Mathematics 3204
Grading Standards June 2003

To view a copy of the exam go to
http://www.gov.nf.ca/edu/k12/pub/sample.htm

1. Pre-Marking Appraisal

All markers thoroughly reviewed the exam. It was considered to be quite fair. The course outcomes were covered well and the exam was of reasonable length and difficulty. The exam was error free and, as a result, no adjustments were necessary. The exam provided a mix of questions that required specific approaches (such as #52 where students had to set up a quadratic function) and questions that were more open-ended with regard to approach (such as #57 where an algebraic solution was not required).

2. Post-Marking Report

(A) Marking Standard and Consistency

On the first day, fifty papers were selected at random. Markers read these papers without putting any marks on the paper. The mark for each question for all fifty exams was recorded on a separate sheet of paper. Each day approximately six papers were inserted into the stack of papers to be corrected that day. When these papers reappeared, the Chief Marker compared the marks with the marks given earlier. Consistency in marking was extremely high.

Throughout the entire marking period, the Chief Marker checked many papers for consistency. Any inconsistencies were brought to the attention of the marker involved and appropriate changes were made.

A computerized printout of reliability analysis using SPSS software was given to the Chief Marker periodically throughout the marking process. The Board always had an alpha value that rounded to 0.9 or above indicating a high reliability in the marking process.

(B) Commentary on Responses

- Many students used a method to factor that is referred to by many names (i.e., "Joe's Method," "Borrow and Pay Back," or AC Method"). The method, as shown in student responses, has no apparent logical thought process evident from step to step:

$$5x^2 - 14x - 24$$
$$x^2 - 14x - 120$$
$$(x - 20)(x + 6)$$
$$(5x - 20)(5x + 6)$$
$$(x - 4)(5x + 6)$$

- The Marking Board suggests that students who factor using this method should do their rough work on scrap paper and transfer only the factors to their exam, or, alternatively, students could show correct workings for the method:

$$5x^2-14x-24$$
$$\frac{5(5x^2-14x-24)}{5}$$
$$\frac{(5x)^2-14(5x)-120}{5}$$
Let $y = 5x$
$$\frac{y^2-14y-120}{5}$$
$$\frac{(y-20)(y+6)}{5}$$
$$\frac{(5x-20)(5x+6)}{5}$$
$$(x-4)(5x+6)$$

Many students:

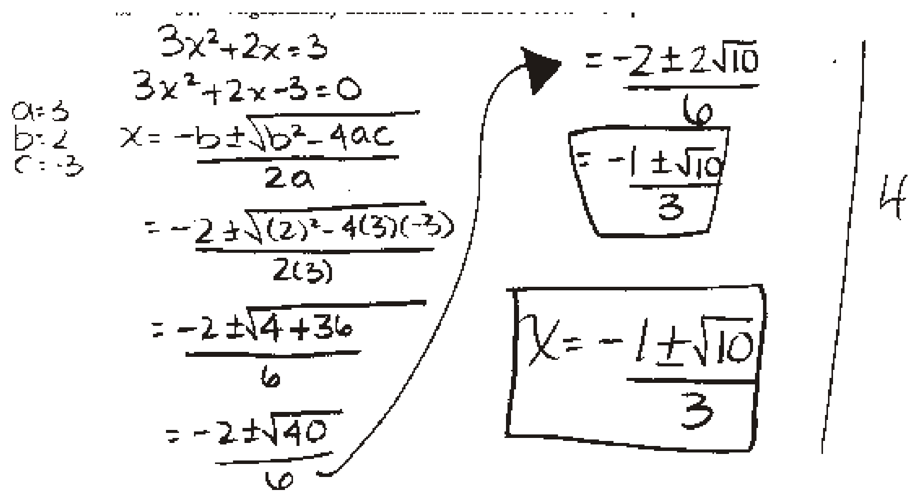
- demonstrated weak basic mathematical skills. For instance, in #51 students said $\frac{-1-1\sqrt{10}}{3} = \frac{-2\sqrt{10}}{3}$ and in #57 students said $40(1.07)^x = 42.8^x$.
- made careless computational errors.
- did not show sufficient workings for some questions.
- did not know the quadratic formula, distance formula, and/or midpoint formula.
- did not follow the instructions in certain questions. For instance, #52 and #58 required functions and many students answered these questions without stating the required function.
- did not distinguish between linear equations, quadratic equations, exponential equations, and circle equations. For instance, in #59 students found the center of the circle and substituted it in for (h, k) in $y = a(x - h)^2 + k$.
- appeared to have memorized techniques for doing certain "types" of problems with little or no understanding of the problem.
- had weak factoring skills.

3. Constructed Response Answers & Common Errors

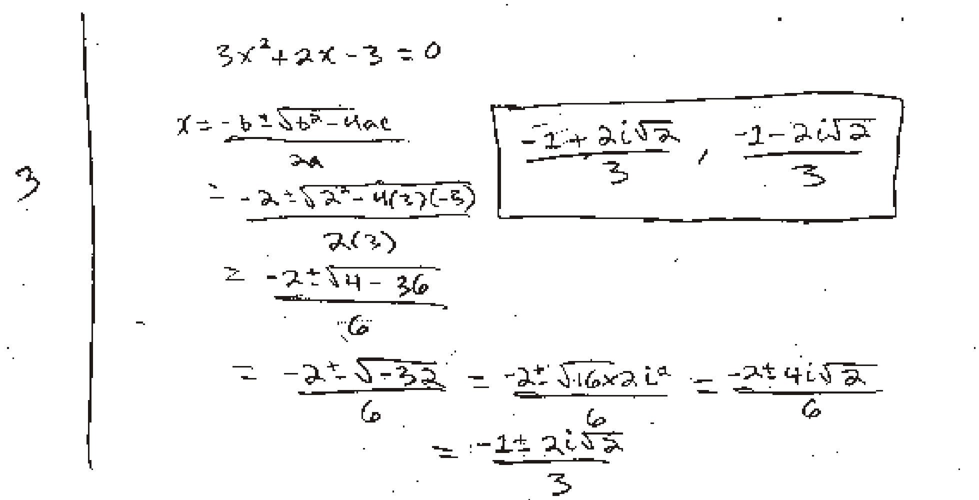
4% 51. Algebraically determine the EXACT roots in simplest form for: $3x^2 + 2x = 3$.

Answer Comment:
The problem required setting a quadratic equation equal to 0 and solving via completing the square or by using the quadratic formula. There were two unequal irrational roots.

Excellent Answer Exemplar:



Good Answer Exemplar:



$\frac{-2\pm\sqrt{2^2-4(3)(-3)}}{6}$ *Should have become* $\frac{-2\pm\sqrt{40}}{6}$, *however* $\frac{-2\pm\sqrt{-32}}{6}$ *was correctly simplified therefore credit was assigned for this simplification.*

Commentary & Common Errors

Many students:

- do not know the quadratic formula.
- do not know how to simplify radicals.
- do not understand the concept of exact roots. For example:

$$\frac{2\pm\sqrt{40}}{6} = \frac{2\pm 6.3}{6}$$

$$\frac{2+6.3}{6} = \frac{8.3}{6} = 1.4$$

$$\frac{2-6.3}{6} = \frac{-4.3}{6} = -0.72$$

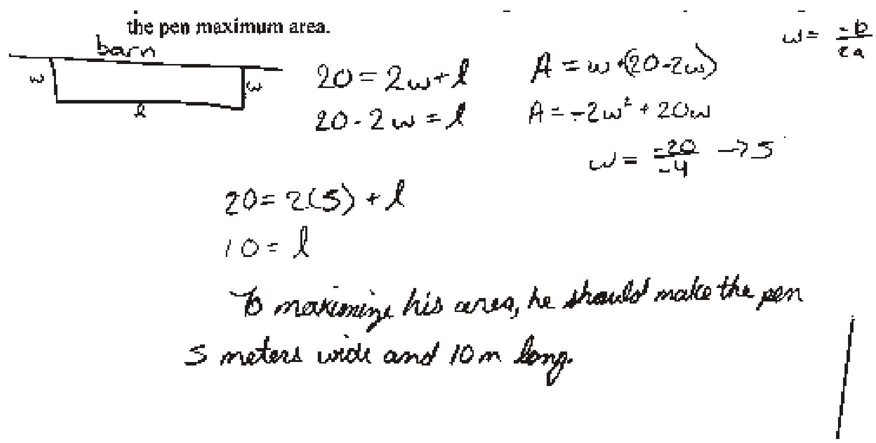
Errors:

- simplifying radicals incorrectly (i.e., $\sqrt{40} = 2\sqrt{20}$ or $\sqrt{40} = 8\sqrt{5}$)
- using $a = 3$, $b = 2$ and $c = 3$ in the quadratic formula
- attempting to factor $3x^2 + 2x - 3$
- $-\frac{-2\pm\sqrt{40}}{6}$ to be $\frac{-1\pm\sqrt{20}}{3}$
- simplifying $\frac{-1+1\sqrt{10}}{3}$ to be $\frac{\sqrt{10}}{3}$
- simplifying $\frac{-1-1\sqrt{10}}{3}$ to be $\frac{-2\sqrt{10}}{3}$
- simplifying $\sqrt{2^2 - 4(3)(-3)}$ to be $\sqrt{4 - 36}$
- not rejecting the negative result

4% 52. A farmer is making a rectangular pen for chickens. One side of a barn will be used as a side of the pen and the farmer has 20m of fencing in total. Set up a quadratic function and use it to find the length and width of the pen that will give the pen maximum area.

