

## Mathematics 3205 Grading Standards June 2003

To view a copy of the exam go to  
<http://www.gov.nf.ca/edu/k12/pub/sample.htm>

### 1. Pre-Marking Appraisal

- #19 - the phrase “is the” was omitted from the question. This, however, had no effect on the interpretation of the question.
- #20 - this question was omitted from the test since no correct answer was among the choices.
- #23 - we felt this question was too long to be included in Part 1 of the exam. In fact, it required more work than #56 in Part II which was valued at 3%. As well there were 2 solutions to this question; only one of which was offered as a choice. We accepted (D) as the answer since it was the only option that contained one of the 2 correct solutions.
- #54 - students should have been asked for the instantaneous rate of change “at” 3 seconds, not “after”. Hence if a student calculated it for a time after 3 seconds, such as 3.5 or 4 seconds, it was agreed they would be awarded credit.
- #58 - realizing there was no common ratio in the adjusted data, we agreed to give student full credit for solutions that reflected use of any one of the ratios found. Also, it was agreed that students would be awarded full marks in the event they recognized the strategy for solving the problem, accounted for the horizontal asymptote, recognized there was no common ratio, stated so, and concluded the function was not exponential.

### 2. Post-Marking Report

#### (a) Marking Standard and Consistency

A trial run of 50 papers was carried out on Day 1. All 50 trial run papers were put back through and checked again. The second round of marking was identical to the first in all but two cases. Papers were also inserted randomly into daily marking and marks compared with the trial runs to ensure reliable marking. The reliability coefficient was very good throughout indicating marking reliability and consistency was quite high.

**(b) Commentary on Response**

Part II questions involving circle geometry and probability were poorly done.

There appeared to be a lot of algebraic errors on the vast majority of papers. Routine skills such as simple factoring, completing the square, use of the distributive property, cross multiplication etc., caused students difficulty.

Students used a variety of strategies to solve many questions.

**NOTE:**

Approximately half way through the Math 3205 marking board, department officials ran a preliminary analysis of Part II results and consulted with the members of the marking board. Board members expressed concern with the lack of opportunity (mainly due to time constraints) teachers had to do higher order activities, particularly in Unit IV and V. This resulted in a superficial treatment of some topics. The preliminary analysis supported this view as there were a number of higher order questions where the majority of students received less than half marks.

**It was decided that:**

- Questions 58, 62, and 64 not be included in the calculation of Part II marks. If a student benefitted from not dropping either of the identified questions, the question or questions were not dropped for the individual student.
- Future Public Examinations would be set to measure outcomes in Unit I - IV only.

### 3. Constructed Response Answers & Common Errors

4% 51. Find the exact value of all roots of  $\frac{1}{x+5} + \frac{3}{x-1} = 2$ .

**Answer Comment:**

*This equation with rational expressions required multiplication by an LCD which created an equation with a non-factorable quadratic. The quadratic formula was necessary and there were two unequal irrational roots.*

**Excellent Answer Exemplar:**

$$\frac{1}{(x+5)} + \frac{3}{(x-1)} = 2$$

$$x-1 + 3(x+5) = 2(x^2+4x-5)$$

$$x-1 + 3x+15 = 2x^2+8x-10$$

$$4x+14 = 2x^2+8x-10$$

$$2x^2+4x-24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-4(2)(-24)}}{2(2)} = \frac{-4 \pm \sqrt{208}}{4} = \frac{-4 \pm 4\sqrt{13}}{4} = -1 \pm \sqrt{13}$$

$$x = -5, 1$$

**Good Answer Exemplar:**

$$\frac{1}{x+5} + \frac{3}{x-1} = 2(x+5)(x-1)$$

$$x-1 + 3x+15 = 2(x^2-1x+5x-5)$$

$$4x+14 = 2x^2+8x-10$$

$$2x^2+4x-24 = 0$$

$$\text{Quadratic Formula}$$

$$a=2, b=4, c=-24$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-4(2)(-24)}}{2(2)} = \frac{-4 \pm \sqrt{16+192}}{4} = \frac{-4 \pm \sqrt{208}}{4} = -1 \pm \sqrt{208} \text{ or } 15\sqrt{208}$$

The student reduced  $\frac{-4 \pm \sqrt{208}}{4}$  incorrectly. In particular, cancelled the 4s. [3 out of 4]

## Commentary & Errors

Most students attempted this question. However, many students lost marks throughout due to careless mistakes. Students generally showed a good understanding of solving the resulting quadratic.

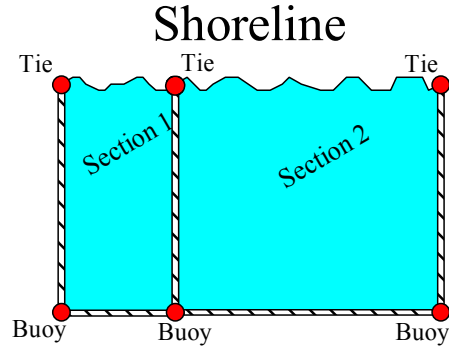
*Errors:*

- not multiplying through correctly by the common denominator
- making mistakes in simplifying which resulted in an incorrect quadratic equation
- attempting to factor the resulting quadratic rather than applying the quadratic formula
- not expressing the radical  $\frac{-2 \pm \sqrt{52}}{2}$  or  $\frac{-4 \pm \sqrt{208}}{4}$  in simplest terms
- using decimals to express their final answer whereas the question required an exact answer

52. A lifeguard must join 3 shoreline ties and 3 anchored buoys (arranged as shown) with single strands of rope to form a rectangular swimming area in 2 sections (no rope runs along the shoreline). The lifeguard uses 600m of rope in total. Find the quadratic function that models this situation, and determine the length and width that will produce a maximum for the entire rectangular swimming area.

**Answer Comment:**

The problem required the quantifying of a diagram with three equal unknown widths and one related unknown length. Then the related quadratic area function would be maximized to determine the length and width requested.



**Excellent Answer Exemplar:**

**Good Answer Exemplar:**

The student had a correct process but the dimensions weren't stated...there was no indication that 30000 was a max.. [3 out of 4]

## Commentary & Errors

This question was poorly answered by the majority of students.

*Errors:*

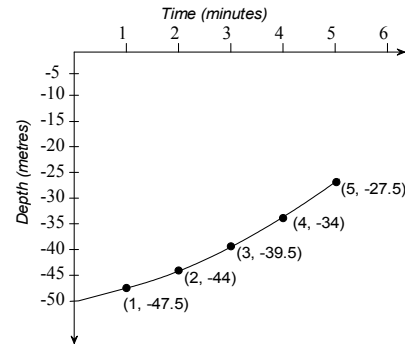
- ignoring the middle section of rope thereby getting an incorrect representation for the area indicating again that students processed this as a ‘problem type’
- solving the quadratic  $A(x) = -3x^2 + 600x$  by making it equal to either 0 or 600 rather than finding maximum value
- making algebraic errors in completing the square
- giving only one dimension as a final response

4%

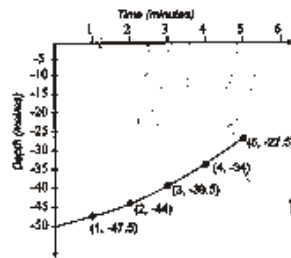
53. The initial depth of a submarine is 50m below sea level. The following graph shows the distance it rises over a 5 minute period. Algebraically determine the quadratic function that defines the path of the submarine.

