Mathematics 3204
Grading Standards
June 2008

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations. A change in value for quadratics and circles was implemented.

Post-Marking Report

a) Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examinations and were held by the Chief Marker for recirculation throughout the marking period. These papers were corrected by the marking board again, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Each marker also made on-going notes regarding partial marks and scoring for their particular question. Whenever a non-common error occurred, it was scored by consensus of the board and made note of, for scoring consistency.

b) Summary

Overall performance in the Math 3204 examination decreased from June 2007 to June 2008, but was similar to performance on the 2006 exam. Students experienced difficulty with questions 53 and 61.

c) Commentary on Responses:

Part II – Constructed Response – Total Value: 50%

Items #53 and #61:

These items were not attempted by many students, and subsequently students scored lower. In item 53, it was decided that giving the vertex may have made this a more appropriate question for Math 3204 students. In item 61, a diagram may have been helpful. These questions were marked on all papers but were not used in determining a student's mark if it was to the student's detriment.

Value

4% 51. Algebraically determine the **EXACT** roots in simplest form for $x = \frac{-13}{x-2}$.

Answer:

$$x = \frac{-13}{x-2}$$

$x(x-2) = -13$ **(0.5 marks)**

$x^2 - 2x + 13 = 0$ **(0.5 marks)**

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(13)}}{2(1)}$ **(1 mark)**

$x = \frac{2 \pm \sqrt{4-52}}{2}$ **(0.5 marks)**

$x = \frac{2 \pm \sqrt{-48}}{2}$ **(0.5 marks)**

$x = \frac{2 \pm 4i\sqrt{3}}{2}$ **(0.5 marks)**

$x = 1 \pm 2i\sqrt{3}$ **(0.5 marks)**

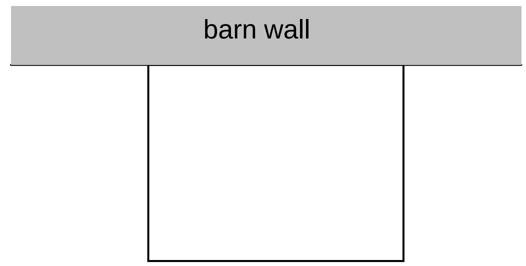
Commentary on Response
Many students scored either a value of zero or a value of 3 or 4. Often students did not recognize the necessity of multiplying both sides by (x–2) (using cross multiplication), or cross multiplied incorrectly. The majority of responses used the quadratic formula to find the roots. Those who used completing the square instead did so incorrectly, or didn't complete the solution.

Common Errors
Students:

- failed to simplify (left answer as $\frac{2 \pm 4i\sqrt{3}}{2}$).
- simplified incorrectly ($\pm 2i\sqrt{3}$).
- did not include brackets under the root when substituting for *b* and as a result $-2^2 = -4$, resulting in a negative discriminant.
- simplified the radical incorrectly (e.g. $\sqrt{-48} = 2i\sqrt{12}$).
- produced the incorrect quadratic function $x^2 - 2x - 13 = 0$ and often wrote this as an expression instead of an equation.
- rejected the negative answer $(1 - 2i\sqrt{3})$.

Value

4 52. A farmer is constructing a pig pen and is using his barn wall as one side of the pen. If he has 32 m of fencing and wants to use it all, write the quadratic function that models the area of the pig pen, and use it to determine the maximum area of the pen.



Answer 1:

$2w + l = 32$	**(0.5 marks)**
$l = 32 - 2w$	**(0.5 marks)**
$A = w(32 - 2w)$	**(0.5 marks)**
$A = -2w^2 + 32w$	**(0.5 marks)**
$w = \frac{-32}{2(-2)}$	**(0.5 marks)**
$w = 8$	**(0.5 marks)**
$A = -2(8)^2 + 32(8)$	**(0.5 marks)**
$A = 128\,\text{m}^2$	**(0.5 marks)**

Answer 2:

$2w + l = 32$	**(0.5 marks)**
$l = 32 - 2w$	**(0.5 marks)**
$A = w(32 - 2w)$	**(0.5 marks)**
$A = -2w^2 + 32w$	**(0.5 marks)**
$w = \frac{-32}{2(-2)}$	**(0.5 marks)**
$w = 8$	**(0.5 marks)**
$l = 32 - 2(8)$	
$l = 16\,\text{m}$	**(0.5 marks)**
$A = (8\,\text{m})(16\,\text{m})$	
$A = 128\,\text{m}^2$	**(0.5 marks)**