Answer Comment:
The problem required quantifying a three-sided rectangular diagram by assigning 'x' to width and '20–2x' to length to determine the maximum value of the related area function.

Excellent
Answer Exemplar:


the pen maximum area.

barn

$20 = 2w + l$

$20 - 2w = l$

$A = w(20 - 2w)$

$A = -2w^2 + 20w$

$w = \frac{-b}{2a}$

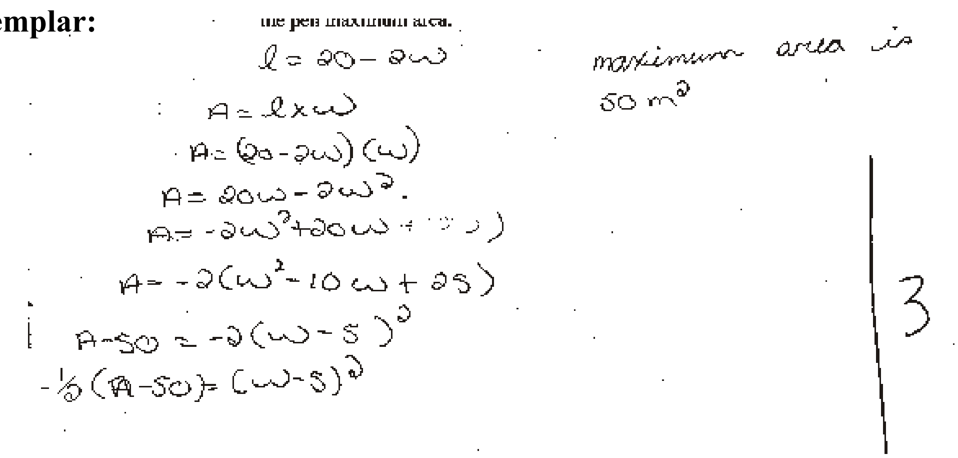
$w = \frac{-20}{-4} \rightarrow 5$

$20 = 2(5) + l$

$10 = l$

To maximize his area, he should make the pen 5 meters wide and 10 m long.

Good Answer
Exemplar:


the pen maximum area.

$l = 20 - 2w$

$A = l \times w$

$A = (20 - 2w)(w)$

$A = 20w - 2w^2$

$A = -2w^2 + 20w$

$A = -2(w^2 - 10w + 25)$

$A - 50 = -2(w - 5)^2$

$-\frac{1}{2}(A - 50) = (w - 5)^2$

maximum area is 50 m^2

3

The student transformed the equation into a necessary form but failed to answer the question.

Commentary & Common Errors

Many students:

- answered the question by completing the square and stated the vertex values for the length and width.
- sated the answer only, showing no workings or explanation.
- did not attempt the question.
- found the function only.

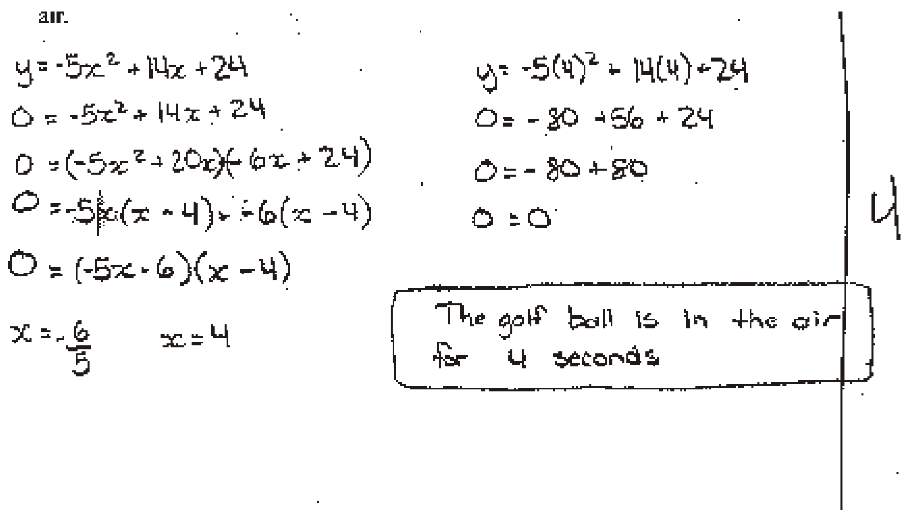
Errors:

- setting the function equal to zero, then solving using the quadratic formula ($-2x^2 + 20x = 0$)
- writing the function incorrectly when rewriting in descending order ($y = 20x - 2x^2$ became $y = 2x^2 - 20x$)
- factoring incorrectly ($-2x^2 + 20x$ became $-2(x^2 + 10x)$)
- stating the area of the rectangle as $2x(20 - 2x)$ instead of $x(20 - 2x)$
- using a table of values to find the answer without setting up a quadratic function as the question specified
- creating the function, but when completing the square, did not factor out (- 2), in fact, ignored it (i.e., $y = -2x^2 + 20x + \underline{100}$)

4% 53. A golf ball is hit from the top of a tower that is 24m high. The ball follows a parabolic path defined by the function $y = -5x^2 + 14x + 24$, where x represents the time in seconds since the ball was hit, and y represents the height of the ball above the ground in metres. Algebraically determine how long the ball is in the air.

Correct Answer:
This problem required determining the zeros of a quadratic function (quadratic formula, completing the square, factoring), rejecting the negative value, and stating the elapsed time from t=0 to the other zero, 4.

Excellent Exemplar Answer:



air.

$y = -5x^2 + 14x + 24$

$0 = -5x^2 + 14x + 24$

$0 = (-5x^2 + 20x) + (-6x + 24)$

$0 = -5x(x - 4) + -6(x - 4)$

$0 = (-5x - 6)(x - 4)$

$x = -\frac{6}{5}$ $x = 4$

$y = -5(4)^2 + 14(4) + 24$

$0 = -80 + 56 + 24$

$0 = -80 + 80$

$0 = 0$

The golf ball is in the air for 4 seconds

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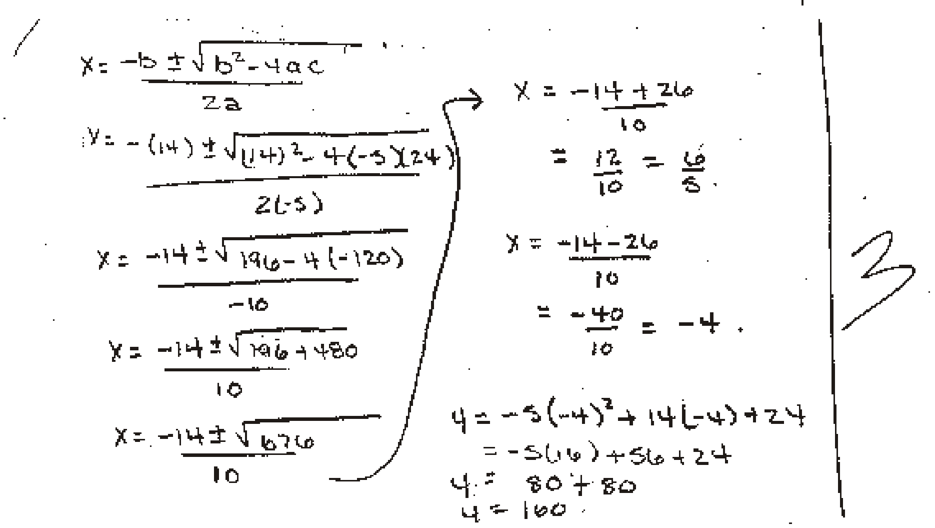
Good Answer Exemplar:



$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(14) \pm \sqrt{(14)^2 - 4(-5)(24)}}{2(-5)}$

$x = \frac{-14 \pm \sqrt{196 - 4(-120)}}{-10}$

$x = \frac{-14 \pm \sqrt{196 + 480}}{10}$

$x = \frac{-14 \pm \sqrt{676}}{10}$

$x = \frac{-14 + 26}{10} = \frac{12}{10} = \frac{6}{5}$

$x = \frac{-14 - 26}{10} = \frac{-40}{10} = -4$

$y = -5(-4)^2 + 14(-4) + 24$

$= -5(16) + 56 + 24$

$y = 80 + 80$

$y = 160$

3

Losing the negative on the denominator caused incorrect signs on the zeros, the time the ball was in the air was not explicitly indicated.

