

Math 3205
Grading Standards
June 2004

Pre-Marking Appraisal

The board considered the exam fair and of the appropriate difficulty and length.

The following notes were made.

- There was no mark deducted in #51 for not rejecting $\frac{-3-\sqrt{29}}{2}$. The Teacher's Resource and Textbook does not provide any examples of rejecting such a solution in this context.
- the board agreed that the widest x -interval to be used in #54 which would still warrant full marks would be 1.9 to 2.1.

Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 35 papers that went through the marker panel and marks were assigned to each question on a separate sheet of paper. The 35 exams were put back into the original stack of exams and corrected again when they appeared. The two values were compared and if there were discrepancies, the chief marker would review the scoring with the individual marker.

Throughout the marking process there was statistical analysis run on item data to enhance reliability and consistency of marking.

Commentary on Response

Students did very well with most questions in the exam.

- The most challenging question was question #63, but it was very appropriate for the exam.
- Students had difficulty in finding the equation in question #52.
- students often left question #54(b) out, however, there are many examples of this question in the text.
- In question #55, students struggled with properties of Logs.
- In question #63, not many students received full marks. They had to apply several properties of circles and check the solution. It was not a difficult question, but it had several steps.

PART 11
Total Value: 50%

Value
4

51. Algebraically determine the EXACT roots in simplest form for $\frac{\sqrt{5-x}}{\sqrt{3x}} = \frac{\sqrt{3x}}{\sqrt{5+x}}$.

$$(\sqrt{5-x})(\sqrt{5+x}) = (\sqrt{3x})(\sqrt{3x})$$

$$5 - x^2 = 3x$$

$$0 = x^2 + 3x - 5$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{29}}{2}, \frac{-3 + \sqrt{29}}{2}$$

Commentary on Response

This question was well done.

Common Errors

- Students got a final answer of $\pm\sqrt{\frac{5}{4}}$ or $\pm\sqrt{\frac{5}{2}}$

- Students made the following computational errors

- $(\sqrt{5-x})(\sqrt{5+2}) = 25 - x^2$ or $5 - x$ or $5 + x^2$

- $(\sqrt{3x})(\sqrt{3x}) = 3x^2$ or $9x$ or $9x^2$

Value

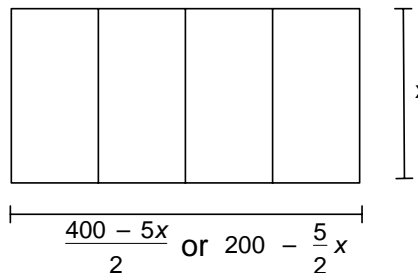
4

52. A farmer uses 400 m of fencing to create a rectangular pig pen and to divide it into four regions of equal area as shown. Algebraically determine the function which gives the area of the pig pen as a function of its width, and state the dimensions that produce maximum area.

$$\boxed{\begin{aligned} A(x) &= x\left(200 - \frac{5}{2}x\right) \\ A(x) &= -\frac{5}{2}x^2 + 200x \end{aligned}}$$

$$x = -\frac{b}{2a} = -\frac{200}{2\left(-\frac{5}{2}\right)} = 40$$

$$A(40) = \frac{400 - 5(40)}{2} = 100$$



Therefore, the dimensions that produce maximum area are:

$$\boxed{40\text{m} \times 100\text{m}}$$

OR

$$-\frac{5}{2}x(x - 80) = 0$$

Zeros are: $x = 0$ or $x = 80$

$$\boxed{\begin{aligned} \text{Symmetry puts Max. at } \frac{0+80}{2} &= 40\text{m} \\ \text{Width is 40m and Length is } 200 - \frac{5}{2}(40) &= 100\text{m} \end{aligned}}$$

Commentary on Response

Well done if they could get the correct equation for the area. If the student obtained the right equation, for the most part he/she earned full marks.

