

**PART I**  
**Total Value: 50%**

**Answer all items. Shade the letter of the correct answer on the computer scorable answer sheet.**

1. Which describes the graph of  $3(y-1) = (x+4)^2$  compared to the graph of  $y = x^2$ ?

- \* (A) vertex  $(-4, 1)$  and vertical stretch factor of  $\frac{1}{3}$
- (B) vertex  $(-4, 1)$  and vertical stretch factor of 3
- (C) vertex  $(4, -1)$  and vertical stretch factor of  $\frac{1}{3}$
- (D) vertex  $(4, -1)$  and vertical stretch factor of 3

2. What is the range of the quadratic function  $y = -2(x-3)^2 + 4$ ?

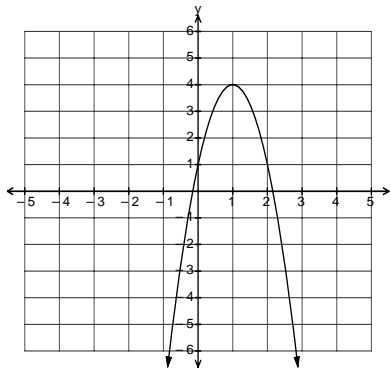
- (A)  $\{y \mid y \leq -4, y \in R\}$
- (B)  $\{y \mid y \geq -4, y \in R\}$
- \* (C)  $\{y \mid y \leq 4, y \in R\}$
- (D)  $\{y \mid y \geq 4, y \in R\}$

3. A quadratic function  $f(x)$  has vertex  $(2, -4)$  and opens downward. What is a possible value of the discriminant for the equation  $f(x) = 0$ ?

- \* (A)  $-3$
- (B)  $0$
- (C)  $3$
- (D)  $3i$

4. What mapping rule has been applied to  $y = x^2$  to result in the graph below?

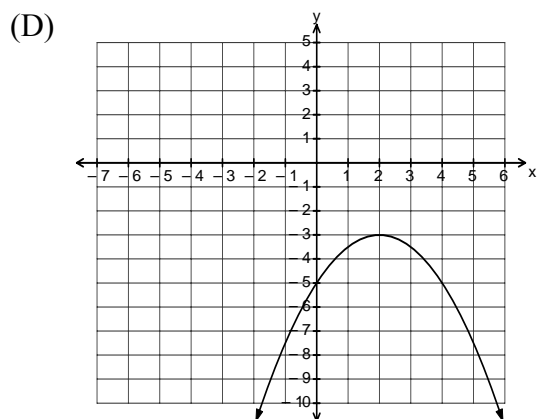
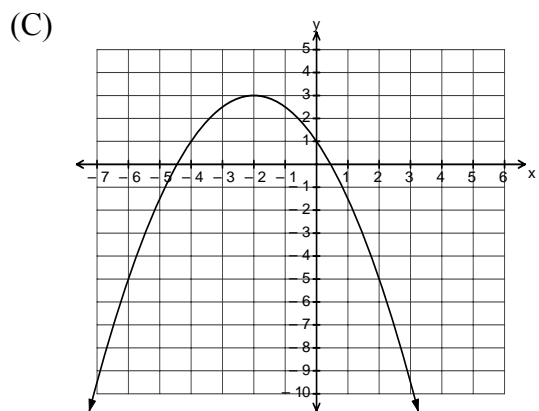
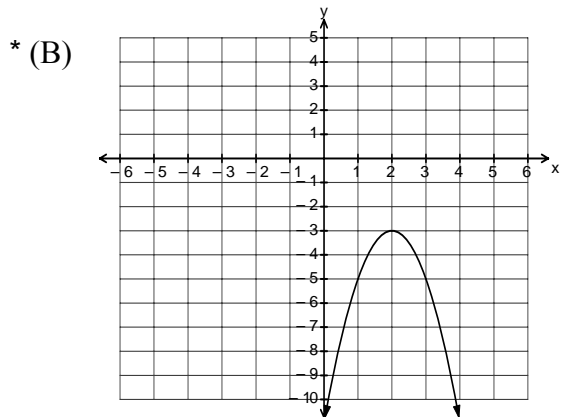
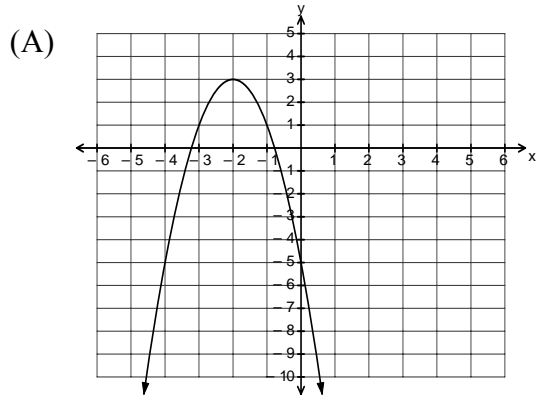
- (A)  $(x, y) \rightarrow (x-1, -\frac{1}{3}y-4)$
- (B)  $(x, y) \rightarrow (x-1, -3y-4)$
- (C)  $(x, y) \rightarrow (x+1, -\frac{1}{3}y+4)$
- \* (D)  $(x, y) \rightarrow (x+1, -3y+4)$



5. Which quadratic function has the greatest vertical stretch factor when compared to  $y = x^2$ ?

- \* (A)  $\frac{3}{4}(y+2) = (x-5)^2$
- (B)  $2(y+2) = (x-5)^2$
- (C)  $\frac{7}{2}(y+2) = (x-5)^2$
- (D)  $5(y+2) = (x-5)^2$