Answer 3:

$2w + l = 32$	**(0.5 marks)**
$l = 32 - 2w$	**(0.5 marks)**
$A = w(32 - 2w)$	**(0.5 marks)**
$A = -2w^2 + 32w$	**(0.5 marks)**
$A = -2(w^2 - 16w)$	**(0.5 marks)**
$A - 128 = -2(w^2 - 16w + 64)$	**(1 mark)**
$-\frac{1}{2}(A - 128) = (w - 8)^2$	
The maximum area is $128\,\text{m}^2$.	**(0.5 marks)**

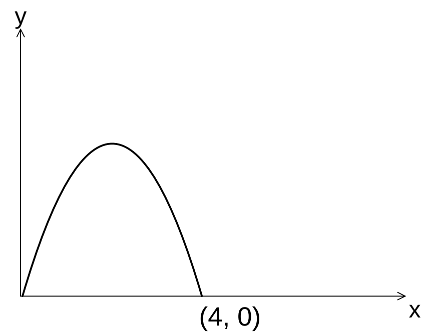
Commentary on Response
Most students attempted the question. Students who remembered the pattern, without the algebra, received some point value for the question.

Common Errors
Students:

- generated the correct equation but then used the quadratic formula to solve for the length and width.
- divided 32 by 3 to get 10.6 (rounded incorrectly) and multiplied 10.6 by 10.6 to get the area.
- generated an incorrect quadratic equation by using one of the following: $A = w(32 - w)$ or $A = w(16 - w)$ or $A = 2w(32 - 2w)$ and then used it to calculate the area.
- completed the question correctly and gave a "dimensions" answer rather than calculating the area as required.
- set the area equal to zero.

Value

4 53. A golf ball is hit from the ground and lands on the green after 4 seconds. If the golf ball reaches a maximum height of 20 m, algebraically determine the quadratic function representing its path, and use it to determine the approximate height of the ball at 3 seconds.



Answer:

Vertex (2,20) **1 mark**

$$\frac{1}{a}(y-20)=(x-2)^2$$

$$\frac{1}{a}(0-20)=(0-2)^2$$

$$\frac{1}{a}=\frac{-1}{5}$$ **1 mark**

$$\frac{-1}{5}(y-20)=(x-2)^2$$ **1 mark**

$$y=-5(3-2)^2+20$$

$$y=15\text{ m}$$ **1 mark**

Commentary on Response

Many students did not attempt this question. Others only identified the vertex.

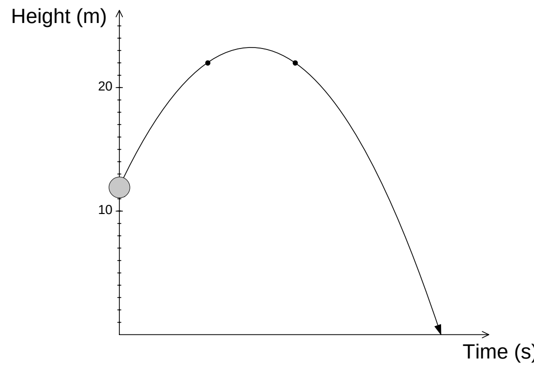
Common Errors

Students:

- wrote the quadratic function without any algebraic development.
- substituted $(4,0)$ for the vertex, resulting in a height of 10 *m*.
- identified the vertex correctly but could proceed no further.

Value

4 54. A cannonball is shot into the air as shown below. The height of the ball above the ground, in metres, t seconds after being shot is approximated by $h(t) = -5t^2 + 15t + 12$. Algebraically determine the times when the ball is at a height of 22 m.



Answer:

$-5t^2 + 15t + 12 = 22$	**(1 mark)**
$-5t^2 + 15t - 10 = 0$	**(0.5 marks)**
$-5(t^2 - 3t + 2) = 0$	**(0.5 marks)**
$-5(t-1)(t-2) = 0$	**(1 mark)**
$t = 1$ and $t = 2$	**(1 mark)**

Commentary on Response

Many students used the quadratic formula rather than factoring. Others used guess and check without any algebra to support the solutions.