Common Errors

- Students used $w = 400 - 5l$ or $400 - l$.
- Students used $2y = 4x + 400$ or $2y = 4x - 400$.
- Students used $5x$ as the width.
- Students used $400 = x\left(200 - \frac{5}{2}x\right)$.
- Students used quadratic formula to obtain answer after area function was found.

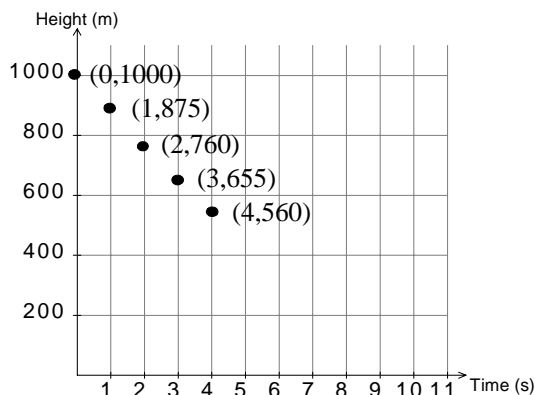
Value

4

53. A small plane begins a parabolic dive from 1000 m. The graph shows the plane's height, in metres, above the ground over time, in seconds, during a parabolic dive. Algebraically determine the quadratic function that defines the path of the plane over time and use it to determine the plane's height at 9.5 seconds.

Note: $c = 1000$

$$\begin{array}{cccc}
 & 875 & 760 & 655 & 560 \\
 & \swarrow & \downarrow & \swarrow & \downarrow \\
 d_1 & -115 & -105 & -95 & \\
 & \swarrow & \downarrow & \swarrow & \\
 d_2 & & 10 & 10 & \\
 & & d_2 = 2a & & \\
 & & 10 = 2a & & \\
 & & 5 = a & &
 \end{array}$$



$$h(t) = 5t^2 + bt + 1000$$

$$875 = 5(1)^2 + b(1) + 1000$$

$$b = -130$$

$$h(t) = 5t^2 - 130t + 1000$$

$$h(9.5) = 216.25\text{m}$$

Alternative way to find a, b, c

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 875 \\ 760 \\ 655 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -130 \\ 1000 \end{pmatrix}$$

Commentary on Response

Very well done!

Common Errors

- Students misinterpreted the points (e.g., (1,875) and (2,760) as the numbers 1,875 and 2,760.
- Students used $D_2 = -10$

Value

- 2 54(a). A diver jumps off a spring board. Her height h , in metres, above the water, t seconds after she jumps, is given by $h(t) = -4.9t^2 + 8t + 5$. Algebraically determine the approximate instantaneous rate of change in her height at 2 seconds.

t	h
1.99	1.51551
2.01	1.28351

Alternative Solution:

$$h'(t) = -9.8t + 8$$

$$h'(2) = -9.8(2) + 8$$

$$h'(2) = -11.6 \text{ m/s}$$

$$\sim \text{I.R.C.} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.28351 - 1.51551}{2.01 - 1.99} = -11.6 \text{ m/s}$$

Commentary on Response

Students did well with this question. There are several variations in the values for “ t ”. This caused a range of answers. As well, variations in rounding throughout the solution caused a range of answers.

Common Errors

- Students arrived at a positive answer
→ 11.6 m/s rather than -11.6 m/s .

- 2 54(b). A ball has a radius of 8 cm and the radius is decreasing at a rate of 15 mm/min. Determine the function, in terms of time, used to calculate the volume of this ball at a given instant ($V = \frac{4}{3}\pi r^3$).

t is in minutes:

$$V_{\text{mm}^3} = \frac{4}{3}\pi(80 - 15t)^3 \text{ or } V_{\text{cm}^3} = \frac{4}{3}\pi(8 - 1.5t)^3$$

Commentary on Response

Students encountered some difficulty with this question. Many proceeded to calculate the instantaneous rate of change, rather than only providing the required function.

