

Grading Standards Mathematics 3204 June 2007

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations.

Post-Marking Report

Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examinations and were held by the Chief Marker for recirculation throughout the marking period. These papers were corrected by the marking board again, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Each marker also made on-going notes regarding partial marks and scoring for their particular question. Whenever a non-common error occurred, it was scored by consensus of the board and made note of, for scoring consistency.

Throughout the marking process there were statistical analysis ran on item data to enhance reliability and consistency of marking.

Summary

Overall performance in the Math 3204 examination improved from June 2006 to June 2007, but was similar to performance on the 2004 and 2005 exams. Students experienced difficulty with questions 53, 61 and 63.

Part II – Constructed Response – Total Value: 50%

Value

- 4% 51. Algebraically determine the **EXACT** roots in simplest form for

$$(3x - 2)(x + 4) = -13.$$

Answer:

$$(3x - 2)(x + 4) = -13$$

$$3x^2 + 12x - 2x - 8 = -13 \quad (0.5 \text{ mark})$$

$$3x^2 + 10x + 5 = 0 \quad (0.5 \text{ mark})$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(5)}}{2(3)} \quad (1 \text{ mark})$$

$$x = \frac{-10 \pm \sqrt{100 - 60}}{6} \quad (0.5 \text{ mark})$$

$$x = \frac{-10 \pm \sqrt{40}}{6} \quad (0.5 \text{ mark})$$

$$x = \frac{-10 \pm 2\sqrt{10}}{6} \quad (0.5 \text{ mark})$$

$$x = \frac{-5 \pm \sqrt{10}}{3} \quad (0.5 \text{ mark})$$

Common Errors

Students:

- did not correctly expand $(3x - 2)(x + 4)$
- did not know the quadratic formula
- did not change the sign when transposing -13
- made calculation errors in the discriminant (e.g., $100 - 60 = 60$ or $100 + 60 = 160$)
- simplified the radical incorrectly
- did not reduce to lowest terms or did it incorrectly (e.g., $\frac{-10 \pm 2\sqrt{10}}{6} = \frac{-5 \pm 2\sqrt{10}}{3}$)

Value

4%

52. A baseball is hit and follows a parabolic path described by the function $h(t) = -3t^2 + 12t + 1$, where t is time in seconds after the ball is hit and $h(t)$ is the height of the ball above the ground in metres. Algebraically determine the maximum height reached by the ball and the time it takes the ball to reach its maximum height.

Answer 1:

$$t = -\frac{b}{2a} = -\frac{-12}{2(-3)} = 2 \quad (1.5 \text{ marks})$$

$$h(2) = -3(2)^2 + 12(2) + 1 \quad (1 \text{ mark})$$

$$h(2) = 13 \quad (0.5 \text{ mark})$$

$$\therefore \text{Max. height is 13m.} \quad (0.5 \text{ mark})$$

$$\text{At } t = 2 \text{ seconds.} \quad (0.5 \text{ mark})$$

Answer 2:

$$h - 1 - 12 = -3(t^2 - 4t + 4) \quad (2 \text{ marks})$$

$$-\frac{1}{3}(h - 13) = (t - 2)^2 \quad (1 \text{ mark})$$

$$\therefore \text{Vertex } (2, 13)$$

$$\text{Max Height is 13 m.} \quad (0.5 \text{ mark})$$

$$\text{When } t = 2 \text{ seconds} \quad (0.5 \text{ mark})$$

Commentary on Response

Students who attempted this question generally solved it by using either method above.

Common Errors

Students:

- used the quadratic formula to find the time
- did not know the formula for finding the x-coordinate of the vertex $-\frac{b}{2a}$
- made calculation errors such as: $-3(2)^2 = 36$ or $\frac{-12}{2(-3)} = 18$
- did not change the sign of $4t$ when factoring out -3
- stated the maximum height as 2 metres at 13 seconds

Value

4%

53.

A rectangular swimming pool has length 30 m and width 20 m. There is a deck of uniform width surrounding the pool. The area of the pool is the same as the area of the deck. Write a quadratic equation to model this situation and use it to determine the width of the deck.

