Math 3204
Grading Standards
June 2005

Pre-Marking Appraisal

The board considered the exam fair and of the appropriate difficulty and length.

Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 40 papers that went through the marker panel and marks were assigned to each question on a separate sheet of paper. The 40 exams were put back into the original stack of exams and corrected again when they appeared. The two values were compared and if there were discrepancies, the chief marker would review the scoring with the individual marker.

Throughout the marking process there was statistical analysis run on item data to enhance reliability and consistency of marking.

Commentary on Responses

- Overall, the test was well done. Average marks seem to have increased over June 2004.
- #57 & 58 students used tables or guess and check. This lead to much discussion over what would warrant full marks. Tables were accepted.
- #59 was done poorly. Most students could sketch the angle in the correct quadrant but could not continue.
- #62 was also poorly done. Students made basic calculation errors and had trouble writing their answer in standard form.

PART 11
Total Value: 50%

Instructions: Answer ALL items in the space provided. Show ALL workings.

Value

4 51. Algebraically determine the **EXACT** roots in simplest form for $\frac{5}{x-3} = \frac{x}{x+1}$.

$$x^2 - 3x = 5x + 5$$
$$x^2 - 8x - 5 = 0$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-5)}}{2(1)}$$
$$x = \frac{8 \pm \sqrt{64 + 20}}{2}$$
$$x = \frac{8 \pm \sqrt{84}}{2}$$
$$x = \frac{8 \pm 2\sqrt{21}}{2}$$
$$\boxed{x = 4 \pm \sqrt{21}}$$

Commentary on Response

This question was generally very well done. Most students used the quadratic formula to answer this question, however some students used completing the square method. Of those who used this method few received full marks due to mistakes made near the end of their solution.

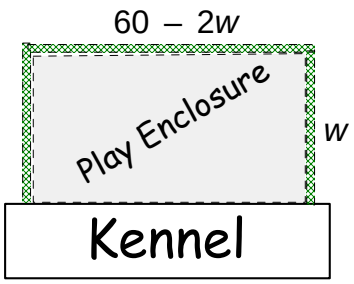
Common Errors

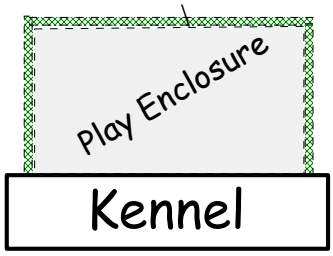
- Students did not write down the correct question when they started. (E.g., $\frac{5}{x+1} = \frac{x}{x+1}$).
- Students made the following errors; $(-8)^2 = 16$,
  $(-8)^2 = -64$ not $+64$, and
- $5(x+1) = 5x+1$ not $5x + 5$.

- Students incorrectly cross multiplied (i.e., multiplying two numerators and then multiplying two denominators (e.g., $5x=(x-3)(x+1)$ or multiplying numerator and denominator of each side together e.g., $5(x-3)=x(x+1)$.
- Students multiplied integers incorrectly $\left(\text{i.e., } (-8)^2-4(1)(-5)=64-20 \text{ rather then } 64+20\right)$.
- Students used incorrect quadratic formula.
- Students found the quad. equation but then stopped.
- Students omitted the "= 0" in the quadratic equation.
- Students divided first before simplifying the radical $\left(\text{i.e., } \frac{8\pm\sqrt{84}}{2}=4\pm\sqrt{42} \text{ or } 4\pm\sqrt{84}=4\pm2\sqrt{21}\right)$.
- Students did not fully simplify $\left(\text{i.e., left as } \frac{8\pm2\sqrt{21}}{2} \text{ or } \frac{-4\pm\sqrt{21}}{-1}\right)$.
- Students rejected the negative value.
- When completing the square was used, many students stopped at $(x-4)^2=21$, or did not add 16 to both sides.

Value

4 52. A rectangular play enclosure for some dogs is to be made with 60 m of fencing using the kennel as one side of the enclosure as shown. Algebraically determine the quadratic function that models the area of the enclosure and use it to find the dimensions that produce the maximum area.





