APPENDIX A

STSE

Science-Technology-Society and the Environment

Important Note

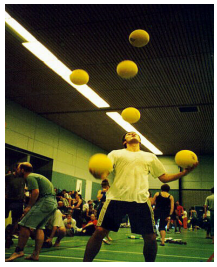
These STSE modules are intended for teacher reference. Each is designed to target specific outcomes within Physics 3204. It should be noted that the activities associated with each module are NOT mandatory. They are suggested activities to be used at the discretion of the teacher.

The Physics of Juggling

Outcomes:

1. Analyze qualitatively and quantitatively the horizontal and vertical motion of a projectile. (325-6)
2. Analyze natural and technological systems to interpret and explain their structure. (116-7)
3. Distinguish between problems that can be solved by the application of physics-related technologies and those that cannot. (118-8)
4. Compile and display evidence and information, by hand or computer, in a variety of formats, including diagrams, flow charts, tables and graphs. (214-3)
5. Analyze and describe examples where technological solutions were developed based on scientific understanding. (116-4)
6. Define and delimit problems, estimate quantities, and interpret patterns and trends in data, and infer or calculate the relationship among variables. (212-2, 213-4, 214-5)

Introduction



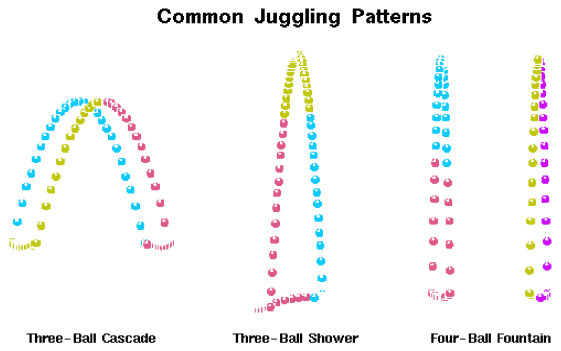
An old riddle tells of a 148-pound man who had to cross a canyon over a bridge that could only support 150 pounds (Beek & Lewbel, 1995). Unfortunately the man was carrying three one-pound cannonballs and only had time for one trip across. The solution to the riddle was that the man would juggle the cannonballs while crossing the bridge. In reality, juggling the balls would not have been much help since catching one of the cannonballs would have exerted a force on the bridge that would have exceeded the weight limit. The poor man would have ended up at the bottom of the canyon! Though not very helpful in this particular case, juggling does have relevance beyond riddles or entertainment.

Beek and Lewbel (1995) suggest the application of juggling in the study of human movement, robotics, and mathematics. Studying the mathematics of juggling became popular in the 1980's though juggling itself is an ancient tradition dating back to Egypt and Rome. The term "juggling" comes from the Latin "joculare" meaning "to jest". Before the mid-twentieth century juggling was mainly a part of magic shows (Juggling). Public interest in juggling as a hobby increased after 1948 when the first juggling convention was held in the United States. That interest has persisted over the years as people continue to test physical limits for the number of objects juggled. Currently, the world record for the greatest number of objects juggled is 13 rings, 12 balls or 9 clubs (List of Numbers Juggling Records, 2002). While these numbers may seem impossibly high, they are in fact attainable with the right combination of physical ability and physics knowledge.

Theory

Good jugglers make juggling look so easy that it is difficult to imagine all the physics that comes into play. Gravity has a significant effect on the number of objects juggled. Each ball must be thrown high enough to allow the juggler time to handle the other balls. While throwing higher gives the juggler extra

time, it also increases the risk of error. Juggling 'low' on the other hand, requires the juggler to catch and throw quickly, also increasing the risk of error. The need for speed or height will also change dramatically as the number of objects being juggled



increases. Beek and Lewbel (1995) assert that learning to juggle three balls can be accomplished in just hours or days. This learning time can increase to weeks or months for four balls, and months or years for five balls.

In the cascade, the hands alternate throwing balls to each other, resulting in a figure eight. In the shower pattern the balls are thrown around in a circle. In the fountain pattern the balls are either thrown (and caught) simultaneously with both hands (in sync), or by catching a ball with one hand and throwing one with the other at the same time (out of sync). Despite the identification of these patterns, it is important to note that due to factors like the oscillation of the jugglers' hands, or individual vision and feel, no two throws or catches are exactly the same.

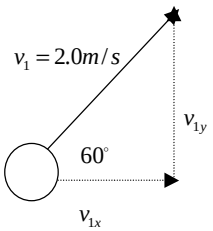
In an attempt to measure juggling consistency, "dwell ratio" has been defined as the "fraction of time that a hand holds on to a ball between two catches (or throws)" (Beek & Lewbel, 1995, p. 3). A large dwell ratio means that the hand cradles the ball for a longer period of time. This means that the juggler has more time to throw accurately. A small dwell ratio means that the balls have a longer time in the air, which allows the juggler time to make corrections to hand repositioning. Novice jugglers typically like larger dwell ratios while professionals tend towards smaller values because they are more interested in shifting patterns.