Common Errors

Students:

- solved the equation $-5t^2 + 15t + 12 = 0$.
- evaluated $h(22) = -5(22)^2 + 15(22) + 12$.
- found the vertex.
- applied the quadratic formula to the correct function, but calculated the discriminant incorrectly (e.g. $\sqrt{15^2 - 4(-5)(-10)}$ became $\sqrt{225 + 200}$).
- solved the equation $22t = -5t^2 + 15t + 12$.

Value

4 55. The motion of a ball thrown upward from the ground is described by $h(t) = -4.9t^2 + 11t$, where h is the height of the ball in metres and t is the time in seconds. Algebraically determine the approximate instantaneous rate of change in the height of the ball at 2 seconds, and describe how the height of the ball is changing at that instant.

Answer:

$$\begin{aligned} IRoC &= \frac{h(2.1) - h(1.9)}{2.1 - 1.9} \\ &= \frac{1.491 - 3.211}{0.2} \qquad \textbf{2 marks} \\ &= \frac{-1.72}{0.2} \\ &= -8.6 \text{ m/s} \qquad \textbf{1 mark} \end{aligned}$$

The ball is falling at a rate of 8.6 m/s. 1 mark

Commentary on Response

Many students did the correct calculation but did not describe how the height of the ball was changing.

Common Errors

Students:

- found only $h(2)$.
- used an incorrect slope formula (e.g. $\frac{h(1.9) - h(2.1)}{2.1 - 1.9}$ or $\frac{2.1 - 1.9}{h(2.1) - h(1.9)}$).
- made errors in calculations and found a positive instantaneous rate of change, but still described the ball's height as *decreasing* at that instant.
- used the quadratic formula to find the roots.
- found the vertex and the maximum height.

Value

4 56. Algebraically solve for x: $32\left(\frac{1}{4}\right)^{x+2} = 8^{\frac{4}{3}}$.

Answer 1:

$2^5\left(2^{-2}\right)^{x+2} = \left(2^3\right)^{\frac{4}{3}}$	**(1.5 marks)**
$2^5\left(2^{-2x-4}\right) = 2^4$	**(1 mark)**
$2^{-2x+1} = 2^4$	**(0.5 marks)**
$-2x + 1 = 4$	**(0.5 marks)**
$x = -\frac{3}{2}$	**(0.5 marks)**

Answer 2:

$32\left(2^{-2}\right)^{x+2} = 16$	**(1.5 marks)**
$2^{-2x-4} = \frac{1}{2}$	**(1 mark)**
$2^{-2x-4} = 2^{-1}$	**(0.5 marks)**
$-2x - 4 = -1$	**(0.5 marks)**
$x = -\frac{3}{2}$	**(0.5 marks)**

Commentary on Response

Most students attempted to answer this question and received partial marks. Many of the problems involved order of operations, the distributive property, and incorrectly applying the exponent rules.

Common Errors

Students:

- multiplied 32 by $\frac{1}{4}$ to get a base of 8.
- multiplied 32 by $\frac{1}{4}$ to get a base of 128.
- multiplied instead of adding exponents when multiplying expressions with the same base.
  $2^5\left(2^{-2x-4}\right) = 2^{-10x-20}$
- incorrectly multiplied exponents.
  $\left(2^{-2}\right)^{x+2} = 2^{-2x+2}$
- combined 2^5 and 2^{-2} to get 2^3.
- some students attempted to use logs to solve the problem, but few were successful.

Value

4 57. Algebraically solve for x: $\log_7 4 + \log_7 (x+3) = 2\log_7 x$.

Answer:

$\log_7 4(x+3) = \log_7 x^2$	**(1 mark)**
$4x + 12 = x^2$	**(1 mark)**
$x^2 - 4x - 12 = 0$	**(0.5 marks)**
$(x-6)(x+2) = 0$	**(0.5 marks)**
$x = 6$, $x = -2$ ***(reject)***	**(1 mark)**

Commentary on Response

Many students solved the quadratic equation using the quadratic formula rather than factoring. The negative answer was often not rejected.