Answer:

$$(20 + 2x)(30 + 2x) = 1200 \quad (1 \text{ mark})$$

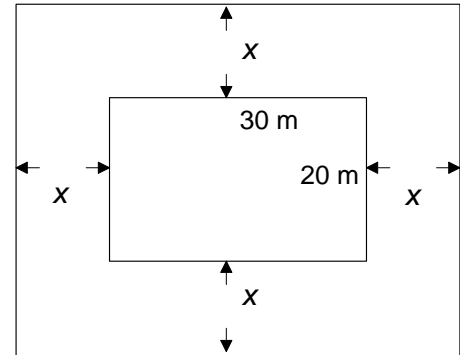
$$4x^2 + 100x - 600 = 0 \quad (1 \text{ mark})$$

$$x^2 + 25x - 150 = 0$$

$$(x - 5)(x + 30) = 0 \quad (1 \text{ mark})$$

$$x = 5, x = -30 \quad (0.5 \text{ mark})$$

$$\therefore \text{Width is } 5\text{m.} \quad (0.5 \text{ mark})$$



Commentary on Response

Students had difficulty writing the equation to model this situation. Most students used the quadratic formula to find the roots instead of factoring. Several students reported the width of the deck to be 30, by substituting 5 into the expression for width. (No marks were deducted for this error)

Common Errors

Students:

- let the total area of the pool + deck = 600 or 0
- multiplied incorrectly (e.g., $30 \times 20 = 60$)
- changed $-b$ to b in the quadratic formula to force the answer to become positive
- gave only one solution when using the quadratic formula
- calculated an incorrect value for the discriminant
- found the x -coordinate of the vertex instead of the roots

Value

- 4% 54. The height of a pebble fired by a sling shot is given by $h(t) = 25t - 4.9t^2$, where $h(t)$ is height in metres and t is time in seconds after the pebble leaves the sling shot. Determine an approximation for the instantaneous rate of change in the height of the pebble at 3 seconds and describe how the height of the pebble is changing at that instant.

Answer:

$$RoC = \frac{h(3.1) - h(2.9)}{3.1 - 2.9} \quad (1.5 \text{ marks})$$

$$RoC = \frac{30.411 - 31.291}{0.2} \quad (1.5 \text{ mark})$$

$$RoC = \frac{-0.88}{0.2}$$

$$RoC = -4.4 \text{ m/s} \quad (0.5 \text{ mark})$$

\therefore height is decreasing at 4.4 m/s (0.5 mark)

Commentary on Response

Students did not describe how the height of the pebble was changing.

Common Errors

Students:

- had difficulty calculating $h(3.1)$ and $h(2.9)$
- found the average rate of change
- calculated $h(3.0)$ only
- used an incorrect slope formula (e.g., $\frac{x_2 - x_1}{y_2 - y_1}$)
- used an inadequate number of decimal places

Value

3% 55. Solve for x : $(\sqrt[3]{16^{2x}}) = 64^{\frac{1}{2}}$.

Answer 1:

$$16^{\frac{2x}{3}} = 64^{\frac{1}{2}} \quad (0.5 \text{ mark})$$

$$2^{\frac{8x}{3}} = (2^6)^{\frac{1}{2}} \quad (1 \text{ mark})$$

$$2^{\frac{8x}{3}} = 2^3 \quad (0.5 \text{ mark})$$

$$\frac{8x}{3} = 3 \quad (0.5 \text{ mark})$$

$$8x = 9$$

$$x = \frac{9}{8} \quad (0.5 \text{ mark})$$

Answer 2:

$$16^{\frac{2x}{3}} = 64^{\frac{1}{2}} \quad (0.5 \text{ mark})$$

$$4^{\frac{4x}{3}} = (4^3)^{\frac{1}{2}} \quad (1 \text{ mark})$$

$$4^{\frac{4x}{3}} = 4^{\frac{3}{2}} \quad (0.5 \text{ mark})$$

$$\frac{4x}{3} = \frac{3}{2} \quad (0.5 \text{ mark})$$

$$8x = 9$$

$$x = \frac{9}{8} \quad (0.5 \text{ mark})$$

Commentary on Response

Some students used logarithms to solve this equation.

