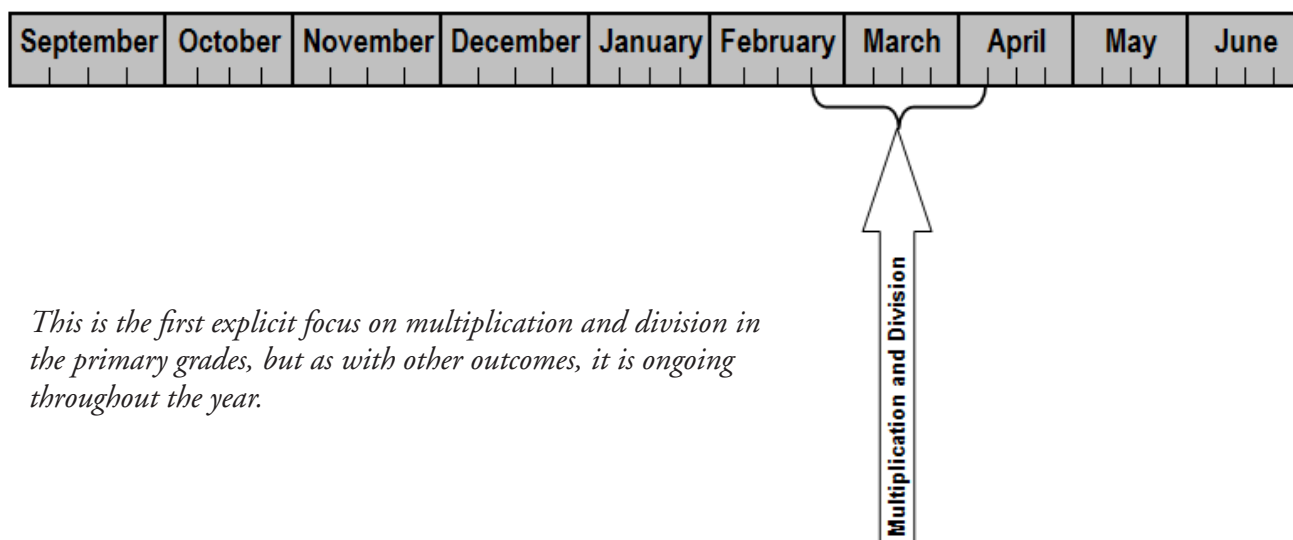


# Multiplication and Division

**Suggested Time: 6 Weeks**



*This is the first explicit focus on multiplication and division in the primary grades, but as with other outcomes, it is ongoing throughout the year.*

# Unit Overview

## Focus and Context

The focus of this unit is to set the stage for the rest of the year with respect to multiplication to  $5 \times 5$  and its related division. This is the first explicit focus on multiplication and division in the primary grades, but as with other outcomes, it is ongoing throughout the year. In Grade 2, students had many opportunities to use mental mathematics strategies for addition such as, using doubles, turnarounds, and using addition to subtract. This knowledge will provide a fundamental basis on which to build, when multiplication is introduced. In Grade 3, the emphasis is on beginning to build students' conceptual understanding of the multiplication operation. Students should focus on the meanings of, and relationship between, multiplication and division. Students should think about multiplication numerically as repeated addition of the same quantities or equal groups, and geometrically as rows and columns in rectangular arrays. Likewise, students should think about division numerically as repeated subtraction, equal sharing, and equal grouping.

Strategies for multiplication facts are a focus in Grade 4 and should not be the emphasis in Grade 3. The focus here is on understanding the meaning of multiplication and division and one to the other. "Modeling multiplication problems with pictures, diagrams or concrete materials helps students learn what the factors and their product represent in various contexts." (Principles and Standards for School Mathematics NCTM, p. 151)

## Math Connects

It is essential that students be given experiences where they see how multiplication and division can be used on a daily basis and how it relates to the world around them. This can be achieved through cross-curricular activities, morning routines, literature connections or through informal lessons. Naturally occurring situations provide students with meaningful contexts and gives them an opportunity to practice multiplication and division in a meaningful way. Whether deciding how many pencils are needed for classroom centers or how many buses are needed to go on a school outing, students can be involved in using multiplication and division daily. Making meaningful connections, challenges and extends students' mathematical thinking and reasoning.

## Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

## Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Number	<p>3N11 Demonstrate an understanding of multiplication to <math>5 \times 5</math> by:</p> <ul style="list-style-type: none"> <li>representing and explaining multiplication using equal grouping and arrays</li> <li>creating and solving problems in context that involve multiplication</li> <li>modelling multiplication using concrete and visual representations, and recording the process symbolically</li> <li>relating multiplication to repeated addition</li> <li>relating multiplication to division.</li> </ul>	[C, CN, PS, R]
Number	<p>3N12 Demonstrate an understanding of division (limited to division related to multiplication facts up to <math>5 \times 5</math>) by:</p> <ul style="list-style-type: none"> <li>representing and explaining division using equal sharing and equal grouping</li> <li>creating and solving problems in context that involve equal sharing and equal grouping</li> <li>modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically</li> <li>relating division to repeated subtraction</li> <li>relating division to multiplication.</li> </ul>	[C, CN, PS, R]

## Strand: Number

## Outcomes

*Students will be expected to*

**3N11 Demonstrate an understanding of multiplication to  $5 \times 5$  by:**

- **representing and explaining multiplication using equal grouping and arrays**
- **creating and solving problems in context that involve multiplication**
- **modelling multiplication using concrete and visual representations, and recording the process symbolically**
- **relating multiplication to repeated addition**
- **relating multiplication to division.**

[C, CN, PS, R]

## Elaborations—Strategies for Learning and Teaching

Prior to this unit, students have worked extensively with the various meanings and principles of addition and subtraction, place value and patterning. This knowledge helps provide the basis for development of multiplication and division.

It is important for students to think about multiplication, numerically, as repeated addition of the same quantities or equal groups, and, geometrically, as rows and columns in rectangular arrays. The numbers being multiplied are the factors and the answer is the product.

Students need conceptual understanding of the multiplication operation rather than simply following a procedure to obtain the product. **Please note, it is not expected that students achieve instant recall of the basic facts**, but rather that they relate repeated addition to multiplication to determine the products up to  $5 \times 5$ .

Students need to be able to interpret a variety of language patterns representing multiplication experiences. Sometimes students learn multiplication facts with little understanding of what they are memorizing. Initially, students need to understand is that multiplication is the process of counting objects by equal groups rather than as single objects. Help students recognize equal groups and help them develop the language of multiplication experiences. Through a variety of teacher-modelled activities, you will demonstrate to students how multiplication can represent equal groups that can be displayed as ‘rows of.’, ‘stacks of...’, ‘piles of...’, etc. It is important not to begin using the term ‘times’ and the corresponding symbol because this may interfere with their understanding of the multiplication situation. It is important for students to understand the following meanings of multiplication:

1. repeated addition
2. equal groups or sets
3. an array

Manipulatives such as buttons, counters, number lines, beans, popsicle sticks, straws, snap cubes, Link-its, cookies, etc. should be used to model various multiplication meanings.