Commentary & Errors

Many students:

- appear to have memorized techniques for doing this type of problem and appear to have little understanding of the problem e.g., many students found the vertex.
- do not know the quadratic formula.
- made careless computational and/or copying errors.

Errors:

- failing to substitute "0" in place of "*y*" in the equation
- replacing *y* with 24
- making errors within the quadratic formula for example:

  (i) $\frac{\sqrt{14^2}-4(-5)(24)}{2(-5)} = \frac{\sqrt{196-480}}{-10}$

  (ii) $\sqrt{676} = \sqrt{26}$

- finding the *x*-coordinate of the vertex and saying it was the solution to the problem
- not factoring - 5 out when completing the square

4%

54. A batter hits a baseball into the air. The ball follows a path described by the equation $h(x) = -4.0x^2 + 36.2x + 3.0$ where h is the height of the ball above the ground in meters and x is the time in seconds since the ball was hit. Algebraically find the approximate instantaneous rate of change of height of the ball at 5.5 seconds.

Answer Comment:
Determining the approximate instantaneous rate of change of height of the ball involves calculating a secant slope on an x–interval no more than 0.1 x–units (seconds here) on either side of the point in question on the quadratic function provided.

Excellent Answer Exemplar:

use 5.4 and 5.6

$h(x) = -4.0x^2 + 36.2x + 3.0$

$h(5.4) = -4.0(5.4)^2 + 36.2(5.4) + 3.0$
$= -116.64 + 195.48 + 3.0$
$= 81.84$

$h(5.6) = -4.0(5.6)^2 + 36.2(5.6) + 3.0$
$= -125.44 + 202.72 + 3.0$
$= 80.28$

$$\frac{f(5.6) - f(5.4)}{(5.6) - (5.4)} = \frac{80.28 - 81.84}{5.6 - 5.4} = \frac{-1.56}{0.2} = -7.8$$

the instantaneous rate of change of height of the ball at 5.5 seconds is -7.8 m/s

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Excellent Answer Exemplar:

seconds.

x_2	$f(x_2)$	x_1	$f(x_1)$	slope $\frac{y_2-y_1}{x_2-x_1}$
5.5	81.1	4.0	83.8	$\frac{81.1-83.8}{5.5-4.0} = \frac{-2.7}{1.5} = -1.8$ m/s
5.5	81.1	4.5	84.9	$\frac{81.1-84.9}{5.5-4.5} = \frac{-3.8}{1} = -3.8$ m/s
5.5	81.1	5.0	84	$\frac{81.1-84}{5.5-5.0} = \frac{-2.9}{0.5} = -5.8$ m/s
5.5	81.1	5.4	81.84	$\frac{81.1-81.84}{5.5-5.4} = \frac{-0.74}{0.1} = -7.4$ m/s
5.5	81.1	5.4976	81.1776	$\frac{81.1-81.1776}{5.5-5.49} = \frac{-0.0776}{0.01}$ -7.76 m/s
5.5	81.1	5.4999	81.10078	$\frac{81.1-81.10078}{5.5-5.4999} = \frac{-7.8\times10^{-4}}{1\times10^{-4}}$ $= -7.8$ m/s

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The instantaneous rate of change of the height of the ball at 5.5 seconds is approximately −8.0 m/s. This is the slope at which its approaching.

Good Answer Exemplar:

inst. rate of change is: $h(5.4) \doteq 81.84$

$h(5.6) \doteq 80.28$

Ball changes height at rate of:

$$\frac{80.28-81.84}{5.4-5.6} \doteq \frac{-1.56}{-0.2} \doteq 7.8 \text{ m/s}$$

at 5.5 seconds.

3

The student subtracted the x–values in the denominator in reverse order against the numerator causing the opposite sign on the answer.

Commentary & Common Errors

Many students:

- mis-read the question and ignored the algebraic procedure. For instance, many only calculated $h(5.5)$
- chose values that were not sufficiently close to 5.5 (i.e., 5 and 6)
- could not relate slope to instantaneous rate of change
- did not attempt the question

Errors:

- writing an incorrect slope formula:

  (i) $\frac{h(x_2)+h(x_1)}{x_2+x_1}$

  (ii) $\frac{x_2-x_1}{h(x_2)-h(x_1)}$

  (iii) $\frac{x_2\left(h(x_2)\right)-x_1\left(h(x_1)\right)}{x_2-x_1}$

- writing the correct slope formula but values in numerator or denominator are calculated in reverse order for example:

  $\frac{h(5.51)-h(5.50)}{5.50-5.51}$

- obtaining an incorrect answer by careless computation (i.e., - 0.78m/s)
- calculating $h(5.6)$ - $h(5.4) = h(0.2)$
- solving the problem as follows:

$$h(\mathrm{T}) = -4\mathrm{T}^2 + 36.2\mathrm{T} + 3$$
$$h(5.5) = 81.1$$
$$\text{answer} = \tfrac{81.1}{5.5}$$
$$= 14.7$$

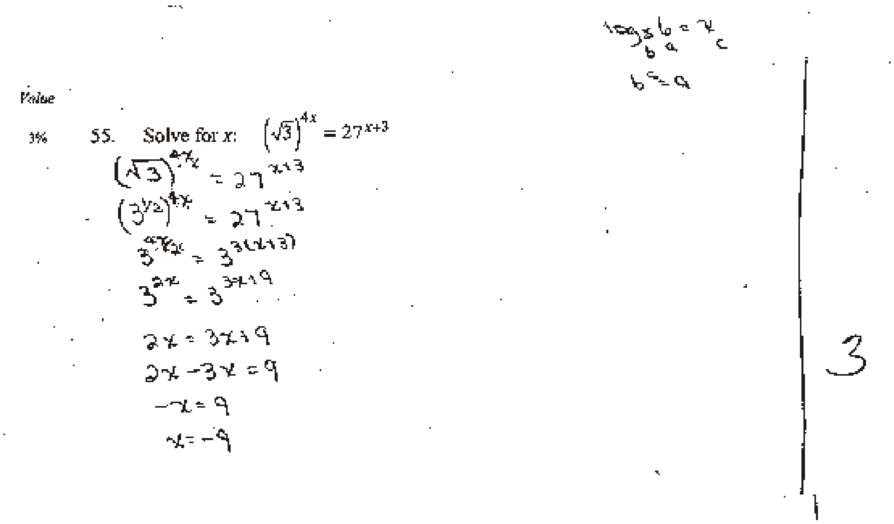
- making careless computational errors in calculating the final answer
- writing either the numbers in the numerator or the numbers in the denominator of the slope formula in reverse order, therefore obtaining an incorrect sign in the final answer
- rejecting the negative value in their final answer

3% 55. Solve for x: $\left(\sqrt{3}\right)^{4x} = 27^{x+3}$

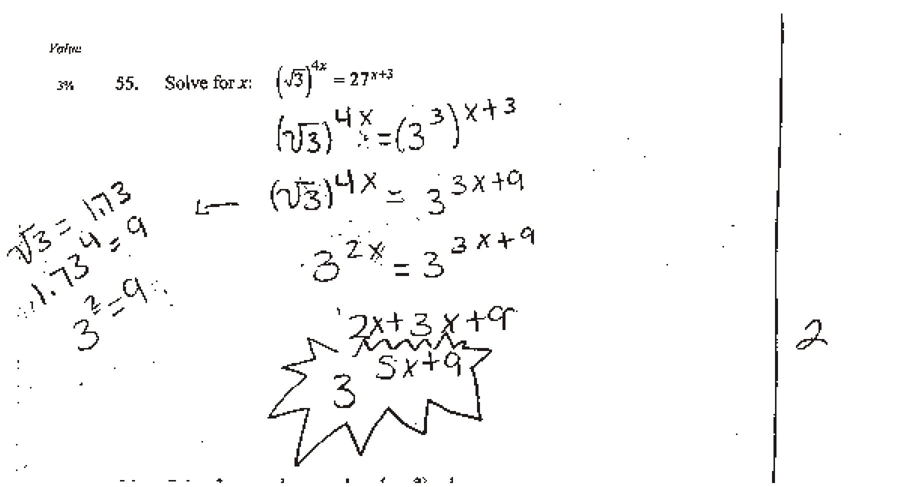
Answer Comment:

Solving this same base exponential equation involves determining the common base, using exponent laws, and solving a linear equation to determine x.

Excellent Answer Exemplar:



Good Answer Exemplar:



The student did not set the exponents equal thus failing to complete the final stage of the problem.