**Answer Comment:**

The problem required the student to determine that the data had a common difference at the second difference level indicating a quadratic model should be determined.



**Excellent Answer Exemplar:**



x	0	1	2	3	4	5
y	-50	-47.5	-44	-39.5	-34	-27.5

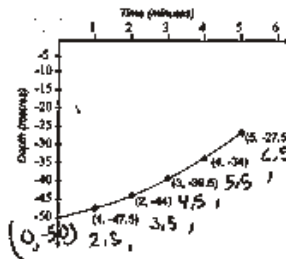
$D_1 = 2.5, 3.5, 4.5, 5.5, 6.5$   
 $D_2 = 1, 1, 1, 1$   
 $D_2 = 2a$   
 $1 = 2a$   
 $\frac{1}{2} = a$   
 $y = \frac{1}{2}x^2 + bx + c$   
 $-50 = \frac{1}{2}(0)^2 + 0 + c$   
 $-50 = c$   
 $y = \frac{1}{2}x^2 + bx - 50$   
 $-47.5 = \frac{1}{2}(1)^2 + b(1) - 50$   
 $-47.5 = 0.5 - 50 + b$   
 $2 = b$

$y = \frac{1}{2}x^2 + 2x - 50$

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**Good Answer Exemplar:**



$D_2 = 2a$   
 $2a = 1$   
 $a = 0.5$   
 Point (0, -50)  
 $y = ax^2 + bx + c$   
 $-50 = 0.5(0)^2 + b(0) + c$   
 $-50 = c$   
 Point (1, -47.5)  
 $y = ax^2 + bx + c$   
 $-47.5 = 0.5(1)^2 + b(1) - 50$   
 $-47.5 = 0.5 + b - 50$   
 $-47.5 + 50 - 0.5 = b$   
 $2 = b$

$A = 0.5$   
 $B = 2$   
 $C = -50$

While the workings seem correct, the function is never explicitly stated. [3½ out of 4]

## Commentary & Errors

This question was well answered by the majority of students. A variety of approaches were used in solving this question. Many students set up a system of equations in three unknowns which would have been developed in Math 2205, rather than using the relationship between the “a” value and  $D_2$ .

### *Errors:*

- assuming initial depth was the vertex of the parabola
- writing a linear or exponential equation for the data even though the question specifically asked to give a quadratic equation
- making simple arithmetic errors when substituting

3%


54. A cube, presently 10 mm on each side, has side lengths that increase at a uniform rate of 5 mm/sec. Find the approximate instantaneous rate of change of volume of the cube after 3 seconds.

**Answer Comment:**

The problem involved quantifying the sides of cube with respect to time, determining the volume relation for the dynamic cube, and using a secant slope near the stated time to approximate the rate of change requested.

**Excellent Answer Exemplar:**

the cube after 3 seconds.



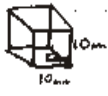
Initial Volume = l · w · h = 10 × 10 × 10 = 1000 mm<sup>3</sup>  
 After 3s each side gains 15mm (3s × 5mm/s)  
 So the new side length is 25mm (10mm + 15mm)  
 $V(3) = s^3 = 25^3 = 15625 \text{ mm}^3$   
 After 3.001s each side gains 15.005mm (3.001s × 5mm/s)  
 So the new side length is 25.005mm (10 + 15.005)  
 $V(3.001) = s^3 = 25.005^3 = 15634.37688 \text{ mm}^3$

Rate of change =  $\frac{V(3.001) - V(3)}{3.001 - 3} = \frac{15634.37688 - 15625}{0.001} = 9376.88 \text{ mm}^3/\text{s}$

The rate of change is 9376.88 mm<sup>3</sup>/s

**Good Answer Exemplar:**

the cube after 3 seconds.



$V = s^3$

At 3 seconds	At 2.999 seconds
3 sec × 5mm/sec = 15mm	2.999 sec × 5mm/sec = 14.995mm
5 + 15 = 20mm	5 + 14.995 = 19.995mm
$V = s^3$	$V = s^3$
$V = 20^3$	$V = (19.995\text{mm})^3$
$V = 8000 \text{ mm}^3$	$V = 7994.0015 \text{ mm}^3$

$\frac{V(3.000) - V(2.999)}{3.000 - 2.999} = \frac{8000 \text{ mm}^3 - 7994.0015 \text{ mm}^3}{0.001 \text{ sec}} = 5998.5 \text{ mm}^3/\text{sec}$

5998.5 mm<sup>3</sup>/sec is the approximate instantaneous rate of change of volume after 3 seconds.

The student was correct in principle except 5mm was used as the initial side length instead of 10mm. [2 out of 3]

## Commentary & Errors

This question was attempted by the majority of students. However, due to a combination of errors, many students did not receive full marks for the question.

*Errors:*

- decreasing the side of the cube rather than increasing
- finding the instantaneous rate of change of the side of the cube rather than the volume of the cube
- students who used  $V = (10 + 5t)^3$  often obtained an incorrect derivative by removing a common factor incorrectly in the process of determining the instantaneous rate of change
- finding the surface area of cube rather than volume

4%

55. Solve for x:  $8^{2x} + 8^x - 12 = 0$

**Answer Comment:**

*This problem involved solving a quadratic equation which utilized a simple exponential substitution. The quadratic was factorable and produced one inadmissible root.*

**Excellent Answer Exemplar:**

Let  $8^x = a$   
 $a^2 + a - 12 = 0$   
 $(a+4)(a-3) = 0$   
 $a = -4 \quad a = 3$   
 ~~$8^x = -4$~~   $8^x = 3$   
 $x = \frac{\log 3}{\log 8}$   
 $= 0.528$

**Good Answer Exemplar:**

4% 55. Solve for x:  $8^{2x} + 8^x - 12 = 0$  [Let  $8^x = y$ ]  
 $y^2 + y - 12 = 0$   
 $(y-3)(y+4) = 0$   
 $y = 3 \quad y = -4$   
 $8^x = 3 \quad 8^x = -4$   
 $x = \frac{\log 3}{\log 8} \quad x = \frac{\log -4}{\log 8}$   
 $x = \pm .5283$   
 $x =$  its not going to go any further because you can't get the log of a negative number. (reject)  
 3% 56. Solve for x:  $\log_7(x-2) + \log_7(x+4) = 1$   
 $\log_7(x-2) + \log_7(x+4) = 1$

*The student correctly rejected  $8^x = -4$ , but introduced an unwarranted  $\pm$  on  $\frac{\log 3}{\log 8}$ . [3½ out of 4]*

## Commentary & Errors

This question was done well by many students. Those who answered it incorrectly did not understand the properties of exponents or tried to use logs incorrectly. Others just did not completely finish the problem to solve for  $x$ .

