APPENDIX A

Important Note

These STSE modules are intended for teacher reference. Each is designed to target specific outcomes within Physics 2204. It should be noted that the activities associated with each module are NOT mandatory. They are suggested activities to be used at the discretion of the teacher.

The Physics of Tailgating

Outcomes:

1. Analyze and describe vertical motion as it applies to kinematics. (116-2)
2. Describe and evaluate the design of technological solutions and the way they function, using scientific principles. (116-6)
3. Analyze and describe examples where scientific understanding was enhanced or revised as a result of the invention of technology. (116-2)
4. Analyze mathematically the relationship among displacement, velocity and time. (325-2)

Introduction

Have you ever been in a rush to get somewhere and wished that the car ahead of you would just hurry up? Have you ever driven a little too close in an attempt to hurry the driver along? If so, you are guilty of tailgating. Tailgating is a dangerous and usually futile practice: "It only takes one crash in a tailgating line to produce a chain reaction" (Frank, n.d.). The laws of physics and of common sense dictate that you cannot go any faster than the slowest car ahead. Also driving too close forces stronger reactions to everything done by the car in front, making the drive much harder on your nerves and your car. An understanding of the physics of tailgating may be crucial in ensuring road safety and in helping tailgaters slow down and enjoy the ride. It might even result in less 'road rage'.

Theory

Tailgating can lead to multiple car crashes if even one car in a line suddenly slows down. The critical question is "how close is too close?" When learning to drive you are usually told to keep a safe distance of at least two seconds behind the car in front of you. As you observe the car ahead of you pass a fixed point, your own car should pass that same point at least two seconds later. This safe distance can also be expressed as one car length per 22 km/h of speed travelled. These rules of thumb are usually given since it is assumed that most people learning to drive do not understand basic physics. But without an understanding of some simple physics, we may all be at increased risk from tailgating. The physics of tailgating is related to motion and the kinematics equations, and includes principles like stopping distance and reaction time.

Reaction Time

If you are driving along the highway at 95 km/h and the car ahead of you suddenly applies the brakes, you must react quickly. Variables like response time become very important. When you first observe that the car ahead of you is stopping, it takes time for the brain to process this information. Reaction time includes the time taken for this processing plus the time for your foot to move to the brake. Reaction time can be determined by utilizing acceleration due to gravity principles (see activity p. 103). Typical reaction times are between 0.2 and 0.7 seconds. Nicklin (1997) tested reaction time with 64 students using computer trials of simulated brake and gas pedals, to find average reaction times of 0.3 to 0.6 seconds.

The reaction times stated above are typically obtained under ideal circumstances where the person being tested is paying attention to the task at hand. In a real situation the driver could possibly be distracted (eg. having a conversation with a friend, or singing along to the radio). Testing reaction time under these conditions might give a more realistic representation of reaction time. An even more realistic estimate would include adding on an estimation of the time it would take to move your foot from the gas pedal to the brake pedal (Alternative Homework Assignment: Tailgating). Since the foot is farther away from the brain than the hand, the reaction time calculation will be increased slightly.

Stopping Distance

A person's reaction time is important in calculating a stopping distance for the vehicle he/she are driving. Initially you are travelling along at some constant velocity before your foot hits the brake. The distance travelled during the reaction time is given by, $d = v_i t$ where v_i is the initial velocity. When the brakes are applied the vehicle begins to decelerate.

During this period of deceleration the distance travelled is given by,

where *a* is negative since the car is decelerating. Thus the total stopping distance for the car is given by,

The following data, originally published in *Popular Science* and *Auto Week* magazines (Nicklin, 1997, p. 78), can be used to solve 'tailgating problems'.

Vehicle	Deceleration (m/s^2) (from 97 km/h)
BMW M3	9.8
Toyota CElica GT	9.2
Lincoln Continental	9
Nissan Maxima	8.3
Chevrolet Blazer	7.5
Dodge Colt GL	7.1

Identical Braking Capacity

Assume that two Lincoln Continentals are travelling along a highway at 97 km/h. The front car slams on its brakes. Knowing the reaction time of the driver we can determine the minimum distance that the second Lincoln should have been behind the first to avoid a rear end collision.

The following calculation shows that the front car will stop in a distance of 41 m.

$$d = \frac{-v_i^2}{2a}$$

$$d = \frac{-\left(27\, {}^{m}\!/_{s}\right)^2}{2\left(-9.0\, {}^{m}\!/_{s^2}\right)}$$

$$d = 41m$$

The second car (using a reaction time of 0.45 s) will stop over a distance of,

$$d = v_i t + \frac{-v_i^2}{2a}$$

$$d = \left(27\, {}^{m}\!/_{s}\right)(0.45s) + \frac{-\left(27\, {}^{m}\!/_{s}\right)^2}{2\left(-9.0\, {}^{m}\!/_{s^2}\right)}$$

$$d = 12m + 41m$$

$$d = 53m$$

Note that 12 m of this distance is travelled before applying the brakes, and the other 41 m is required to stop. Thus a safe distance behind the first car would be at least 12 m. Given that the average car length is about 5.0 m, this safe distance translates into about 2.4 car lengths behind. A constant speed of 27 m/s over this 12 m translates into a 'safe time' that is equal to the reaction time.

$$t = \frac{d}{v}$$

$$t = \frac{12m}{27\,m/s}$$

$$t = 0.44s$$

The only factor affecting the required separation distance is the reaction time (when both cars are travelling at the same speed and have the same deceleration).

At this point it might appear that the two second rule is overly cautious. However the situation described is an idealized one where both cars have the same braking ability and the tailgater has a reasonably good reaction time. The situation could be much worse if the tailgater had a poor reaction time, if the road conditions were wet or icy, if the lead car were travelling slower than the tailing car, or if the braking capacity of the cars were different.

$9.8\,m/s^2$

$$2ad = v_f^2 - v_i^2 \quad \text{where } v_f = 0$$

$$d = \frac{-v_i^2}{2a}$$

Different Braking Capacity

The situation with different braking capacities can also be illustrated using data from the table given (Nicklin, 1997). Nicklin describes a situation where two cars are travelling at 121 km/h with a separation distance of 5 car lengths (24.38 m). Car A decelerates at (a BMW), while car B decelerates at $7.5\,m/s^2$ (a Chevrolet Blazer). If the driver of car B has a reaction time of 0.45 s, the following calculations show that car B will in fact hit car A even at 5 car lengths away.

