Mathematics 3204 Grading Standards
June 2006

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations.

Post-Marking Report

a) Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examination and were held by the Chief Marker for recirculation throughout the marking period. These papers were re-corrected by the marking board, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Significant discrepancies were rare. In addition, each marker made on-going notes regarding partial marks and scoring schemes for their particular questions. Whenever a non-common error presented, it was scored by consensus of the board and made note of for scoring consistency.

b) Summary

The performance on Part II of the examination was lower than 2005, however the results were quite similar to 2003 and 2004. A decline in performance was most noticeable from #59 to #63 (with the exception of #60). Questions from the 'Circles' section were poorly done.

c) Commentary on Responses:

Part I – Selected Response – Total Value: 50%

Item #13 – Both B and D are correct answers for this item.

Item #26 – If this question been asked using specific values instead of parameters *b* and *c*, performance may have been better.

Item #50 – There was an error in the prompt (' $\overline{AB}$ and $\overline{CD}$ are tangents' should have read ' $\overline{AB}$ and $\overline{AC}$ are tangents'). This question was dropped.

Part II – Constructed Response – Total Value: 50%

4% 51. Algebraically determine the **EXACT** roots in simplest form for $16x(x+1) = -13$.

Answer:

$16x^2 + 16x + 13 = 0$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(16)(13)}}{2(16)}$$

$$x = \frac{-16 \pm \sqrt{256 - 832}}{32}$$

$$x = \frac{-16 \pm \sqrt{-576}}{32}$$

$$x = \frac{-16 \pm 24i}{32} = \frac{-2 \pm 3i}{4}$$

1 mark: Expansion of the product and collection of terms to one side.
1 mark: Values placed in quadratic formula.
1 mark: Calculation of radicand and other values in quadratic formula.
½ mark: Simplifying of radical.
½ mark: Final answers in simplest form.

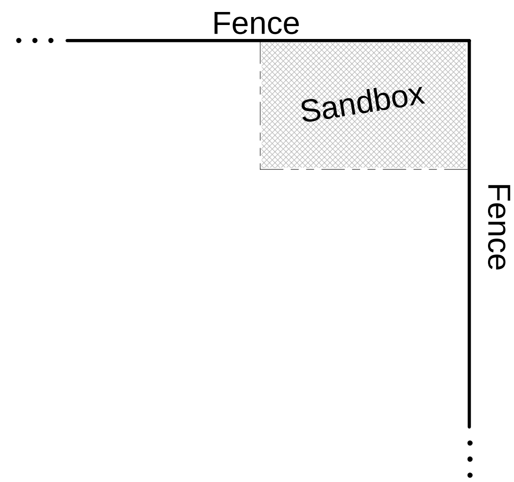
Alternatively, students used completing the square.

Common Errors

Students:

- expanded $16x(x+1)$ incorrectly.
- did not handle negative radicands correctly (e.g., $\sqrt{-576} = i\sqrt{24}$).
- did not write the final answer in simplest form.
- did not know the quadratic formula.

4% 52. A day care centre bought 20 m of board to form two sides of a rectangular sandbox against a corner in a fenced yard as shown. If all 20 m of board is used, write the quadratic function that models the area of the sandbox and use it to determine the maximum area the sandbox can have.



Answer:

Length + Width = 20

Length = x

Width = $20 - x$

A = $(x)(20-x)$

A = $-x^2 + 20x$

$x = -\frac{b}{2a} = -\frac{-20}{2(-1)} = 10$

Length = 10 m and Width = 10 m.

$\therefore$ Max. area is 100 m^2.

1 mark: Area function with correct variable dimensions.
1 mark: Expansion of Area function.
1 mark: *x*-value for maximum area
1 mark: Maximum area.

Alternatively, students used completing the square.

Commentary on Responses

Students who attempted this question solved it by using either $x = -\frac{b}{2a}$ or by completing the square.

Common Errors

Students:

- used $x(20-2x)$ instead of $x(20-x)$.
- got -10 instead of 10 using $x = -\frac{b}{2a}$.
- got the correct set up and width, but failed to determine the maximum area.
- treated this as a three-sided instead of two-sided diagram.