6. Which graph represents  $-\frac{1}{2}(y+3) = (x-2)^2$ ?



7. Which function has  $x = \frac{-k}{4p}$  as its axis of symmetry?

(A)  $y = \frac{1}{2} px^2 - kx + q$

(B)  $y = \frac{1}{2} px^2 + kx + q$

(C)  $y = 2px^2 - kx + q$

\* (D)  $y = 2px^2 + kx + q$

8. Find the roots of  $\sqrt{2}n(5\sqrt{2} - \sqrt{2}n) = -28$ .

- \* (A)  $\{7, -2\}$
- (B)  $\{-7, 2\}$
- (C)  $\left\{\frac{5 \pm i\sqrt{3}}{2}\right\}$
- (D)  $\left\{\frac{-5 \pm i\sqrt{3}}{2}\right\}$

9. What is(are) all possible value(s) of  $b$  that make  $x^2 + bx + \frac{49}{4}$  a perfect square?

- (A)  $\pm\frac{7}{2}$
- \* (B)  $\pm 7$
- (C)  $\frac{7}{2}$
- (D)  $7$

10. What is the standard form of  $2(y+3) = (x-4)^2$ ?

- (A)  $y = 2(x-4)^2 + 3$
- (B)  $y = 2(x-4)^2 - 3$
- (C)  $y = \frac{1}{2}(x-4)^2 + 3$
- \* (D)  $y = \frac{1}{2}(x-4)^2 - 3$

11. What is the function rule for the sequence  $\{7, 3, -1, -5, -9, \dots\}$ ?

- (A)  $t_n = -4n + 3$
- (B)  $t_n = 4n + 3$
- \* (C)  $t_n = -4n + 11$
- (D)  $t_n = 4n + 11$

12. What is the transformational form of  $y = 3x^2 - 12x + 1$ ?

- (A)  $\frac{1}{3}(y-13) = (x-2)^2$
- \* (B)  $\frac{1}{3}(y+11) = (x-2)^2$
- (C)  $3(y-13) = (x+2)^2$
- (D)  $3(y+11) = (x+2)^2$

13. What is the sum of the roots of  $\sqrt{3}qx^2 + \sqrt{2}qx + q = 0$ ?

- \* (A)  $-\frac{\sqrt{6}}{3}$
- (B)  $-\frac{\sqrt{3}}{3}$
- (C)  $\frac{\sqrt{3}}{3}$
- (D)  $\frac{\sqrt{6}}{3}$

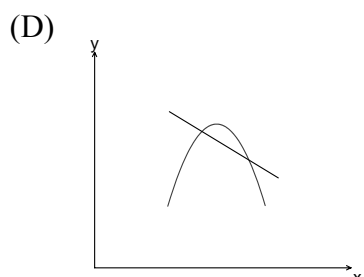
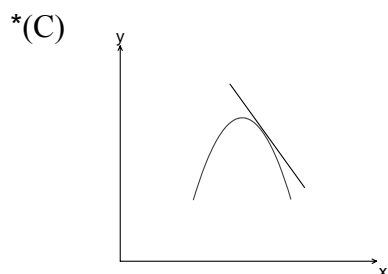
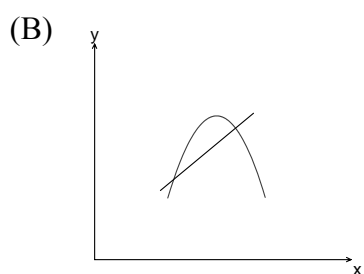
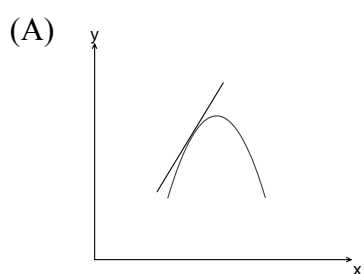
14. The roots of  $x^2 + kx + k + 8 = 0$  are real and equal. What are the possible values of  $k$ ?

- \* (A)  $-4$  and  $8$
- (B)  $-8$  and  $4$
- (C)  $-4 < k < 8$
- (D)  $k < -4$  or  $k > 8$

15. What is the leading coefficient of the function represented by the sequence  $\{2, 6, 26, 59, 102, 152\}$ ?

- (A)  $-3$
- (B)  $-2$
- (C)  $-\frac{3}{2}$
- \* (D)  $-\frac{1}{2}$

16. Which line illustrates a negative instantaneous rate of change?



17. The table shown illustrates gas prices over a five-week period. What is the average rate of change, in cents/week, of the price of gas from week 3 to week 5?

- (A)  $-1.2$   
 \* (B)  $-0.6$   
 (C)  $0.6$   
 (D)  $1.2$

Week	Price per Litre (cents)
1	103.5
2	102
3	99.1
4	98.7
5	97.9

18. What type of function is illustrated by the table shown?

- (A) cubic  
 (B) exponential  
 \* (C) linear  
 (D) quadratic

x	2	3	4	5
y	-4	-1	2	5

19. Which has a y-intercept of (0,5)?

- (A)  $y = 2(5)^x$   
 \* (B)  $y = 5(2)^x$   
 (C)  $y = (2)^x + 5$   
 (D)  $y = (2)^x + 3$

20. Which function models a situation where the market value of an investment depreciates by 5% annually?

- (A)  $y = 1000(0.05)^x$   
 \* (B)  $y = 1000(0.95)^x$   
 (C)  $y = 1000(1.05)^x$   
 (D)  $y = 1000(1.5)^x$

21. Which represents an exponential sequence?

- (A)  $\{2, 2+3, 2+6, 2+9, 2+12, \dots\}$   
 (B)  $\{2, 2 \cdot 3, 2 \cdot 6, 2 \cdot 9, 2 \cdot 12, \dots\}$   
 \* (C)  $\{2, 2 \cdot 3, 2 \cdot 3^2, 2 \cdot 3^3, 2 \cdot 3^4, \dots\}$   
 (D)  $\{2, 2+3, 2+3^2, 2+3^3, 2+3^4, \dots\}$