Common Errors

Students:

- incorrectly changed 64 to 16^2 or 32^2
- wrote $\sqrt[3]{16^{2x}}$ as $16^{\frac{3x}{2}}$
- multiplied incorrectly (e.g., $(2^4)^{\frac{2x}{3}} = 2^{\frac{2x}{12}}$ or $2^{\frac{8x}{12}}$)
- multiplied incorrectly (e.g., $\frac{4x}{3} = \frac{3}{2}$ become $4x = 6$)

Value

4% 56. Solve for x : $\log_2(x-6) + \log_2(x) = 4$.

Answer:

$$\log_2(x^2 - 6x) = 4 \quad (1 \text{ mark})$$

$$2^4 = x^2 - 6x \quad (1 \text{ mark})$$

$$0 = x^2 - 6x - 16 \quad (0.5 \text{ mark})$$

$$0 = (x-8)(x+2) \quad (0.5 \text{ mark})$$

$$x = 8 \text{ or } x = -2 \quad (0.5 \text{ mark})$$

$$x = 8 \quad x \neq -2 \quad (0.5 \text{ mark})$$

Commentary on Response

Students had difficulty converting from logarithm form to exponential form.

Common Errors

Students:

- did not reject the negative root
- ignored the logs, resulting in $x-6+x=4$ or $x(x-6)=4$
- used the quadratic formula to solve the equation and made errors in doing so
- simplified the equation into log form correctly but then ignored the \log_2 and set up the quadratic as $x^2 - 6x - 4 = 0$

Value

- 4% 57. Tom bought a car for \$14 000. The value of the car depreciates at a rate of 21% every two years. Write an equation that models the value of the car over time and use it to determine the approximate value of the car after five years.

Answer:

$$y = 14000(0.79)^{\frac{x}{2}} \quad (2.5 \text{ marks })$$

$$y = 14000(0.79)^{\frac{5}{2}} \quad (0.5 \text{ mark })$$

$$y = 7765.97 \quad (1 \text{ mark })$$

Commentary on Response

Most students knew the base equation $y = a(b)^c$ but substituted the values incorrectly.

Common Errors

Students:

- made computational errors
- multiplied the base by the initial amount
- used $\frac{2}{x}$ instead of $\frac{x}{2}$
- used 0.21 or 1.21 as the base instead of 0.79
- substituted the value 5 for y and solved using logs

Value

4%

58.

A laboratory assistant decided to observe the reproductive properties of a new strain of bacteria. The assistant started observing a population of 300 bacteria and noted that the bacteria population doubled every 5 minutes. Write a function to model this situation and use it to determine the time it will take for the population to reach 18 000 bacteria.

Answer:

$$y = 300(2)^{\frac{x}{5}} \quad (1.5 \text{ mark})$$

$$\frac{18000}{300} = \frac{300(2)^{\frac{x}{5}}}{300} \quad (0.5 \text{ mark}) \text{ * setting equal to 18000}$$

$$60 = 2^{\frac{x}{5}} \quad (0.5 \text{ mark})$$

$$\log 60 = \log 2^{\frac{x}{5}} \quad (0.5 \text{ mark})$$

$$\frac{\log 60}{\log 2} = \frac{x \log 2}{5 \log 2} \quad (0.5 \text{ mark})$$

$$5.9 = \frac{x}{5}$$

$$x = 5(5.9) = 29.5 \quad (0.5 \text{ mark})$$

Commentary on Response

Some students used common bases (e.g., $2^{5.91} = 2^{\frac{x}{5}}$) instead of logarithms.

Common Errors

Students:

- multiplied the initial value by the base $300(2)^{\frac{x}{5}} = 600^{\frac{x}{5}}$
- got to $60 = (2)^{\frac{x}{5}}$ and could not finish
- got to $60 = (2)^{\frac{x}{5}}$ and divided each side by 2 and then continued with $30 = \frac{x}{5}$
- subtracted 300 from both sides to get $17700 = 2^{\frac{x}{5}}$.