Common Errors

- Unit conversion errors were very common.
- Students didn't convert mm to cm (or cm to mm)
(e.g., $V = \frac{4}{3}\pi(8-15t)^3$).
- Students incorrectly converted mm to cm (or cm to mm)
(e.g., $V = \frac{4}{3}\pi(8-0.15t)^3$).

3 55. Solve for x : $\log_2(x^2 - 8) = \log_2 x + \frac{1}{2}\log_2 4$.

$$\begin{aligned}\log_2(x^2 - 8) &= \log_2 x + \log_2 4^{\frac{1}{2}} \\ \log_2(x^2 - 8) &= \log_2 2x \\ (x^2 - 8) &= 2x \\ x^2 - 2x - 8 &= 0 \\ (x + 2)(x - 4) &= 0 \\ \boxed{\cancel{x = -2} \text{ or } x = 4}\end{aligned}$$

Alternative Solution

$$\begin{aligned}\log_2(x^2 - 8) &= \log_2 x + \log_2 4^{\frac{1}{2}} \\ \log_2(x^2 - 8) &= \log_2 2x \\ \log_2(x^2 - 8) - \log_2 2x &= 0 \\ \log_2 \frac{(x^2 - 8)}{2x} &= 0 \\ \frac{(x^2 - 8)}{2x} &= 2^0 \\ (x^2 - 8) &= 2x \\ x^2 - 2x - 8 &= 0 \\ (x + 2)(x - 4) &= 0 \\ \boxed{\cancel{x = -2} \text{ or } x = 4}\end{aligned}$$

Commentary on Response

Students had difficulty correctly applying the properties of logs.

Common Errors

- Students dropped logs immediately: $\log_2(x^2 - 8) = \log_2 x + \frac{1}{2}\log_2 4$
becomes: $x^2 - 8 = x + 2$.
- Students changed $\log_2 x + \frac{1}{2}\log_2 4$ to $\frac{1}{2}\log_2 4x$.
- Students added the arguments rather than multiplying, giving
 $\log_2(x^2 - 8) = \log_2 x + 2$.
- Students failed to reject the negative solution.

Value

4 56. Solve for x : $2(2)^{2x} - 5(2^x) - 12 = 0$.

$$\begin{aligned} \text{Let } y &= 2^x \\ 2y^2 - 5y - 12 &= 0 \\ (2y + 3)(y - 4) &= 0 \\ y &= -\frac{3}{2} \text{ or } y = 4 \end{aligned}$$

$$\begin{aligned} \text{Alternative Solution} \\ (2(2^x) + 3)((2^x) - 4) &= 0 \\ \cancel{2^x = -\frac{3}{2}} \text{ or } 2^x &= 4 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} \cancel{2^x = -\frac{3}{2}} \text{ or } 2^x &= 4 \\ x &= 2 \end{aligned}$$

Commentary on Response

This question was done well.

Common Errors

- Students factored $2y^2 - 5y - 12$ incorrectly.
- Students made mistakes when using the quadratic formula to solve the quadratic.
- Students solved for “ y ” correctly, but did not substitute back 2^x to find the value of “ x ”.
- Students did not reject $2^x = -\frac{3}{2}$.

4 57. An element has a half-life of 120 years. If its initial mass is 42 grams, algebraically determine how long it will take to decrease to 5 grams.

$$\begin{aligned} y &= a \cdot b^{\frac{t}{h}} \\ y &= 42 \left(\frac{1}{2}\right)^{\frac{t}{120}} & 5 &= 42 \left(\frac{1}{2}\right)^{\frac{t}{120}} \\ & & \frac{5}{42} &= \left(\frac{1}{2}\right)^{\frac{t}{120}} \\ \log\left(\frac{5}{42}\right) &= \frac{t}{120} \log\left(\frac{1}{2}\right) \end{aligned}$$