4% 53. An osprey dives toward the water to catch a salmon. Its height above the water, in metres, t seconds after it begins its dive, is approximated by $h(t) = 5t^2 - 30t + 45$. Algebraically determine the time it takes the osprey to reach a return height of 20 m as shown.

Answer:

$20 = 5t^2 - 30t + 45$

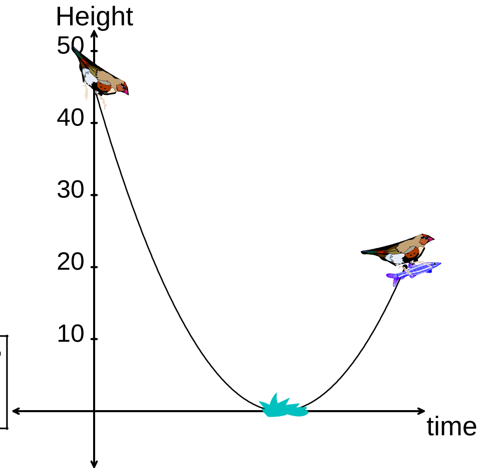
$0 = 5t^2 - 30t + 25$

$0 = t^2 - 6t + 5$

$0 = (t-5)(t-1)$

$t = 1$ or $t = 5$

So, the osprey is again at 20 m above the water 5 seconds after beginning the dive.



½ mark:	Equating 20 and $h(t)$.
½ mark:	Setting the above line to 0.
1 mark:	Factoring quadratic (or quadratic formula values).
1 mark:	Identifying the two times.
1 mark:	Identifying the return height time.

Commentary on Responses

Many students used tables or guessed at $t = 5$ and then verified $h(5) = 20$. Most students who used the quadratic formula did so correctly.

Common Errors

Students:

- found $h(20)$ instead of $h(t) = 20$.
- found only the vertex (3, 0) using $x = -\frac{b}{2a}$.
- set $5t^2 - 30t + 45 = 0$ and solved.
- made computational errors in using the correct quadratic formula.
- set up as: $20t = 5t^2 - 30t + 45$.

4% 54. The volume of water at a given time in a 2000 L tank is represented by the formula $V = 2000\left(1 - \frac{t}{45}\right)^2$, where t is time in minutes. Determine the average rate of change in the volume of water in the tank from minute 0 to minute 10, and use it to describe how the volume of water in the tank is changing during that time.

Answer:

$$\begin{aligned}\text{Average RoC} &= \frac{V(10) - V(0)}{10 - 0} \\ &= \frac{2000(1 - \frac{10}{45})^2 - 2000}{10} \\ &= \frac{1209.9 - 2000}{10} \\ &= \boxed{-79.01 \text{ L/minute.}}\end{aligned}$$

On average, the tank is losing 79.01 L of water each minute.

1 mark:	Difference quotient set up.
2 marks:	Values from difference quotient.
½ mark:	-79.01 L/minute.
½ mark:	How volume of water is changing during that time.

Commentary on Responses

Many students had difficulty calculating V(0) and V(10). Many students found the instantaneous rate of change rather than average rate of change. Another common response was to calculate V(0) and V(10) and average them.

Common Errors

Students:

- misused the slope formula (e.g., $\frac{y_2 - y_1}{x_1 - x_2}$ or $\frac{x_2 - x_1}{y_2 - y_1}$).
- had difficulty calculating $2000(1 - \frac{10}{45})^2$.
- calculated $\frac{V(0) + V(10)}{2}$.
- incorrectly used 200 instead of 2000.

3% 55. Algebraically solve: $\left(\frac{1}{4}\right)^{3-x} = 64^{x+1}$.

Answer:

$$(4)^{x-3} = 64^{x+1}$$
$$4^{x-3} = 4^{3x+3}$$
$$x-3 = 3x+3$$
$$-2x = 6$$
$$\boxed{x = -3}$$

½ mark: Reciprocal of ¼ as 4^{-1}.
½ mark: Same base with correct exponents.
1 mark: Equating exponents.
1 mark: Solution.