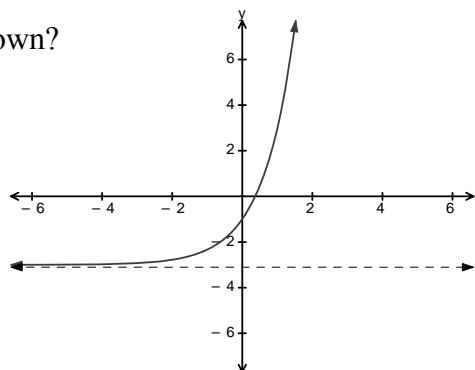
22. Solve:  $\log_3 x + \log_3 2 = 2 \log_3 4$ .

- (A) 4  
 \* (B) 8  
 (C) 14  
 (D) 32

23. What is the range of the function  $y = 3 \cdot 2^{-x} + 4$ ?

- \* (A)  $\{y \mid y > 4, y \in R\}$   
 (B)  $\{y \mid y \geq 4, y \in R\}$   
 (C)  $\{y \mid y < 4, y \in R\}$   
 (D)  $\{y \mid y \leq 4, y \in R\}$

24. Which equation best represents the graph shown?

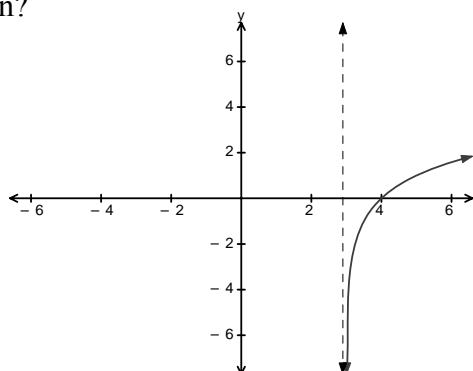


- \* (A)  $y = A(B)^x - 3, A > 0, B > 1$
- (B)  $y = A(B)^x - 3, A > 0, 0 < B < 1$
- (C)  $y = A(B)^x + 3, A > 0, B > 1$
- (D)  $y = A(B)^x + 3, A > 0, 0 < B < 1$

25. What mapping rule was applied to  $y = 4^x$  to result in the function  $\frac{1}{2}(y - 3) = 4^{2(x+1)}$ ?

- (A)  $(x, y) \rightarrow (\frac{1}{2}x - 1, \frac{1}{2}y - 3)$
- \* (B)  $(x, y) \rightarrow (\frac{1}{2}x - 1, 2y + 3)$
- (C)  $(x, y) \rightarrow (2x - 1, \frac{1}{2}y + 3)$
- (D)  $(x, y) \rightarrow (2x - 1, 2y + 3)$

26. What is the equation of the graph shown?



- (A)  $y = \log_2(x - 4)$
- \* (B)  $y = \log_2(x - 3)$
- (C)  $y = \log_2(x + 4)$
- (D)  $y = \log_2(x + 3)$

27. What is the simplest form of  $\sqrt{\frac{9^{x+2}}{25^{1-2x}}} \cdot (125^{x+2})$ ?

- (A)  $3^{x+1} \cdot 5^{2x+1}$
- (B)  $3^{x+1} \cdot 5^{4x+1}$
- (C)  $3^{x+2} \cdot 5^{x+5}$
- \* (D)  $3^{x+2} \cdot 5^{5x+5}$

28. Solve:  $3 \cdot 5^{x-1} = 12$ .

- (A)  $\frac{\log 5 + 1}{\log 4}$
- (B)  $\frac{\log 4 + 1}{\log 5}$
- (C)  $\frac{\log 5}{\log 4} + 1$
- \* (D)  $\frac{\log 4}{\log 5} + 1$

29. Given  $7^x + 5 = 25$ , what is the approximate value of  $x$ ?

- (A) 0.83
- (B) 1.30
- \* (C) 1.54
- (D) 1.75

30. The population,  $P$ , of a bacteria culture is described by the function  $P = 400(2)^{\frac{t}{20}}$ , where  $t$  is time in minutes. How long, in minutes, will it take the population to reach 1600?
- (A) 20  
 \* (B) 40  
 (C) 200  
 (D) 400
31. Simplify:  $(a^{-1} + b^{-1})^{-2}$ .
- (A)  $a^2 + b^2$   
 (B)  $a^2 + 2ab + b^2$   
 \* (C)  $\frac{a^2b^2}{a^2 + 2ab + b^2}$   
 (D)  $\frac{a^2 + 2ab + b^2}{a^2b^2}$
32. Which is equivalent to  $\log_3 \sqrt{\frac{A^3B}{C}}$ ?
- \* (A)  $\frac{1}{2}[3\log_3 A + \log_3 B - \log_3 C]$   
 (B)  $\frac{1}{2}\left[\frac{3\log_3 A \cdot \log_3 B}{\log_3 C}\right]$   
 (C)  $\frac{3}{2}\log_3 A + \frac{1}{2}\log_3 B - \log_3 C$   
 (D)  $\frac{3}{2}\log_3 A + \log_3 B - \log_3 C$
33. Solve:  $5^{3x-1} = \sqrt{2^x}$ .
- (A) 0.24  
 (B) 0.31  
 \* (C) 0.36  
 (D) 0.47
34. The mapping rule  $(x, y) \rightarrow (x, -2y)$  is applied to the function given by  $2(y-3) = 5^{3(x+4)}$ . What is the equation of the new function?
- (A)  $-4(y-3) = 5^{3(x+4)}$   
 (B)  $-4(y-6) = 5^{3(x+4)}$   
 (C)  $-1(y - \frac{3}{2}) = 5^{3(x+4)}$   
 \* (D)  $-1(y-3) = 5^{3(x+4)}$
35. The equation  $y = 5^x$  is transformed such that its new focal point is  $(-3, 4)$ . Which mapping rule describes this transformation?
- (A)  $(x, y) \rightarrow (2x-3, 3y-7)$   
 \* (B)  $(x, y) \rightarrow (2x-3, 3y+1)$   
 (C)  $(x, y) \rightarrow (2x+3, 3y-7)$   
 (D)  $(x, y) \rightarrow (2x+3, 3y+1)$

36. The function  $y = 4^x$  is transformed so that the focal point is  $(3, -5)$  and the vertical stretch is 2. What is the equation of the new function?