Commentary & Common Errors

Many students:

- were aware of the procedure necessary to complete this question, but had difficulty executing the process.
- chose a more difficult but equally correct approach to this question. For example, $\left(\sqrt{3}\right)^{4x}$ became $3^{\frac{4x}{2}}$ and students proceeded to find a common denominator to solve $\frac{4x}{2} = 3x + 9$ rather than first simplifying $\frac{4x}{2}$ to $2x$.
- changed $\sqrt{3}$ to 1.73 and then took the log of both sides of the equation. This resulted in the use of decimals, leading to answers that were not exact. Students were given full credit for approximate solutions. However, it is possible on future public exams for students to be asked for the exact solution.

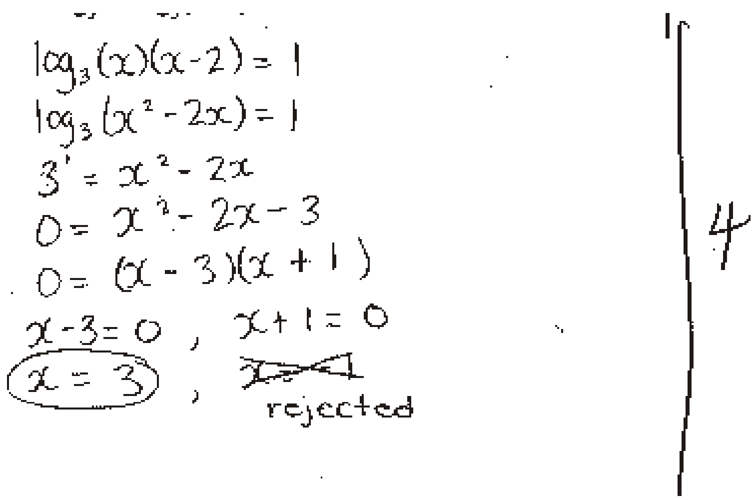
Errors:

- incorrectly changing $\sqrt{3}$ to 3^2 or $3^{\frac{1}{3}}$
- incorrectly changing 27 to 3^9 or $3\sqrt{3}$
- making exponents equivalent without getting common bases for example: $\left(\sqrt{3}\right)^{4x} = 27^{x+3}$ led to $4x = x + 3$ or $\left(\sqrt{3}\right)^{4x} = 3^{3x+9}$ led to $4x = 3x + 9$.
- not using the distributive property for example: $\left(3^3\right)^{x+3}$ became 3^{3x+3}
- multiplying incorrectly for example: $\left(3^3\right)^{x+3}$ became 3^{3x+6}
- concluding if $-x = 9$, then $x = 9$ or $x = \frac{1}{9}$

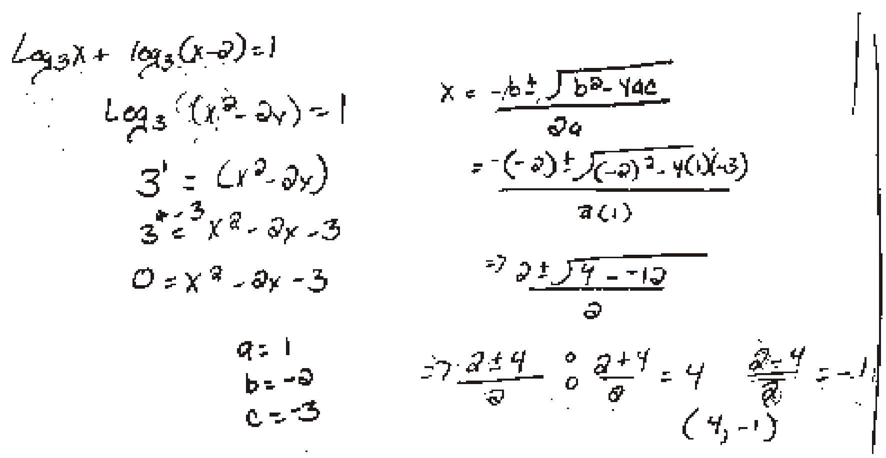
4% 56. Solve for x: $$\log_3 x + \log_3 (x-2) = 1$$

Answer Comment:
This base 3 logarithm equation involved combining two logarithms, rewriting in exponential form, solving the resulting quadratic equation, and rejecting the inadmissible $x = -1$.

Excellent Answer Exemplar:



Good Answer Exemplar:



The student made an error in the final step of the quadratic formula. Specifically, $\frac{2+4}{2}$ became 4 rather than 3. Also, the negative root was not rejected.

Commentary & Common Errors

Many students:

- changed 1 to $\log_3 3$ resulting in $\log_3 x^2 - 2x = \log_3 3$, leading to equal arguments rather than converting to exponential form.
- did not attempt to change to exponential form.
- chose to solve the quadratic equation by completing the square. Often, when they used this method, they only indicated the positive square root. As a result, the negative root was lost in the process.

Errors:

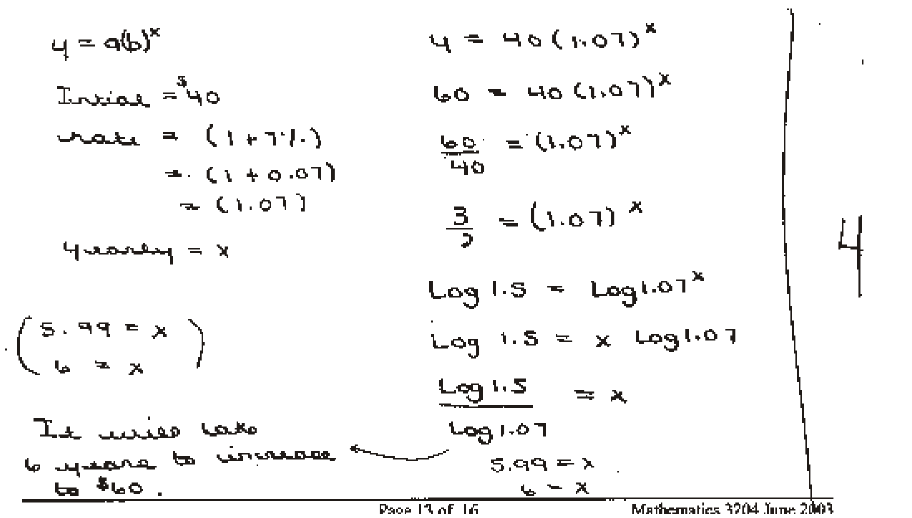
- difficulty changing to exponential form. In many instances $\log_3 x^2 - 2x = 1$ became $x^2 - 2x = 1$
- not rejecting the negative root
- ignoring the logs, resulting in $x + x - 2 = 1$ or $x(x - 2) = 1$
- dividing arguments rather than multiplying
- adding arguments rather than multiplying
- factoring incorrectly or executing the quadratic formula incorrectly
- changing $\log_3 x$ to 0.4771, which is the value of $\log 3$

4% 57. Suppose the cost of a ticket to an NHL hockey game increases by 7% yearly. If the cost of a ticket now is \$40, how long will it take to increase to \$60?

Answer Comment:

The problem required determining the correct exponential model for ticket cost and then solving for a specific cost using logarithms.

Excellent Answer Exemplar:



$y = a(b)^x$

Initial = \$40

rate = (1 + 7%)

= (1 + 0.07)

= (1.07)

years = x

$y = 40(1.07)^x$

$60 = 40(1.07)^x$

$\frac{60}{40} = (1.07)^x$

$\frac{3}{2} = (1.07)^x$

$\log 1.5 = \log 1.07^x$

$\log 1.5 = x \log 1.07$

$\frac{\log 1.5}{\log 1.07} = x$

$5.99 = x$

$6 = x$

It will take 6 years to increase to \$60.

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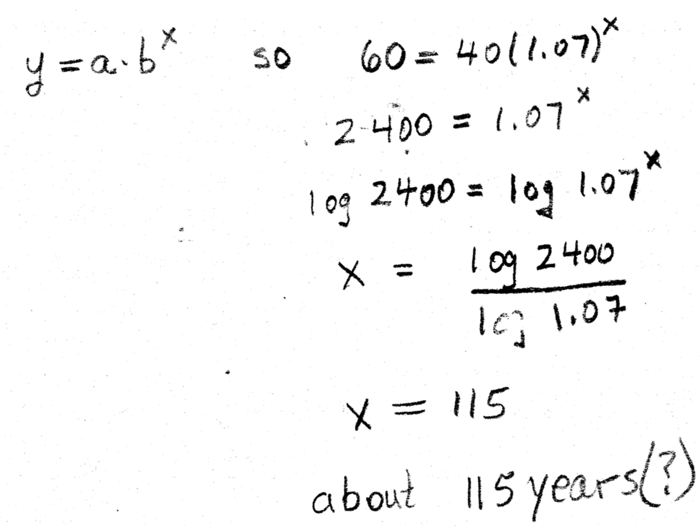
Good Answer Exemplar:



$y = a \cdot b^x$ so $60 = 40(1.07)^x$

$2400 = 1.07^x$

$\log 2400 = \log 1.07^x$

$x = \frac{\log 2400}{\log 1.07}$

$x = 115$

about 115 years(?)