Common Errors

Students:

- changed ¼ to 4^{-1} but did not change the exponent to (x – 3).
- changed 64^{x+1} to 4^{3x+1} instead of 4^{3x+3}.
- incorrectly changed 64 to 4^{16} or 2^{32}.
- incorrectly changed $\left(\frac{1}{4}\right)^{3-x}$ to -4^{3-x}.

4% 56. Write as a single logarithm and approximate the value to the nearest hundredth: $2\log_5 3 + \log_5 96 - \frac{1}{3}\log_5 64$.

Answer:

$\log_5 3^2 + \log_5 96 - \log_5 64^{\frac{1}{3}}$

$\log_5 9 + \log_5 96 - \log_5 4$

$\log_5\left(\frac{(9)(96)}{4}\right)$

$\log_5 216$

$\frac{\log 216}{\log 5} \doteq 3.34$

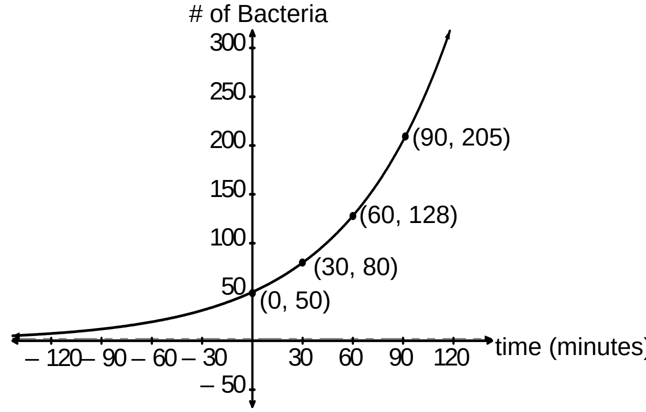
1 mark:	Putting coefficients to exponents.
1 mark:	Converting exponents to 9 and 4.
½ mark:	Combining three log terms into one.
½ mark:	Combining to get 216
½ mark:	Solution 3.34.
½ mark:	Correct rounding in solution (2 decimals).

Common Errors

Students:

- did not equate the 2 and $\frac{1}{3}$ as an exponent on the argument of the logs.
- did equate the 2 and $\frac{1}{3}$ as an exponent on the argument of the logs but evaluated incorrectly (i.e., $3^2 = 6$ or $64^{\frac{1}{3}} = 21.3$).
- stopped at $\log_5 216\ldots$ did not approximate to the nearest 100th.
- did not correctly round answer to nearest hundredth.
- eliminated the logarithm without regard for the base (i.e., 216 was the answer).
- incorrectly simplified $\log_5 9 + \log_5 96 - \log_5 4$ to $\log_5 101$.

4% 57. A university student studied and recorded the population of a bacterial culture every 30 minutes, as shown on the graph. Determine the exponential function that models the bacteria population during the study and use it to find the bacteria population 180 minutes after the study began.



Answer:

50 80 128 205

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1.60 1.60 1.60

x-increment is 30 (minutes)

$y = 50(1.6)^{\frac{x}{30}}$

180 minutes later:

$50(1.6)^{\frac{180}{30}} = 50(1.6)^6 \doteq 838.86$ so 838 bacteria were present

½ mark: $a = 50$
1 mark: $b = 1.6$
1 mark: $x = 180$
½ mark: $c = 30$
½ mark: Workings.
½ mark: Final Answer.

Commentary on Responses

Some students who determined *a*, *x* and *c* were unable to determine the common ratio.

Common Errors

Students:

- did not identify this situation as exponential and worked through a common difference approach (i.e., D_1, D_2, …).
- used a common ratio of $\frac{50}{80}$ and not $\frac{80}{50}$.
- found slope of two points and used that as the common ratio.
- attempted to extrapolate to 180 from the graph only.

4% 58. Technetium-99, a radioactive isotope used in nuclear medicine, has a half-life of 6 hours. Set up an equation and use it to determine how long it would take for 500 micrograms of Technetium-99 to reduce to 100 micrograms.