Stopping distance of car A:

Stopping distance of car B:

$$d = \frac{-\left(33.6\,m/s\right)^2}{2\left(-9.8\,m/s^2\right)}$$

$$d = 57.6m$$

where the $v_i t$ portion corresponds to the distance travelled during the reaction time.

$$d = \left(33.61\,m/s\right)(0.45s) + \frac{-\left(33.61\,m/s\right)^2}{2\left(-7.5\,m/s^2\right)}$$

$$d = 15.1m + 75.3m$$

$$d = 90.4m$$

Thus when car A has stopped, it would be 24.38 m (5 car lengths) + 57.6 m = 81.98 m from where car B started. If Car A has come to a complete stop, it will still be hit by Car B since Car B requires 90.4 m to stop (it can be shown that Car B will actually collide with Car A 3.4 s after Car A starts to brake). Car B would have been decelerating for 81.98 - 15.1 m = 66.88 m before reaching car A. The final velocity of car B at 66.88 m is,

$$v_f^2 = 2ad + v_i^2$$

$$v_f^2 = 2\left(-7.5\,m/s^2\right)(66.88m) + \left(33.61\,m/s\right)^2$$

$$v_f^2 = -1003.2\,m^2/s^2 + 1129.63\,m^2/s^2$$

$$v_f = 11.2\,m/s$$

Under these conditions when car A has better brakes and can stop faster, car B will collide with car A even with a good reaction time and a separation distance of five car lengths.

The situation is even more complicated when there is a line of tailgating cars. If the car ahead of you is also tailgating, you have no way of knowing how much they have reduced their own safety margin. As a driver you can roughly tell your own reaction time, velocity, and braking ability. Unfortunately you know nothing about the other driver's reaction time or braking conditions. This lack of knowledge further increases the risk of tailgating.

Getting Ahead?

Traffic lights can be particularly frustrating especially when trying to reach a destination in a hurry. Many drivers think that tailgating and driving as fast as possible between lights will get them there faster than somebody who obeys the speed limit. However this is not necessarily the case. In the case of heavy traffic, tailgating can actually slow you down. How many times have you observed a car whiz by you by weaving in and out of traffic, only to find that four or five lights later they are still only slightly ahead of you? Traffic lights are timed to ensure easy flow of traffic. One way of doing this allows a person following the speed limit to get every green light (once they get one). Tailgaters however are forced to slow down or stop every time a car ahead slows or takes a turn. Getting back up to speed leaves a larger gap in front of the car than if they had been travelling along at a constant speed at a safe distance. This gap is quickly filled in heavy traffic, so the tailgater doesn't get much further ahead. Also, having to get up to speed at every red light causes the slowdown of trailing lines of traffic that would ordinarily have made the light, thus contributing to traffic congestion.

Conclusion

In our fast-paced world it is often difficult to slow down when there is so much to do in so little time. Tailgating may give the perception of getting ahead, but a basic understanding of motion shows that this is not the case. So, how close is close enough? In the case of tailgating the answer to this question is 'too close for comfort'.

Questions

1. In a realistic model of tailgating what factors should be considered that would increase the safe stopping distance?
2. What is the stopping distance of a Toyota Celica (a = -9.2 m/s^2) from 97 km/h where the driver has a reaction time of 0.55 s?
3. A Chevrolet Blazer travelling at 97 km/h can stop in 48 m. Given that the actual stopping distance for a certain driver is 54 m, what was the driver's reaction time?
4. An automobile is travelling at 25 m/s on a country road when the driver suddenly notices a cow in the road 30 m ahead. The driver attempts to brake the automobile but the distance is too short. With what velocity would the car hit the cow if the car decelerated at 7.84 m/s^2 and the driver's reaction time was 0.75 s?
5. Research: Look in car magazines to determine stopping distances and deceleration rates for your own or family car.

References

Alternative Homework Assignment: Tailgating. http://www.physics.umd.edu/rgroups/ripe/perg/abp/aha/tail.htm

Frank, L.: http://www.leefrank.com/books/margins/asides/keepaway.htm

Kinematics of Driving: Some "Real" Traffic Considerations. http://www.dctech.com/physics/features/physics_0700a.htm

Kinematics of Tailgating: http//www.ecu.edu/si/cd

Nicklin, R.C. (1997). Kinematics of tailgating. *The Physics Teacher*, 35, p. 78-79.

Activities

Reaction Time

Purpose: To determine a person's reaction time.

Materials: • Meter stick

Procedure:

1. Have a partner hold a meter stick while you position your thumb and forefinger just at the 0 mark. Have your partner release the ruler while you try to catch it as quickly as possible. You can then record the distances of several trials and take an average distance.
2. The meter stick will fall at a rate of 9.8 m/s^2 toward the ground from an initial velocity of 0 m/s. Given this data, reaction time can be calculated from the kinematics formula,

$$d = v_i t + \frac{1}{2}at^2$$

where $v_i = m\,0$ do,

$$d = \frac{1}{2}at^2$$

$$at^2 = 2d$$

$$t^2 = \frac{2d}{a}$$

$$t = \sqrt{\frac{2d}{a}}$$

This activity can be repeated under more realistic conditions by having your partner distract you as you try to catch the ruler.

The Physics of Karate

Outcomes:

1. Analyze natural and technological systems to interpret and explain their structure and dynamics (116-7).
2. Describe the functioning of a natural technology based on principles of momentum (116-5).
3. Apply Newton's Laws of motion to explain the interaction of forces between two objects (325-8).
4. Apply quantitatively the law of conservation of momentum to one-dimensional collisions and explosions (326-3).
5. Interpret patterns and trends in data, and infer or calculate linear and nonlinear relationships among variables (214-5).
6. Compile and display evidence and information, by hand or computer, in a variety of formats, including diagrams, flow charts, tables, graphs, and scatter plots (214-3).
7. Use appropriate language and conventions when describing events related to momentum and energy (114-9).