- (A)  $y = \frac{1}{2}(4)^{x-3} - 7$
- (B)  $y = \frac{1}{2}(4)^{x-3} + 7$
- \* (C)  $y = 2(4)^{x-3} - 7$
- (D)  $y = 2(4)^{x-3} + 7$

37. The circle  $(x-1)^2 + (y+2)^2 = 1$  has been stretched vertically by a factor of 3 and horizontally by a factor of 4. What is the equation of the ellipse formed?

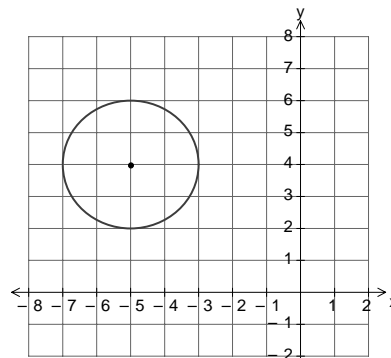
- (A)  $[3(x-1)]^2 + [4(y+2)]^2 = 1$
- (B)  $[\frac{1}{3}(x-1)]^2 + [\frac{1}{4}(y+2)]^2 = 1$
- (C)  $[4(x-1)]^2 + [3(y+2)]^2 = 1$
- \* (D)  $[\frac{1}{4}(x-1)]^2 + [\frac{1}{3}(y+2)]^2 = 1$

38. "If a point lies on the bisector of an angle, then that point is equidistant from the sides of the angle." What is the converse of this statement?

- (A) If a point does not lie on the bisector of an angle, then that point is equidistant from the sides of the angle.
- (B) If a point does not lie on the bisector of an angle, then that point is not equidistant from the sides of the angle.
- (C) If a point is equidistant from the sides of an angle, then that point does not lie on the bisector of the angle.
- \* (D) If a point is equidistant from the sides of an angle, then that point lies on the bisector of the angle.

39. Which mapping rule when applied to  $x^2 + y^2 = 1$  would generate the graph shown?

- \* (A)  $(x, y) \rightarrow (2x-5, 2y+4)$
- (B)  $(x, y) \rightarrow (2x+5, 2y-4)$
- (C)  $(x, y) \rightarrow (4x-5, 4y+4)$
- (D)  $(x, y) \rightarrow (4x+5, 4y-4)$



40. What is the radius of the circle with equation  $x^2 + y^2 - 10x + 12y + 36 = 0$ ?

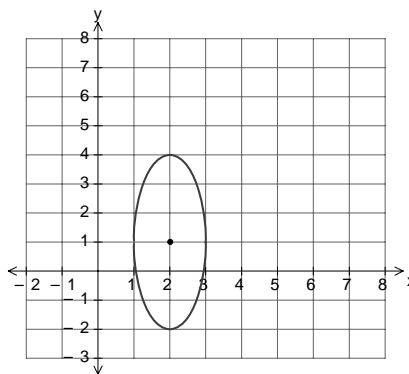
- \* (A) 5
- (B) 6
- (C) 10
- (D) 12

41. If point P is in standard position on a circle of radius 3, which angle of rotation, in degrees, places P at  $(-\frac{3}{2}, \frac{3\sqrt{3}}{2})$ ?

- (A) 30
- (B) 60
- \* (C) 120
- (D) 150

42. What is the equation of the graph shown?

- \* (A)  $[(x-2)]^2 + [\frac{1}{3}(y-1)]^2 = 1$
- (B)  $[(x-2)]^2 + [3(y-1)]^2 = 1$
- (C)  $[(x+2)]^2 + [\frac{1}{3}(y+1)]^2 = 1$
- (D)  $[(x+2)]^2 + [3(y+1)]^2 = 1$

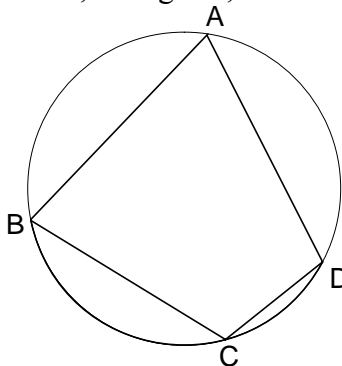


43. What is the exact value of  $2\sin 150^\circ + \cos 30^\circ$ ?

- (A)  $\frac{-2 + \sqrt{3}}{2}$
- \* (B)  $\frac{2 + \sqrt{3}}{2}$
- (C)  $1 + \sqrt{3}$
- (D)  $\sqrt{3}$

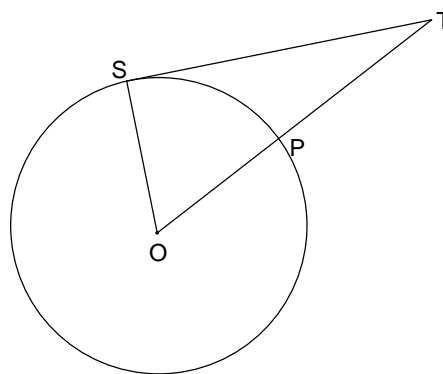
44. Given the circle shown, what is the measure of  $\angle BCD$ , in degrees, if  $\widehat{BCD} = 110^\circ$ ?