Answer:

$$y = 500\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$100 = 500\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\tfrac{1}{5} = \left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\log\left(\tfrac{1}{5}\right) = \log\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\log\left(\tfrac{1}{5}\right) = \tfrac{x}{6}\log\left(\tfrac{1}{2}\right)$$

$$x = \frac{6\log\left(\frac{1}{5}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\boxed{x \doteq 13.93 \text{ or about 14 hours}}.$$

½ mark: $a = 500$
½ mark: $b = ½$
½ mark: Exponent as $\frac{x}{6}$.
½ mark: Setting the above equal to 100.
½ mark: $\frac{1}{5} = \left(\frac{1}{2}\right)^{\frac{x}{6}}$.
½ mark: Log. Equation from above.
½ mark: Solving log. Equation above symbolically.
½ mark: Determining the time as about 13.9 or 14 hours.

Commentary on Responses

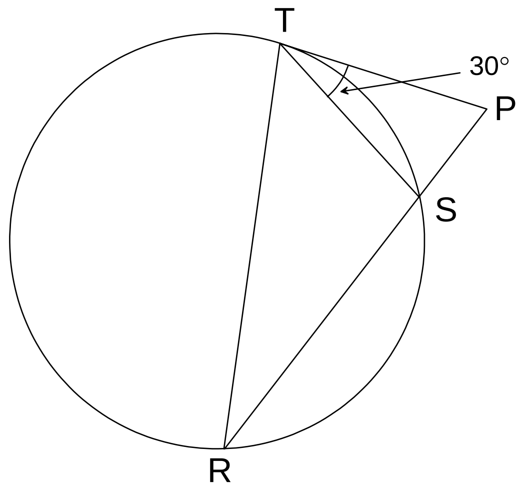
Most students were able to set up the equation $100 = 500\left(\frac{1}{2}\right)^{\frac{x}{6}}$. Some students interpreted 'use it' to mean 'guess & check'; however, given that this was a problem solving question, full marks were awarded, provided students demonstrated that the LHS was equal to the RHS through arriving at about 13.9 hours.

Common Errors

Students:

- multiplied 500 by ½ .
- divided $\log\frac{1}{2}$ by $\log\frac{1}{5}$.
- divided 2.32 by 6 instead of multiplying.

3% 59. If $\overline{TP}$ is tangent to the circle, $\overset{\frown}{RS} = 100°$, and $\angle PTS = 30°$, determine the measure of $\angle TRS$, $\angle RTS$, and $\angle TPS$.



Answer:

$\angle TRS = \angle PTS$

$\angle TRS = 30°$

$\angle RTS = 50°$

$$\angle TPS = 180° - 80° - 30°$$
$$= 70°$$

1 mark: $\angle TRS = 30°$
1 mark: $\angle RTS = 50°$
1 mark: $\angle TPS = 70°$

Common Errors

Students:

- assumed $\overline{TR}$ was a diameter thus concluding that $\angle RTP = 90°$.
- believed that $\overline{TS} \perp \overline{PR}$
- incorrectly named or misinterpreted names of angles.

4% 60. Write $x^2 + y^2 + 6x - 8y + 9 = 0$ in standard form and state the radius and centre.

Answer:

$$x^2 + y^2 + 6x - 8y + 9 = 0$$
$$(x^2 + 6x + 9) + (y^2 - 8y + 16) = 9 + 16 - 9$$
$$\boxed{(x+3)^2 + (y-4)^2 = 16}$$

Radius $r = 4$

Centre $C(-3, 4)$

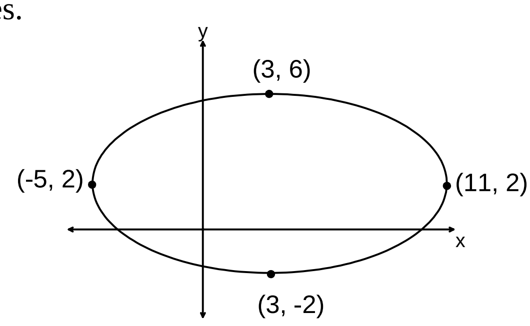
1 mark:	Completing the squares on the left hand side…
1 mark:	…followed by correct balancing of the equation on the right hand side.
1 mark:	Standard form of the circle.
1 mark:	½ each for the radius and centre.