Value

- 4% 59. Find the centre of the circle defined by $x^2 + y^2 + 10x - 16y - 11 = 0$ and prove that it lies on the line $3x - 4y + 47 = 0$.

Answer:

$$(x^2 + 10x) + (y^2 - 16y) = 11 \quad (0.5 \text{ mark})$$

$$(x^2 + 10x + 25) + (y^2 - 16y + 64) = 11 + 25 + 64 \quad (1 \text{ mark})$$

$$(x + 5)^2 + (y - 8)^2 = 100 \quad (1 \text{ mark})$$

$$\text{Centre } (-5, 8) \quad (0.5 \text{ mark})$$

$$3(-5) - 4(8) + 47 \stackrel{?}{=} 0 \quad (0.5 \text{ mark})$$

$$-15 - 32 + 47 \stackrel{?}{=} 0$$

$$0 = 0 \quad (0.5 \text{ mark})$$

\therefore It lies on the line

Commentary on Response

The question did not specify writing the transformational form of the function first to find the coordinates of the centre. Full marks were given for stating the coordinates of the centre and substituting it into the equation of the line to verify. No marks were deducted for incorrectly stating the radius.

Common Errors

Students:

- transposed incorrectly (e.g., $(x^2 + 10x) + (y^2 - 16y) = -11$)
- completed the square incorrectly
- factored each trinomial incorrectly (e.g., $(y^2 - 64y + 64) = (y - 4)^2$)
- found the centre but did not verify that it lies on the line
- failed to make a final statement or state that $0 = 0$

Value

- 3% 60. Write $4x^2 + y^2 - 8x + 4y - 8 = 0$ in transformational form and state the coordinates of the centre.

Answer:

$$4(x^2 - 2x) + (y^2 + 4y) - 8 = 0 \quad (0.5 \text{ mark})$$

$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4 \quad (1 \text{ mark})$$

$$4(x-1)^2 + (y+2)^2 = 16 \quad (0.5 \text{ mark})$$

$$\left[\frac{1}{2}(x-1)\right]^2 + \left[\frac{1}{4}(y+2)\right]^2 = 1 \quad (0.5 \text{ mark})$$

$$\text{Centre } (1, -2) \quad (0.5 \text{ mark})$$

Common Errors

Students:

- incorrectly factored the 4 from $4x^2 - 8x$
- did not balance the equation properly after completing the square
- did not complete the square properly
- did not factor the trinomial properly
- did not get the square root of 4 and 16 as the final step in writing in transformational form
- did not change the signs when stating the centre
- did not state the coordinates of the centre

Value

- 4% 61. Given the square ABCD with vertices A(-1, -1), B(3, 2), C(6, -2) and D(2, -5), prove that the diagonals are congruent..

Answer:

$$d_{BD} = \sqrt{[2 - (-5)]^2 + (3 - 2)^2} \quad (1 \text{ mark})$$

$$= \sqrt{49 + 1}$$

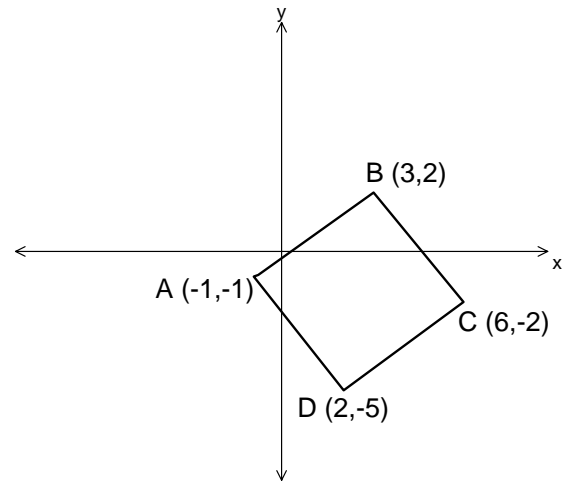
$$= \sqrt{50} \quad (1 \text{ mark})$$

$$d_{AC} = \sqrt{(-1 - 6)^2 + [-1 - (-2)]^2} \quad (1 \text{ mark})$$

$$= \sqrt{49 + 1}$$

$$= \sqrt{50} \quad (1 \text{ mark})$$

Both Equal $\sqrt{50}$



Commentary on Response

Students did not identify the diagonals of the square.