*Errors:*

- simplifying:  $8^{2x} + 8^x$  to be  $8^{3x}$
- using logs incorrectly:  $8^{2x} + 8^x = 12$   
 $\log 8^{2x} + \log 8^x = \log 12$
- factoring incorrectly
- not finishing the problem
- stating  $y = -4$ ,  $y = -3$  **or**  $8^x = -4$ ,  $8^x = -3$
- not recognizing there is no solution to  $8^x = -4$
- changing  $8^x = -4$  to  $8^x = 4$  and solving
- incorrectly solving  $8^x = 3$ ,  $x = \frac{\log 8}{\log 3}$

56. Solve for x:  $\log_7(x-2) + \log_7(x+4) = 1$

**Answer Comment:**

The problem involved combining two logarithms, re-writing the equation in exponential form, and solving the resulting quadratic equation (the quadratic was factorable). There was an inadmissible root.

**Excellent  
Answer  
Exemplar:**

$$\begin{aligned} \log_7(x-2) + \log_7(x+4) &= 1 \\ \log_7[(x-2)(x+4)] &= 1 \\ \log_7[x^2 - 2x + 4x - 8] &= 1 \\ \log_7[x^2 + 2x - 8] &= 1 \\ 7^1 &= x^2 + 2x - 8 \\ 7 &= x^2 + 2x - 8 \\ 0 &= x^2 + 2x - 8 - 7 \\ 0 &= x^2 + 2x - 15 \\ 0 &= (x+5)(x-3) \\ 0 &= x+5 \quad 0 = x-3 \\ x &= -5 \quad x = 3 \\ \text{reject} \end{aligned}$$

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**Good  
Answer  
Exemplar:**

$$\begin{aligned} \log_7(x-2)(x+4) &= 1 \\ \log_7(x^2+2x-8) &= 1 \\ \log_7(x^2+2x-8) &= \log_7 7 \\ x^2+2x-8 &= 7 \\ x^2+2x-15 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2} \\ x &= \frac{-2 \pm \sqrt{64}}{2} \\ x &= \frac{-2 \pm 8}{2} \\ \boxed{x = 5} \quad x = -3 &\text{ reject} \end{aligned}$$

The student lost the negative sign on  $-2$  near the end of the workings causing a sign change on the roots although was aware the negative should be rejected here. [2 out of 3]

## Commentary & Errors

Most students performed well on this question. It was a routine logarithmic equation and those who did the question incorrectly did not understand the properties of logs.

*Errors:*

- dropping log on LHS:  $(x - 2)(x + 4) = 1$
- factoring incorrectly
- not rejecting  $x = -5$  as a solution

4%

57. Two ATVs were bought at the same time. One ATV cost \$10 000 and depreciates at 22% yearly. The other ATV cost \$8000 and depreciates at 18% yearly. Algebraically determine the number of years it will take for the two ATVs to be of equal value.

**Answer Comment:**

The problem involved correctly modeling two yearly depreciations, equating them, and solving the exponential equation for the time requested.

**Excellent**

**Answer**

of equal value.

**Exemplar:**

$$y = ab^x$$

$$ATV_1 = 10000(0.78)^x$$

$$ATV_2 = 8000(0.82)^x$$

It will take 5 years for the ATVs to be of equal value

$$10000(0.78)^x = 8000(0.82)^x$$

$$0.78^x = 0.8(0.82)^x$$

$$x \log 0.78 = \log 0.8 + x \log 0.82$$

$$-0.10791x = -0.09691 - 0.08619x$$

$$-0.02172x = -0.09691$$

$$x = 4.46$$

**Good**

**Answer**

**ATV 1**

Algebraically determine the number of years it will take for the two ATVs to be of equal value.

**ATV 2**

**Exemplar:**

$$y = \frac{10000(0.78)^t}{8000} = \frac{8000(0.82)^t}{8000}$$

$$1.25(0.78)^t = (0.82)^t$$

$$\log 1.25 + t \log(0.78) = \log(0.82)^t$$

$$\log 1.25 + t \log(0.78) = t \log(0.82)$$

$$\log 1.25 = t \left( \frac{\log 0.82}{\log 0.78} \right)$$

$$\log 1.25 = t(0.799)$$

$t = \frac{\log 1.25}{0.799}$   
 $t = 0.121$   
 It will take 0.121 years.

The student incorrectly grouped the 't log' terms producing an incorrect value for 't'. Also, the answer is clearly too small and unreasonable. [3 out of 4]

## Commentary & Errors

While most students attempted this question, many just set up the equations to solve the problem. These students were unable to solve the equation algebraically and many found an answer using their calculator instead.

*Errors:*

- not solving algebraically to find solution
- using log properties incorrectly (ie., shown below)

$$\log 1.25(0.78)^t = \log 0.82^t$$

$$t \log 1.25(0.78) = t \log 0.82$$

or

$$10000(0.78)^t = 8000(0.82)^t$$

$$7800^t = 6560^t$$

4%

58. A cup of hot water was left to cool in a room. The room temperature was  $21^{\circ}\text{C}$ . The temperature of the water was recorded as shown in the table. Find the exponential function that models the cooling behaviour of the water and use it to determine the temperature of the water after 14 minutes.

**Answer Comment:**

The problem involved exponential decay data which was subject to a non-zero limiting value. The correct decay model would be calculated and used to determine the temperature at the time in question.

Time (minutes)	Temperature ( $^{\circ}\text{C}$ )
0	70
4	68
8	65
12	63

**Excellent Answer Exemplar:**

Time (minutes)	Temperature ( $^{\circ}\text{C}$ )
0	70
4	68
8	65
12	63

$$y = 49(0.959)^{x/4} + 21$$

$$y = 49(0.959)^{14/4} + 21$$

$$y = 62.6$$

It was  $62.6^{\circ}\text{C}$  after

14 mins

$$y = a b^x$$

Because of the horizontal asymptote at  $y = 21$ , we must subtract 21 from the  $y$ -values to find our common ratio.