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General Outcome: Develop Number Sense

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Suggested Assessment Strategies

Resources/Notes

*Math Makes Sense 3*

**Launch:** Sports Day

TG pp. 2 - 3

**Lesson 1:** Investigating Equal Groups

3N11

TG pp. 4 - 7

**Game:** Closest to Twelve

TG: p. 8

**Additional Activity:**

Equal-omimoes

TG: p. v

**Children's Literature**

(not provided):

Aker, Suzanne and Karlin, Bernie

*What Comes in 2s, 3s and 4s*

ISBN 0671671731

Strand: Number

Outcomes

*Students will be expected to*

**3N11 Continued**

**Achievement Indicator:**

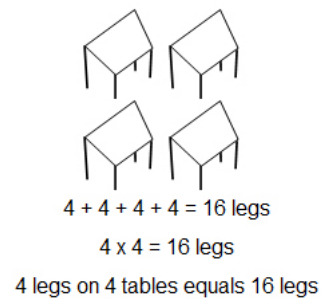
*3N11.1 Identify events from experience that can be described as multiplication.*

*3N11.2 Represent a given story problem, using manipulatives or diagrams, and record the problem in a number sentence.*

Elaborations—Strategies for Learning and Teaching

Challenge students to think of real-life objects that come in equal groups, such as wheels on bicycles/tricycles, legs on chairs, legs on stools, 3-leaf clovers, animal legs, pairs of shoes, cookies on a tray, people’s eyes or fingers on hands. Provide a real-life multiplication problem for students to solve. For example, ask students to find out how many eyes there are in the classroom or how many legs four desks have. When students have an answer, ask them if there is a faster way to solve the problem besides adding them up. Record all suggestions and introduce multiplication as a faster way of adding equal groups.

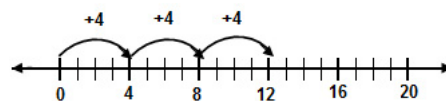
When students solve simple multiplication story problems before learning about multiplication symbolism, they will most likely write repeated-addition equations. This is an opportunity to introduce the multiplication sign and explain what the two factors mean.



$4 + 4 + 4 + 4 = 16 \text{ legs}$   
 $4 \times 4 = 16 \text{ legs}$   
 4 legs on 4 tables equals 16 legs

One of the most meaningful ways to apply and practice multiplication is in a problem solving context. Solving real life problems is a means to help deepen their understanding of number sense.

It is essential to provide students with visual representations such as manipulatives, pictures, diagrams and storyboards when representing and solving multiplication problems. Hundreds charts and number lines should be readily available to help students visualize the connection between repeated addition and multiplication. Students have used these manipulatives to skip-count in Grade 2. When you skip-count, you are saying multiples of a number. It is important for students to have a conceptual understanding of the meaning of multiples. Using language such as 1 group of 3 is 3, or 2 groups of 3 is 6 and 3 groups of 3 is 9 helps students develop this understanding that multiples of a number are the products of that number. To find 3 multiples of 4, students can use a number line or a hundreds chart.



## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Student –Teacher Dialogue*

- Present a contextual multiplication problem to students and ask students to explain how they solved the problem. For example, “How many fingers are on 4 hands?” or “How many legs on 3 chairs?”

(3N11.1, 3N11.3)

*Performance*

- Provide various magazines, grocery flyers or books for students to collect pictures of items that are displayed in equal groups. Ask students to explain how multiplication can be used to find the total number of items.



$$3 + 3 + 3 = 9 \text{ juice boxes}$$

$$3 \times 3 = 9 \text{ juice boxes}$$

3 groups of 3 juice boxes equals 9 juice boxes

( 3N11.1, 3N11.6, 3N11.5)

- Tell a multiplication story problem that students can act out, either with classroom objects or with counters on a storyboard. E.g.,  
Claire stacks books into two piles. She put four books in each pile.

As students are acting out the problem encourage them to use the language patterns representing the multiplication experience. E.g.,

How many stacks is Claire making?

2 stacks

How many books are in each pile?

2 stacks of 4 books in each pile

How many books all together?

2 stacks of 4 books = 8 books (3N11.2, 3N11.5, 3N11.6)

*Paper and Pencil*

- Provide students with a variety of personal story problems, including student names, interests, hobbies, etc. Ask them to represent the problem using manipulatives, numbers, pictures, and words.

E.g., Luke has a collection of hockey cards. He has 5 pages with 4 hockey cards on each page. How many hockey cards does Luke have in all? (3N11.2, 3N11.6, 3N11.7)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 1 (Cont'd):** Investigating Equal Groups

3N11

TG pp. 4 - 7

Strand: Number

Outcomes

*Students will be expected to*

**3N11 Continued**

**Achievement Indicators:**

*3N11.2 Continued*

Elaborations—Strategies for Learning and Teaching

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In this hundreds chart, some multiples of 4 are circled.

The first 3 multiples of 4 are 4, 8 and 12. Ask students to continue circling the remaining multiples.

Integrate physical activities into your daily routines to reinforce math concepts such as skip-counting and find multiples of a number.

1. Ask students to count by 2s, 3s, 4s or 5s when they are doing activities such as jumping jacks or toe touches. (whole group)
2. Ball Toss: Ask students to toss a ball to each other. E.g., Child A starts with 4, and throws the ball to Child B. Child B says 8 and throws the ball to Child C. Child C says 12 and throws the ball to Child D; and so on.

Snack Activity - In groups of two or three, provide students with various snack items that come in equal groups of 2s, 3s, 4s or 5s. E.g., 3 packages of cheese & crackers. Without opening the packages, ask students to find out how many snack items they have in their group altogether. Ask students to record the results of the “Snack Activity” using pictures, numbers and words. Ask students to share the results and strategies used to solve the problem. (Some students may use repeated addition and others may use multiplication. Discuss how the two operations are related.)

Some possible snack items are cheese & crackers, packaged cookies, junior juices, gum, etc.



$$4 + 4 + 4 + 4 = 16$$

$$4 \times 4 = 16$$

My group has 4 packages of gum. There are 4 pieces in each package. 4 equal groups of 4 make 16.