Common Errors

Students:

- failed to reject $x = -2$.
- solved $x^2 + 4x + 12 = 0$ or $4x + 12 = 2x$.
- combined the logs by dividing the arguments.
- factored the quadratic incorrectly: $(x+6)(x-2)$.
- made mistakes with the discriminant when using the quadratic formula.

Value

4 58. A toxin is accidentally released into a pond, affecting the trout population. Initially there were 1500 trout in the pond, but after 5 months this number was reduced to 1050. After another 5 months, there were 735 trout. If this pattern continues, algebraically determine the exponential equation that models this situation, and use it to determine the trout population one year after the toxin was released.

Answer:

common ratio = 0.7	**(1 mark)**
$y = 1500(0.7)^{\frac{x}{5}}$	**(1.5 marks)**
$y = 1500(0.7)^{\frac{12}{5}}$	**(0.5marks)**
$y = 637.27$	**(1 mark)**

Commentary on Response

Students experienced difficulty in finding the common ratio.

Common Errors

Students:

- used 0.3 as the common ratio (1 subtract 0.7).
- used 1.43 as the common ratio $(1500 \div 1050)$.
- sometimes failed to change 1 year to 12 months.
- reversed the *x* and *c* values in the exponential equation.
- set up a table and found the common difference instead of the common ratio.

Value

4 59. A student invests $2 000 with a bank that promises to double the investment in 7 years. Write an exponential equation that models this situation and use it to determine when the investment will be worth $10 000.

Answer:

$y = 2000(2)^{\frac{x}{7}}$	**(1.5 marks)**
$10000 = 2000(2)^{\frac{x}{7}}$	**(0.5 marks)**
$5 = (2)^{\frac{x}{7}}$	**(0.5 marks)**
$\log(5) = \frac{x}{7}\log(2)$	**(0.5 marks)**
$x = 7\left(\frac{\log 5}{\log 2}\right)$	**(0.5 marks)**
$x = 16.25$	**(0.5 marks)**

Commentary on Response

Some students ended the solution with $5 = 2^{\frac{x}{7}}$ and did not take the log of both sides.

Common Errors

Students:

- incorrectly divided 10000 by 2000 to get 5000.
- reversed the *x* and *c* values in the exponential equation.
- incorrectly calculated $\frac{\log 5}{\log 2}$.
- used incorrect common ratios ($\frac{1}{2}$, 1.2, 1.02, 1.5, 1.07).
- incorrectly substituted values into the exponential equation.
- multiplied *a* and *b* resulting in $10000 = 4000^{\frac{x}{7}}$.
- subtracted 2000 from 10000 instead of dividing.

Value

3 60. Write $4x^2 + y^2 - 8x - 12 = 0$ in transformational form, and state the coordinates of the centre and the length of the major axis.

Answer:

$4(x^2 - 2x + 1) + y^2 = 12 + 4$ **(1 mark)**

$4(x-1)^2 + y^2 = 16$ **(0.5 marks)**

$\frac{1}{4}(x-1)^2 + \frac{1}{16}y^2 = 1$

$\left[\frac{1}{2}(x-1)\right]^2 + \left[\frac{1}{4}y\right]^2 = 1$ **(0.5 marks)**

centre (1,0) **(0.5 marks)**

major axis = 8 units **(0.5 marks)**

Commentary on Response

Many students encountered difficulty in completing the square.

Common Errors

Students:

- did not factor the 4 correctly.
- divided only the x^2 by 16 (not y^2) when making the equation equal 1.
- did not change the sign of the constant term when moving it to the right side of the equation $(4x^2 + y^2 - 8x = -12)$.
- made the computational error $(-1)^2 = -1$.
- stated the length of the major axis from standard form rather than transformational form (32 units).
- grouped the constant term with y^2 and completed the square.

Value

3 61. Determine the equation of the circle, in general form, with centre $(-3,4)$ and passing through the point (3,12).