- (A) 70
- (B) 110
- \* (C) 125
- (D) 220



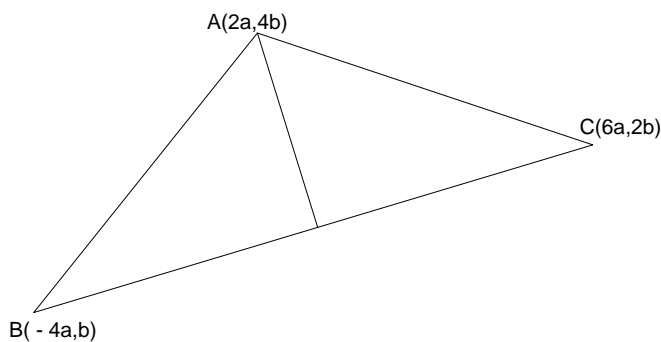
45. In the circle with centre O shown,  $\overline{TS}$  is tangent to the circle at S. If the circle has diameter 4, and  $\overline{TS} = 4$ , what is the length of  $\overline{PT}$ ?

- (A)  $4\sqrt{2} - 4$
- (B)  $4\sqrt{2}$
- \* (C)  $2\sqrt{5} - 2$
- (D)  $2\sqrt{5}$



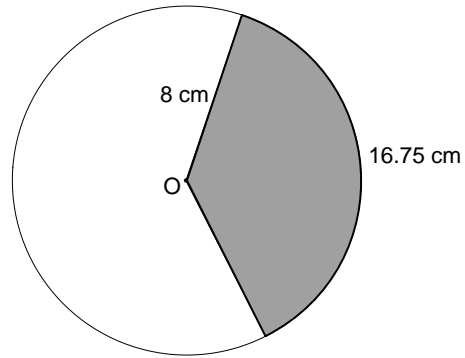
46. What is the slope of the median from A to  $\overline{BC}$  in the diagram shown?

- (A)  $\frac{2}{5ab}$
- (B)  $\frac{2a}{5b}$
- (C)  $\frac{5ab}{2}$
- \* (D)  $\frac{5b}{2a}$



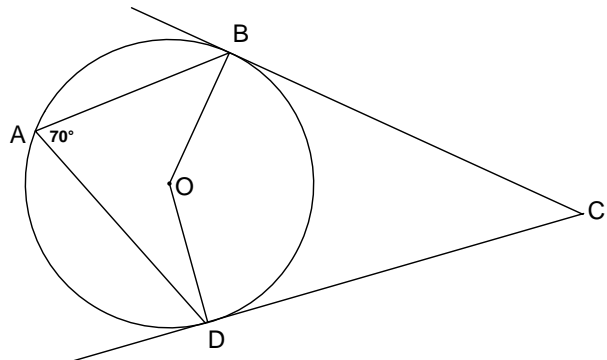
47. What is the area, in square cm, of the shaded region in the circle with centre O shown?

- (A) 16.8  
 (B) 33.5  
 \* (C) 67.0  
 (D) 134.0



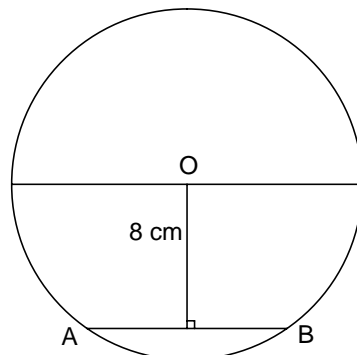
48. In the circle with centre O shown,  $\overline{BC}$  and  $\overline{DC}$  are tangent to the circle at points B and D respectively. What is the measure, in degrees, of  $\angle BCD$ ?

- (A) 20  
 (B) 35  
 \* (C) 40  
 (D) 70



49. The circle shown with centre O has a diameter of 20 cm. If chord  $\overline{AB}$  is 8 cm from the centre, find the length, in centimeters, of  $\overline{AB}$ .

- (A) 6.0  
 \* (B) 12.0  
 (C) 12.8  
 (D) 25.6



50. The unit circle is transformed to an ellipse using the mapping rule  $(x, y) \rightarrow (2x - a, 4y + b)$ . What is the equation of the ellipse?

- \* (A)  $\frac{1}{4}(x+a)^2 + \frac{1}{16}(y-b)^2 = 1$   
 (B)  $\frac{1}{2}(x+a)^2 + \frac{1}{4}(y-b)^2 = 1$   
 (C)  $\frac{1}{4}(x-a)^2 + \frac{1}{16}(y+b)^2 = 1$   
 (D)  $\frac{1}{2}(x-a)^2 + \frac{1}{4}(y+b)^2 = 1$

**PART II**  
**Total Value: 50%**

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

4    51.    Algebraically determine the **exact** roots in simplest form for  $\frac{3}{x+1} + 4 = \frac{5x}{x-1}$ .

$$(x+1)(x-1)\left[\frac{3}{x+1} + 4 = \frac{5x}{x-1}\right] \quad 0.5 \text{ pts.}$$

$$3(x-1) + 4(x-1)(x+1) = 5x(x+1) \quad 0.5 \text{ pts.}$$

$$3x - 3 + 4x^2 - 4 = 5x^2 + 5x \quad 0.5 \text{ pts.}$$

$$0 = x^2 + 2x + 7 \quad 0.5 \text{ pts.}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(7)}}{2(1)} \quad 1.0 \text{ pt.}$$

$$x = \frac{-2 \pm \sqrt{4 - 28}}{2}$$

$$x = \frac{-2 \pm \sqrt{-24}}{2} \quad 0.5 \text{ pts.}$$

$$x = \frac{-2 \pm 2i\sqrt{6}}{2} \quad 0.5 \text{ pts.}$$

$$x = -1 \pm i\sqrt{6} \quad 0.5 \text{ pts.}$$

4    52.    A total of 102 m of fencing is used to make a divided rectangular region as shown. Algebraically determine the maximum area that could be enclosed.