Introduction

What kind of person would intentionally bring their hand or foot crashing down onto a slab of wood or concrete? A daredevil? A Hollywood stuntperson? As it turns out, that kind of person is simply someone who understands the physics of karate - someone like you!

Karate means "open or empty hand", and began as a form of weaponless combat in 17th century Japan. In recent years it has become popular in our culture, as a form of fitness, self-defense and self-expression. Karate participants - called Karateka - often break concrete or wooden boards as a demonstration of the strength developed through training. Surprisingly there are no tricks involved in accomplishing such a feat. What is involved is a physics-based knowledge of how to do it properly. "Few things offer more visceral proof of the power of physics than a karate chop. Punch a brick with your bare hand, untutored in the martial arts, and you may break a finger. Punch it with the proper force, momentum and positioning and you'll break the brick instead" (Rist, 2000).

Theory

Force, Speed and Area

Karateka agree that the secret to karate lies in the force, speed and focus of the strike. The more quickly a board is hit, the harder the strike. Maximum hand velocity is actually achieved when the arm reaches 75-80% of extension. Since the hand cannot move forward a distance greater than the length of the arm, it must have a velocity of 0 at full arm's extension. To get the hardest hit, contact must be made with the object before this slowdown begins. Thus a good karate chop has no follow-through (as would a good tennis or golf swing). The hand is typically in contact with the object for fewer than five milliseconds.

How fast can a karate punch actually move? Experiments done with a strobe light on karateka throwing punches found that beginners can throw a punch at about 6.1 m/s (20 feet/sec), while black belts could chop at 14 m/s (46 feet/sec). At the latter speed a black belt can deliver about 2800 N to the object being hit. (Splitting a typical concrete slab requires only about 1900 N). A concrete slab could probably support a force of 2800 N if it were not concentrated into such a small area. Minimizing the striking surface of the hand, and therefore the area of the target being hit, maximizes the amount of force and energy transferred per unit area. To understand why speed and focus are so important, the principles of momentum and impulse must also be considered.

Momentum and Impulse

Momentum ( ) is defined as an object's mass x velocity. Change in momentum, ( ) is defined as impulse (symbol), and is given by force x time. According to Newton's third law momentum is a conserved quantity. The third law states that for every action force on an object in a given time, there is an equal and opposite reaction force by that object for the same amount of time. Thus, any momentum lost by the first object is exactly gained by the second object. Momentum is transferred from one object to the other. Using,

we can see that if remains fixed, then force and time are inversely proportional. This means that if force increases, than time decreases and vice versa. It follows that a fixed amount of momentum can then be transferred with a small force for a long time or with a large force for a short time.

The quicker the karateka can make the chop, the larger the force transferred to the target. According to Newton's second law ($\vec{F} = m\vec{a}$) the part of the object struck with this force will begin to accelerate or oscillate. Breakage occurs if the small area hit accelerates enough relative to the stationary ends of the object. The object will experience strain and begin to crack from the bottom up.

What about the strain experienced by the hand or foot? Fortunately bone can withstand about forty times more force than concrete. Hands and feet can withstand even more than that due to the skin, muscles and ligaments which absorb much of the impact. Despite possessing these "natural shock absorbers", breaking wood, concrete or bricks should not be attempted without proper training. Such training would include toughening up the hand and knowing exactly how and where to hit the object with maximum speed. Over time the knife edge of the hand, called the "shuto", develops a callous which acts to absorb the collision force. As well, experts know to only hit things that can actually be broken. Sihak Henry Cho, a grand master at the Karate Institute in Manhattan sums it up nicely: "Being good at karate is a lot like being good at telling a joke. It's not what you break; it's how you break it" (Rist, 2000).

Questions

1. Why is it important to hit a concrete slab quickly when attempting to break it?
2. Karate black belts often advise beginners before their first attempt at breaking, not to try to break the board, but to aim for the floor underneath the board. How would this advice help?
3. Research: Karate practitioners usually yell "Kiai" when striking an object. Research the meaning of this term.

References

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Activities

Activity 1: How Much Weight does it take to break a board?

Materials:
- masses (2 kg)
- board
- supports (bricks)
- meter stick

Procedure:

Design a procedure to see how much weight must be placed on the board in order to get the board to break. As the weight is added to the board, measure how far the board bends.

Analysis:

1. Graph applied force (y-axis) versus bending distance (x-axis).
2. Find the slope of your graph. Describe how the applied force is related to the bending distance.
3. Recall that work can be done to a system to change the energy of the system. The work done by a force F can be determined by finding the area under the curve of a force versus distance graph. From your data determine the work that was done to break the board.

Activities

Activity 2: Pretzel

Purpose: To use pretzel sticks to better understand what causes materials to break.

Materials:

- pretzel sticks of varying thickness
- pieces of uncooked spaghetti
- rolls of 50 pennies each, plus 50 loose pennies
- thick string or wire about 6 cm long
- paper cup
- empty plastic film container
- scissors or craft knife
- tweezers

Procedure:

1. Build a pretzel strength-testing machine. Start by cutting a large hole in the bottom of the paper cup. Set the cup on the table, bottom side up. Rest a pretzel stick across the center of the cup.
2. Next create a weight bucket to hang on the pretzel. Take the empty plastic film container and make two holes about 1 cm from the top rim and directly across from each other. Thread the string or wire through the holes and tie the end at each hole. The bucket should hang on the pretzel without touching the table.
3. Begin testing. With the bucket hanging on the pretzel stick begin adding pennies. See how many pennies the pretzel can hold without breaking. Find the average number of pennies one type of pretzel stick can hold.
4. Gaining momentum: Test to see if it makes a difference if you drop the pennies in the bucket or you place them in gently using the tweezers.
5. Breaking point: Test to see if the weakest point of the pretzel is really at the center.
6. Length and width test: Try pretzels of various lengths and widths to see what size and length hold the most and least pennies.
7. Compare with other materials: Do you think a pretzel or an uncooked piece of spaghetti is stronger when bent? Try testing uncooked spaghetti to see how it holds up in comparison to the pretzel sticks.