$A(x)=w(60-2w)$

$A(x)=-2w^2+60w$

Max. is where w = $\frac{-b}{2a}=\frac{-60}{2(-2)}=15$

So greatest area is where the enclosure is 15 m wide by 30 m long.

Commentary on Response

Equal numbers of students solved the question using $x=\frac{-b}{2a}$ and completing the square. Less errors occurred by those students choosing $x=\frac{-b}{2a}$. This question was not well done.

Common Errors

- Students used $\ell = 2x - 60$ instead of $\ell = 60 - 2x$.
- Students wrote the function as $A = 2x(60 - 20x)$ or $60 = x(60 - 2x)$.
- Students solved the equation using quadratic formula instead of finding the vertex.
- When completing the square, students often neglected to remove -2 from the linear term, resulting in
  $A = -2(x + 15)^2 + 450$ instead of $A = -2(x - 15)^2 + 450$.
- Students neglected the answer of -15 and wrote it as 15 and then proceeded to get an apparently correct answer of 30 for other dimension.

Value

4 53. A baseball player hits a ball into the air. The ball's height above the ground, in metres, t seconds after being hit, is approximated by $h(t) = 1 + 13t - 5t^2$. Algebraically determine the time when the ball first reaches a height of 9 m

$9 = 1 + 13t - 5t^2$

$0 = -5t^2 + 13t - 8$

$0 = 5t^2 - 13t + 8$

$0 = (5t - 8)(t - 1)$

$t = \frac{8}{5}$ or $t = 1$

The ball initially reaches 9 m at 1 s.

Commentary on Response

This question was not well done. Most students attempted to use the quadratic formula with an assortment of various errors arising. There were very few students who successfully solved for t using completing the square. A number of students found the vertex (1.3, 9.45) and guessed at t =1 to get the height of 9.

Common Errors

- Students found the vertex only.
- Students substituted 9 m for time.
- Students did not substitute 9 at all, but used the quadratic formula on the given equation $h(t) = 1 + 13t - 5t^2$.
- Students substituted $9(t)$ for $h(t)$ producing the wrong quadratic
  $9(t) = 1 + 13t - 5t^2$
  $0 = 1 + 4t - 5t^2$
- Students used guess and check to get the time.

4 54. The power P, in watts, supplied to a circuit by a battery is given by the formula $P = 6I - 0.5I^2$, where I is the current in amperes. What is the approximate instantaneous rate of change of power when the current is 4 amperes?

Inst. Roc:

$\frac{P(4.1)-P(3.9)}{4.1-3.9} = \frac{16.2-15.8}{0.2} = \frac{0.4}{0.2} = 2$

About 2 watts per ampere.

Commentary on Response

This question was done very well overall. Some students took the first derivative at 4. Some students did the problem 2 or 3 times by using
P(4) + P(3.9), or P(4) + P(4.1), etc.
Some students then averaged the 2 or 3 answers. Some did P(4), P(3.9), P(3.999) like the investigation in the text.

Common Errors

- Students only found P(4) = 16 and stated IROC is 16.
- Students used 3 and 5 instead of 3.9 and 4.1.
- Students used 3.99 and 4.11, thinking 4.11 was close to 4.
- Students incorrectly rearranged the formula: $0.5I^2 - 6I$
- Students used $-0.5I^2 - 6I$ calculating P(4.1) as 16.2 and continuing by multiplying as follows $\frac{16.2(4.1) - 15.8(3.9)}{4.1 - 3.9}$.
- Students treated $6I$ as 61.
- Students used an incorrect difference quotient (e.g.; $\frac{16.2 - 4.1}{15.8 - 3.9}$).
- Students used the reciprocal of the difference quotient.

Value

3 55. Solve: $25^{2x+1} - 5 = 120.$

$$\left(5^2\right)^{2x+1} = 125$$
$$5^{4x+2} = 5^3$$
$$4x + 2 = 3$$
$$4x = 1$$
$$x = \tfrac{1}{4}$$

Commentary on Response

Student responses illustrated knowledge of exponential equations, creating like bases, and equating exponents. Some responses included the use of logarithms and $\log_B A = \frac{\log A}{\log B}$.