Common Errors

Students:

- made sign errors when transposing terms.
- balanced the equation by adding 9 and 16 to LHS twice instead of LHS and RHS once.
- believed the radius was 16.
- did sign reversal when identifying the centre.
- gave the final answer in transformational form.
- did not balance equation at all following completing the square.
- subtracted 9 and 16 from LHS despite adding them to the RHS.
- changed signs between terms prior to completing the square.

4% 61. Determine the equation of the ellipse shown in transformational form, and state the lengths of the major and minor axes.



Answer:

Centre: $\left(\frac{-5+11}{2},\frac{2+2}{2}\right)=(3,2)$

Equation: $\left[\frac{1}{8}(x-3)\right]^2+\left[\frac{1}{4}(y-2)\right]^2=1$

Major Axis length: 16 units
Minor Axis Length: 8 units

1 mark: Centre.
1 mark: Stretch factors.
1 mark: Transformational form.
1 mark: ½ each for the major and minor axis length.

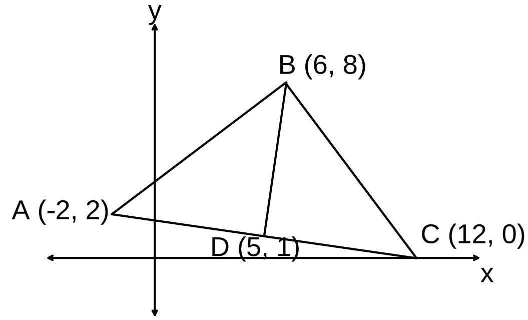
Common Errors

Students:

- did not use brackets in transformational form.
- used quadratic function instead of ellipse equation.
- made sign reversals on centre when substituting into the relation.
- ignored the vertical and horizontal stretch factors.
- stated 8 and 4 as the major and minor axes (they are the semi-major and minor).
- set the relation equal to 0 and not 1.
- reversed the applicable stretch factors.

4% 62. Given ΔABC as shown, use coordinate geometry to prove $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

Answer:



slope $\overline{BD} = \frac{8-1}{6-5} = 7$

slope $\overline{AC} = \frac{2-0}{-2-12} = -\frac{1}{7}$

$\therefore\ \overline{BD} \perp \overline{AC}$

Midpoint of $\overline{AC}$:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+12}{2}, \frac{2+0}{2}\right) = (5,1) \text{ i.e., Point D}$$

$\therefore\ \overline{BD}$ bisects $\overline{AC}$.

Since, $\overline{BD} \perp \overline{AC}$, and $\overline{BD}$ is the midpoint of $\overline{AC}$,
$\therefore \overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

3 marks: Proving slopes are opposite reciprocals.
1 mark: Proving D is the midpoint and final statement.

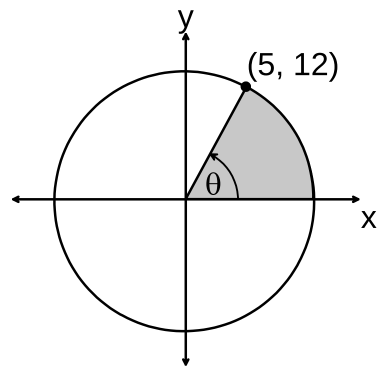
Alternatively, some students found lengths of sides in the smaller triangles and demonstrated via the Pythagorean Theorem they were right triangles and AD = DC.

Common Errors

Students:

- used the incorrect slope formula.
- used slopes of AB and BC not understanding that BD and AC were the target of the prompt.
- used the distance formula to determine the lengths of BD and AC with no logical conclusion.
- calculated $\frac{-2}{14}$ in the slope calculation as -7.
- used the distance formula to determine all sides and stated that this proved perpendicular bisector with no logical conclusion.
- proved perpendicular but not bisector.
- proved the bisector but not perpendicularity.