$$3w + 2l = 102 \quad 0.5 \text{ pts.}$$

$$2l = 102 - 3w$$

$$l = \frac{102 - 3w}{2} \quad 0.5 \text{ pts.}$$



$$A = l \times w$$

$$A = \left(\frac{102 - 3w}{2}\right)w \quad 0.5 \text{ pts.}$$

$$A = \left(\frac{102w - 3w^2}{2}\right)$$

$$A = -\frac{3}{2}w^2 + 51w \quad 0.5 \text{ pts.}$$

$$w = -\frac{b}{2a} = \frac{-51}{2\left(-\frac{3}{2}\right)} \quad 0.5 \text{ pts.}$$

$$\text{width} = \frac{-51}{-3} = 17\text{m.} \quad 0.5 \text{ pts.}$$

$$A = -\frac{3}{2}(17)^2 + 51(17) \quad 0.5 \text{ pts.}$$

$$\text{Area} = 433.5\text{m}^2 \quad 0.5 \text{ pts.}$$

Value

- 4 53. A golf ball is thrown up in the air such that its height,  $h$ , in metres above the ground,  $t$  seconds after being thrown is shown below. Algebraically determine the function that defines the height of the golf ball above the ground and use it to determine the height of the ball at 3.5 seconds.

$t$	0	1	2	3	4	5
$h$	3	24.1	35.4	36.9	28.6	10.5

$$d_2 = -9.8 \quad 0.5 \text{ pts.}$$

$$2a = -9.8$$

$$a = \frac{-9.8}{2} = -4.9 \quad 0.5 \text{ pts.}$$

$$c = 3 \quad 0.5 \text{ pts.}$$

$$h(t) = -4.9t^2 + bt + 3$$

$$24.1 = -4.9(1)^2 + b(1) + 3 \quad 0.5 \text{ pts.}$$

$$b = 26 \quad 0.5 \text{ pts.}$$

$$\therefore h(t) = -4.9t^2 + 26t + 3 \quad 1.0 \text{ pt.}$$

$$h(3.5) = -4.9(3.5)^2 + 26(3.5) + 3$$

$$h(3.5) = 34\text{m} \quad 0.5 \text{ pts.}$$

- 2 54.(a) A rectangular picture is reduced in size using a photocopier. The area of the picture is given by  $A = -2w^2 + 60w$  where  $A$  is the area of the picture in  $\text{cm}^2$ , and  $w$  is the width of the picture in cm. Algebraically determine the approximate instantaneous rate of change of area of the picture when the width is 3 cm.

$$\text{Inst. Rate of Change} = \frac{A(3.1) - A(2.9)}{3.1 - 2.9} \quad 1.0 \text{ pt.}$$

$$\text{Inst. Rate of Change} = \frac{166.78 - 157.18}{0.2} \quad 0.5 \text{ pts.}$$

$$\text{Inst. Rate of Change} = 48 \text{ cm/s} \quad 0.5 \text{ pts.}$$

- 2 (b) A cylindrical oil tank of radius 5 m, and height 20 m, is leaking oil at a constant rate. The tank is initially full and the depth of the oil in the tank is changing by 15 cm/hour. Algebraically determine the function in terms of  $t$  that models the volume of oil in the tank  $t$  hours after the leak began. ( $V = \pi r^2 h$ )

$$15\text{cm} = 0.15\text{m}$$

$$h = 20 - 0.15t \quad 0.5 \text{ pts.}$$

$$V = \pi r^2 h$$

$$V = \pi(5)^2(20 - 0.15t) \quad 1.0 \text{ pts.}$$

$$V = 25\pi(20 - 0.15t) \text{ m}^3/\text{h} \quad 0.5 \text{ pts.}$$

Value

3 55. Algebraically solve:  $3^{2x+1} - 5 \cdot 3^x - 2 = 0$ .

$$3^{2x+1} - 5 \cdot 3^x - 2 = 0 \quad \text{let } y = 3^x$$

$$3^1 \cdot 3^{2x} - 5 \cdot 3^x - 2 = 0 \quad 0.5 \text{ pts.}$$

$$3y^2 - 5y - 2 = 0 \quad 0.5 \text{ pts.}$$

$$(3y+1)(y-2) = 0 \quad 0.5 \text{ pts.}$$

$$y = -\frac{1}{3} \quad \text{or } y = 2$$

$$3^x \neq -\frac{1}{3} \quad \text{or } 3^x = 2 \quad 1.0 \text{ pt.}$$

$$\text{No Soln. or } x \log 3 = \log 2$$

$$x = \frac{\log 2}{\log 3}$$

$$x \approx 0.631 \quad 0.5 \text{ pts.}$$

4 56. Algebraically solve:  $\log_7(2x+2) - \log_7(x-1) = \log_7(x+1)$ .

$$\log_7(2x+2) - \log_7(x-1) = \log_7(x+1)$$

$$\log_7\left(\frac{2x+2}{x-1}\right) = \log_7(x+1) \quad 0.5 \text{ pts.}$$

$$\frac{2x+2}{x-1} = x+1 \quad 0.5 \text{ pts.}$$

$$2x+2 = x^2 - 1 \quad 0.5 \text{ pts.}$$

$$x^2 - 2x - 3 = 0 \quad 0.5 \text{ pts.}$$

$$(x-3)(x+1) = 0 \quad 0.5 \text{ pts.}$$

$$x = 3 \quad \text{or } x = -1 \quad 1.0 \text{ pts.}$$

$$x \neq -1 \quad 0.5 \text{ pts.}$$

4 57. A new car was purchased for \$32 000 and sold 4 years later for \$12 500. If the car continues to depreciate at the same rate, how long, to the nearest tenth of a year, will it take for the car to depreciate from \$12 500 to \$5400.