## General Outcome: Develop Number Sense

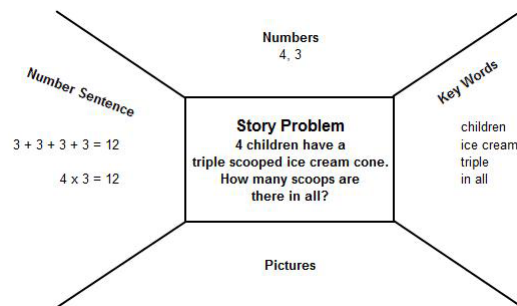
## Suggested Assessment Strategies

*Performance*

- Using storyboards (e.g., picture of the ocean) ask students to create story problems for multiplication. Ask students to use manipulatives (e.g., fish crackers or counters) to represent their story problem and write the corresponding number sentence. (3N11.4, 3N11.5)

*Presentation*

- Mathematician's Chair - Designate a special chair as the 'Mathematician's Chair'. Ask students to create and solve their own multiplication problem. Provide a graphic organizer, such as the one below, to assist students in solving their problem. Ask individual students to sit in the 'Mathematician's Chair' to share their problem and how they solved it.



(3N11.4, 3N11.5, 3N11.3)

## Resources/Notes

*Math Makes Sense 3***Lesson 1 (Cont'd):** Investigating Equal Groups**3N11**

TG pp. 4 - 7

Strand: Number

Outcomes

*Students will be expected to*

**3N11 Continued**

**Achievement Indicators:**

*3N11.3 Solve a given multiplication problem.*

*3N11.4 Create and illustrate a story problem for a given number sentence.*

*3N11.5 Represent, concretely or pictorially, equal groups for a given number sentence.*

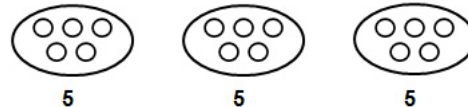
Elaborations—Strategies for Learning and Teaching

Morning Routines – Present a multiplication sentence and ask students to represent a story problem to match, on a storyboard. Ask students to share their stories.

Provide pictures that display collective nouns (e.g., school of fish, pod of whales or any picture with a group of items). Have multiplication sentences printed on the back of each picture up to a product of  $5 \times 5$ . Ask students to use these cards to create their own word problem. For example, “There are 3 pods of whales and 5 whales in each pod. How many whales in all?”

$5 + 5 + 5 = 15$

$3 \times 5 = 15$



Multiplication Game – Provide pairs of students with 5 multiplication expressions on index cards such as,  $5 \times 1$ ,  $2 \times 4$ ,  $3 \times 3$ ,  $0 \times 2$  and  $5 \times 5$ . Ask students to represent each expression symbolically (the product), pictorially (as equal groups or an array) and as repeated addition on separate index cards. E.g., for  $2 \times 4$  the cards will be 8,  $\begin{bmatrix} : & : & : & : \\ : & : & : & : \end{bmatrix}$  and  $4 + 4$ . Shuffle the cards and place face down or in a 20-pouch pocket chart. Ask students to take turns choosing a pair of cards to try and find a match. When a match is found, the student keeps the pair and takes another turn. Continue until all matches are found. The player with the most cards is declared the winner.

Use everyday situations to present multiplication problems to students. For example, “We used 4 boxes of granola bars for our Breakfast Program this morning. Each box had 5 bars. How many students were served if each student received 1 bar?”

Display various riddles from *The Best of Times* by Greg Tang as lesson starters. Using the riddle clues and the visuals, give students time to mentally solve the problem. Discuss individual problem solving strategies used by students.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Performance/Paper and Pencil*

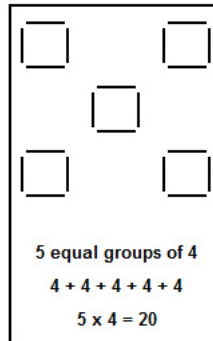
- Class Book Activity – Students can create picture books to illustrate multiplication facts. Have each student choose a multiplication sentence and draw a picture to match. Record the multiplication sentence below each picture. Collect all pages and assemble into a class book. (3N11.5, 3N11.8)

*Journal/Performance*

- Provide students with toothpicks. Ask them to use the toothpicks to make 5 squares. Glue or draw the arrangement in their journals. Label the arrangement in 3 different ways, as shown.

Ask students to explain in words the meaning of  $5 \times 4 = 20$  in their journals.

If necessary, repeat the activity and use a different set of objects each time.



(3N11.4, 3N11.5)

*Performance*

- Centres - Create centers using manipulatives and number cubes (0 - 5) to find ‘how many groups of...’. At each center, students will roll the number cube to determine how many groups to make. Roll the number cubes again to determine how many will be in each group. Make those groups. The student then determines how many altogether and records the information on a recording sheet.

How many (cups, groups, rows, piles)?	How many in each?	How many altogether?

(3N11.5)

Resources/Notes

*Math Makes Sense 3*

**Lesson 1 (Cont'd):** Investigating Equal Groups

3N11

TG pp. 4 - 7

**Children’s Literature**

(not provided):  
Tang, Greg. *The Best of Times*

ISBN 9780439210447

## Strand: Number

## Outcomes

*Students will be expected to*

**3N11 Continued****Achievement Indicators:**

*3N11.6 Represent a given multiplication expression as repeated addition.*

*3N11.7 Represent a given repeated addition as multiplication.*

*3N11.8 Represent a given multiplication expression, using an array.*

## Elaborations—Strategies for Learning and Teaching

It is essential that students view multiplication as an alternate and more efficient form of repeated addition. Students should recognize that addition and multiplication describe how many there are in all when combining groups of objects. For addition, combined groups do not have to be equal. For multiplication, combined groups must be equal.

Bingo Game - Call out repeated addition expressions. Ask students to use counters to cover the matching multiplication expression on individual game boards or charts. The first player to cover a row, column or diagonal wins. This game can also be modified to match the products to the multiplication sentences.

When multiplying, students can think in two ways, numerically as repeated addition of the same amounts, and geometrically in rectangular arrays as rows and columns. An array is an arrangement of objects in equal rows.

Literature Connection - Use the book *Amanda Bean's Amazing Dream*, as a springboard for connecting multiplication to real-life situations and showing the difference between organizing items in equal groups and arrays.

Window Pane Discussion – Ask students to look at the second page and observe the 6 pane window. Note that there are a number of windows on this page. Ask, “What do you notice about the windows?” Bring attention to the panes. Draw a window with 3 rows of 2 panes. Ask students, how they could find out how many panes there are on the window without counting. Ask how they could figure this out through multiplication. ( $3 \times 2$ ) There are 3 rows of 2 panes. Introduce students to ‘rows’ and ‘columns’.