Common Errors

- Students never transposed the 5 to R.H.S.
- Students wrote 125 as 25^3 or 5^5 or 5^{25} or 25^5.
- Students wrote 25^{2x+1} as $5^{5(2x+1)}$ or $5^{\frac{1}{2}(2x+1)}$ or 25^{2x-4} or subtracted 5 from the exponent
  i.e., $25^{2x+1} - 5 = 25^{2x+1-5}$.
- Students who correctly wrote $5^{2(2x+1)}$ mistakenly wrote 5^{4x+1}, and they did not distribute the 2 inside the brackets.
- Students used a calculator to get $25^{2x+1} - 5^1 = 5^{2.9746}$ then had $2x + 1 - 1 = 2.9746$. They showed that they needed to make exponents equal, but did not make bases equal or recognize exponents cannot be equated here due to the operation (–) involved.

Value

4 56. Solve: $\frac{1}{3}\log_3 27 + \log_3 x = 4^{\frac{1}{2}}$.

$$\log_3 27^{\frac{1}{3}} + \log_3 x = 2$$
$$\log_3 3 + \log_3 x = 2$$
$$\log_3 3x = 2$$
$$3x = 3^2 \therefore \boxed{x = 3}$$

Commentary on Response

Most students were able to simplify the RHS knowing what the exponent $\frac{1}{2}$ meant.

Common Errors

- Students did not move $\frac{1}{3}$ as an exponent of 27 before recombining the logs.
- Students who moved the $\frac{1}{3}$ back as an exponent evaluated the expression incorrectly (i.e., $\log_3 27^{\frac{1}{3}} = \log_3 9$) taking one-third of 27 instead of raising 27 to the exponent of $\frac{1}{3}$
- Students who answered the question incorrectly did not convert correctly to its exponential form. They wrote
  $\log_3 3x = 2$ gives $3x = 3$ instead of $3^2 = 3x$.
- Students wrote the variable '*x*' as an exponent of 3 so
  $\log_3 3x = 2$ became $3^2 = 3^x$ and therefore the answer was 2.

4 57. If a Honda Prelude bought in 2000 cost \$32 000 and depreciates at a rate of 20% per year, and a Chevrolet Corvette bought in1994 cost \$43 000 and depreciates at a rate of 15% per year, algebraically determine which care is worth more in 2005.

$$\text{Honda}: \text{Value} = 32000\,(0.80)^5 \doteq \$10485.76$$
$$\text{Chevrolet}: \text{Value} = 43000\,(0.85)^{11} \doteq \$7195.76$$
$$\therefore \text{Prelude is worth more in 2005.}$$

Commentary on Response

Marks were not given for the answer alone if done without any supporting work.

Common Errors

- Students had a problem with placing the number of years in formula (e.g., $y = 32000(0.80)^{\frac{2005}{2000}}$.
- Students used depreciation factor incorrectly $e.g., y = 32000(0.2)^5$.

Value

4 58. Suppose there are 25 coyotes in one area of NL right now. If this population is known to triple every 4 years, how long will it take for the population to reach 258 animals?

$258 = 25(3)^{\frac{t}{4}}$ $\log 10.32 = \frac{t}{4}\log 3$

$\frac{258}{25} = 3^{\frac{t}{4}}$ $t = \frac{4\log 10.32}{\log 3}$

$10.32 = 3^{\frac{t}{4}}$ $t \doteq 8.5$ years

Commentary on Response

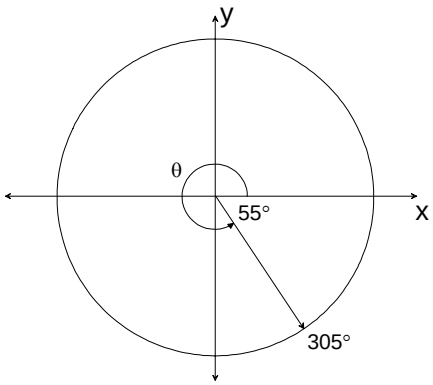
The question did not specify the need to create an exponential function or to use logs, therefore many students simply tripled the initial value and estimated an answer somewhere between 8 and 12 years. Many students created the equation and then used "guess & check" to obtain the correct answer