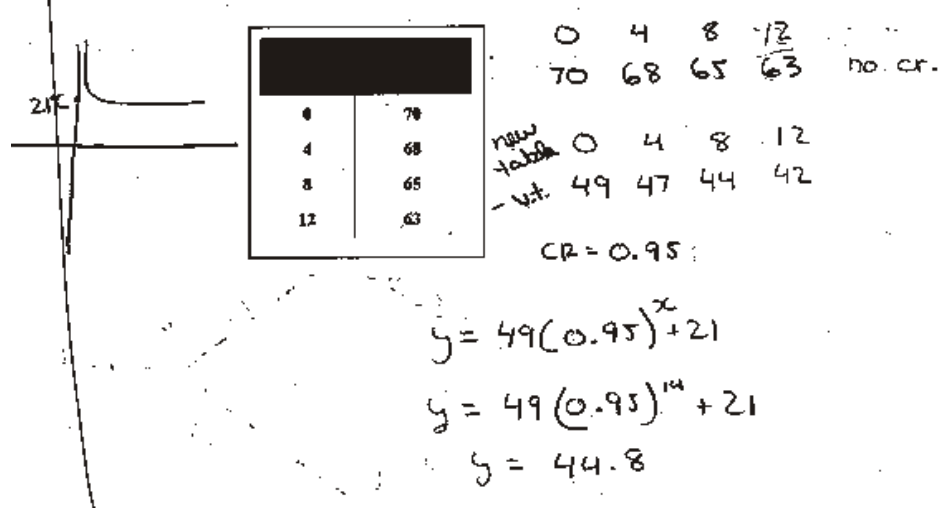
$$b = (63 - 21) / (65 - 21)$$

$$b = 0.959$$

To get our  $a$  value we subtract  $70 - 21$  because of the horizontal asymptote

$$a = 49$$

**Good  
Answer  
Exemplar:**



*The student forgot the temperatures were taken at 4 minute intervals..the exponent 'x' should have been 'x / 4'. [3 out of 4]*

**Commentary & Errors**

This question was poorly answered by the majority of students. Although the common ratio was not exact between all values, this did not cause difficulty with the problem and any student who recognized this as a problem was given full marks if they demonstrated an understanding of the asymptotic behavior of the function and the adjusted initial value. Most students who answered this question incorrectly did so because they did not recognize the asymptotic behavior of the graph of the exponential function.

*Errors:*

- not recognizing asymptotic behavior of data
- using 70 as the initial value "a" instead of  $70 - 21 = 49$
- using values from the table to find the common ratio instead of subtracting 21 to make an adjusted table of values
- using quadratic or cubic functions to model the cooling behavior of the water

3%

59. Algebraically show that the line  $5x - 2y - 11 = 0$  passes through the centre of  $2x^2 + 3y^2 - 4x + 18y - 7 = 0$  ?

**Answer Comment:**

The question required the determination of the centre of an ellipse given in general form and demonstrating that the centre satisfied the equation of a given line.

**Excellent Answer Exemplar:**

$$2x^2 + 3y^2 - 4x + 18y - 7 = 0$$

$$(2x^2 - 4x) + (3y^2 + 18y) = 7$$

$$2(x^2 - 2x + 1) + 3(y^2 + 6y + 9) = 7 + 2 + 27$$

$$2(x - 1)^2 + 3(y + 3)^2 = 36$$

center is  $(1, -3)$

To show that the line passes through the center

$$5x - 2y - 11 = 0$$

$$5(1) - 2(-3) - 11 = 0$$

$$5 + 6 - 11 = 0$$

$$0 = 0$$

thus the line does pass through the center.

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**Good Answer Exemplar:**

$$2x^2 + 3y^2 - 4x + 18y - 7 = 0$$

$$(2x^2 - 4x) + (3y^2 + 18y) = 7$$

$$2(x^2 - 2x + 1) + 3(y^2 + 6y + 9) = 7 + 2 + 27$$

$$\frac{2(x-1)^2}{24} + \frac{3(y+3)^2}{24} = \frac{34}{24}$$

$$\frac{(x-1)^2}{12} + \frac{(y+3)^2}{8} = 1$$

$$\left[\frac{1}{\sqrt{12}}(x-1)\right]^2 + \left[\frac{1}{\sqrt{8}}(y+3)\right]^2 = 1$$

Center:  $(1, -3)$

$$5x - 2y - 11 = 0$$

$$5(1) - 2(-3) - 11 = 0$$

$$5 + 6 - 11 = 0$$

$$11 - 11 = 0$$

$$0 = 0$$

True

The line  $5x - 2y - 11$  does pass through the center of  $2x^2 + 3y^2 - 4x + 18y - 7 = 0$ .

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The student made several arithmetic and procedural errors in completing the square yet produced the correct centre. The remainder of the process was correct. [2 out of 3]

### Commentary & Errors:

This question was done well by students.

*Errors:*

- incorrectly completing the square shown in the example below:

$$2(x^2 - 2x + 1) + 3(y^2 + 6y + 9) = 7 + \underset{2}{\downarrow} 1 + \underset{27}{\downarrow} 9$$

- choosing the centre incorrectly shown in the example below:

$$2(x - 1)^2 + 3(y + 3)^2 = 36$$

*Centre*(-1, 3)

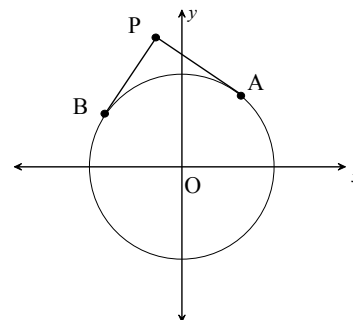
- making errors when simplifying such as:

$$\begin{array}{l|l} 5(1) - 2(-3) - 11 & 0 \\ 5 - 6 - 11 & 0 \\ -12 & \neq 0 \end{array}$$

- not verifying that the centre is on the line.

3%

60. A(3, 4) and B(-4, 3) are points on the circle shown having centre O and tangents  $\overline{PA}$  and  $\overline{PB}$ . If the slope of  $\overline{AO}$  is  $\frac{4}{3}$ , and the slope of  $\overline{BO}$  is  $-\frac{3}{4}$ , what are the coordinates of point P?



**Answer Comment:**

The problem involved developing a coordinate geometry proof. It was necessary to know that tangents to a circle meet its radii at right angles. This provides enough information to determine the slope of PA and PB. There are several methods that would work here to determine the coordinates of P. One such method would be to determine the equation of the lines PA and PB using the slope information mentioned and the given coordinates, and then intersect those lines algebraically.