The student multiplied the initial amount by the base rather than dividing.

Commentary & Errors

Many students:

- chose to use trial and error to solve $60 = 40(1.07)^t$ or the trace/table features on a graphing calculation for this question. As the wording of the question did not specify the approach to be used, or that an equation was required, students were given full marks for the approach, provided sufficient workings were shown.
- did not consider the reasonableness of their solution. For instance, responses of 1 year and over 2000 years were common.

Errors:

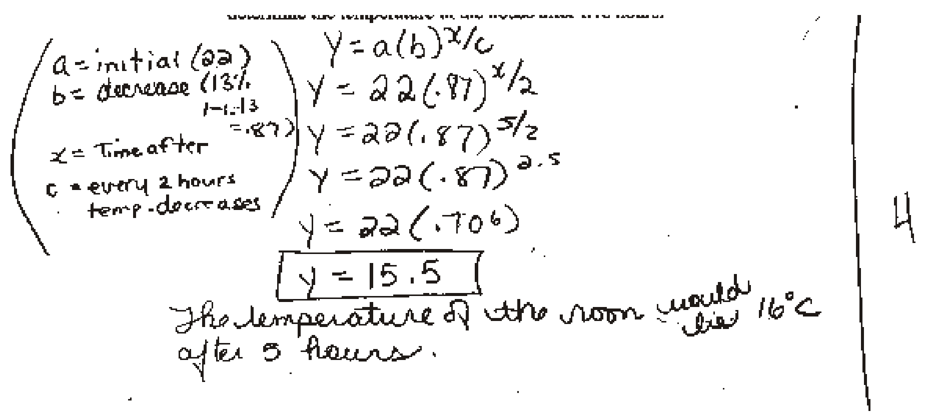
- not recalling the technique of taking the log of both sides to solve $60 = 40(1.07)^t$
- incorrectly rearranging the equation to solve for time (i.e., $60 = 40(1.07)^t$ became $20 = 1.07^t$)
- multiplying the base by the initial amount (i.e., $60 = 40\ (1.07)^t$ became $60 = 42.8^t$)
- neglecting to show sufficient workings (i.e., $60 = 40(1.07)^t$ therefore $t = 6$)
- using calculators incorrectly $\left(\frac{\log\left(\frac{3}{2}\right)}{\log(1.07)}\right)$ was entered as $\log\left(\frac{\frac{3}{2}}{1.07}\right)$

4% 58. The temperature in a house at the time of a power failure was 22°C. The outside temperature was 0°C. If the temperature inside the house decreases at a rate of 13% every 2 hours, create a function to model the situation and use it to determine the temperature in the house after five hours.

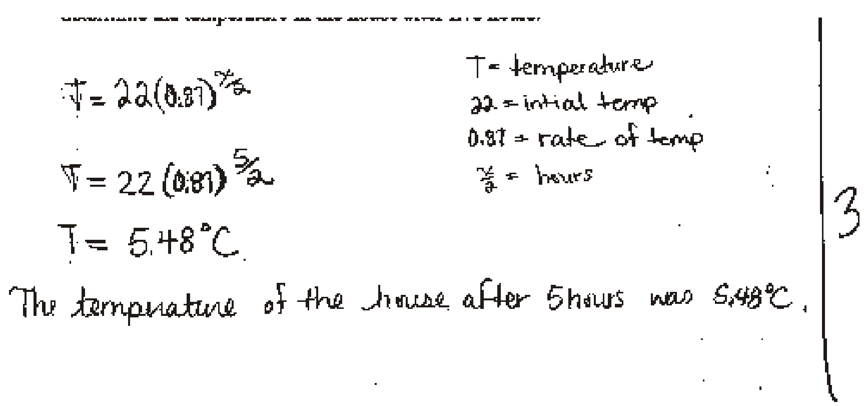
Answer Comment:

The problem required determining the correct exponential function to model the temperature and evaluating it for a specific time.

Excellent Answer Exemplar:



Good Answer Exemplar:



The student made a calculation error in the final step.

Commentary & Errors

Many students:

- did not consider the reasonableness of their solution. For instance, a decrease to 0.13°C in just 5 hours or having the temperature actually increase.

Errors:

- writing the exponential expression $22(0.87)^{\frac{t}{2}}$ instead of the exponential function $y = 22(0.87)^{\frac{t}{2}}$ or $f(t) = 22(0.87)^{\frac{t}{2}}$
- not realizing that this was an exponential situation and therefore setting it up as linear
- setting up a growth situation instead of a decay situation
- forgetting to use $\frac{x}{2}$ as the exponent to compensate for the rate of change happening every two hours
- using $\frac{2}{x}$ as the exponent instead of $\frac{x}{2}$
- not using the correct base (i.e., using 0.13 or - 0.87 instead of 0.87)
- setting up the equation properly, yet being unable to calculate a fractional exponent properly, or entering it improperly into the calculator

3% 59. A circle has a diameter with endpoints (–7, 4) and (1, –2). Find the equation of the circle in standard form.

Answer Comment:
This problem requires the midpoint formula on diameter endpoints to determine the centre, and the distance formula on an appropriate pair of points to determine the radius, then re-writing that information as a circle in standard form.

Excellent Answer Exemplar:

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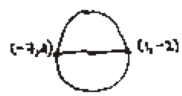


(i) Use Midpoint formula to calculate the centre.

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-7+1}{2}, \frac{4+-2}{2}\right) = (-3,1)$

(ii) Use distance formula to calculate distance b/w centre and an endpoint. (-7,4)

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-3--7)^2+(1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$

So, the radius is 5.

(iii) $(x+3)^2+(y-1)^2 = 25$ is the equation of the circle.

Good Answer Exemplar:

2

the circle in standard form.

$d_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(1+7)^2+(-2+4)^2}$
$= \sqrt{(8)^2+(-6)^2}$
$= \sqrt{64+36}$
$= \sqrt{100}$
$= 10$

∴ radius is $\frac{1}{2}$ of 10
$r = 5$

$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{-7+1}{2}, \frac{4-2}{2}\right)$
$= \left(\frac{-6}{2}, \frac{2}{2}\right)$
$= (-3, 1)$

Equation is:
$\frac{1}{5}(y-1) = (x+3)^2$

The student did not give the correct equation of the circle. The equation provided was a quadratic.

Commentary & Errors

Many students:

- did not know the standard form of a circle; many wrote their answer in transformational form.

Errors:

- writing final answers in the form $y = a(x - h)^2 + k$ where (h, k) was the centre of the circle
- putting the final answer in standard form for a quadratic, even after doing all other workings correctly
- simplifying $\frac{-7+1}{2}$ to 3
- using $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ for the midpoint formula
- using the wrong radius for example; $\left(\text{i.e., } 10 \text{ or } \sqrt{10}\right)$
- subtracting incorrectly in distance formula, especially in computing $^-3 - {}^-7$
- using incorrect formulas for distance and midpoint
- computational errors

4%

60. Prove that the line $x - 4y + 30 = 0$ passes through the centre of $x^2 + y^2 + 12x - 12y + 27 = 0$.

Answer Comment:
The problem required determining the centre of the circle from the general form provided, and demonstrating that the centre satisfied the equation of the line given.

Excellent Answer Exemplar:

$x^2 + 12x + \_\_ + y^2 - 12y + \_\_ = -27$

$(x+6)^2 + (y-6)^2$ So centre is $(-6, 6)$

and since $(-6) - 4(6) + 30 \overset{?}{=} 0$ 4

the line $x - 4y + 30 = 0$ contains the centre of the circle

Good Answer Exemplar:

$x^2 + y^2 + 12x - 12y + 27 = 0$

$(x^2 + 12x) + (y^2 - 12y) + 27 = 0$

$x^2 + 12x + 36 + y^2 - 12y + 36 + 27 = 0$

$(x+6)^2 + (y-6)^2 - 36 - 36 + 27 = 0$

$(x+6)^2 + (y-6)^2 = 45$

$(6, -6)$ 3

Check

$6 - 4(-6) + 30$

$6 - (-24) + 30$

$6 - 6$

$= 0$

$0 = 0$

The student stated the centre of the circle incorrectly but demonstrated a correct approach for testing the given condition.

Commentary & Errors

Many students:

- did not attempt the question.

Errors:

- not stating the coordinates of the centre
- stating the centre as (6, -6) instead of (-6, 6)
- collecting x terms and factoring out x
- not finishing the question (for example, just finding the circle's centre)
- stating $x^2 + y^2 + 12x - 12y + 27 = x - 4y + 30$
- not substituting the coordinates of the centre of the circle into the equation $x - 4y + 30 = 0$
- rearranging/manipulating the equation to complete the square
- careless computational and/or copying errors

4% 61. Determine the centre and lengths of the two axes of the ellipse

$$16x^2 + 9y^2 - 32x + 36y - 92 = 0.$$

Correct Answer:

The problem involved completing the square from the general form of a given ellipse and identifying and using values correctly to determine the centre and semi-major and semi-minor axis lengths, doubling to find major and minor axis lengths,.