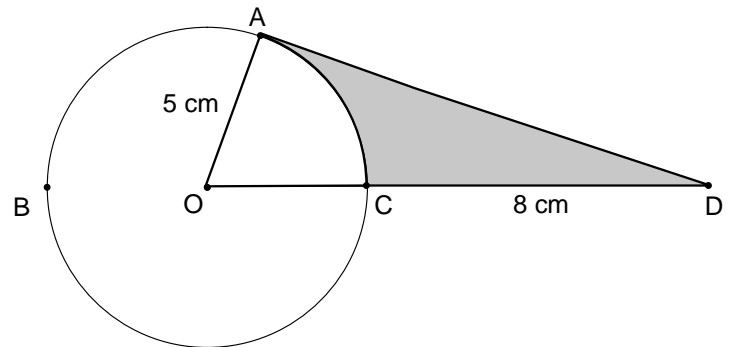
Common Errors

Students:

- found the lengths of the sides of the square
- found the slopes of the sides of the square
- found the slopes of the diagonals
- found the midpoints of the diagonals

Value

- 4% 62. The circle shown has centre O and radius 5 cm. If \overline{AD} is tangent to the circle, $\widehat{ABC} = 293^\circ$, and $\overline{CD} = 8$ cm, determine the area of the shaded region.



Answer:

$$\text{Central Angle} = 67^\circ \quad (0.5 \text{ mark})$$

$$A_{\text{sect}} = \frac{67^\circ}{360^\circ} (\pi)(5)^2 = 14.6 \quad (1.5 \text{ marks})$$

$$A_{\Delta} = \frac{1}{2} (5)(13)(\sin 67^\circ) = 30 \quad (1.5 \text{ marks})$$

or

$$AD = 12,$$

$$A_{\Delta} = \frac{1}{2} (12)(5)(\sin 90^\circ) = 30 \quad \text{or} \quad A_{\Delta} = \frac{1}{2} (5)(12)$$

$$\therefore \text{Area Shaded} = \text{Area } \Delta - \text{Area Sector}$$

$$= 30 - 14.6$$

$$= 15.4 \text{ cm}^2 \quad (0.5 \text{ mark})$$

Common Errors

Students:

- used 293° as the central angle instead of 67°
- used 13 as the height of the triangle instead of 12
- used incorrect values in calculating the area of the triangle
- used the Pythagorean theorem incorrectly
- calculated the central angle to be $\frac{1}{2}(67^\circ) = 33.5^\circ$
- omitted $\frac{1}{2}$ in calculating the area of the triangle

Value

- 4% 63. \overline{AB} is a chord with midpoint M (6, 5) in the circle having centre O (5, 8) as shown. Determine the equation of the line containing chord \overline{AB} .

Answer:

$$M_{OM} = \frac{8-5}{5-6} = \frac{3}{-1} = \frac{-3}{1} \quad (1 \text{ mark})$$

$$\therefore M_{AB} = \frac{1}{3} \quad (1 \text{ mark})$$

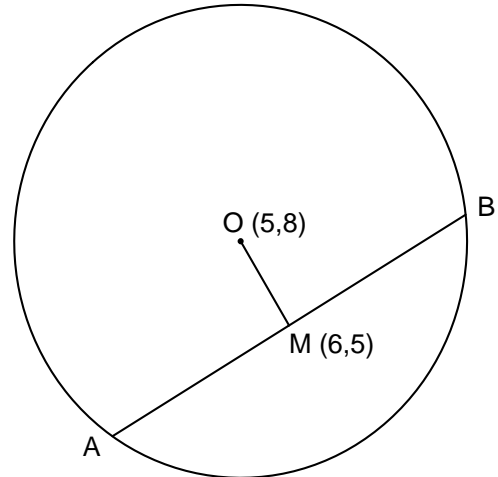
$$\therefore y = \frac{1}{3}x + b$$

$$5 = \frac{1}{3}(6) + b \quad (0.5 \text{ mark})$$

$$5 = 2 + b$$

$$b = 3 \quad (1 \text{ mark})$$

$$\therefore \boxed{y = \frac{1}{3}x + 3} \quad (0.5 \text{ mark})$$



Commentary on Response

Many students did not attempt this question.