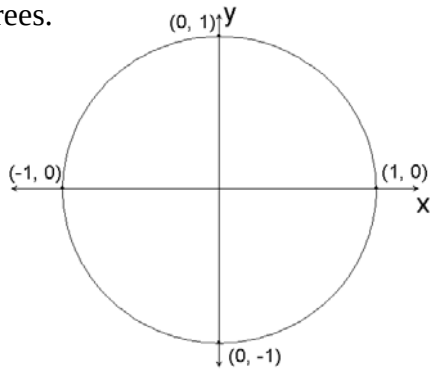
Common Errors

- Students multiplied 25 and 3, then proceeded to do the rest correctly.
- Students divided by log 25 instead of just 25.
- Students mixed up the 3 and the 4.
- Students divided by log 10.32 instead of by log 3.

Value

3 59. A point P having coordinates (0.5736, −0.8192), lies on the terminal arm of an angle in standard position on a unit circle. Sketch this situation on the axes provided and determine the value of θ in degrees.





$$\cos\alpha = 0.5736$$
$$\alpha = \cos^{-1}(0.5736)$$
$$\alpha \doteq 55^{o}$$
$$\theta = 360^{o} - 55^{o} = 305^{o}$$

Commentary on Response

This question did not stipulate $0^{o} \le \theta < 360^{o}$ so -55^{o} is acceptable.

Common Errors

- students had real difficulty with this problem. Many got the sketch or part of it but could not go beyond that point.
- Students gave the reference $\angle$ as the answer.
- Students used the complement of 55 and subtracted it from 360 $\left(\text{i.e., } 360^{o} - 35^{o} = 325^{o}\right)$.

Value

4 60. Write $x^2 + y^2 - 6x + 4y - 12 = 0$ in transformational form and state the radius.

$x^2 - 6x\underline{+9} + y^2 + 4y\underline{+4} = 12\ \underline{+9}\ \underline{+4}$

$(x-3)^2 + (y+2)^2 = 25$

$\frac{1}{25}(x-3)^2 + \frac{1}{25}(y+2)^2 = 1$

$\boxed{\left[\frac{1}{5}(x-3)\right]^2 + \left[\frac{1}{5}(y+2)\right]^2 = 1}$ $\boxed{r = 5}$

Commentary on Response

Some students stopped at Standard Form and gave the correct answer. Students also had trouble completing the square/balancing the equation. Students also wrote the correct transformational form but failed to complete the question and state the radius.

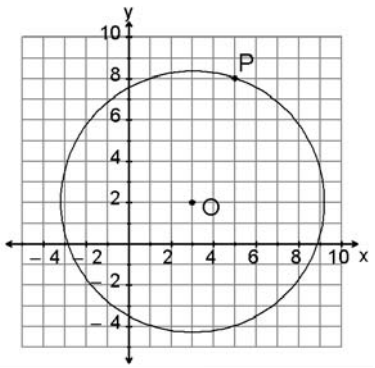
Common Errors

- Students made the following errors;

- $(x^2 - 6x) + (y^2 + 4y) = -12$, i.e., forgot to change sign when transposing
- $\left[\frac{1}{5}(x-3)\right]^2$ no sign written throughout the solution $\left[\frac{1}{5}(y+2)\right]^2 = 1$
- $\left[\frac{1}{5}(x-3)\right]^2 + \left[\frac{1}{5}(y+2)\right]^2$ not equal 1 $(\text{sometimes equal } 0)$
- $\left[\frac{1}{5}(x-6)\right]^2 + \left[\frac{1}{5}(y+4)\right]^2 = 1$, i.e., incorrectly factoring trinomials
- $\left(x^2 - 6x + 9\right) + \left(y^2 + 4y + 4\right) = 12$, i.e., equation not balanced
- $\frac{1}{5}(x-3)^2 + \frac{1}{5}(y+2)^2 = 1$, i.e., 2nd bracket is omitted enclosing the squares
- $\left[\frac{1}{25}(x-3)\right]^2 + \left[\frac{1}{25}(y+2)\right]^2 = 1$, i.e., did not take square root of number when writing as squares

Value

4 61. Determine the equation of the circle in standard form having centre O, and point P on the circumference, as shown.