A knowledge of projectile motion can give a juggler valuable information on the time available to throw and catch balls in a juggling pattern. Let us consider some numbers for throwing one ball in the air at around 2.0 m/s. The equations for projectile motion can tell us how long the juggler has to catch the ball, how high it will rise and about how far apart to keep the hands. In the following calculations assume that the ball is being thrown from the left hand to the right at an angle of 60° to the horizontal (neglecting air resistance on the ball).

Using trigonometry we see that,

$$v_{1x} = v_1 \cos 60^\circ$$

$$v_{1y} = v_1 \sin 60^\circ$$



At the peak of its ascent the velocity of the ball in the y-direction will be 0 m/s. The time taken to reach that maximum height can be found from the equation,

$$a = \frac{v_{2y} - v_{1y}}{t} \quad .$$

Solving for t gives,

$$t = \frac{v_{2y} - v_{1y}}{a}$$

where $a = -9.8m/s^2$

$$v_{1y} = v_1 \sin 60^\circ$$

$$v_{2y} = 0m/s \cdot$$

Therefore, $$t = \frac{0m/s - v_1 \sin 60^\circ}{-9.8m/s^2}$$

$$t = \frac{0m/s - (2.0m/s)(\sin 60^\circ)}{-9.8m/s^2}$$

$$t = 0.18s$$

Since the ball traces out a parabolic path, doubling this time will allow us to figure out how far the ball

travels in the x-direction (i.e. the range). This distance is how far the hands should be apart and is given by:

$$d_x = v_{1x}t + \frac{1}{2}a_x t^2$$

where acceleration in the horizontal direction is zero (i.e., $a_x = 0m/s^2$). Thus,

$$d_x = v_{1x}t$$
$$d_x = v_1 \cos 60^\circ t$$
$$d_x = (2.0m/s)(\cos 60^\circ)(2x0.18s)$$
$$d_x = 0.36m$$

We can also figure out how high the ball will travel using the following equation:

$$d_y = v_{1y}t + \frac{1}{2}a_y t^2$$
$$d_y = (2.0m/s)(\sin 60^\circ)(0.18s) + \frac{1}{2}(-9.8m/s^2)(0.18s)^2$$
$$d_y = 0.15m$$

These calculations reveal that a ball initially travelling at 2.0 m/s will reach a height of 0.15 m in 0.18 s, and that horizontally it will travel 0.36 m. This information will be extremely useful to a juggler (especially a novice juggler).

To juggle two balls under these conditions would entail throwing the second ball just as the first is reaching its peak (at around 0.18 s). For three balls, you would start with two balls in one hand and one in the other. The third ball would then be thrown when the second is at its peak and the first ball again when the third reaches its peak. The trick is to throw the third ball and catch the first one with the same hand, in the limited time available.

Conclusion

A basketball or volleyball coach would probably advise his/her players to keep his/her eye on the ball. The best advice for a juggler, however, would be to keep his/her eye *off* the ball, since a jugglers' attention must continually shift from one ball to the next. Possessing information on the timing and height of ball flight would make this job a little easier. Becoming a good juggler then requires patience, practice *and* physics.

Questions

1. A ball is thrown at an initial velocity of 3.0 m/s upward at an angle of 80° to the horizontal. How high will the ball rise? How far apart should the juggler hold his/her hands?
2. A juggler throws a ball at an angle of 70° to the horizontal. If the ball took 0.20 s to reach its maximum height, at what initial velocity was it thrown?
3. How is knowledge of projectile motion principles useful to a juggler?
4. **Research:** Who holds the current world record for greatest number of balls juggled?

References

Beek, P.J., & Lewbel, A. (1995). The science of juggling. *Scientific American, 273,* p. 92-97.

Dudzik, J. Projectile motion. Available: http://www.iit.edu/~smile/ph9204.html.

Giancoli, D.C. (1998). *Physics*. Prentice-Hall Canada Inc.: Toronto.

It's all up in the air: The mathematics of juggling. (2001). Available: http://www.teachingtools.com/GoFigure/UpInAir.htm.

Juggling. Available: http://www.unc.edu/~lgaudio/juggling.htm.

Juggling 101. (2001). Available: http://www.teachingtools.com/GoFigure/Activity-Juggle.htm.

Juggling – its history and greatest performers. (1997). Available: http://www.juggling.org/books/alvarez/part1.html.

List of numbers juggling records. (2002). Available: http://www.juggling.org/records/records.html.

The instant jugglers' manual. (1993). Available: http://yoyoguy.com/info/ball/index2.html.

Activities

Activity 1: Projectile Motion

Adapted from: http://www.iit.edu/~smile/ph9204.html.

Purpose: To explore projectile motion.