$$-0.9243 \doteq \frac{t}{120} (-0.3010)$$

$$\frac{-0.9243}{-0.3010} \doteq \frac{t}{120}$$

$$3.0708 \doteq \frac{t}{120}$$

$$t \doteq 368.5 \text{ years}$$

$$\begin{aligned} \text{Alternative Solution} \\ 5 &= 42 \left(\frac{1}{2}\right)^{\frac{t}{120}} \\ \log 5 &= \log 42 + \frac{t}{120} \log \frac{1}{2} \\ \frac{\log 5 - \log 42}{\log \frac{1}{2}} &= \frac{t}{120} \\ t &\doteq 368.5 \text{ years} \end{aligned}$$

Commentary on Response

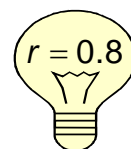
This question was answered well. However, some students provided unreasonable solutions (e.g., a negative number of years). In an advanced course, they should be able to determine what is realistic in the context of this question.

Common Errors

- Students multiplied 42 by $\frac{1}{2}$ to get $5 = (21)^{\frac{t}{120}}$.
- Students subtracted 5 from 42 giving $37 = 42\left(\frac{1}{2}\right)^{\frac{t}{120}}$.
- Students substituted 5 in for t .

- 4 58. In a stadium, air temperature 5°C , the temperature of hot chocolate is recorded in the table shown. Determine the equation of the exponential function that models the temperature of the hot chocolate as it relates to time and find the temperature of the hot chocolate after 45 minutes.

Time(min)	0	5	10	15	20
Temp($^{\circ}\text{C}$)	45	37	30.6	25.48	21.384
Adj.Temp	40	32	25.6	20.48	16.384



$$T(t) = 40(0.8)^{\frac{t}{5}} + 5$$

$$T(45) = 40(0.8)^{\frac{45}{5}} + 5 = 10.4^{\circ}\text{C}$$

Commentary on Response

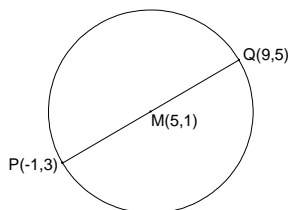
Most students knew something about this question and were able to earn partial marks though not many scored full marks.

Common Errors

- used regression on calculator to get best possible match but in the form of $y = ab^x$.
- Students forgot to use $\frac{t}{5}$.
- Students forgot to subtract 5 from Temperature (not adjusting the table for room temperature) to start process.
- Students did not recognize illogical answer (ex: -300°C).

- Students used 45 as start temperature: $45(0.8)^{\frac{t}{5}}$.
- Students subtracted 5 on end of process: $45(0.8)^{\frac{t}{5}} - 5$.
- Students forgot to find temperature at 45 minutes.

- 3 59. The endpoints of a diameter of a circle are P(1, 3) and Q(9,5). Find the equation of the circle in standard form.



$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

$$M = \left(\frac{9 + 1}{2}, \frac{5 + 3}{2} \right)$$

$$M = (5, 1)$$

$$(x - 5)^2 + (y - 1)^2 = r^2$$

$$(9 - 5)^2 + (5 - 1)^2 = r^2$$

$$(4)^2 + (4)^2 = r^2$$

$$32 = r^2$$

$$\boxed{(x - 5)^2 + (y - 1)^2 = 32}$$

Commentary on Response

This question not as well done as it should have been for advanced math students.

Common Errors

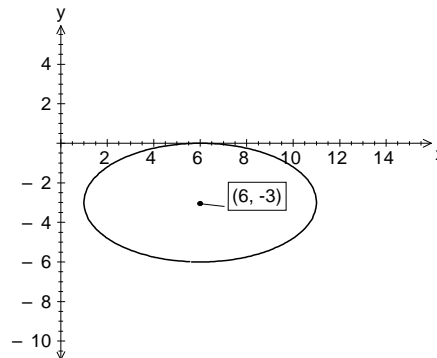
- Students did not know (or did not know how to use) the midpoint formula and/or distance formula.
- Students used diameter instead of radius.
- Students did not state answer in standard form.