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Calculator Activity - Give students a calculator. Ask them to choose a number between 1 and 5 to repeatedly add and choose the number of times (up to 5 times) they will add the number. Then they will use the calculator's repeat function to add the number that many times. Students can predict the number that will appear on the display each time the equal (=) key is pressed. Observe and listen to students' predictions. (3N11.6, 3N11.7)
- Find Your Partner Activity: Make up repeated addition sentence cards with the corresponding multiplication sentence to match. E.g.,  $3 + 3 = 6$  matches  $2 \times 3 = 6$ . Shuffle the cards, pass out to students and ask them to circulate to find their matching partner. Pairs can sit together until all matches are made. This activity can also be used to find the product. Extension: Matches could include two equal multiplication expressions such as  $2 \times 2$  and  $4 \times 1$ . (3N11.6, 3N11.7)

*Journal/Student-Teacher Dialogue*

- After reading the book, *Amanda Bean's Amazing Dream*:
  - (i) Use illustrations in the book that show equal groups. Ask questions such as: How many cookies are on each tray in the bakery window? How many lollipops are stuck in each block? How many lollipops are there altogether? How many stripes are there in a loaf of bread? How many bushes are in the rectangle in the centre of the park? Ask students to respond to the following questions in their journals. What is the difference between multiplying as repeated addition (such as the groups of lollipops) and using arrays (such as the cookies on the tray)? When would it be difficult to use repeated addition? (3N11.8)
  - (ii) Ask students to estimate how many kernels of popcorn Amanda is pouring into the bowl. (Note that the popcorn is not organized in any particular way.) Ask students to write about how they could organize it by grouping into 2s, 3s or 5s, etc. (3N11.8)

## Resources/Notes

*Math Makes Sense 3***Lesson 2:** Relating Multiplication and Repeated Addition

3N1

TG pp. 9 – 11

**Lesson 3:** Arrays to Multiply

3N1

TG pp. 12 - 15

**Additional Activity:**

Amazing Arrays

TG: p. v

**Children's Literature** (provided):Neuschwander, Cindy. *Amanda Bean's Amazing Dream*,

## Strand: Number

## Outcomes

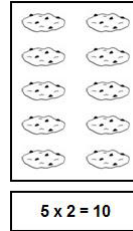
*Students will be expected to*

**3N11 Continued****Achievement Indicator:**

*3N11.8 Continued*

## Elaborations—Strategies for Learning and Teaching

“Array for Cookies”— Give each student a sheet of paper representing a cookie tray. Ask them to choose a multiplication sentence written on an index card. Have the students arrange real cookies or counters in an array to match the multiplication sentence they chose. Choose a new card and repeat.



Literature Connection - *The Visit* by Helen Chapman - Ask students to identify the arrangements of the chairs in arrays in each picture in the book. Ask students to count the number of chairs in each array. Ask students to record the different arrays for 24.

The book *One Hundred Hungry Ants* is an excellent resource to give third graders informal experience with multiplication and division.

The book *Minnie's Diner* is also an excellent resource for exploring multiplication patterns. The story says repeatedly that the Mcfay brothers “ordered twice as much as the brother before.” An activity to try: Change the word twice to three times more. How would that change the multiplication pattern? How many of each item would Papa receive?

Which Has More? - Ask students to make up their own game to play with a partner called, “Which Has More?” Students create question cards such as, “Which has more? 4 rows with 4 donuts in a row or 3 rows of 5?”.

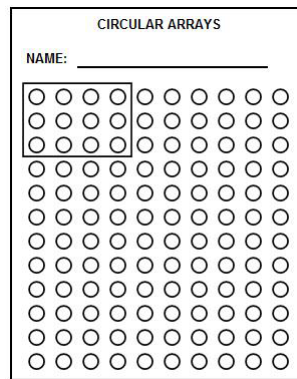
## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Paper and Pencil/Portfolio*

- Array Game - Provide students with a circular array recording sheet as shown below and two 0-5 number cubes. With a partner, take turns rolling the cubes and outline an array on the recording sheet. Use the two numbers for the dimension of the array. Record the multiplication equation inside the outline.

E.g., write  $3 \times 4 = 12$  and say, "Three rows of 4 is 12." Once a circle has been used in an array, it cannot be used again. There cannot be overlapping. A player loses a turn if the array will not fit in the area remaining on the recording sheet. After six rounds, add the area of each rectangle outlined. The player with the greater total wins.



(3N11.8)

- Multiplication Constellations - Ask students to choose a multiplication sentence such as  $4 \times 5 = 20$ . Give one star shaped sponge to each student to dip in yellow or metallic poster paint. Ask them to sponge an array of stars onto a large sheet of black construction paper to match their multiplication sentence. If every multiplication sentence from  $1 \times 1 = 1$  to  $5 \times 5 = 25$  were used, an entire multiplication sky can be displayed on the ceiling.

(3N11.8)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 3 (Cont'd):** Arrays to Multiply

**3N1**

TG pp. 12 - 15

**Children's Literature** (provided for Grade 2 Implementation):

Chapman, Helen. *The Visit*

**Children's Literature**

(not provided):

Pinczes, Elinor J. *One Hundred Hungry Ants*

ISBN 0046442971232

**Children's Literature** (provided):

Dodds, Dayle. *Minnie's Diner*

Strand: Number

Outcomes

*Students will be expected to*

**3N11 Continued**

**Achievement Indicator:**

*3N11.9 Create an array to model the commutative property of multiplication.*

Elaborations—Strategies for Learning and Teaching

Using the book, *Amanda Bean’s Amazing Dream*, draw a window to show 3 rows of 2 panes ( $3 \times 2$ ). This time turn it around to make 2 rows of 3 panes ( $2 \times 3$ ). Ask, “Does this change the number? Why or why not?” In pairs ask students to use graph paper to make their own window panes. Switch with a partner to write two multiplication sentences on each window.

Five Times The Fun Game - Give each pair two number cubes (0-5), and a copy of a recording sheet similar to the one below.

**FIVE TIMES THE FUN**

Player 1: \_\_\_\_\_ Player 2: \_\_\_\_\_

	T		I		M		E		S	
	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2
Roll 1										
Roll 2										
Total										

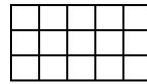
Player 1 rolls the number cubes and multiplies the values together. Repeat this step and record the second product. Add both products together for the total score of the first round. Player 2 then takes a turn. The player with the higher sum circles the number and wins the round. The player who wins the most rounds wins the game.