Answer 1:

$(x+3)^2+(y-4)^2=r^2$ **(0.5 marks)**

$(3+3)^2+(12-4)^2=r^2$ **(0.5 marks)**

$36+64=r^2$

$100=r^2$ **(0.5 marks)**

$(x+3)^2+(y-4)^2=100$ **(0.5 marks)**

$x^2+6x+9+y^2-8y+16=100$ **(0.5 marks)**

$x^2+y^2+6x-8y-75=0$ **(0.5 marks)**

Answer 2:

$r=\sqrt{(-3-3)^2+(4-12)^2}$ **(0.5 marks)**

$r=\sqrt{100}$ **(0.5 marks)**

$r^2=100$ **(0.5 marks)**

$(x+3)^2+(y-4)^2=100$ **(0.5 marks)**

$x^2+6x+9+y^2-8y+16=100$ **(0.5 marks)**

$x^2+y^2+6x-8y-75=0$ **(0.5 marks)**

Commentary on Response

Many students did not attempt this question.

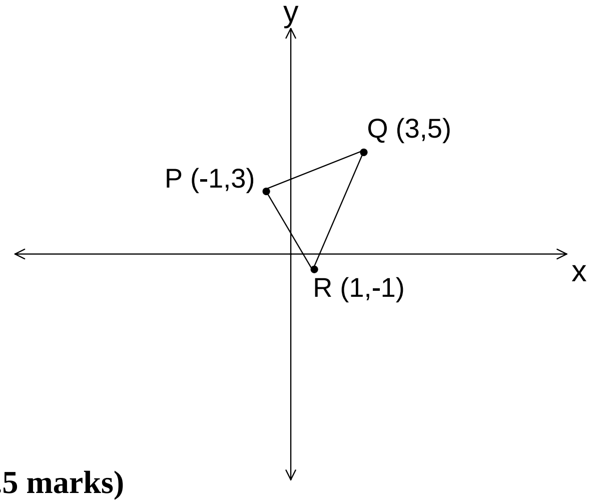
Common Errors

Students:

- generated a quadratic function.
- found the slope and equation of a line using the points given.
- did not convert the equation to general form.
- made basic computational mistakes in numeracy and polynomials (e.g. $3+3=9$; $(x-4)^2=x^2-8x-16$).
- attempted to balance the equation, as if completing the square, after multiplying polynomials.
- set up an equation for an ellipse using the point on the circle as the length of the major and minor axes.

Value

4 62. Using coordinate geometry, prove that ΔPQR is both a right and an isosceles triangle.



Answer:

$$m_{PQ} = \frac{5-3}{3+1}$$

$$m_{PQ} = \frac{1}{2}$$ **(0.5 marks)**

$$m_{PR} = \frac{-1-3}{1+1}$$
$$m_{PR} = -2$$ **(0.5 marks)**

Since slopes are negative reciprocals,
PQ is perpendicular to PR.
$\therefore \Delta PQR$ is a right triangle. **(1 mark)**

$$d_{PQ} = \sqrt{(3+1)^2 + (5-3)^2}$$
$$d_{PQ} = \sqrt{20}$$ **(0.5 marks)**

$$d_{PR} = \sqrt{(1+1)^2 + (-1-3)^2}$$
$$d_{PR} = \sqrt{20}$$ **(0.5 marks)**

$$d_{PQ} = d_{PR} = \sqrt{20}$$
$\therefore \Delta PQR$ is an isosceles triangle. **(1 mark)**

Commentary on Response

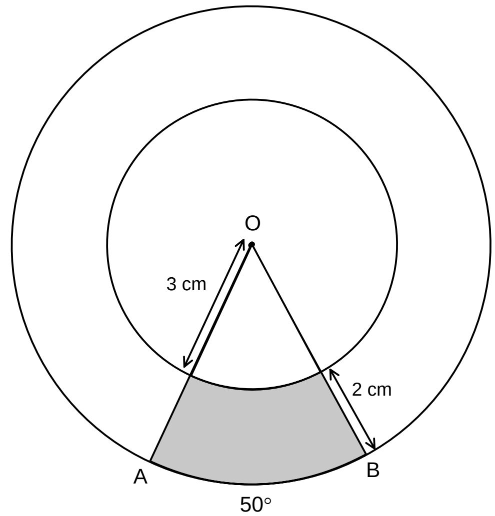
This question required a two part proof. Students often proved either an isosceles triangle or a right triangle, but not both. Many did not attempt a response. Some students approached the right triangle proof from a different, but correct, perspective. Once the lengths of the sides were calculated, they then applied the Pythagorean Theorem to the triangle. Others used trigonometric ratios to show that $\angle Q = \angle R = 45°$.