Questions:

1. Look at the ends of a broken pretzel with a magnifying glass. Does its structure tell you anything about its bending strength?
2. Can you figure out a way to spread weight out across the entire length of the pretzel? Can it hold more weight when the weight is distributed over a larger area?

(Activity designed by Jane Copes, Science Museum of Minnesota and adapted from Newton's Apple Teacher's Guide: Karate)

Activities

Activity 3: (taken from Pushing Air located at http://www.schools.ash.org.au/paa/downloads/actbook.pdf)

Purpose: To relate a successful karate chop to air pressure.

Materials:

- sheet of newspaper
- ruler

Procedure:

1. Place a ruler or flat stick on a bench top with about a quarter of its length overhanging.
2. Give the overhanging part of the ruler a quick karate chop from above.
3. Repeat the above steps with a piece of newspaper covering the nonoverhanging part of the ruler.

Questions:

1. Why do you think the ruler snaps during the second part of the experiment?

Explanation:

Air is all around us pushing on everything. It pushes on our skin and on the bench top. The ruler has a small surface area, so the air pushing down on it is not enough to hold the ruler in place when you hit it. The newspaper has a large surface area. The force of the air acts over the whole area. The result is that air holds down the paper which holds the ruler in place. Unable to lift quickly enough when the overhanging part of the ruler is struck, the ruler has no option but to snap.

The Physics of Bungee Jumping

Outcomes:

1. Analyze natural and technological systems to interpret and explain their structure. (116-7)
2. Describe and evaluate the design of technological solutions and the way they function, using energy principles. (116-6)
3. Analyze and describe examples where technological solutions were developed based on scientific understanding. (116-4)
4. Distinguish between problems that can be solved by the application of physics-related technologies and those that cannot. (118-8)
5. Analyze and describe examples where energy-related technologies were developed and improved over time. (115-5, 116-4)
6. Analyze the risks and benefits to society and the environment when applying scientific knowledge or introducing a particular technology (118-2)
7. Construct and test a prototype of a device and troubleshoot problems as they arise. (212-14)
8. Analyze quantitatively the relationships among mass, height, gravity, spring constant, gravitational potential energy and elastic potential energy. (326-1)
9. Solve problems using the law of conservation of energy, including changes in elastic potential energy.

Introduction

Would you plunge off a bridge attached only by a soft springy cord that could stretch three to four times its free length? If you understood the physics behind such a daring feat you just might! Bungee jumping involves attaching oneself to a long cord and jumping from extreme heights. It is related to a centuries old practice from the Pentecost Island in the Pacific Archipelago of Vanuatu. On this island, the men jump to show their courage and to offer thanks to the gods for a good harvest of yams. In 1979, members of the Oxford University Dangerous Sport Club jumped off a bridge near Bristol, England, apparently inspired by a film about "vine jumpers". In the early 1990's, the sport gained popularity in the United States and Canada. Today it is still dubbed the "ultimate adrenaline rush" (Menz, 1993).

Equipment

The old adage of "less is more" certainly applies to bungee jumping. The only equipment required is a springy cord and a harness. However it is very important that the equipment used be strong and secure. The harnesses are similar to those used in mountain climbing, including the caribiner which is the main link between the cord and the harness. The cord itself is soft and springy and is secured tightly to the jumper's body. Jumpers today are typically aided by double hookups. If an ankle jump is chosen, the body harness is used as a backup. If the body harness is chosen, a chest/shoulder harness becomes the backup.

Though there have been some accidents related to bungee jumping (three deaths in France in 1989), they can be traced to human error in attachment, total height of jump available, or a mismatch between the cord and jumper. Minor injuries like skin burn or being hit by the cord happen when jumpers do not follow instructions. Skin burn for example is caused by gripping the cord. Understanding and adhering to some basic physics principles would prevent such problems.

Theory

Energy Distribution

The main physics concepts involved in bungee jumping are the gravitational potential energy of the jumper and the elastic potential energy of the stretched cord. Initially the jumper is attached to the cord which is attached to a supporting structure on the same level as the jumper's center of mass. Standing on the platform, the jumper possesses gravitational potential energy given by,

where *h* is the height from the top to the bottom extremity of the jump. At the beginning of the jump (before the cord reaches maximum length) the jumper experiences free fall. In free fall the only force acting on the jumper (neglecting air friction) is the force of gravity which causes the person to accelerate downward at 9.8 m/s^2. Free fall is a funny sensation in that the jumper experiences no outside forces and thus their internal organs are not pushing on each other. The free fall typically lasts between one and two seconds. During this time the bungee cord is not yet stretching and some of the original gravitational potential energy is transferred into kinetic energy $\frac{1}{2}$. The distribution of energy at a certain height "d" is then given by,

$$E_{total} = mgd +$$

When the cord reaches its full length it begins to stretch and applies an upward force that begins to slow the jumper. At this point some of the jumper's energy is stored in the bungee cord
and the total energy is given by,

$$\frac{1}{2}$$

When the jumper reaches the bottom extremity of the jump the velocity of the jumper, and therefore the kinetic energy, is zero. At that point the gravitational potential energy possessed at the top has been totally converted into the elastic potential of the cord. Since energy is conserved in the jump, the gravitational potential energy of the jumper must equal the elastic potential energy of the cord.

$$E_{total} = E_{bottom}$$

$$mgd = \frac{1}{2} kx^2$$

The elastic potential energy refers to the energy stored in the cord by virtue of stretching it. The jumper will realize that there is stored energy in the cord when it rebounds to its equilibrium shape. The restoring force of the cord is used to decelerate and eventually stop the jumper.

The figure below (Nowikow & Heimbecker, 2001) shows how the different types of energy change during the jump. Note that as the gravitational potential energy decreases during the fall, the kinetic energy increases. At the bottom extremity of the fall as the cord tightens, the loss in gravitational potential energy is matched by a corresponding increase in the elastic potential energy of the bungee cord. At any point in the fall, the sum of the kinetic and elastic potential energies is equal to the gravitational potential energy lost during the fall.