**Excellent**

**Answer**

**Exemplar:**

coordinates of point P? -

Because  $\overline{OA}$  and  $\overline{PA}$  are perpendicular the slope of  $\overline{PA}$  is  $-\frac{3}{4}$ . Because  $\overline{OB}$  and  $\overline{PB}$  are perpendicular the slope of  $\overline{PB}$  is  $\frac{4}{3}$ .

line $\overline{PB}$	line $\overline{PA}$		
$y = mx + b$	$y = mx + b$		
$3 = \frac{4}{3}(-4) + b$	$4 = -\frac{3}{4}(3) + b$	$\frac{4}{3}x + \frac{25}{3} = -\frac{3}{4}x + \frac{25}{4}$	
$3 = -\frac{16}{3} + b$	$4 = -\frac{9}{4} + b$	$\frac{4}{3}x + \frac{3}{4}x = \frac{25}{4} - \frac{25}{3}$	Find y:
$3 + \frac{16}{3} = b$	$4 + \frac{9}{4} = b$	$\frac{16}{12} + \frac{9}{12}x = \frac{75}{12} - \frac{100}{12}$	$y = \frac{4}{3}(-1) + \frac{25}{3}$
$\frac{25}{3} = b$	$\frac{25}{4} = b$	$\frac{25 + 25}{12} = -\frac{25}{12} = \frac{25}{12}$	$y = -\frac{4}{3} + \frac{25}{3}$
$y = \frac{4}{3}x + \frac{25}{3}$	$y = -\frac{3}{4}x + \frac{25}{4}$	$x = \frac{-300}{300}$ $x = -1$	$y = \frac{21}{3}$

The coordinates of P are (-1, 7)

**Good  
Answer  
Exemplar:**

coordinates of point P

$$m_{PA} = m_{BO}$$

$$\frac{4-y}{3-x} = -\frac{3}{4}$$

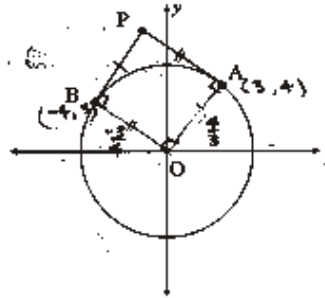
$$4-y = -\frac{3}{4}(3-x)$$

$$y = 7$$

$$3-x = 4$$

$$x = -1$$

$$P = (-1, 7)$$



*While the student did arrive at the correct coordinates requested, the method invoked a special case since the equation the student used have infinitely many such points as solutions. [2 out of 3]*

### Commentary & Errors

This question was done poorly by the majority of students and often left out completely.

*Errors:*

- when creating an equation cross-multiplying incorrectly as shown:

$$-\frac{3}{4} = \frac{y-4}{x-3}$$

$$-3x + 3 = 4y - 4$$

- making algebraic errors, which are often very simple mistakes, in solving the system of equations as shown:

$$-\frac{25}{12}x = \frac{25}{12}$$

$$x = 1$$

- using the slope of the perpendicular line incorrectly

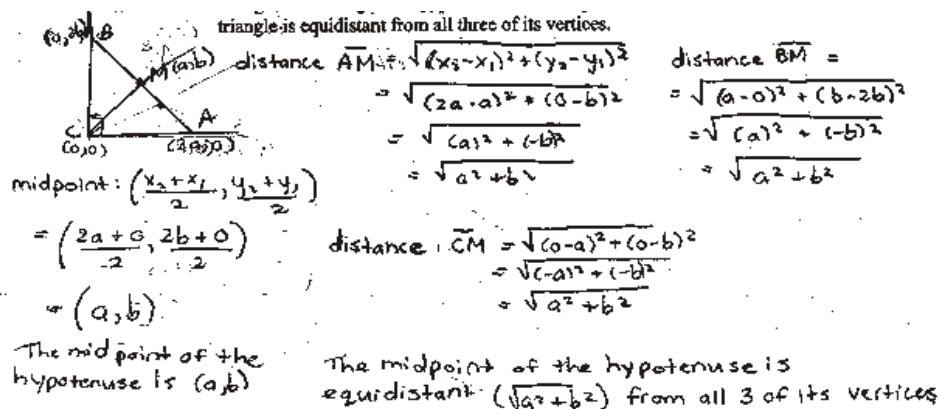
$$PA = \frac{3}{4} \text{ instead of } -\frac{3}{4}.$$

61. Using coordinate geometry, prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three of its vertices.

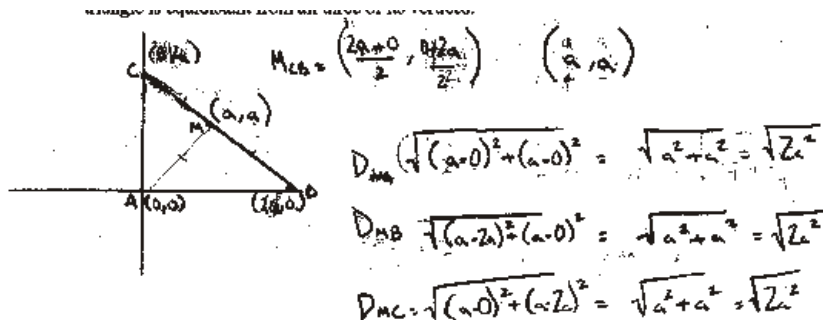
**Answer Comment:**

The problem involved developing a coordinate geometry proof. Quantifying the vertices on a right triangle, establishing the midpoint of the hypotenuse, and showing the distance from that midpoint to each vertex was equal were necessary. A diagram would assist and friendly coordinates, (e.g., using  $2a$  etc.) would demonstrate insight.

**Excellent Answer Exemplar:**



**Good Answer Exemplar:**



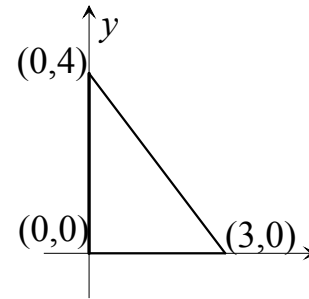
The student used a special case, an isosceles right triangle.  
**[3 out of 4]**

## Commentary & Errors

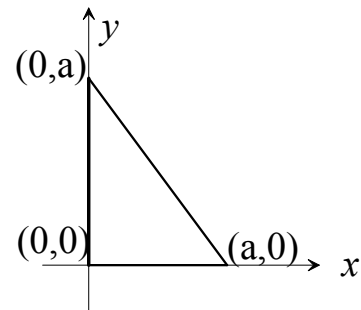
Students completed the proof for particular cases as opposed to the general case as required.

*Errors:*

- using numerical values instead of variables, thus completing a proof of a specific case.



- using variables but still completing a proof of a more specific case. (An isosceles right triangle)



- incorrectly taking a square root.

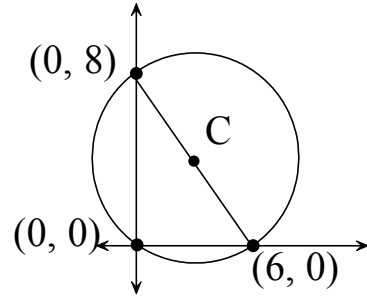
$$\sqrt{a^2 + b^2} = a + b$$

3%

62. A circle passes through the point (0, 0) and intersects the x-axis at 6 and the y-axis at 8. What is the equation of the circle?

**Answer Comment:**

The problem involved identifying three points on the given circle. One clear way to then access the equation (i.e., establishing the centre and radius) was to note that information provided included a diameter (since the inscribed angle, vertex the origin, was a right angle).