Common Errors
Students:

- proved either a right triangle or an isosceles triangle.
- did not include a statement of proof.
- did not know the definition of an isosceles triangle.
- did not relate perpendicular to negative reciprocal slopes.
- made errors within the distance and slope formulae.

Value

4 63. In the concentric circles with centre O shown, minor $\overarc{AB} = 50°$. Determine the area of the shaded region.



Answer:

$A_{shaded} = \frac{50}{360}(25\pi) - \frac{50}{360}(9\pi)$ **(2 marks)**

$A_{shaded} = 10.91 - 3.93$ **(1 mark)**

$A_{shaded} = 6.98\,\text{units}^2$ **(1 mark)**

Common Errors

Students:

- found the area of a segment.
- used $25°$ or $310°$ as the central angle.
- used incorrect area formula $(2\pi r)$.
- found the difference of the areas of two triangles.
- found only the area of the larger sector.

TABLE 1
MATHEMATICS 3204 ITEM ANALYSIS
SELECTED RESPONSE (PART I)[1]

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	D	3.4	15.3	9.0	72.2
2	B	21.2	64.5	11.2	2.9
3	D	6.4	17.6	20.6	55.3
4	C	24.5	20.0	55.0	0.3
5	A	92.4	3.4	3.1	1.1
6	C	10.6	10.0	75.7	3.7
7	D	9.2	4.5	5.4	80.9
8	B	6.6	69.1	18.4	5.5
9	B	2.5	69.3	10.1	17.8
10	D	9.2	9.5	36.2	44.7
11	C	5.8	22.0	54.1	17.7
12	A	68.1	17.2	4.2	10.4
13	A	63.6	7.9	12.3	16.0
14	B	5.3	83.8	6.8	4.0
15	A	33.5	40.1	8.0	18.0
16	B	4.1	57.2	34.8	3.9
17	C	11.1	15.9	58.6	14.0
18	B	6.7	89.8	1.4	2.0
19	A	70.1	10.9	5.2	13.5
20	B	19.1	48.4	13.7	18.4
21	B	40.4	48.4	9.2	1.8
22	B	8.7	26.5	23.3	41.3
23	C	6.2	6.2	74.0	13.5
24	D	8.2	17.6	11.6	62.4
25	A	55.8	8.0	29.2	6.9
26	C	20.3	4.2	71.5	3.9
27	C	16.9	3.2	66.2	13.6
28	C	28.1	7.8	58.7	5.0
29	A	71.0	14.6	8.9	5.3
30	D	8.2	15.0	11.7	65.0
31	D	11.3	39.6	12.9	36.1
32	B	15.0	43.8	32.3	8.7
33	D	8.0	16.5	17.5	57.4
34	D	9.8	36.8	5.9	47.3
35	D	15.9	6.7	27.6	49.6

[1] Note: Percentages may not add to 100% due to multiple responses, missing values, or rounding.

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
36	D	9.2	13.0	14.9	62.7
37	A	85.3	2.3	1.8	10.5
38	A	57.2	23.0	13.0	6.7
39	A	41.3	11.5	9.1	38.0
40	A	53.3	11.0	19.2	16.3
41	D	4.4	28.0	8.3	59.2
42	B	19.7	42.9	24.8	12.2
43	B	11.3	40.5	24.5	22.7
44	C	29.3	6.9	60.4	3.0
45	B	41.4	25.1	20.8	12.4
46	B	16.9	46.5	20.8	15.4
47	C	7.0	6.0	52.2	34.3
48	B	15.4	39.7	38.4	5.7
49	B	3.3	78.9	10.3	7.2
50	C	15.1	25.8	26.8	31.6

TABLE 2
MATHEMATICS 3204 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	2941	4	2.3
52	2941	4	2.1
53	2941	4	0.9
54	2941	4	2.4
55	2941	4	2.8
56	2941	4	1.9
57	2941	4	2.2
58	2941	4	1.9
59	2941	4	2.8
60	2941	3	1.4
61	2941	3	0.7
62	2941	4	1.9
63	2941	4	2.1