Excellent Answer Exemplar:

$16x^2 + 9y^2 - 32x + 36y - 92 = 0.$

(I) Line up Variables:

$16x^2 + 32x + 9y^2 + 36y = 92$

(II) Complete the Square:

$16(x^2 - 2x) + 9(y^2 + 4y) = 92$

$16(x^2 - 2x + 1) + 9(y^2 + 4y + 4) = 92 + 16 + 36$

$16(x-1)^2 + 9(y+2)^2 = 144$

(III) Change to transformational form:

$\frac{16(x-1)^2}{144} + \frac{9(y+2)^2}{144} = \frac{144}{144}$

$\frac{1}{9}(x-1)^2 + \frac{1}{16}(y+2)^2 = 1 \rightarrow [\frac{1}{3}(x-1)]^2 + [\frac{1}{4}(y+2)]^2 = 1$

the length of the major (y) axis is 8

the length of the minor (x) axis is 6.

the centre is located at (1, -2)

4

Good Answer Exemplar:

$16x^2 + 9y^2 - 32x + 36y - 92 = 0.$

$16x^2 - 32x + 9y^2 + 36y = 92$

$16(x^2 - 2x + 1) + 9(y^2 + 4y + 4) = 92 + (16 \times 1) + (9 \times 4)$

$\frac{16(x-1)^2}{144} + \frac{9(y+2)^2}{144} = \frac{144}{144}$

$\frac{1}{9}(x-1)^2 + \frac{1}{16}(y+2)^2 = 1$

$[\frac{1}{3}(x-1)]^2 + [\frac{1}{4}(y+2)]^2 = 1$

minor axis = 3

major axis = 4

centre = (1, -2)

3

The student represented the semi-axis lengths of 3 & 4 as the axis lengths instead of the actual lengths 6 & 8.

Commentary & Errors

Many students:

- did not read the question carefully (i.e., students did not state the centre and the lengths of the two axes).
- have weak factoring skills.

Errors:

- not factoring out the leading coefficients prior to completing the square
- not compensating on the right-hand side with 16 and 36 (i.e., using 1 and 4 instead)
- subtracting instead of adding 16 and 36 on the right-hand side
- dividing 144 by 144 and getting 0
- copying part of the question incorrectly (i.e., -36y instead of +36y)
- factoring incorrectly (i.e., sign errors)
- not taking the square root of 9 and 16 when expressing the equation in transformational form, resulting in $\left[\frac{1}{9}(x-1)\right]^2 + \left[\frac{1}{16}(y+2)\right]^2 = 1$
- not correctly changing from standard form to transformational form
- making copying and/or computational errors

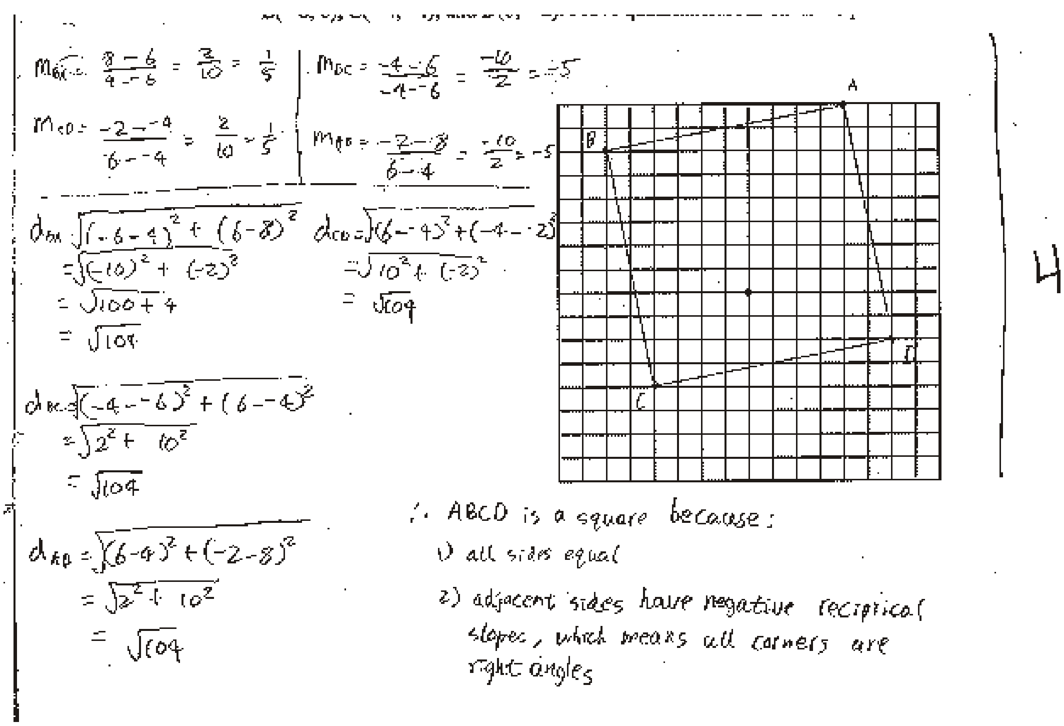
4% 62. Pat is building a garage. The four corners of the garage are located at A(4,8), B(– 6, 6), C(– 4, –4), and D(6, – 2). Prove quadrilateral ABCD is a square.

Correct Answer:
One method would include demonstrating that all sides were equal via the distance formula and demonstrating that at least 2 adjacent sides have opposite reciprocal slopes. The concluding statement would essentially be, "Since all sides are equal, and adjacent sides are perpendicular, ABCD is a square."

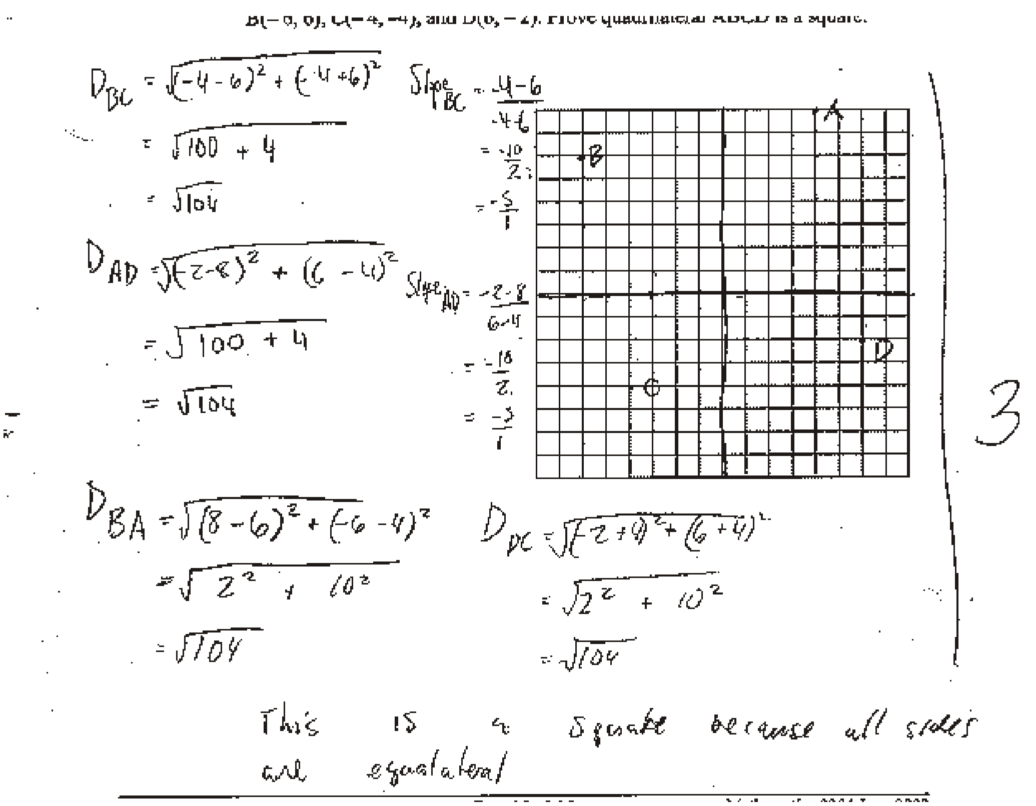
Another method would include showing via the distance formula that the diagonals were equal, showing via the midpoint formula that the diagonals bisected each other (shared a common midpoint), and showing via slopes that the diagonals are perpendicular. The concluding statement would essentially be, "Since the diagonals are equal and perpendicular bisectors of each other, ABCD is a square."

Other sufficient conditions were given full credit, such as, showing all angles are right angles and two adjacent sides are equal.