Hooke's Law and Elastic Potential Energy

The work done to stop the jumper is related to the stiffness of the bungee cord. The cord acts like a spring that obeys Hooke's Law. Hooke's Law is given by,

where F is the restoring force, k is the spring constant and x is the stretch of the cord. The elastic potential energy possessed by the cord at the bottom of the fall is given by, ½

Thus we can write that,

Potential energy at the top relative to the bottom of the fall. = *Elastic potential energy of cord at the bottom extremity of the fall.*

or mathematically, ½

$$mg\left(L+x\right)=\tfrac{1}{2}kx^2$$

where , h = (L + x), L is the length of the bungee cord and x is the stretch of the bungee cord.

This relationship allows the correct matching of cord with person or of jump height with person. If for example a given jump height (L + x) is to be matched with a given person of mass m, we can determine what stiffness (k) of cord should be used for that jump.

$$kx^2 = 2mg\left(L+x\right)$$

$$k = \frac{2mg\left(L+x\right)}{x^2}$$

If, however, a given cord of length L and stiffness k is to be matched with a person of mass m, then the amount of stretch can be determined as follows,

½

$$mgL + mgx = \frac{kx^2}{2}$$

$$2mgL + 2mgx = kx^2$$

$$kx^2 = 2mgx - 2mgL = 0$$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 8kmgL}}{2k}$$

$$x = \frac{2mg \pm \sqrt{4mg\left(mg + 2kL\right)}}{2k}$$

$$x = \frac{mg \pm \sqrt{mg\left(mg + 2kL\right)}}{k}$$

In most cases the latter method is the way the match would be made so that the total fall (L + x) will fit the jumping facility.

Hooke's Law can also be applied to determine the maximum force experienced by a jumper. If for example a 68 kg person is to jump using a 9.0 m cord which will stretch 18 m, we get the following.

$$k = \frac{2mg\left(L+x\right)}{x^2}$$

$$= \frac{2\left(68kg\right)\left(9.8\,{}^{m}\!/\!_{s_2}\right)\left(9.0+27m\right)}{\left(27m\right)^2}$$

$$= 111\,{}^{N}\!/\!_{m}$$

Therefore,

$$F = kx$$

$$= \left(111\,{}^{N}\!/\!_{m}\right)\left(18m\right)$$

$$= 1998N$$

Thus the force is about three times the person's weight. A cord with more stretch would give a "softer" ride. If for example the stretch of the 9.0 m cord were 27 m,

$$k = \frac{2mg\left(L+x\right)}{x^2}$$

$$= \frac{2\left(68kg\right)\left(9.8\,{}^{m}\!/\!_{s^2}\right)\left(9.0m+27m\right)}{\left(27m\right)^2}$$

Therefore,

$$= 1998N$$

Thus the force is about three times the person's weight. A cord with more stretch would give a "softer" ride. If for example the stretch of the 9.0 m cord were 27 m,

$$= \frac{2(68kg)(9.8 \quad)(9.0m = 27m)}{(27m)^2}$$
$$= 66$$

and

$$N/m^2$$

This exerts a lesser force on the jumper for a more comfortable jump. In reality of course, one must consider that given facilities will have a limited number of cords of differing length and stiffness. Also, bungee cords have been found to demonstrate variable stiffness over their range of use (i.e. k does not remain constant).

Oftentimes it is a matter of choosing the best fit available for the jump. It seems reasonable however to match heavier people with stiffer cords and lighter people with softer cords. The following table (Menz, 1993) illustrates this idea.

Menz (1993) recommends that a proper match of cord and jumper should produce maximum accelerations of the order of 3 g's (where g=9.8 m/s^2).

Conclusion

Bungee jumping then deals with the conversion of gravitational potential energy into the elastic potential energy of a stretched cord. It is an extreme sport that requires courage, daring and a knowledge of physics – at least by the people organizing the jump.

Questions

1. A person of mass 65 kg is to bungee jump from a platform that is 18.5 m above the ground. If the bungee cord used has a stiffness of 204 N/m and a length of 9.5 m, is it safe for the person to jump?
2. A 75 kg person is to bungee jump with a cord of length 8.0 m that will stretch 10.0 m. What force will be exerted on the person?
3. Describe the energy conversions that take place as a person bungee jumps.
4. Research: What were bungee cords originally designed for?

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Activities

Activity 1: Bungee Egg

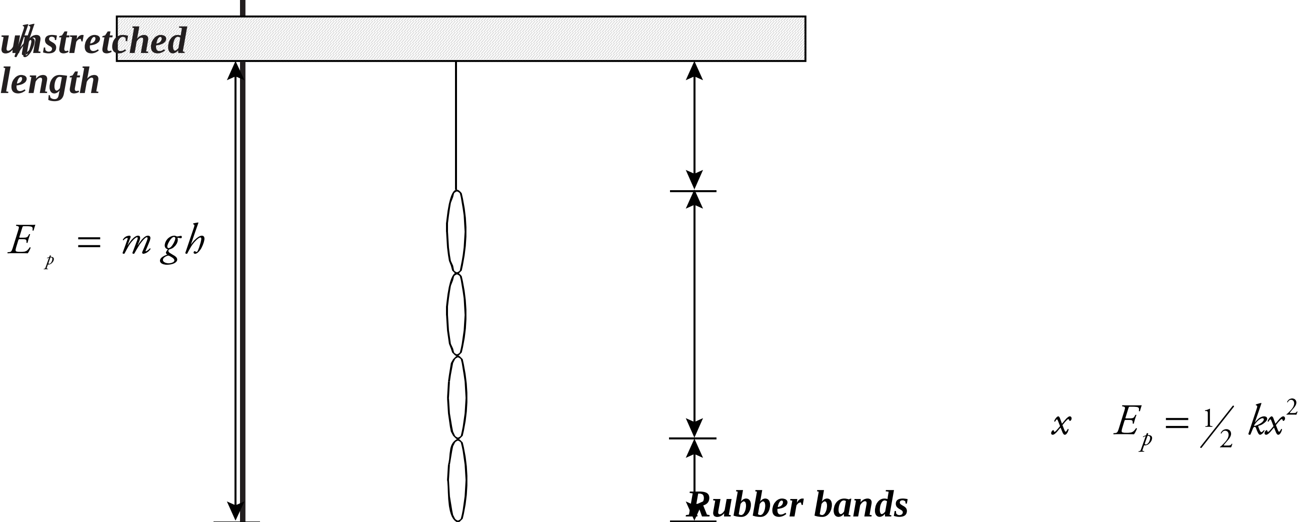
Purpose: To drop an egg attached to a bungee cord and have it come as close to the ground as possible without breaking.