$O(3,2)$ $P(5,8)$

$radius^2: (5-3)^2+(8-2)^2=r^2$

$40=r^2$

$\boxed{(x-3)^2+(y-2)^2=40}$

Commentary on Response

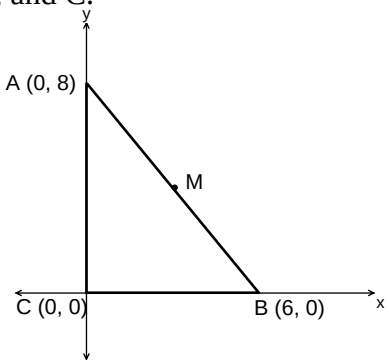
This question was done poorly. Poor performance seems to be a result of poor basic skills such as problems subtracting or squaring. Students had trouble writing their answer in standard form. Many students determined the radius by counting blocks and estimating an answer of 6.

Common Errors

- Students used the equation of a parabola.
- Students used transformational form or general form.
- Students counted blocks to approximate the radius.
- Students misread the coordinates of points O and P.
- Students determined the equation of the line forming the radius or the tangent line at point P.
- Students found the slope of $\overline{OP}$ and used it as the radius.
- Students did not square the radius.
- Students subtracted improperly when determining the radius.
- Students determined the midpoint of $\overline{OP}$ and used this point for the center.

Value

4 62. Given the diagram as shown, use coordinate geometry to prove that the midpoint of the hypotenuse, M, is an equal distance from A, B, and C.



Midpoint $\overline{CM}$: $\left(\frac{0+6}{2}, \frac{8+0}{2}\right)$

$(3, 4)$

$AM = \sqrt{(3-0)^2 + (4-8)^2} = \sqrt{9+16} = 5$

$BM = \sqrt{(6-3)^2 + (0-4)^2} = \sqrt{9+16} = 5$

$CM = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16} = 5$

Since AM = BM = CM, M is equidistant from A, B, and C.

Commentary on Response

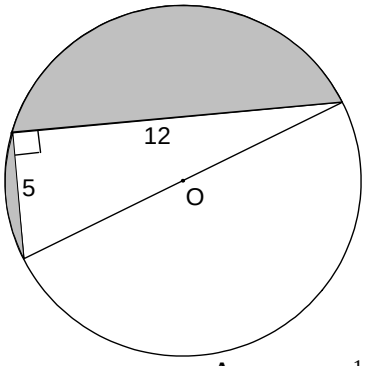
Students seemed to do fairly well on this question.

Common Errors

- Students used slope formula instead of midpoint formula.
- Students found distances for AB, AC and BC.
- Students used the wrong distance formula
  i.e., $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$
- Students indicated a midpoint of (4, 3) not (3, 4).

Value

4 63. In this circle with centre O, determine the total area of the shaded regions.



$c^2 = a^2 + b^2$

$c^2 = 5^2 + 12^2$

$c = 13$

$\therefore$ Radius is $\frac{1}{2}(13) = 6.5$

$A_{\text{Triangle}} = \frac{1}{2}(5)(12) = 30 \text{ sq.units}$

$A_{\text{Semi-Circle}} = \frac{1}{2}\pi(6.5)^2 \doteq 66.4 \text{ sq.units}$

$A_{\text{Shaded region}} \doteq (66.4 - 30) \text{sq.units}$

$= 36.4 \text{ sq.units}$

Commentary on Response

Students seemed to have problems with this question. Many did not see that the area required was simply the area of the semi-circle minus the area of the triangle.

Common Errors

- Students used 90^o not 180^o for the portion of the circle involved in the shading.
- Students used 13 not 6.5 for radius.
- Students attempted to get the area of triangle by using $2\pi r$ or $2\pi r^2$ or $b \times h$.
- Students added $A_{semi-circle}$ and $A_{triangle}$ rather than subtracting.