## General Outcome: Develop Number Sense

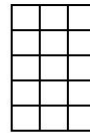
## Suggested Assessment Strategies

*Journal*

- Ask students to use graph paper to cut out two arrays (window panes) of equal size. Then ask them to glue the arrays in their journals showing the turnaround or commutative property of multiplication. Label each array with the matching multiplication sentence. Ask students to write their observations. For example, turning the array does not change the product.



$$3 \times 5 = 15$$

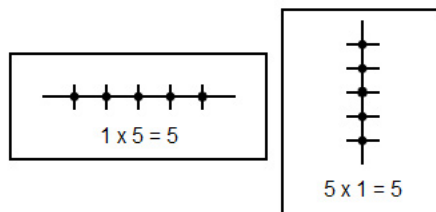


$$5 \times 3 = 15$$

(3N11.9)

*Performance*

- Toothpick Arrays - Put students in pairs. Give each pair a number of toothpicks/craft sticks. Ask students to arrange the sticks in arrays. The vertical sticks can represent one factor and horizontal sticks represent another factor. Ask students to demonstrate the commutative property by making two arrays while labeling them with the correct multiplication sentence. For example, if a pair is using six sticks, they would arrange the sticks as pictured below.



Students can glue their arrays on index cards to be shared with others or draw a picture of their created arrays in their journals with an explanation. (3N11.9)

- Exit Cards – Exit cards can be used to assess the understanding of any concept taught. Students independently respond to a question or any problem posed related to the current concept. These questions can be written on index cards, flashcards, etc. As students respond, they leave their cards in a designated area before they exit the room. This is an excellent tool to use to quickly assess if there is need for reteaching. For example, when assessing the understanding of the commutative property of multiplication, the question could be, “Draw 2 arrays to represent 6 and label with the matching multiplication sentences.” (3N11.9)

## Resources/Notes

*Math Makes Sense 3***Lesson 3 (Cont'd):** Arrays to Multiply

3N1

TG pp. 12 - 15

*Math Makes Sense 3***Lesson 4:** Relating Multiplication Sentences

3N1

TG pp. 16 - 18

Strand: Number

Outcomes

*Students will be expected to*

**3N11 Continued**

**Achievement Indicator:**

*3N11.9 Continued*

Elaborations—Strategies for Learning and Teaching

Connect Four Multiplication Game - Each pair will need two paper clips and 16-20 counters. 8-10 of the counters should be one color and the other 8-10 should be a different color. Alternatively, they can mark directly on the game board, with each player using a different color pencil or one person using X's and the other using O's. The player that starts places the paper clips on two numbers on the strip of factors below the game board. That player then uses one of his/her colored counters to cover the product of those two numbers on one square of the game board. The second player moves exactly one of the paper clips to make a second product. The second player then places his/her counter on the product of the two factors on the game board. Play alternates until one player connects four of his/her own color either horizontally, vertically or diagonally. Of course, players will want to block each other, and this will require that they practise strategies that have learned.

0	8	5	8	4
20	1	24	2	15
6	4	3	18	25
18	25	5	20	9
1	10	2	10	24
15	0	9	6	12

**Strip of factors**

0	1	2	3	4	5
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General Outcome: Develop Number Sense

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Suggested Assessment Strategies

Resources/Notes

*Math Makes Sense 3*

**Lesson 4 (Cont'd):** Relating  
Multiplication Sentences

**3N1**

TG pp. 16 - 18

Strand: Number

Outcomes

*Students will be expected to*

**3N12 Demonstrate an understanding of division (limited to division related to multiplication facts up to  $5 \times 5$ ) by:**

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

[C, CN, PS, R]

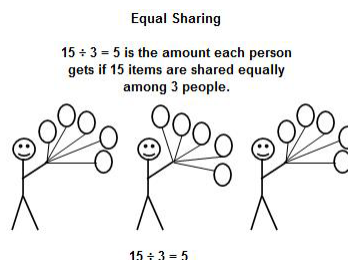
Elaborations—Strategies for Learning and Teaching

Traditionally, multiplication and division were taught separately. It is important, however, to combine multiplication and division shortly after multiplication has been introduced. “Multiplication and division “undo” each other. They are related inverse operations. For example, if  $12 \div 3 = 4$ , then  $3 \times 4 = 12$ .” (*Making Math Meaningful*, Small 2008, p. 123)

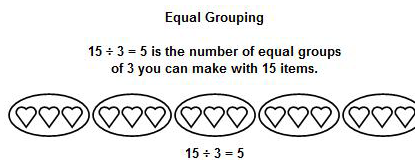
When one number is divided by another, the number being divided is the dividend. The other number is the divisor. The result is the quotient.

It is important for students to understand the three meanings of division:

1. Division as Equal Sharing – “In the sharing situation, some known quantity (amount) is shared equally among a known number of entities (people, boxes, packages, etc.). What is not known in a sharing situation is the amount of the given quantity per share. The quotient in this situation represents the amount per share, the size of each share or the unit rate.” ([http://www.utdanacenter.org/mathtoolkit/instruction/lessons/7\\_divide.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/7_divide.php))



2. Division as Equal Grouping – “In a grouping situation, the unknown is the number of groups of a given size that can be made from a given quantity (amount). The quotient in this situation tells how many groups of the specified size can be made from the given quantity.” ([http://www.utdanacenter.org/mathtoolkit/instruction/lessons/7\\_divide.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/7_divide.php))



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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Journal*

- To find out about students' prior knowledge of division, ask them to write a response to, "What is division?" What does it mean and what kinds of things can be divided? The focus is to find out what they understand about division and the mathematical notation used, equal grouping and equal sharing. Take note of student misconceptions. (3N12)

*Performance*

- Tell a division story problem that students can act out, either with actual classroom objects or with counters on a story board. E.g., There are 8 bottles of glue. Each table of students will get two of those bottles. How many tables will get glue? (3N12.8)
- Present a division sentence and ask students to represent a story problem to match, on a story board. Ask students to share their stories. 3N12.4)
- Using story boards, ask students to create story problems for division. Ask students to use manipulatives to represent their story problem and write the corresponding number sentence. (3N12.1, 3N12.7)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 5:** Division as Grouping  
3N12

TG pp. 19 - 22

**Children's Literature:**Feely, Jenny. *Dazzling Division*Feely, Jenny. *Fair Share*

(provided for Grade Two Implementation)

## Strand: Number

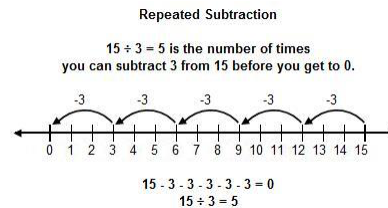
## Outcomes

*Students will be expected to*

**3N12 Continued**

## Elaborations—Strategies for Learning and Teaching

3. Division as Repeated Subtraction – To divide using repeated subtraction, subtract equal groups from the total until you reach 0. Division is a shortcut for repeated subtraction.



Manipulatives such as buttons, counters, number lines, beans, popsicle sticks, straws, snap cubes, link-its, cookies, etc., should be used to model various division meanings.