Common Errors

Students:

- applied random formulae (e.g., midpoint, distance and Pythagorean theorem)
- found the equation of the circle
- did not use the negative reciprocal
- found the equation of \overline{OM}
- found the equation of a parabola using the two points
- used the centre when determining the y-intercept of \overline{AB}

TABLE 1
MATHEMATICS 3204 ITEM ANALYSIS
SELECTED RESPONSE (PART I)

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	D	10.8	23.8	15.3	50.0
2	B	4.9	74.5	5.5	14.9
3	D	12.0	21.0	26.5	40.5
4	A	64.2	5.0	19.1	11.6
5	A	45.5	15.1	13.9	25.1
6	C	2.8	1.0	93.2	2.9
7	A	90.6	0.9	0.4	8.0
8	A	87.3	7.4	4.2	1.2
9	B	4.5	68.3	16.1	11.0
10	B	6.9	79.3	12.7	1.1
11	C	12.8	2.5	72.2	12.4
12	D	11.9	22.7	18.6	46.5
13	C	15.4	12.0	57.1	15.0
14	D	11.4	21.2	14.7	52.2
15	A	86.8	6.9	3.8	2.5
16	D	1.2	1.9	2.0	94.9
17	A	58.3	17.9	2.9	20.6
18	D	3.5	25.9	4.7	65.8
19	A	80.7	1.0	18.1	0.2
20	B	16.6	56.7	16.8	9.4
21	C	5.2	15.1	65.0	14.4
22	D	7.9	50.4	8.5	33.1
23	C	2.0	3.5	80.6	14.0
24	C	20.7	6.2	61.9	10.9
25	B	21.2	67.7	2.6	8.4
26	A	71.7	9.4	6.2	12.6
27	A	46.4	19.9	14.6	18.8
28	A	63.4	15.8	15.2	5.2

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
29	C	2.2	15.5	71.4	10.8
30	D	16.6	5.6	27.4	50.1
31	D	3.4	21.4	11.3	63.8
32	B	6.2	69.6	11.0	13.0
33	C	12.2	21.1	52.7	13.7
34	A	88.6	5.6	4.7	1.1
35	A	80.2	7.3	6.0	6.4
36	C	1.5	1.0	81.6	15.9
37	B	16.2	69.3	6.9	7.4
38	A	84.1	5.6	8.1	2.1
39	D	7.4	25.7	14.8	51.7
40	B	2.0	74.8	16.1	7.0
41	D	10.0	25.0	11.6	53.3
42	D	2.4	34.6	7.2	55.8
43	C	10.9	28.7	30.9	29.0
44	B	7.8	64.2	3.8	23.9
45	C	30.1	17.3	41.6	10.8
46	A	68.0	16.6	11.1	3.9
47	C	4.5	24.1	62.4	8.5
48	B	6.4	70.0	12.3	10.9
49	B	4.8	69.9	9.2	15.6
50	C	41.4	6.1	47.5	4.4

NOTE: Percentages may not add to 100% due to multiple responses or missing values.

TABLE 2
MATHEMATICS 3204 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students	Value	Average
51	3264	4	3.0
52	3264	4	2.7
53	3264	4	1.6
54	3264	4	2.8
55	3264	3	2.0
56	3264	4	2.2
57	3264	4	3.3
58	3264	4	3.0
59	3264	4	2.7
60	3264	3	1.7
61	3264	4	1.8
62	3264	4	2.3
63	3264	4	0.9