Excellent Answer Exemplar:



Good Answer Exemplar:



The student used the distance formula and stated correctly that all sides were equal. The student also found the slopes of sides BC and AD. However, the student did not state what the slopes meant with respect to the diagram. They did not prove the square contained a right angle.

Commentary & Errors

Many students:

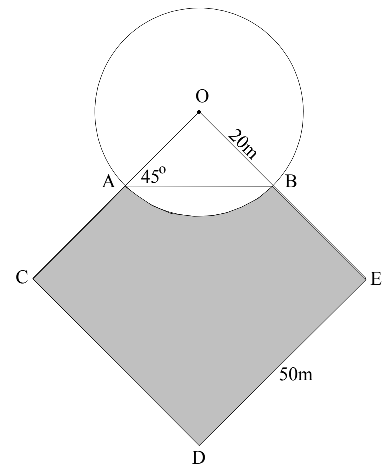
- used incorrect terminology when making conclusions. For example, when finding slopes to be negative reciprocals, students stated that the lines were $\perp$ bisectors, instead of just $\perp$.
- occasionally did not use the word diagonal...instead, students referred to it as the distance from corner to corner.

Errors:

- simplifying $\frac{-2}{10}$ to be -5 or stating that $\frac{-10}{2}$ was $\frac{-1}{5}$
- using the incorrect slope formula for example: $\frac{x_2-x_1}{y_2-y_1}$
- using the incorrect distance formula for example: $\frac{(x_2-x_1)^2+(y_2-y_1)^2}{\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}}$
- reducing $\sqrt{104}$ to be $\sqrt{100}\cdot\sqrt{4} = 10\cdot 2 = 20$
- not stating a conclusion after completing the necessary workings
- not concluding that sides of the square formed right angles or $\perp$ lines after finding the slopes were negative reciprocals
- leaving off the negative sign when reducing fractions for example: $\frac{-2}{10} = \frac{1}{5}$
- stating that sides of the square were $\perp$ bisectors of each other, rather than stating there were $\perp$
- stating that all sides equal was enough to guarantee the figure was a square
- stating that since the figure had four right angles, it must be a square
- stating that it was sufficient to show all sides equal to guarantee the quadrilateral was a square
- calculating the slopes of all four sides of the quadrilateral and stating that since all angles were right angles; the figure was a square
- using an incorrect formula for slope such as $\frac{x_2-x_1}{y_2-y_1}$
- using an incorrect (or not knowing any) distance formula...or leaving off the $\sqrt{\ }$ symbol.
- making errors in simplifying radicals for example $\sqrt{104}$
- making errors in simplifying fractions
- not writing a concluding statement in their proof
- thinking it was sufficient to sketch the figure ABCD and state that all sides were equal and all angles were 90° without showing any workings

4% 63. A circle with centre O, and the square OCDE, share some of the same area as shown. If $\angle OAB = 45°$, OB = 20m, and DE = 50m, find the area of the shaded region.

Answer Comment:
This problem involved determining the area of the circle and taking one-quarter of it to reflect the portion of the circle taken up by a sector; determining the area of the square; and then subtracting the area of the sector from the square.

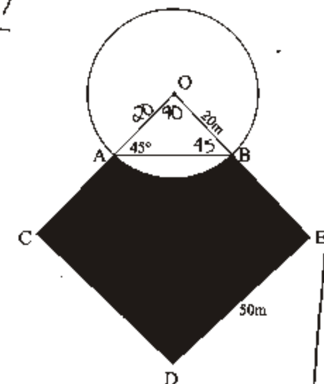


Excellent Answer Exemplar:

A(shaded region) = A(square) − A(sector)

Step 1: Area of Square

$A = lw$
$A = (50)(50)$
$A = 2500m^2$



Step 2: Area of Sector

$A = \frac{\theta}{360} \cdot \pi r^2$

$A = \frac{90}{360} \cdot \pi (20)^2$

$A = (.25)(1256.64)$

$A = 314.16m^2$

Step 3: Area of Shaded Region:

$A(S.R) = A(\text{square}) - A(\text{sector})$
$= 2500m^2 - 314.16m^2$
$= 2185.84m^2$

The Area of the Shaded Region is $2185.84m^2$

4

Good
Answer
Exemplar:

region.

area of square OCDE $= l \times w$

$= 50\text{m} \times 50\text{m}$

$= 2500\ \text{m}^2$

area of sector $= \frac{45°}{360°} \times \pi(20)^2$

$= 50\pi$

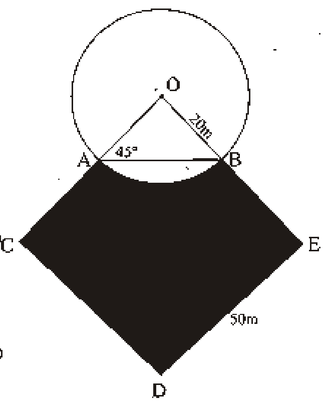
$\doteq 157.1\ \text{m}^2$

area of shaded = area square − area sector

$= 2500\ \text{m}^2 - 157.1\ \text{m}^2$

$= 2342.9\ \text{m}^2$

The area of the shaded region is $2342.9\ \text{m}^2$.



$3\frac{1}{2}$

The student used a central angle of 45° *instead of* 90°

Commentary & Errors

Many Students:

- found the area of ΔAOB due to the unnecessary inclusion of chord AB in the diagram and then incorporated this area in the response to the question. Many incorrect approaches were employed such as
  $A_{square} - A_{sector} + A_{triangle}$
  $A_{square} - A_{sector} - A_{triangle}$
  $A_{square} - A_{sector} - A_{segment}$
  $A_{sector} - A_{triangle}$
  $A_{square} - A_{sector} + A_{segment}$
- found the areas of the triangle, square, circle, sector and segment and then correctly concluded that only the square and sector areas were needed in the final step of the solution

Errors:

- calculating the area of a square incorrectly
  (i.e., 30 × 50, 20 × 50, 40 × 50, 2(50) + 2(50), $\frac{1}{2}(50)(50)$)
- using an incorrect central angle for the sector
  (i.e., 45°, 120°, 270°, and 100^0)
- using $2\pi r$ or $2\pi r^2$ to find the area of a circle
- using incorrect circle terminology (i.e., segment for sector and arc for segment)

Part I: Cognitive Level & Key [50%]

Q= Quadratics; R=Rate of Change; E=Exp. And Log.; G=Circle Geometry; P=Probability

Item	Outcome	C	P1	P2	PS
1-D	C32	Q			
2-A	A4	Q			
3-A	C8	Q			
4-D	C9		Q		
5-A	C8	Q			
6-D	A4		Q		
7-A	C31		Q		
8-D	C29		Q		
9-B	C4	Q			
10-C	B1, C22		Q		
11-A	C4		Q		
12-D	C15		Q		
13-B	C3		Q		
14-B	A7	Q			
15-C	C8		Q		
16-C	A4, B1		Q		
17-D	C16	R			
18-D	B4		R		
19-B	C11	E			
20-A	C34	E			
21-A	C33	E			
22-A	A7	E			
23-D	C11	E			
24-A	C19	E			
25-C	B2	E			

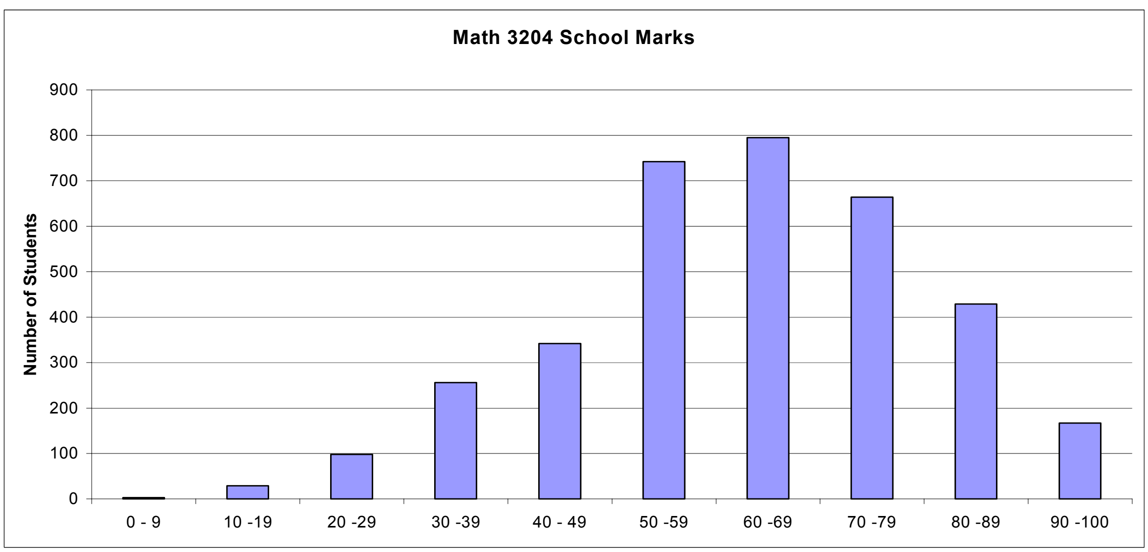
Item	Outcome	C	P1	P2	PS
26-B	B12		E		
27-D	A5		E		
28-B	B1, B12,C24		E		
29-D	C4,C11		E		
30-B	B13		E		
31-C	C24, B12		E		
32-D	C34		E		
33-B	C19	E			
34-B	C24, B12		E		
35-D	B1, B12, A5		E		
36-B	A5, B1		E		
37-C	E13	G			
38-D	E12	G			
39-C	E13	G			
40-D	E3		G		
41-C	A3, C36		G		
42-C	E4		G		
43-B	D1		G		
44-A	E4, E8, E7		G		
45-A	E3		G		
46-A	D1		G		
47-B	C36		G		
48-A	E13		G		
49-A	E4		G		
50-B	E4		G		