Materials:

- ten rubber bands
- meter stick
- mass balance
- sandwich baggie (to minimize mess)
- masses
- egg (or eggs)

Procedure:

1. The procedure for this activity will be student-designed. Their goal is to use energy calculations to determine the height of drop that will allow the egg to land safely. Students should be aware that the design of bungee jumps involves calculating the point at which the gravitational potential energy lost during the fall will equal the elastic potential energy gained by the elastic cord (or rubber bands). The following diagram may be useful in helping them visualize what they have to do. (Hint: Don't forget to take into account the height of the "egg in bag" attachment).



Where h is the total drop height, l is a fixed string (optional), and x is the amount of stretch.

Raw eggs are dropped in a harness made from a sandwich baggie. Evaluate students on how close they come to the floor without breaking the egg.

Activity 1: Bungee Egg (continued)

Notes to Teacher:

1. The bungee cord can be constructed from rubber bands or a bungee cord if one is available. The piece of string of length L, may or may not be used. The rubber bands (or cord) could be attached directly to an adjustable platform (eg. ring stand). If the string is used we will ignore its stretch in the calculations.

2. Students will first determine the spring constant k for their bungee cord by hanging masses from the cord and measuring the amount of stretch. The slope of a graph of force versus stretch will then give the value of k. This is Hooke's Law.

3. Energy conservation principles can then be used to figure out the stretch of the cord at the bottom extremity of its fall:

   $$mgh = \tfrac{1}{2} kx^2$$
   $$mg\left(L + x\right) = \quad kx^2$$

   where L is the length of the unstretched bungee cord and x is the amount of stretch.

4. Use the amount of stretch to determine what height the egg can be safely dropped from (by either adjusting the platform or adding a string to adjust the length).

5. Students should present calculations supporting their proposed drop height before any actual testing takes place.

Activities

Activity 2: Bungee Egg Drop 2

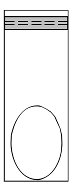
Refer to the following website (or see below) for another type of bungee egg drop experiment using a graphical analysis: http://www.physics.ucok.edu/~chughes/~plrc/Labs/BungeeEgg2.

Introduction:

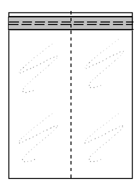
You may have seen the bungee egg apparatus in a previous experiment: Bungee Egg Drop 1. The apparatus is constructed from a minimum 2 meter length of unstretchable string or cord, a minimum of 6 standard size rubber bands, one grade AA large egg, one small safety pin, and one “half size” ziplock bag. Briefly, the egg is placed in the plastic bag which is then attached to a string of rubber bands tied together. This is, in turn, attached to the unstretchable length of cord. Detailed instructions for constructing this object are presented below.

The egg is dropped by attaching the unstretchable cord to a support arm a chosen height above the floor. During the first part of the jump, only gravity acts on the egg causing it to fall faster and faster. When the egg has fallen a distance greater than the unstretched cord’s length, the cord pulls upward causing the egg to come to a stop somewhere above the ground (hopefully). This maximum height through which the egg is dropped can be adjusted by changing the length of the cord.

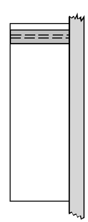
Constructing the Bungee Apparatus



The egg holder is constructed by taking a normal ziplock bag and cutting it lengthwise down the middle with a pair of sharp scissors:



One of the resulting half size bags will be used as the egg holder (the other one should be kept as a spare). To keep the egg from rolling out, it is necessary to tape the open side of the bag with transparent tape. This should be done carefully to make sure that the top still opens and closes with the ziplock.

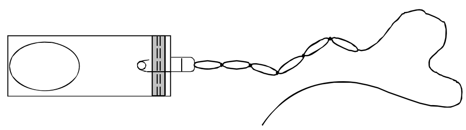


Finally, a hole should be punched in the top of the bag with a hole punch. After placing the egg in the bag, a safety pin is attached to the bag through the hole.

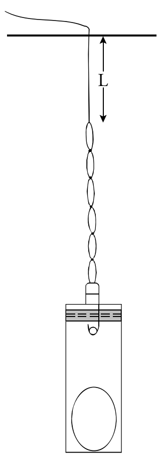
Activities

Activity 2: Bungee Egg Drop 2 (continued)

Next the "bungee" part of the apparatus must be constructed. At least six standard rubber bands should be tied end-to-end to produce an elastic chain. One end of the rubber band chain is attached to the top of the safety pin. The other is tied to the piece of unstretchable cord.



The exact number of rubber bands, and the length of the string needed, must be chosen so that the bungee apparatus can cover the range of distances when the egg is dropped: from 100 cm to500 cm



The different maximum distances that the egg falls through are achieved by changing the length of the unstretchable part of the cord, that is, the length between the topmost rubber band and the point where the cord is tied to the support arm. As this distance gets bigger, the egg will fall through a larger distance.

It may take a little experimentation to find a workable combination of cord length and number of rubber bands that will effectively cover the range of values needed. One group of students may choose to have more rubber bands and shorter string lengths while another may opt for less rubber bands and a longer string length. The only restriction is that each group must have a minimum of six rubber bands and a minimum of 2 meters of string.

Springs

A rubber band doesn't look much like a spring, but it really has behavior very similar to that of a spring.

Taking and Analyzing the Data

Ultimately you will have to use your measurements to make predictions. With this in mind, you might wonder about the best way to keep track of the data from the measurements.