**Excellent Answer Exemplar:**

y-axis at 8. What is the equation of the circle?

midpoint of AB  $\left(\frac{0+6}{2}, \frac{0+8}{2}\right)$  midpoint of AC  $\left(\frac{0+0}{2}, \frac{0+8}{2}\right)$  center of circle  $(3, 4)$

$(3, 0)$   $(0, 4)$

radius  $\sqrt{(3-0)^2 + (4-0)^2}$   
 $= \sqrt{9 + 16}$   
 $= \sqrt{25}$   
 $= 5$

$(x-h)^2 + (y-k)^2 = r^2$   
 $(x-3)^2 + (y-4)^2 = 5^2$   
 $(x-3)^2 + (y-4)^2 = 25$

**Good Answer Exemplar:**

$(6, 0)$   $(0, 8)$   
 $\frac{6}{2}, \frac{8}{2}$   $3, 4$

centre = 3, 4  
 distance or radius  
 $= \sqrt{(6-3)^2 + (0-4)^2}$   
 $= \sqrt{9 + 16}$   
 $= \sqrt{25}$   
 $= 5$

$\left[\frac{1}{\sqrt{5}}(y-4)\right]^2 + \left[\frac{1}{\sqrt{5}}(x-3)\right]^2 = 1$

Workings here are essentially correct but  $\frac{1}{\sqrt{5}}$  should be  $\frac{1}{5}$ .

[2 out of 3]

## Commentary & Errors

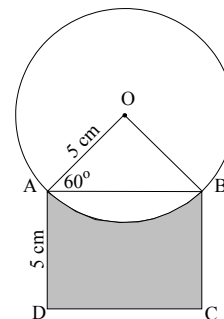
This question was done poorly by most students. The students who completed this question used two different methods. They solved by using a 3-4-5 right triangle or a system of equations.

*Errors:*

- assuming centre at (0, 0)
- using the diameter as the radius
- not squaring the radius in the circle equation
- using the equation of a line
- using the equation of a parabola
- using the major or minor axis to find the equation of an ellipse

4%

63. A sector of a circle, centre O, and a square ABCD, share points A and B as shown. If  $\angle OAB$  is  $60^\circ$ , and  $OA = AD = 5\text{cm}$ , what is the area of the shaded region to the nearest  $\text{cm}^2$ ?



**Answer Comment:**

*This problem required the determination of three separate areas...a square, sector and triangle...and correct relating of those areas in calculating the shaded region requested. Some trigonometry was necessary.*

**Excellent Answer Exemplar:**

region to the nearest  $\text{cm}^2$ ?

pythagoras theorem  
 $5^2 = 2.5^2 + x^2$   
 $25 = 6.25 + x^2$   
 $25 - 6.25 = x^2$   
 $x = 4.3$

$A_{\text{circle}} = \pi r^2$   
 $= 3.14(5)^2$   
 $= 3.14(25)$   
 $A_{\text{circle}} = 78.5\text{cm}^2$

$A_{\text{small sector}} = \frac{\pi r^2 \times 60}{360}$   
 $= \frac{\pi(5)^2 \times 60}{360}$   
 $= 78.5 \times \frac{60}{360}$   
 $A_{\text{small sector}} = 13.08$

$A_{\text{big sector}} = 78.5\text{cm}^2 - 13.08$

$A_{\text{triangle}} = \frac{1}{2}bh$   
 $= \frac{1}{2}(5)(4.3)$   
 $A_{\text{triangle}} = 10.8\text{cm}^2$

$A_{\text{square}} = 1 \times w$   
 $= 5 \times 5$   
 $= 25\text{cm}^2$

$A_{\text{segment}} = 13.08 - 10.8$   
 $A_{\text{segment}} = 2.28\text{cm}^2$

$A_{\text{shaded}} = 25\text{cm}^2 - 2.28\text{cm}^2$   
**Area Shaded = 23cm<sup>2</sup>**

the area of the shaded region is 23cm<sup>2</sup>.

**Good  
Answer  
Exemplar:**

region to the nearest  $\text{cm}^2$ :

$$\begin{aligned} A_{\text{square}} &= 25\text{cm}^2 \\ A_{\text{sector}} &= \pi r^2 \cdot \frac{60}{360} \\ &= \pi \cdot 5^2 \cdot \frac{60}{360} \\ &= 25\pi \cdot \frac{60}{360} \\ &= 13.09\text{cm}^2 \end{aligned}$$

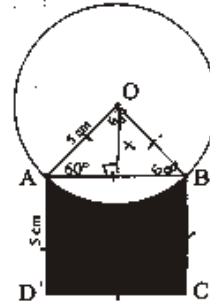
$$\begin{aligned} S^2 &= 2.5^2 + x^2 \\ 25 &= 6.25 + x^2 \\ 18.75 &= x^2 \\ 4.33 &= x \end{aligned}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2}(2.5)(4.33) \\ &= 5.4\text{cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{segment}} &= A_{\text{sector}} - A_{\text{triangle}} \\ &= 13.09 - 5.4 \\ &= 7.69\text{cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{shaded}} &= A_{\text{square}} - A_{\text{segment}} \\ &= 25\text{cm}^2 - 7.69\text{cm}^2 \\ &= \underline{17.31\text{cm}^2} \end{aligned}$$

The area of the shaded region is about  $17\text{cm}^2$ .



*Triangle area took only half the base. [3 out of 4]*

**Commentary & Errors**

This question was done well by many students.

*Errors:*

- not knowing area of a triangle
- using  $2\pi r^2$  as area of circle
- rounding 13.09 to 13.9 for area of a sector
- not rounding the final answer
- leaving out the area of the triangle or the area of the sector
- not knowing a square has equal sides
- finding the area for half the triangle only

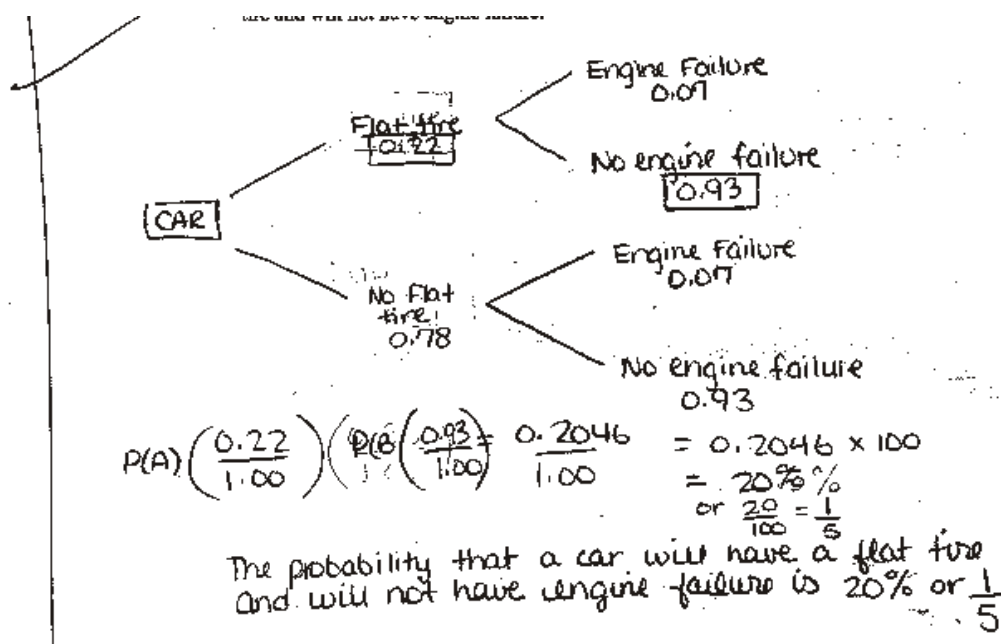
3%

64. An engineer calculates that, for a one year period, the probability of a certain car having a flat tire is 22%, and having engine failure is 7%. Using a tree diagram, calculate the probability that a randomly selected car of this type will have a flat tire and will not have engine failure.