The National Council of Teachers of Mathematics recommends that “students in grade three should develop a stronger understanding of the various meanings of multiplication and division, encounter a wide range of representations and problem situations that embody them, learn about the properties of these operations, and gradually develop fluency in solving multiplication and division problems.” (NCTM 2000, p. 149).

**Achievement Indicator:**

*3N12.1 Identify events from experience that can be described as equal grouping.*

Ask students to brainstorm real-life situations where equal groups are necessary. Remind students that each group has the same number of items. E.g., counters in baggies, cookies on trays, tennis balls in packages, etc. Place 20 counters on an overhead. Ask: How many groups of 4 counters can we make if we have 20 to work with? Most students will be able to solve the problem mentally. After receiving several answers, ask a student to demonstrate how to verify the answer of 5 groups of 4. Ask: What number sentence could we write for the groups formed? Possible number sentences include:  $4 + 4 + 4 + 4 + 4 = 20$ ,  $5 \times 4 = 20$ . This is a good opportunity to introduce the division symbol and the corresponding number sentence. We say, “20 divided by 4 is 5.” We write the division sentence  $20 \div 4 = 5$ . It is also important for students to act out story problems using real classroom objects that involve remainders. Remainders are a natural part and students should deal with them as they begin to learn how to divide. Allow students to deal with the concept of ‘leftovers’ or remainders, beginning with these very first experiences.

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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Performance/Paper and Pencil*

- Small Group Activity - *Amanda Bean's Amazing Dream*. If possible, ask students to group themselves in groups of 4. Provide each group with various numbers of counters that can be grouped equally with 4 in each group (8, 12, 16, 20). How many groups of 3 counters can you make with the total number of counters in your group? Ask students to represent their groupings concretely, pictorially and symbolically with number sentences on chart paper. Encourage the use of a division sentence. Give time for each group to share with the whole class. (3N12.1, 3N12.2)

*Student-Teacher Dialogue*

- Ask students to use counters to act out story division problems with and without remainders. Provide opportunity for students to discuss how they solved the problems.
  - (i) Theresa has twenty-five pieces of paper to hand out for booklet covers. Each student needs two pieces of paper to make a cover. How many students can have two pieces?
  - (ii) Craig collects stamps. He has twenty two stamps. Four stamps fit on each page of his stamp collection book. How many pages can he fill?
 (3N12.1)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 5 (Cont'd):** Division as Grouping

3N12

TG pp. 19 - 22

**Children's Literature**

(provided):

Neuschwander, Cindy. *Amanda Bean's Amazing Dream*

## Strand: Number

## Outcomes

*Students will be expected to*

**3N12 Continued****Achievement Indicators:**

*3N12.2 Illustrate, with counters or a diagram, a given story problem, presented orally, that involves equal grouping; and solve the problem.*

*3N12.3 Listen to a story problem; represent the numbers, using manipulatives or a sketch; and record the problem with a number sentence.*

*3N12.4 Create and illustrate, with counters, a story problem for a given number sentence; e.g.,*

$$6 \div 3 = 2$$

*3N12.5 Solve a given problem involving division.*

## Elaborations—Strategies for Learning and Teaching

Literature Connection - *A Remainder of One* by Elinor J. Pinczes. Although the outcome does not deal with remainders, sometimes when dividing, students will discover that there will be remainders.

This book reviews the relationship between multiplication and division while realizing, at times, there may be a remainder when putting objects into equal groups or sets.

Prediction Activity - Read the story and stop at the sentence “The troop had divided by two for the show”. Ask students to predict how many bugs would be in each line. Would there be any bugs left out? How many? Why? Stop and make predictions as above throughout the story. Ask students what they think, for example, when oddball bug Joe is thinking throughout the night that a fourth bug line would work out. Would a fourth line solve a problem? How do you know?

Using this book as a springboard, have pairs of students create their own division problem for other pairs to solve later. Problems may or may not contain a remainder.

Hamster Vacation - Present the following problem to the students:

The Grade 3 class has 20 hamsters. Susan must put them into 5 cages to go home for the summer vacation. How many hamsters will be in each cage? Ask students to explain why they put the hamsters in the cages this way.

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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Performance*

- “Mystery Object Riddle”: Have pairs of students choose a mystery object and make up a riddle about it using a division sentence as a clue. For example, I am thinking of something in this room that shows  $20 \div 4 = 5$ . (20 chairs in the room with 4 chairs per table will be 5 tables or groups.) (3N12.2, 3N12.5)

*Journal*

- Principal Visit/Pretend Scenario - Have the school principal visit the class and notice that students are learning about equal grouping and sharing. Have him/her ask the class to help with a school problem. Some lockers have been removed because they are old. Now there must be three students share a locker instead of two. How many lockers will the Grade 3 class need with three students to a locker? If there are 15 students in grade three, how many lockers will your class need? (3N12.3)
- Ask students to record a solution to the Principal Visit problem using pictures, numbers and words. Ask students to label their sketch with an appropriate number sentence. (3N12.3, 3N12.2, 3N12.5)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 5 (Cont'd):** Division as Grouping

3N12

TG pp. 19 - 22

**Children’s Literature**

(not provided):

Pinczes, Elinor J. *A Remainder of One*

ISBN 9780618250776

Strand: Number

Outcomes

*Students will be expected to*

**3N12 Continued**

**Achievement Indicators:**

*3N12.6 Identify events from experience that can be described as equal sharing.*

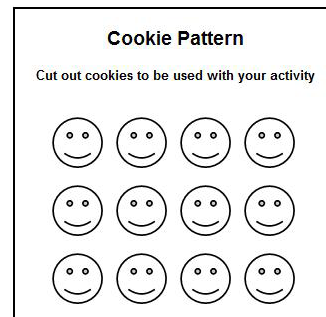
*3N12.7 Illustrate, with counters or a diagram, a given story problem, presented orally, that involves equal sharing; and solve the problem.*

Elaborations—Strategies for Learning and Teaching

Brainstorm real-life situations where equal sharing is necessary. For example, balloons in loot bags, money, cards in a game, a box of chocolates, a carton of strawberries, etc. Have volunteers model the following problem in front of the class using real strawberries and plates. “There are 24 strawberries in the carton. Share the strawberries equally among 6 students. How many strawberries will each student get? Discuss results.

Literature Connection - *The Doorbell Rang* by Pat Hutchins. In this story, students get a chance to use division skills in predictable situations.