$$\text{rate} = \frac{12500}{32000} = 0.391$$

$$5400 = 12500(0.391)^{t/4} \quad 2.0 \text{ pts.}$$

$$\frac{5400}{12500} = \frac{12500}{12500}(0.391)^{t/4}$$

$$0.432 = (0.391)^{t/4} \quad 0.5 \text{ pts.}$$

$$\log 0.432 = \frac{t}{4} \log(0.391) \quad 0.5 \text{ pts.}$$

$$\frac{t}{4} = \frac{\log 0.432}{\log 0.391} \quad 0.5 \text{ pts.}$$

$$t = (4) \frac{\log 0.432}{\log 0.391}$$

$$t = 3.6 \text{ years} \quad 0.5 \text{ pts.}$$

Value

- 4 58. Determine the equation of the exponential function that best represents the graph shown and use it to find the value of  $y$  where  $x = 3$ .

$$y = -0.26, -1.44, -2.33, -3, -3.5$$

$$y + 5 = 4.74, 3.56, 2.67, 2, 1.5 \quad 0.5 \text{ pts.}$$

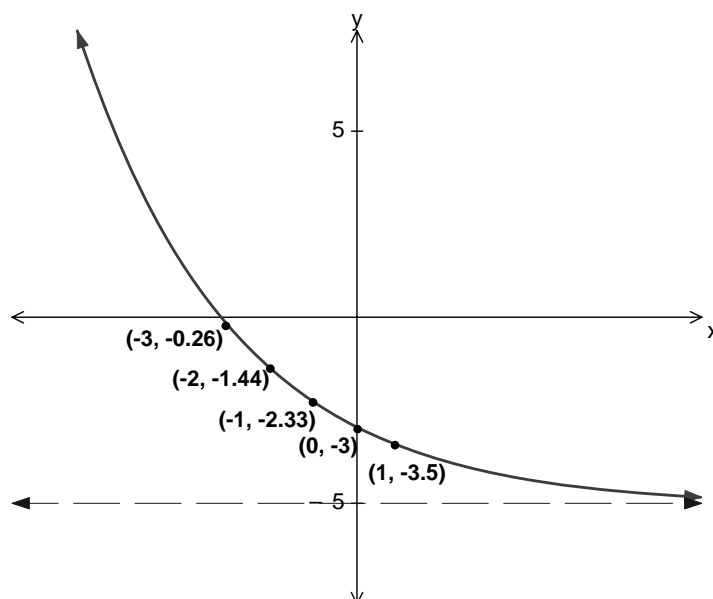
$$\text{ratio} = \frac{3.56}{4.74} = 0.75 \quad 1.0 \text{ pt.}$$

$$\therefore y = 2(0.75)^x - 5 \quad 1.5 \text{ pts.}$$

When  $x = 3$

$$y = 2(0.75)^3 - 5 \quad 0.5 \text{ pts.}$$

$$y = -4.16 \quad 0.5 \text{ pts.}$$



- 3 59. The points  $(2, 7)$  and  $(2, -1)$  are endpoints of the minor axis of an ellipse. If the major axis is 10 units long, find the equation of the ellipse in transformational form.

$$\text{minor axis} \rightarrow y - \text{axis} = 7 - (-1) = 8 \quad 0.5 \text{ pts.}$$

$$\text{centre } (2, 3) \quad 0.5 \text{ pts.}$$

$$\left[ \frac{1}{5}(x-2) \right]^2 + \left[ \frac{1}{4}(y-3) \right]^2 = 1 \quad 2.0 \text{ pts. See breakdown below}^*$$

\* (0.5 for  $\frac{1}{5}$ , 0.5 for  $\frac{1}{4}$ , 0.5 for  $(x-2)$  and  $(y-3)$  and 0.5 for the correct form.

- 4 60. Write  $x^2 + 5y^2 - 10x + 20y + 40 = 0$  in transformational form. State the coordinates of the centre and, if it represents a circle, state the radius; if it represents an ellipse, state the length of the major axis and the minor axis.

$$(x^2 - 10x + \underline{25}) + 5(y^2 + 4y + \underline{4}) = -40 + \underline{25} + \underline{20} \quad 2.0 \text{ pts.}$$

$$(x-5)^2 + 5(y+2)^2 = 5$$

$$\frac{(x-5)^2}{5} + \frac{5(y+2)^2}{5} = \frac{5}{5}$$

$$\left[ \frac{1}{\sqrt{5}}(x-5) \right]^2 + [y+2]^2 = 1 \quad 0.5 \text{ pts.}$$

$$\text{Centre } (5, -2) \quad 0.5 \text{ pts.}$$

$$\text{Major axis} = 2(\sqrt{5}) = 2\sqrt{5} \quad 0.5 \text{ pts.}$$

$$\text{Minor axis} = 2(1) = 2 \quad 0.5 \text{ pts.}$$

Value

- 4 61. Algebraically show that the centre of the circle  $x^2 + y^2 - 8x + y + \frac{49}{4} = 0$  lies on the parabola defined by  $y = -\frac{1}{2}x^2 + 3x - \frac{9}{2}$ .