You probably will want to start with a data table:

d(cm)	L(cm)

Activities

Activity 2: Bungee Egg Drop 2 (continued)

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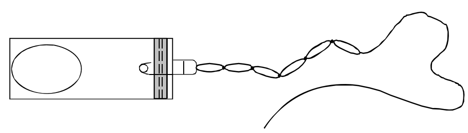

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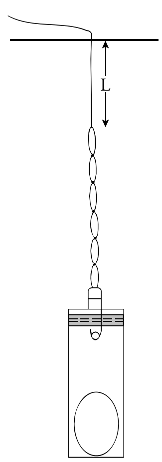

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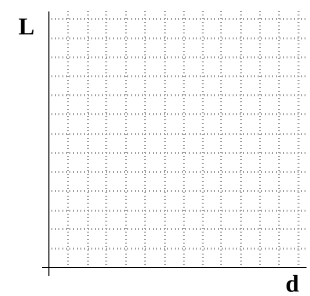
d(cm)	L(cm)

Activities

Activity 2: Bungee Egg Drop 2 (continued)

1. You should take enough data points so that the length L can be easily predicted to produce a given distance d in the range necessary for the contest. If the data points are too far apart, it becomes difficult to interpolate between the data.

2. The data table represents an insight into the real physics behind this apparatus. As such, it only gives a few pieces of information that point the way to explaining the relationship between L and d. The best way to use the data to explore this relationship is with a graph (as on the right). A graph of the data will show a collection of points on the graph. If enough points are found, a "curve" can be sketched through the data. While the line through the data is called a "curve," the line can actually be curved or straight. Its shape depends on the underlying physical law that relates the quantities being plotted. The curve is the first clue about the nature of that physical law. It tells a theoretical physicist that "this is the information which your theory must match."



   For our purposes, the curve simply means an infinite number of data points, each representing how far an egg will drop for a given choice of string length. The curve should be a smooth line drawn through the data. Some data points might lie on either side of the final curve because of errors in measurement. This gives an indication of the uncertainty in your measurements and should be taken into account when selecting a string length for the contest.

The Contest

The graph of L vs d provides a visual description of the physics of your bungee apparatus. If you made careful measurements and took care to draw a neat curve through your data, you should be able to predict the length of string needed to cause the egg to drop exactly the distance d.

Your bungee apparatus should be able to accurately predict the length of string necessary to make the egg fall, between 100 cm and 500 cm

When you arrive for the contest, you will be shown the area where the egg will be dropped. You will then have 5 minutes to measure the height of the drop with a meter stick, determine the proper length of the string for your apparatus, and drop your egg. The winner will be the group of students with the egg that gets closest to the ground without cracking. Closeness to the ground will be judged by your instructor (whose opinion is final). Eggs must be raw and may not be cushioned in any way. The bag is merely to hold the egg. The eggs will be "tested for rawness" at the end of the competition (so don't get too attached to your egg).

The Physics of Karate

Outcomes:

1. To describe and evaluate the design of technological solutions and the way they function, using scientific principles. (116-6)
2. To analyze natural and technological systems to interpret and explain their structure. (116-7)
3. To analyze and describe examples where technological solutions were developed based on scientific understanding. (116-4)
4. To analyze society's influence on scientific and technological endeavours. (117-2)
5. To analyze why and how a particular technology was developed and improved over time. (115-5)
6. To analyze and describe examples where scientific understanding was enhanced as a result of the invention of a technological device. (116-2)
7. To describe what is meant by a vibration, and give examples from technology.
8. To explain how standing waves are produced on a stretched string.
9. Given the fundamental frequency and fundamental wavelength of a vibrating string, produce diagrams of various overtones labelled to show wavelength and frequency of each.
10. Describe how sound as a form of energy is produced and transmitted.

Introduction

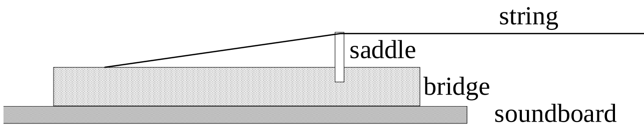
Chris Griffiths of St. John's, Newfoundland has had a lifelong interest in music, beginning guitar lessons at the age of twelve. He began building guitars at the age of seventeen. Since then he has turned his interest into the successful guitar making business known as Griffith's Guitar Works - a 20 000 square foot, multimillion dollar high tech acoustic guitar factory. Though Griffiths may not have chosen physics as a career, a knowledge of physics was certainly important in producing great sounding guitars like his latest creation - the Garrison. The Garrison guitar line includes a full range of acoustic guitars, beginning with the G-10 and following through to the top of the line G-50. Through innovative construction techniques, these guitars offer "superb playability and clarity of tone" that is setting a new standard for acoustic guitars (Garrison Guitars).

Construction

There are many different types of acoustic guitars, producing varying qualities of sound. However they all share some basic construction features. The three main parts of any guitar are the hollow body, the neck and the head.

Body

The guitar body includes the soundboard, a wooden piece mounted on the front of the guitar. The soundboard should be made so that it can vibrate up and down relatively easily. It is usually made of spruce or another light springy wood. Griffith's Garrison guitars are constructed from all solid wood including East Indian rosewood, sapele, englemann spruce, sitka spruce, Canadian birch and western red cedar. There is a large hole in the soundboard called the sound hole. Also attached to the soundboard is the bridge. The bridge anchors one end of the six strings. On the bridge is a saddle which the strings rest against.



When the strings are plucked they vibrate. The vibrations travel through the saddle and bridge to the soundboard. The hollow body of the guitar then amplifies the vibrations of the soundboard. These vibrations then disturb the air producing a sound wave reaching our ears. Without the amplification of sound produced by the hollow body, these vibrations would be barely audible. Bracing refers to the internal reinforcement of a guitar that must add strength where necessary but still allow the top to vibrate as freely as possible. Garrison guitars boast a single-unit brace that allows the resonant sound to travel uninterrupted through the guitar no matter where the vibration is created.