Discuss what ‘equal’ or ‘fair shares’ mean. Read the story, but stop at the sentence “Share them between yourselves.” and insert the word ‘equally’ each time. Ask students to predict how many cookies the two children will have, the four children, the six children, and so on. Ask students to work in pairs and give each pair a sheet of paper cookies as shown below. Reread the story but stop each time the cookies must be shared and ask students to use their paper cookies to chart the various arrangements. Use play dough or take students to a kitchen if one is available in your school. Students can work in groups. Give them a chocolate cookie recipe that yields 12 cookies. Ask students to shape the dough into 12 round cookies. Give each group 36 chocolate chips to go into the dough. Ask students to estimate how many chocolate chips there should be per cookie, if they were shared equally. After the cookies are made, ask students to equally divide their cookies up with each member of their group.



Ask students to total the number of cookies made by the whole class. If this number of cookies were equally shared among the total number of teachers in the school, how many cookies would your teacher get?

Graphing - This lesson can also include the concept of graphing. For example, students could do a class survey for favourite cookies and graph the results.

The Doorbell Rang Game - Choose 12 students to play the part of the *(continued)*

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance/Paper and Pencil*

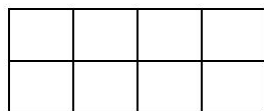
- Arrange students into groups of 4. Provide each student with a loot bag and each group with 20 stickers. Ask students to share the stickers equally within the group. How many stickers will be in each loot bag? On chart paper ask students to record the results concretely, pictorially and symbolically using number sentences. Encourage students to use a division sentence. Allow time for groups to compare results. (3N12.6, 3N12.7, 3N12.3, 3N12.5)

*Journal*

- Ask students to show how to share 20 cookies among 5 children. Ask them to explain their thinking using pictures, numbers and words. (3N12.7, 3N 2.3, 3N12.5)

*Performance*

- Packing Up Shoes - Ask students to remove their shoes and place all of them in a pile. Provide 5 large boxes and ask students to equally share the shoes among the boxes. Should there be leftovers, or remainders, observe how students deal with them. Note: In contextual problems, there are often leftovers. Ask students to record their results in their journals. (3N12.7, 3N12.3, 3N12.5)
- Paper Folding Activity: Show students how to fold a sheet of 12-by-18-inch paper into eight sections as shown below.



Explain that in the first box, students should write *The Doorbell Rang* and their own name. They retell the story in the next five boxes, writing five mathematical sentences that are presented in the story. In the last two boxes students can make up their own final parts of the story. (3N12.7, 3N12.3, 3N12.5)

## Resources/Notes

*Math Makes Sense 3***Lesson 6:** Division as Sharing

3N12

TG pp. 23 – 25

**Additional Activity:**

Division Tag

TG: p. v

**Children's Literature** (not provided):

Hutchins, Pat *The Doorbell Rang*  
ISBN 9780688092344

Strand: Number

Outcomes

*Students will be expected to*

**3N12 Continued**

**Achievement Indicators:**

*3N12.7 Continued*

*3N12.8 Represent a given division expression as repeated subtraction.*

*3N12.9 Represent a given repeated subtraction as a division expression.*

Elaborations—Strategies for Learning and Teaching

children. Also, choose students for the mother, grandmother, doorbell and narrator. Role play the story using real cookies.

Ask students to start with 25 and repeatedly subtract 5. Counters or a calculator may be used. Ask him/her to record these subtractions on paper until they reach 0. Have him/her explain the results, encouraging correct mathematical vocabulary to explain division as repeated subtraction.

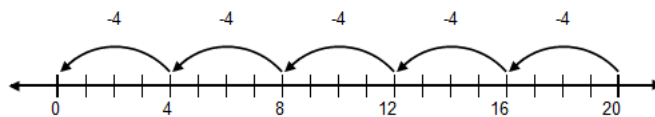
Snap Cube Activity - Have pairs of students link together 5 groups of 4 snap cubes (each group must be a different color). Write on the board the repeated subtraction sentence  $20 - 4 - 4 - 4 - 4 - 4 = 0$ . Then, ask students to role play this with their partner until each colored group is removed. Ask students to record their findings using a picture, the repeated subtraction sentence and the division sentence.



$$20 - 4 - 4 - 4 - 4 - 4 = 0$$

$$20 \div 4 = 5$$

Number Line Activity: Use a number line to show repeated subtraction.



The repeated subtraction sentence,  $20 - 4 - 4 - 4 - 4 - 4 = 0$ , can be written as  $20 \div 4 = 5$ .

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance*

- Around the World Game - Create a set of question and answer cards that include a variety of multiplication and division tasks. See sample cards below.

Distribute cards to the students. The student with the start 'card' begins and he/she reads aloud, "I have the start card", and the first question. Each student then looks at his/her card to see if he/she has the matching answer. Then, the student with the matching answer reads the answer and then poses his/her question. All students then look for the match and the game continues until you get to the student with the end card.

**Multiplication and Division Tasks for up to 5 x 5 (15 cards)**

I have the START card	I have 7
Who has the quotient of $25 \div 5$ ?	Who has the product of $0 \times 3$ ?
I have 5.	I have 0
Who has $12 \div 3$ ?	Who has the repeated addition sentence for $4 \times 5$ ?
I have 4.	I have $5 + 5 + 5 + 5 = 20$
Who has 5 multiplied by 2?	Who has the repeated addition sentence for $5 \times 3$ ?
I have 10.	I have $3 + 3 + 3 + 3 + 3 = 15$
Who has $6 \div 3$ ?	Who has the repeated subtraction sentence for $10 \div 5$ ?
I have 2.	I have $10 - 5 - 5 = 0$
Who has the product of $5 \times 5$ ?	Who has the repeated subtraction sentence for $16 \div 4$ ?
I have 25.	I have $16 - 4 - 4 - 4 - 4 = 0$
Who has 2 equal groups of 4?	Who has 9 shared equally among 3 groups?
I have 8.	I have 3.
Who has $3 \times 2$ ?	I have the END card.
I have 6	
Who has 14 divided by 2?	

Note:

1. You will need ALL cards to play the game.
2. Students can work together if you do not have enough cards for every student. If there are too many cards, then some students can have 2 cards each.

(3N12.8, 3N12.9, 3N11.7, 3N11.2)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 6 (Cont'd):** Division as Sharing

3N12

TG pp. 23 – 25

Strand: Number

Outcomes

*Students will be expected to*

**3N12 Continued**

**Achievement Indicator:**

*3N12.10 Relate division to multiplication by using arrays and writing related number sentences.*

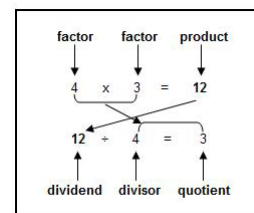
**3N11 Continued**

**Achievement Indicator:**

*3N11.10 Relate multiplication to division by using arrays and writing related number sentences.*

Elaborations—Strategies for Learning and Teaching

Understanding the meaning of multiplication and division, and the connection between the operations, is crucial as the students develop their multiplication and division facts. Students will discover that division is the inverse of multiplication, just as subtraction is the inverse of addition. “Multiplication problems arise from joining equal groups of objects; division problems arise from separating a set of objects into equal groups. Students develop an understanding of division as the inverse of multiplication by separating a whole into equal groups.” (NCTM, 2009, p. 16, 17)



Related multiplication and division equations

If there are 5 groups of 3, how many are there in all? The answer is represented by  $5 \times 3 = 15$ . To learn about a related division situation students can use the arrangement of 15 separated into 5 equal groups with 3 in each group,  $15 \div 5 = 3$ .