$$(x^2 - 8x + 16) + \left(y^2 + y + \frac{1}{4}\right) = -\frac{49}{4} + 16 + \frac{1}{4} \quad 1.0 \text{ pts.}$$

$$(x-4)^2 + \left(y + \frac{1}{2}\right)^2 = 4 \quad 1.0 \text{ pts.}$$

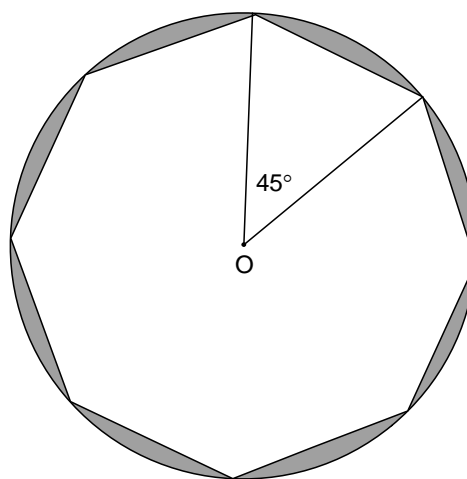
$$\text{Centre } \left(4, -\frac{1}{2}\right) \quad 0.5 \text{ pts.}$$

$$-\frac{1}{2} = -\frac{1}{2}(4)^2 + 3(4) - \frac{9}{2} \quad \text{Verify} \quad 0.5 \text{ pts.}$$

$$-\frac{1}{2} = -\frac{16}{2} + 12 - \frac{9}{2}$$

$$-\frac{1}{2} = -\frac{1}{2} \quad \therefore \text{The centre lies on the parabola.} \quad 1.0 \text{ pts.}$$

- 4 62. A regular octagon is inscribed in a circle as shown. If the area of the shaded region is  $5.6 \text{ cm}^2$ , find the radius of the circle to the nearest tenth of a cm.



$$\text{Area of circle} = \pi x^2 \quad 0.5 \text{ pts.}$$

$$\text{Area of triangle} = \frac{1}{2}(x)(x) \sin 45^\circ \quad 0.5 \text{ pts.}$$

$$\text{Area of triangle} = \frac{\sqrt{2}x^2}{4} \quad 0.5 \text{ pts.}$$

$$\text{Area of shaded} = \text{Area of circle} - 8(\text{Area of triangle})$$

$$5.6 = \pi x^2 - 8\left(\frac{\sqrt{2}x^2}{4}\right) \quad 1.0 \text{ pts.}$$

$$5.6 = \pi x^2 - 2\sqrt{2}x^2$$

$$5.6 = x^2(\pi - 2\sqrt{2}) \quad 0.5 \text{ pts.}$$

$$\frac{5.6}{(\pi - 2\sqrt{2})} = \frac{x^2(\pi - 2\sqrt{2})}{(\pi - 2\sqrt{2})}$$

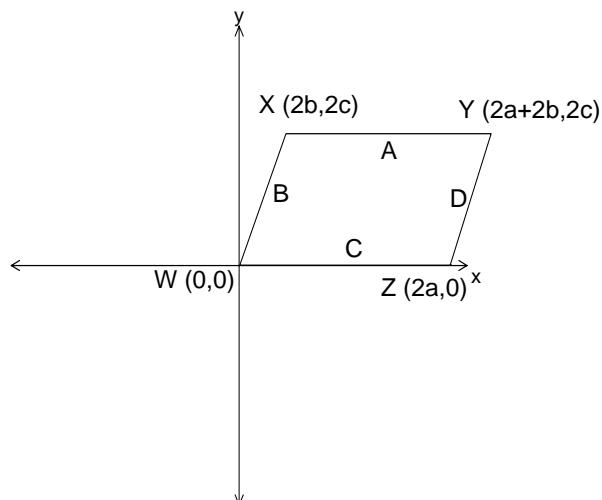
$$\frac{5.6}{(\pi - 2\sqrt{2})} = x^2 \quad 0.5 \text{ pts.}$$

$$17.88 = x^2$$

$$4.2 = x \quad 0.5 \text{ pts.}$$

Value

- 4 63. Using coordinate geometry, prove that the segments joining the midpoints of adjacent sides of WXYZ form a parallelogram.



$$\text{Midpt}_{xy} = \left( \frac{2b+2a+2b}{2}, \frac{2c+2c}{2} \right)$$

$$\text{Midpt}_{xy} = \left( \frac{2a+4b}{2}, \frac{4c}{2} \right)$$

$$\text{Midpt}_{xy} = (a+2b, 2c)$$

$$\text{Midpt}_{wz} = \left( \frac{0+2a}{2}, \frac{0+0}{2} \right)$$

$$\text{Midpt}_{wz} = (a, 0)$$

$$\text{Midpt}_{yz} = \left( \frac{2a+2b+2a}{2}, \frac{2c+0}{2} \right)$$

$$\text{Midpt}_{yz} = \left( \frac{4a+2b}{2}, \frac{2c}{2} \right)$$

$$\text{Midpt}_{yz} = (2a+b, c)$$

$$\text{Midpt}_{xw} = \left( \frac{2b+0}{2}, \frac{2c+0}{2} \right)$$

$$\text{Midpt}_{xw} = (b, c)$$

\* 0.5 pts. for each midpoint.

$$m_{AB} = \frac{2c-c}{a+2b-b} = \frac{c}{a+b}$$

$$m_{CD} = \frac{c-0}{2a+b-a} = \frac{c}{a+b}$$

$$m_{AC} = \frac{2c-c}{a+2b-2a-b} = \frac{c}{b-a}$$

$$m_{BD} = \frac{c-0}{b-a} = \frac{c}{b-a}$$

$$m_{AB} = m_{CD} \therefore AB \text{ is parallel to } CD.$$

$$m_{AC} = m_{BD} \therefore AC \text{ is parallel to } BD.$$

\* 0.5 pts. for each slope.