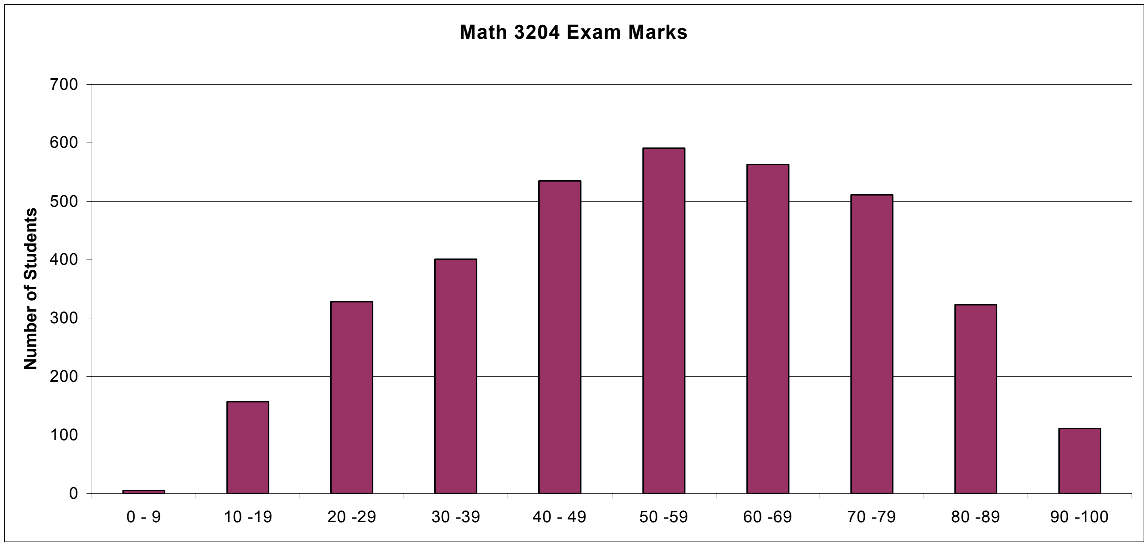
Part II: Cognitive Level & Value [50%]

Q= Quadratics; R=Rate of Change; E=Exp. And Log.; G=Circle Geometry; P=Probability

Item	Outcome	C	P1	P2	PS
51 [4]	A3, C22, A9			Q	
52 [4]	C1, C23				Q
53 [4]	C23, C31				Q
54 [4]	C28				R
55 [3]	C24, B12			E	
56 [4]	B13,C24			E	
57 [4]	C2, C25				E
58 [4]	C2, C34				E

Item	Outcome	C	P1	P2	PS
59 [3]	D1, E4			G	
60 [4]	E4, E14			G	
61 [4]	E15				G
62 [4]	E11, D1				G
63 [4]	E15				G




Math 3204 Exam Marks
Number of Students
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600
500
400
300
200
100
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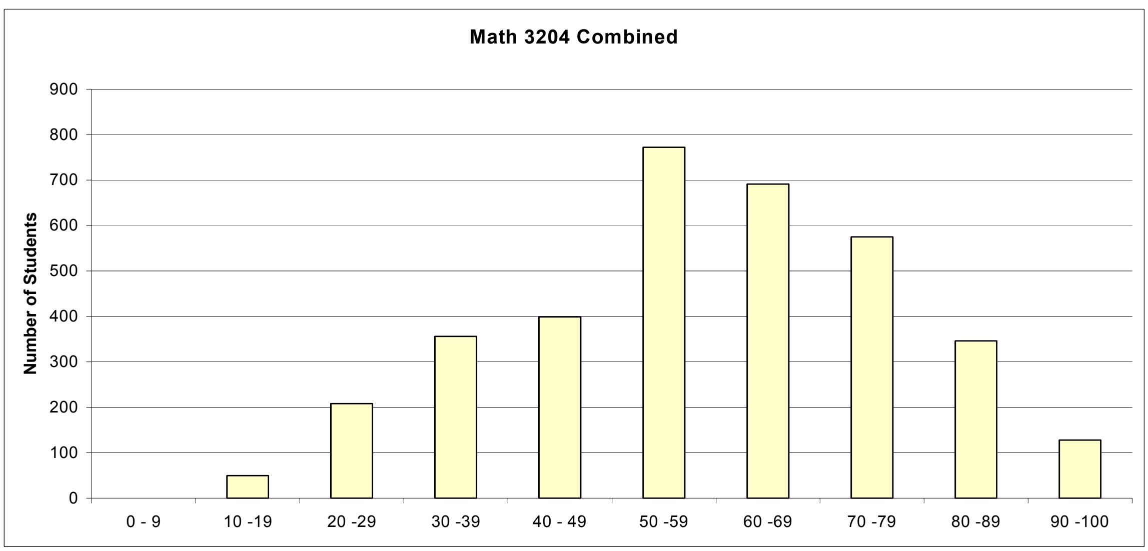
0 - 9
10 -19
20 -29
30 -39
40 - 49
50 -59
60 -69
70 -79
80 -89
90 -100


Math 3204 Combined
Number of Students
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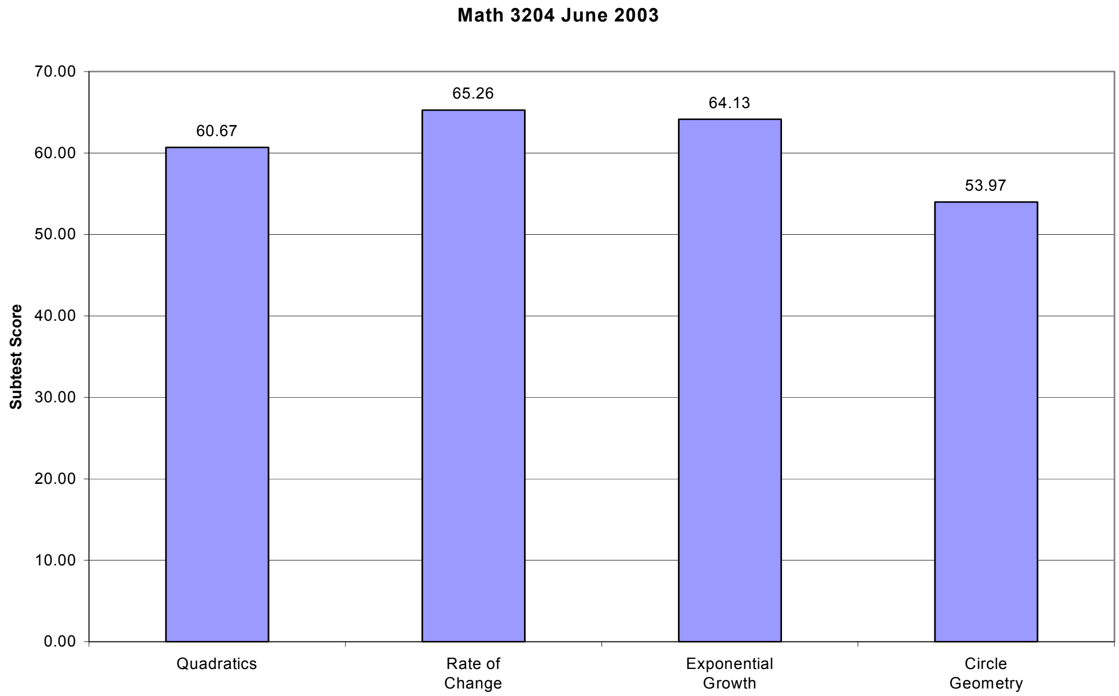
40 - 49
50 -59
60 -69
70 -79
80 -89
90 -100


Math 3204 June 2003
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Subtest Score
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65.26
64.13
53.97
Quadratics
Rate of Change
Exponential Growth
Circle Geometry