Materials:

Meter stick	Spring
Tape	Protractor

Procedure and Analysis:

1. Hold a meter stick against the wall. Mark a 1 meter mark with tape and then measure up 2 more meters and mark with tape.
2. Stretch a spring on a meter stick: hold it parallel to the wall and then shoot the spring straight up. Do this several times until you can repeat a shot to go exactly 2 m several times. Write down how far you stretched the spring. Get an assigned angle for your launch from your teacher. (TEACHER: assign 15°, 30°, 45°)
3. Calculate the speed at which the spring is fired, using the equation $v^2 = 2ad$.
4. Determine the vertical and horizontal components of the original velocity by using $v_{vertical} = v\sin(angle)$ and $v_{horizontal} = v\cos(angle)$.
5. Use the vertical velocity to find the flight time, $t = \frac{2v_{vertical}}{a}$.
6. Use the flight time and the horizontal velocity to calculate the range $d = v_{horizontal}\ t$.
7. Measure off this distance and shoot the spring using the same stretch. See how close you come to the measured distance.

Activities

Activity 2: Learn to Juggle

Adapted from: http://yoyoguy.com/info/ball/index2.html.

These instructions will teach you exercises so you can juggle 3 balls. The first few exercises are not complete juggling patterns, but are exercises that will teach you to juggle.

One Ball Exercise: Start with one ball. Throw the ball in an arc from hand to hand about eye level. The pattern will be an arc, not a circle.

Two Ball Exercise: Start with one ball in each hand. First toss the ball in your right hand (1) in the arc to about eye level to your left hand. When this ball (1) reaches the highest point in its arc throw ball (2) in an arc from your left hand to your right. Catch (1) in your left hand. Then catch (2) in your right hand. Stop. Do this same exercise, except start with your left hand instead of your right. Practice until you can do this smoothly. Common mistakes include throwing two balls in a circle, or throwing both balls at the same time.

Juggling 3 Balls: Start with 2 balls in one hand (1&3) (in this case the right hand, but if you are a lefty, use your left hand) and one ball (2) in the other. Start by throwing the ball in the front of your right hand in an arc to your left hand. When ball (1) reaches its highest point, throw the ball in your left hand (2) in an arc to your right hand. Catch (1) in your left hand. This is like the two ball exercise. When the ball thrown to your right hand reaches its height, throw the ball from your right hand (3) in an arc to your left hand. Catch (2) in your right hand. This move can be difficult. It is often helpful to roll the ball (3) in your right hand to the front of your hand with a slight downward motion of the hand before you throw it. When that ball (3) reaches its highest point, throw the ball in your left hand (1) in an arc to your right hand. Catch (3) in your left hand. And so on . . .

Problems and Solutions:

I move foward as I juggle.

This is a common problem. Stand in front of a wall, or a bed to keep you from moving forward.

I can't throw ball number (3), I just catch ball number (2).

Concentrate on throwing ball number (3). Do not even try to catch ball (2).

The balls keep hitting/there isn't time for to make the throws.

Concentrate on making your throws an even height at eye level.

The Half Shower: Instead of having the balls cross in the standard 3 ball pattern, throw a ball from the right hand over the rest of the pattern. When it comes down, continue juggling. Do this for a throw or every throw; from either hand, or both hands.

Activities

Activity 3

Adapted from: http://www.teachingtools.com/GoFigure/Activity-Juggle.htm.

Juggling takes skill, control and a will to learn. Here are instructions for the basic three-ball juggle (the cascade) that may have you and your students juggling with abandon! Instead of balls, you might begin with beanbags or wadded up socks. They're easier to manage. Another tip–practising a foot or two from a wall can help if your balls seem to be wandering forward and getting out of control. The balls can glance off the wall and you can continue without dropping the balls.

Juggling One Ball

First, learn with one ball. Throw the ball from hand to hand so it peaks at about eye level. The ball should be thrown from the palm of the hand and the hands should be at about hip level and on either side of the body. After you have the throw and catch perfected with one ball to and from both hands, move on to two balls.

Juggling Two Balls

Hold one ball in each hand. Throw the first ball (it doesn't matter which one) as you did in the first exercise but when it reaches its peak, throw the second ball. Catch the first and then the second. Practise this until you are comfortable with starting with either your left or right hand.

Juggling Three Balls

Now you go to three balls. Hold two in one hand (either hand, whichever is more comfortable) and one in the other. Start by practising three throws.

Throw a ball from the hand holding two balls (we'll say the right hand). As this first ball reaches its peak, throw the ball from your left hand. The first ball will be caught before the second reaches its peak. As the second ball peaks, throw the ball from your right hand. This is quite difficult, especially catching the balls. Don't be disheartened if it seems as though the balls spend more time on the ground than in the air. All you need is practice. When you can get three consecutive throws and catches, then add another throw. After four throws and catches, make it five and then just keep going.

Before you know it, you'll be juggling!