4% 63. In the circle with centre at the origin as shown, determine the measure of angle θ and use it to find the area of the shaded region.



Answer:

$$5^2 + 12^2 = r^2$$
$$r = 13$$

$$\tan\theta = \tfrac{12}{5}$$
$$\boxed{\theta \doteq 67.4^\circ}$$

$$\text{Area} = \tfrac{67.4}{360} \cdot \pi(13)^2$$
$$\boxed{\text{Area} \doteq 99.4 \text{ units}^2}$$

1 mark: Radius.
1 mark: Theta.
1 mark: Area formula with values.
1 mark: Approximate area.

Common Errors

Students:

- did not calculate theta but used a visual approximation (e.g., 60°).
- did not find the radius at all.
- used 12 or 5 for the radius instead of the Pythagorean Triple {5, 12, 13}.
- subtracted the area of the sector from the whole circle despite being only asked the area of the sector.

TABLE 1
MATHEMATICS 3204 ITEM ANALYSIS
SELECTED RESPONSE (PART I)[1]

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	A	37.6	39.4	10.0	12.9
2	A	19.4	23.3	7.9	49.1
3	C	3.8	2.9	77.8	15.5
4	C	5.2	16.7	60.8	17.0
5	D	11.7	18.6	21.0	48.6
6	B	2.1	90.5	5.1	2.2
7	D	7.1	13.2	8.5	70.8
8	B	4.5	86.5	2.8	6.2
9	C	4.5	26.1	60.1	9.1
10	D	14.6	18.3	20.2	46.7
11	D	10.2	45.8	6.7	37.3
12	B	9.1	70.5	14.2	6.2
13	B or D	19.0	29.8	16.5	34.2
14	C	23.8	16.2	38.9	20.9
15	D	12.2	12.2	19.7	55.9
16	D	21.8	8.2	11.4	58.6
17	D	3.0	1.8	14.1	80.9
18	D	7.0	35.6	7.5	49.8
19	A	54.2	38.1	4.6	3.1
20	D	12.7	2.6	23.9	60.8
21	B	9.3	62.2	9.6	18.9
22	A	48.4	45.4	4.4	1.6
23	A	31.2	47.2	12.0	9.6
24	D	3.0	38.0	10.5	48.2
25	A	49.9	5.0	35.6	9.5
26	D	20.2	22.1	18.3	39.2
27	D	11.6	26.3	12.8	49.1
28	B	11.5	56.3	23.0	8.9
29	C	8.1	12.5	69.7	9.7
30	B	4.3	85.0	9.5	1.3
31	D	18.1	13.0	15.8	52.6
32	D	6.0	8.3	11.2	74.4
33	C	12.9	19.5	50.6	16.3
34	C	4.9	1.6	86.6	6.9
35	C	6.6	12.0	68.4	12.7

[1] Note: Percentages may not add to 100% due to multiple responses, missing values, or rounding.

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
36	B	2.8	89.9	5.0	2.2
37	C	6.5	24.3	63.5	5.5
38	A	77.3	7.1	9.9	5.6
39	B	11.5	72.4	6.3	9.7
40	C	5.2	12.6	78.5	3.6
41	D	17.2	20.2	20.7	41.5
42	B	9.3	84.2	3.6	2.9
43	B	23.2	41.9	17.0	17.3
44	C	7.0	10.6	72.9	9.1
45	B	30.2	47.8	15.5	6.3
46	B	16.9	64.4	9.3	9.2
47	B	7.1	47.5	39.5	5.3
48	B	3.8	85.0	6.9	4.1
49	C	6.3	10.3	62.7	20.3
50	Dropped				

TABLE 2
MATHEMATICS 3204 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	3048	4	2.84
52	3048	4	1.92
53	3048	4	1.73
54	3048	4	2.27
55	3048	3	1.81
56	3048	4	2.05
57	3048	4	1.76
58	3048	4	2.43
59	3048	3	1.04
60	3048	4	2.59
61	3047	4	1.82
62	3047	4	1.46
63	3047	4	1.27