Neck

The neck of the guitar joins the body to the head. On the face of the neck (called the fingerboard) are metal pieces called frets that are cut at specific intervals. When a string is pressed onto a fret, the length of the string is changed. Changing the length changes the sound that is produced. The frequency of sound produced is inversely proportional to lengt $\left(f \propto 1/L\right)$. As length decreases frequency increases. The six strings on guitars also have different weights which affect the sound produced. The first string is as fine as a thread while the sixth is wound much heavier and thicker. More massive strings vibrate more slowly. The frequency of sound produced is inversely proportional to the square root of the density of the string ($f \propto 1/\sqrt{P}$) . As the density decreases the frequency increases. The frequency is also inversely proportional to the diameter of the string ($f \propto 1/d$). This means that as the diameter decreases, the frequency increases.

The strings themselves do not make much noise when plucked since they do not cause a large disturbance to the air around them. It is the vibrations of the bridge and body that produce such pleasing sounds.

Head

Joining the neck to the head is a piece called the nut. The nut has grooves to hold the strings. From the nut the strings are connected to the tuning pegs on the head. Turning these pegs allows the tension in the strings to be increased or decreased. These pegs are used to tune the guitar. The tighter the string the higher the pitch and frequency of sound produced. In fact, frequency varies directly as the square root of the tension ($f \propto \sqrt{T}$).

Electric Guitars

The major difference between electric guitars and acoustic guitars is in the body. Electric guitars have a solid body with no sound hole. A string plucked on an electric guitar makes almost no sound if not connected to an amplifier. This is because without a hollow body there is nothing to amplify the sound. Electric guitars therefore rely on amplifiers and speakers to produce sound. Vibrations are sensed electronically and then sent to the amplifier and speaker.

Theory

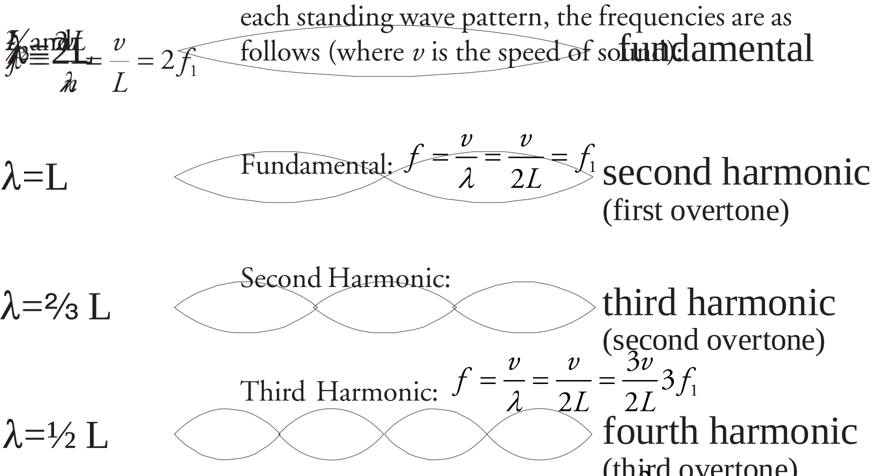
Standing Waves

Guitar strings are fixed on both ends by the saddle and the nut. The body of the guitar will resonate when standing waves are set up on the strings. A string will resonate when its length is equivalent to $\frac{1}{2}$ etc. This is the same pattern of resonant lengths that exist in an open air column. The standing waves in the strings are illustrated in the following diagrams:

Note that since the string is fixed at both ends, any vibration of the string will have nodes at each end. This limits the possible vibrations that can be achieved on a given length of string. We can see that for each of the diagrams, the wavelengths are $2L, L,$. In general this is written as

where n is the harmonic number. Thus for each standing wave pattern, the frequencies are as follows (where v is the speed of sound):

$f = \frac{v}{L} = 2f_1$



Fundamental: $f = \frac{v}{\lambda} = \frac{v}{2L} = f_1$

Second Harmonic:

Third Harmonic: $f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{3v}{2L} 3f_1$

Fourth Harmonic: $f = \frac{v}{\lambda} = \frac{v}{L/2} = \frac{2v}{L} 4f_1$

Since all waves in the same string travel with the same speed, then waves with these different wavelengths must have different frequencies. The frequencies f_1, $2f_1$, $3f_1$, $4f_1$, etc. are referred to as the harmonic series. It is the rich variety of harmonics that make a guitar or any stringed instrument interesting to hear.

Conclusion

Guitar construction is really a combination of art and science. Physics principles dictate the kind of sound produced in terms of frequency and wavelength. However it is the craftsman's artistry in constructing the shape of the body and soundboard that give each guitar its distinctive sound. For Griffiths there has been a "brilliant blend of technology, art and craftsmanship" which has set a new standard for acoustic guitars worldwide.

Questions

1. A guitar string of length 0.60 m has a frequency of 395 Hz. If the string is shortened to 0.30 m, what is its new frequency?
2. A standing wave is set up on a guitar string of length 0.60 m. If the string vibrates in the third harmonic, what is the wavelength of the sound produced?
3. What is the main function of the body of the guitar in producing the music we hear?
4. A guitar string has an original frequency of 146 Hz. How would the tension have to change to have the string vibrate with a frequency of 292 Hz?
5. Research: How has Griffith's guitars upped the standard for worldwide acoustic guitar construction?

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Activity

Activity:

Purpose: To experimentally determine whether the soundboard really amplifies sound.

Materials:
- large bowl
- tape
- plastic wrap
- rubber band

Procedure:

1. Tightly seal a large bowl with plastic wrap (you may need to use tape to wrap the plastic tightly to the sides).
2. Tape a rubber band to the center of the taut plastic wrap and twang the rubber band.
3 Compare the sound heard to the twang of an identical rubber band not taped to the plastic wrap. You should notice a big difference. The plastic wrap greatly increases the amount of surface area that is vibrating, so the sound is much louder. (This activity is taken from the web site "How stuff works" http://www.howstuffworks.com/guitar1.htm)
4. Demonstrate how to produce standing waves for students. A very effective way to do this is to attach one end of a string to an electric drill, and the other securely to some immovable object. When the drill is turned on at varying speeds, students can clearly see standing waves at the fundamental frequency and various overtones.
5. Refer to the activity at the following web site for an activity on making standing wave patterns on a guitar: http://scienceworkshop.freeyello.com/sound_guitar.htm