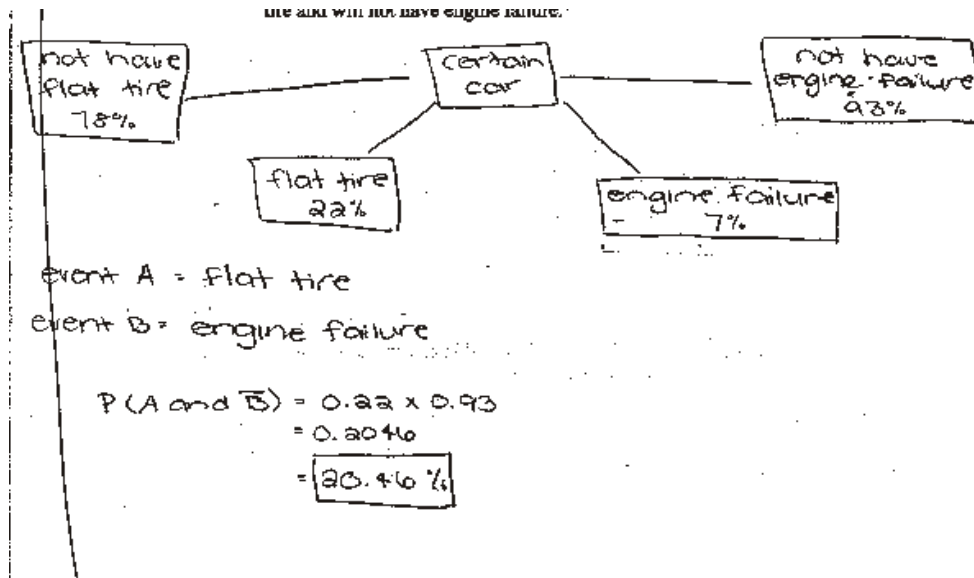
**Answer Comment:**

*The problem involved determining probability of dependent events utilizing a tree diagram (a specific outcome in the course).*

**Excellent  
Answer  
Exemplar:**



**Good  
Answer  
Exemplar:**



*The tree diagram was inaccurate and did not reflect probability of dependent events. [2 out of 3]*

**Commentary & Errors**

This question was done poorly by the majority of students. Many students did not attempt to do this question. It seems that students did not have an understanding of drawing a tree diagram.

*Errors:*

- not drawing a tree diagram
- subtracting 22 from 100 to obtain 88
- not knowing the difference between  $P(a \text{ and } \bar{b})$  and  $P(a|\bar{b})$
- not finding the complement of 7%

# Part I: Cognitive Level & Key [50%]

Q= Quadratics; R=Rate of Change; E=Exp. And Log.; G=Circle Geometry; P=Probability

Item	Outcome	C	P1	P2	PS
1-D	C31	Q			
2-C	A7	Q			
3-B	C15	Q			
4-B	A9	Q			
5-A	C32	Q			
6-B	C31		Q		
7-D	A4		Q		
8-C	C9		Q		
9-B	B10		Q		
10-C	C8		Q		
11-B	A3		Q		
12-B	B11		Q		
13-B	C4		Q		
14-A	C17	R			
15-A	B4		R		
16-D	F1	E			
17-C	C34	E			
18-C	C4	E			
19-C	A7	E			
20-A	C19	E			
21-D	C33	E			
22-D	B2	E			
23-D	B13, C24		E		
24-C	B1, B12, C24		E		
25-C	A5		E		

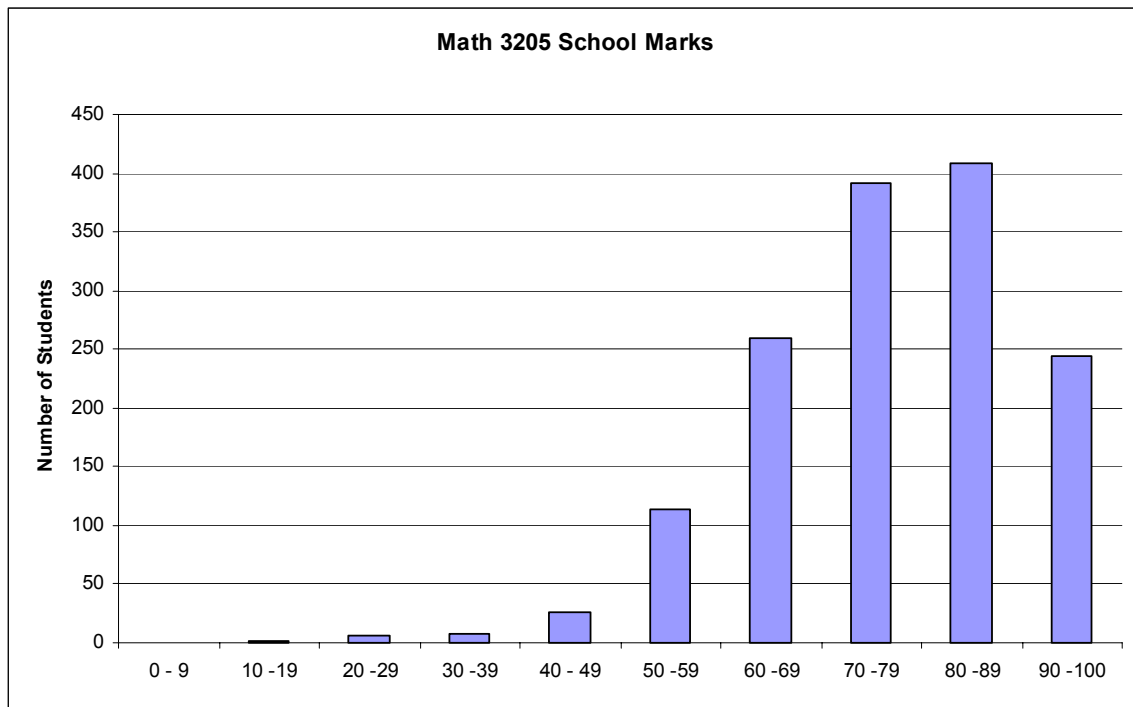
Item	Outcome	C	P1	P2	PS
26-A	B12		E		
27-B	C11		E		
28-A	C35Z		E		
29-B	B12		E		
30-A	C34, C33, C35Z		E		
31-C	E12	G			
32-C	E16	G			
33-C	C36	G			
34-B	D1		G		
35-A	E14		G		
36-D	E13		G		
37-B	E3		G		
38-B	E4		G		
39-D	E4		G		
40-B	E4		G		
41-C	C36		G		
42-B	D1		G		
43-B	E4		G		
44-B	G7	P			
45-B	A6, G8		P		
46-A	B8, A6		P		
47-B	G3, A6		P		
48-C	G5Z		P		
49-B	G3		P		
50-D	G3, G2		P		