MATHEMATICS 3204
PART 1
SELECTED - RESPONSE ITEM ANALYSIS

Item	A	B	C	D
	%	%	%	%
1. Correct answer is B	2.7	93.9	1.9	1.5
2. Correct answer is D	10.6	27.0	14.0	48.4
3. Correct answer is C	33.5	5.1	44.3	17.0
4. Correct answer is D	25.1	16.0	30.2	28.7
5. Correct answer is C	9.8	2.5	76.7	10.9
6. Correct answer is C	10.1	13.4	50.6	25.8
7. Correct answer is A	70.5	5.1	8.4	16.1
8. Correct answer is C	3.1	4.1	82.6	10.1
9. Correct answer is B	15.0	47.0	21.8	15.8
10. Correct answer is B	20.3	72.4	4.9	2.4
11. Correct answer is C	29.2	26.2	37.2	7.3
12. Correct answer is A	44.2	19.9	25.5	9.9
13. Correct answer is A	67.3	11.0	10.4	11.2
14. Correct answer is D	3.2	6.8	8.8	81.2
15. Correct answer is B	26.0	27.7	12.7	33.4
16. Correct answer is A	46.5	14.1	6.7	32.6
17. Correct answer is A	63.9	10.4	10.4	14.8
18. Correct answer is B	28.2	52.0	15.0	4.7
19. Correct answer is D	5.7	15.5	9.1	69.7
20. Correct answer is C	10.6	20.6	35.1	33.6
21. Correct answer is A	85.3	6.1	5.5	3.1
22. Correct answer is B	15.9	43.6	27.5	13.0
23. Correct answer is C	11.6	14.4	47.6	26.2
24. Correct answer is A	48.2	6.8	32.8	12.1
25. Correct answer is A	67.5	10.1	9.7	12.8

MATHEMATICS 3204
PART 1
SELECTED - RESPONSE ITEM ANALYSIS

Item	A	B	C	D
	%	%	%	%
26. Correct answer is A	89.6	3.9	4.9	1.5
27. Correct answer is B	24.1	53.1	10.9	11.8
28. Correct answer is B	1.7	73.4	22.2	2.4
29. Correct answer is D	4.1	3.2	3.4	89.3
30. Correct answer is D	9.7	16.5	10.7	62.9
31. Correct answer is C	18.3	9.1	46.4	26.0
32. Correct answer is A	44.1	28.2	14.2	12.8
33. Correct answer is D	4.2	10.6	7.4	77.7
34. Correct answer is C	25.6	7.6	61.2	5.5
35. Correct answer is B	6.2	67.0	22.2	4.3
36. Correct answer is C	2.5	1.5	82.3	13.7
37. Correct answer is A	56.5	16.2	8.1	19.0
38. Correct answer is B	9.3	68.1	15.0	7.6
39. Correct answer is D	2.1	7.8	11.0	79.1
40. Correct answer is C	49.8	10.4	34.5	5.1
41. Correct answer is A	52.8	26.2	12.7	8.2
42. Correct answer is B	12.0	63.0	13.8	11.0
43. Correct answer is A	28.7	30.0	13.7	27.1
44. Correct answer is A	54.9	13.3	13.4	18.1
45. Correct answer is B	28.3	38.8	22.8	9.6
46. Correct answer is B	8.1	80.2	7.8	3.9
47. Correct answer is D	5.9	36.4	10.0	47.5
48. Correct answer is C	8.4	3.8	84.6	3.1
49. Correct answer is C	31.3	39.6	20.7	7.6
50. Correct answer is C	36.6	12.7	39.2	10.8

MATHEMATICS 3204
PART II
CONSTRUCTED - RESPONSE ITEM ANALYSIS

Item	Students Completing Item	Value	Average	Average % Per Item
PART II		**50**		
51	3499	4	2.8	70.0
52	3499	4	1.8	45.0
53	3499	4	1.7	42.5
54	3499	4	3.0	75.0
55	3499	3	2.2	73.3
56	3499	4	2.1	52.5
57	3499	4	2.9	72.5
58	3499	4	2.9	72.5
59	3499	3	1.0	33.3
60	3499	4	2.8	70.0
61	3499	4	1.9	47.5
62	3499	4	2.6	65.0
63	3499	4	1.9	47.5