Introduce relating multiplication and division with a situation such as: Susan bought a pack of 20 stickers. She wanted to give each of her four friends the same number of stickers. How many stickers will Susan give to each friend?

What operation can you use to find the answer to this problem? (division) Remind students that if they know their multiplication facts, they will already know the answer because just like addition and subtraction are related ( $8 - 2 = 6$  so  $6 + 2 = 8$ ), multiplication and division are also related. ( $4 \div 2 = 2$  so  $2 \times 2 = 4$ )

Put students in pairs. Ask them to draw a picture to illustrate the sharing of 20 stickers equally among 4 friends. Monitor pairs and reinforce that division separates the whole into groups with the same number in each group.



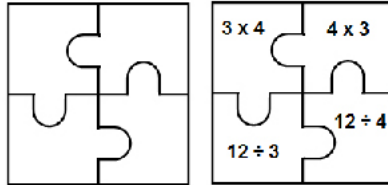
20 stickers for 4 friends

General Outcome: Develop Number Sense

Suggested Assessment Strategies

*Performance*

- Fact Family Puzzle - Create a template for a 4 piece puzzle. Write the related multiplication and division facts on the puzzle pieces. Cut apart the puzzles and ask students to assemble the fact family puzzles.



(3N12.10)

Resources/Notes

*Math Makes Sense 3*

**Lesson 8:** Relating Multiplication and Division Using Arrays

3N12

TG pp. 30 - 32

Strand: Number

Outcomes

*Students will be expected to*

**3N12 Continued**

**Achievement Indicator:**

*3N12.10 Continued*

**3N11 Continued**

**Achievement Indicator:**

*3N11.10 Continued*

Elaborations—Strategies for Learning and Teaching

Ask students to write both a division and a multiplication sentence that describes the problem they pictured. ( $20 \div 4 = 5$ ;  $4 \times 5 = 20$ ) Ensure that students can explain 20 means the total number of stickers, 4 means the number of groups (friends) and 5 means the number of stickers in each group.

Number Cube Game – Provide students with two 0 - 5 number cubes.

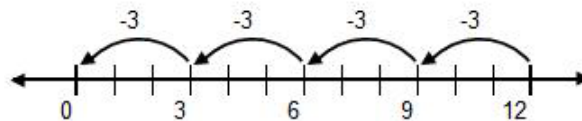
Provide or ask students to make a chart like the one below to record each roll.

Cube 1	Cube 2	Multiplication Sentence	Division Sentence
3	5	$3 \times 5 = 15$	$15 \div 3 = 5$

Player 1 rolls both number cubes.

Each player writes down the numbers and uses the two numbers to make both a multiplication and division sentence. Repeat with Player 2 in the same manner. Each player receives a point for every correctly written number sentence.

A number line also helps students see the relationship between division and multiplication



Ask students what division and multiplication sentences might this number line be showing:

$4 \times 3 = 12$      $12 \div 3 = 4$

## General Outcome: Develop Number Sense

## Suggested Assessment Strategies

*Performance/Paper and Pencil*

- Building Number Relationships Game - Provide students with a group of numbers that can be related. E.g., 3, 4, 5, 12, 15, 20. Ask students to pick three of the numbers given and use the numbers with addition, subtraction, multiplication, or division, in any way they like to show a relationship. Ask them to make a list of the different relationships they find and share with a partner. (3N12.10, 3N12.5)

*Performance*

- Circle and Stars - Roll the number cube (0 -5) and ask students to draw that number of circles. Roll the number cube again and draw that number of stars in each circle. Write the multiplication sentence that describe the drawing. Ask students to write the related division sentence that would describe their drawing. E.g.,



$$3 \times 2 = 6$$

$$6 \div 3 = 2$$

(3N12.10)

## Resources/Notes

*Math Makes Sense 3*

**Lesson 8 (Cont'd):** Relating Multiplication and Division Using Arrays

3N12

TG pp. 30 - 32

*Math Makes Sense 3*

**Lesson 9:** Relating Multiplication and Division Using Groups

3N12

TG pp. 33 – 36

**Additional Activity:**

Fishing for Partners

TG: p. v

## Strand: Number

## Outcomes

*Students will be expected to*

**Problem Solving Strategy:**

Make a Chart

## Elaborations—Strategies for Learning and Teaching

Strategy Focus: Make a Chart - Some word problems are challenging but can be simplified with the use of a chart. Encourage students to use a chart to organize information as this helps to ensure that they have thought of all possibilities when solving problems.

“Problem solving is an integral part of all mathematics learning, and as such it should not be an isolated part of the mathematics program. When problem solving is integrated into all aspects of the mathematics curriculum, teachers and students can experience the energy and excitement of learning mathematics. Problem solving and problem posing, when students are pushed beyond simply finding a right answer to questioning the answer, can be one of the most pleasurable and powerful ways to learn mathematics. Learning to question the answers by posing additional questions when solving the original problem is one way that teachers and students can develop mathematical power.”  
(NCTM, Sept. 2006, p. 79)

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 General Outcome: Develop Number Sense
 

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## Suggested Assessment Strategies

*Performance*

- What is the Question? - Use the following template to help students write a division story problem that has a quotient of 5. E.g., “I had 20 apples and I shared them among 4 of my friends. How many will each friend receive?”

A Problem-Writing Format	
	_____’s Word Problem
My Story:	_____
	_____
My Question:	_____
My Number Sentence:	_____
My Illustration:	

*Portfolio*

- “Students who develop a habit of self-assessment will also develop their potential for continued learning.” (Mathematics Assessment NCTM, 1991, p.6). Ask students to think about their own learning. Ask them to write about the strategy they used to solve the problem (make a chart, guess and check, use a pattern, draw a picture, use a model, solve a simpler problem, make an organized list, work backward or another strategy). Ask: Do you think about using strategies at all? Do you look at a strategy list but not try one? Do you look at a strategy list and pick one to try? Do you just pick a strategy you think of trying? Can you think of a situation where you found one strategy to be particularly helpful?

## Resources/Notes

*Math Makes Sense 3***Lesson 10:** Strategies Toolkit

TG pp. 36 - 37