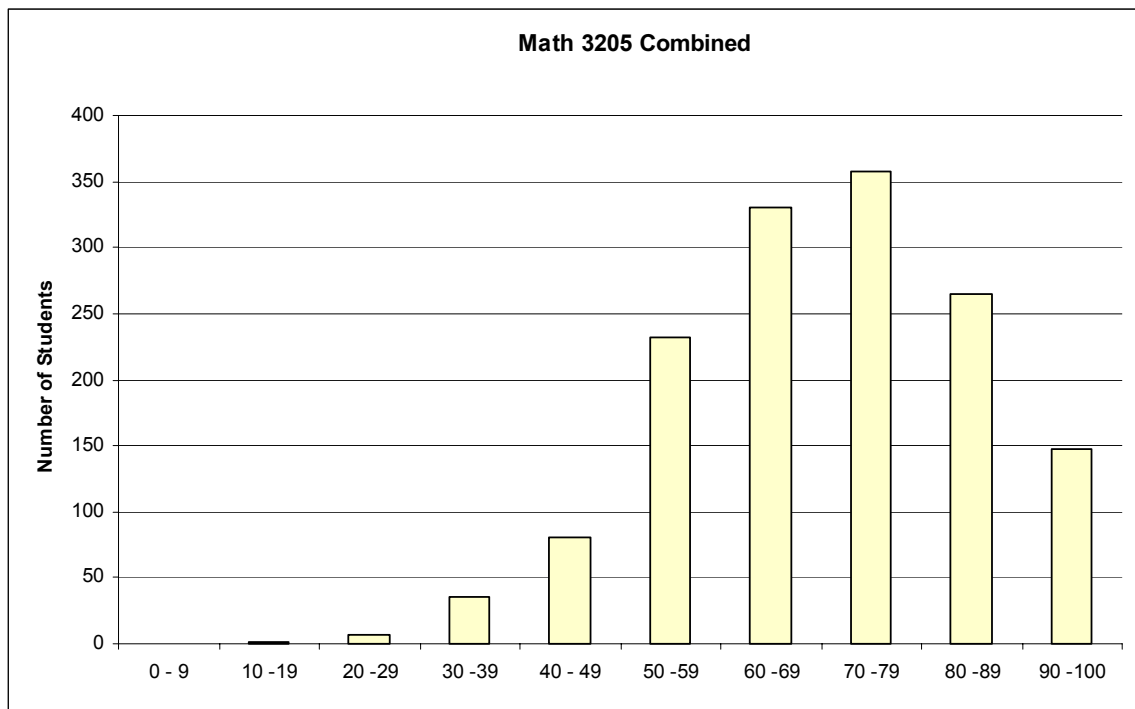
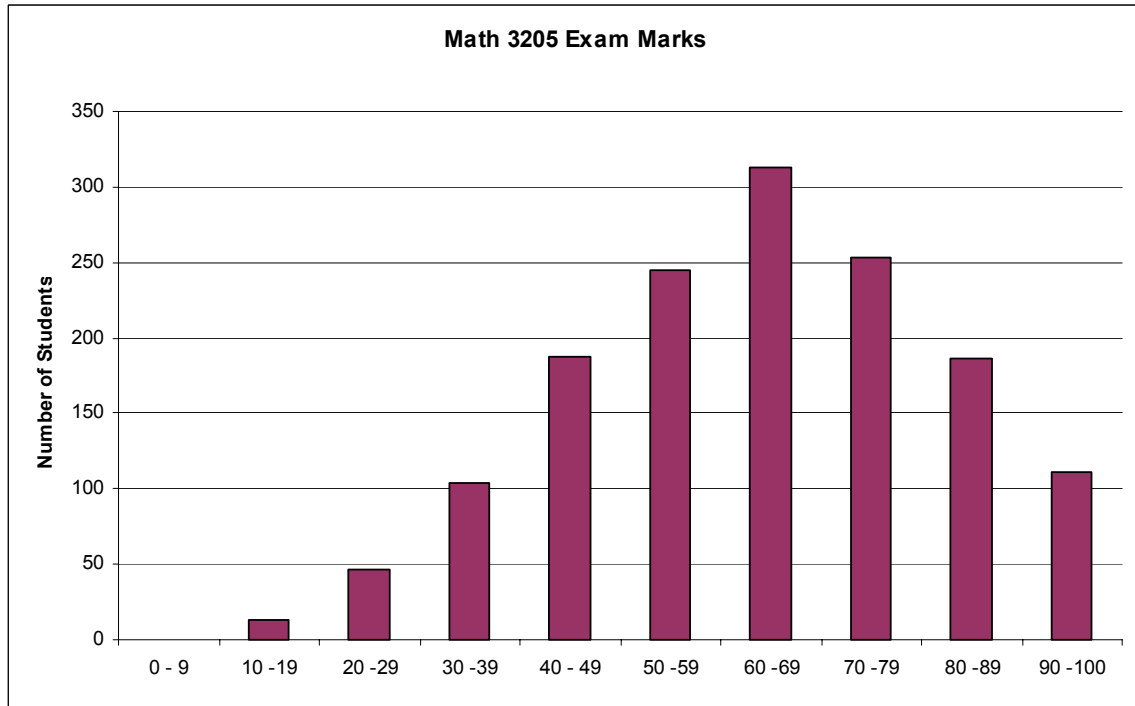
## Part II: Cognitive Level & Value [50%]

Q= Quadratics; R=Rate of Change; E=Exp. And Log.; G=Circle Geometry; P=Probability

Item	Outcome	C	P1	P2	PS
51 [4]	C22			Q	
52 [4]	C23/C1				Q
53 [4]	C1/C10				Q
54 [3]	C28				R
55 [4]	C24/B12			E	
56 [3]	C24			E	
57 [4]	C2/C25				E
58 [4]	C34				E

Item	Outcome	C	P1	P2	PS
59 [3]	E15			G	
60 [3]	E15			G	
61 [3]	D1, E11				G
62 [4]	E4, E15				G
63 [4]	E15				G
64 [3]	G5				P





Math 3205 June 2003