Value

4

60. The centre of an ellipse is (6, -3), the major horizontal axis is 10 units long and the minor vertical axis is 6 units long. Find its equation in transformational form and sketch the graph of the ellipse.

$$\left[\frac{1}{5}(x-6)\right]^2 + \left[\frac{1}{3}(y+3)\right]^2 = 1$$



Commentary on Response

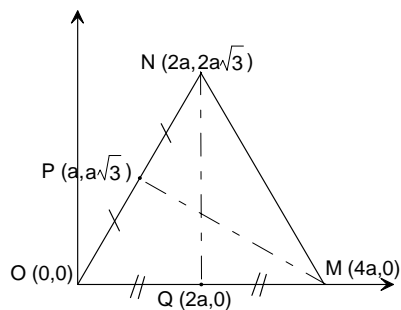
This question was well done.

Common Errors

- Students completed wrong form for equation.
- Students made slight mistakes in form for equation
(e.g., $\frac{1}{5}(x-6)^2 + \frac{1}{3}(y+3)^2 = 1$).
- Students did not apply vertical and horizontal correctly.
- Students did not set equal to 1; and used 0 instead
(e.g., $\left[\frac{1}{5}(x-6)\right]^2 + \left[\frac{1}{3}(y+3)\right]^2 = 0$).
- Students had no labels on grid.

Value

- 4 61. Given $\triangle MNO$ as illustrated, prove median \overline{MP} is equal in length to median \overline{NQ} .



$$d_{MP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{MP} = \sqrt{(4a - a)^2 + (0 - a\sqrt{3})^2}$$

$$d_{MP} = \sqrt{(3a)^2 + (-a\sqrt{3})^2}$$

$$d_{MP} = \sqrt{9a^2 + 3a^2}$$

$$d_{MP} = \sqrt{12a^2}$$

$$d_{MP} = 2a\sqrt{3}$$

$$d_{NQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{NQ} = \sqrt{(2a - 2a)^2 + (2a\sqrt{3} - 0)^2}$$

$$d_{NQ} = \sqrt{(0)^2 + (2a\sqrt{3})^2}$$

$$d_{NQ} = \sqrt{4a^2(3)}$$

$$d_{NQ} = \sqrt{12a^2}$$

$$d_{NQ} = 2a\sqrt{3}$$

Since, $d_{MP} = d_{NQ}$, medians are equal.

Commentary on Response

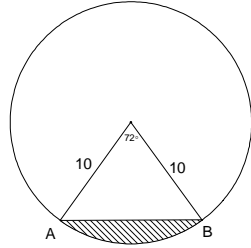
This question was well done but the vertex $N(2a, 2a\sqrt{3})$ was often mistaken.

Common Errors

- Students made errors in use or form of midpoint formula and/or distance formula.
- Students computed incorrectly $(3a)^2$ or $(-a\sqrt{3})^2$.
- Students computed incorrectly $\sqrt{12a^2}$.
- Students didn't state the medians were equal.

Value

- 4 62. \overline{AB} is a chord of a circle with radius 10 cm. If $m\widehat{AB} = 72^\circ$, find the area of the region enclosed by \overline{AB} and \widehat{AB} .



$$Area_{\Delta} = \frac{1}{2}(10)(10) \sin 72^\circ \doteq 47.6 \text{cm}^2$$

$$Area_{\text{O}} = \pi r^2 = \pi(10)^2 \doteq 314.16 \text{cm}^2$$

$$Area_{\text{sector}} = \frac{72}{360} \pi r^2 = \frac{72}{360} \cdot 314.16 \text{cm}^2 \doteq 62.83 \text{cm}^2$$

$Area_{\text{shaded region}} \doteq 62.83 \text{cm}^2 - 47.6 \text{cm}^2 \doteq 15.23 \text{cm}^2$
--

Commentary on Response

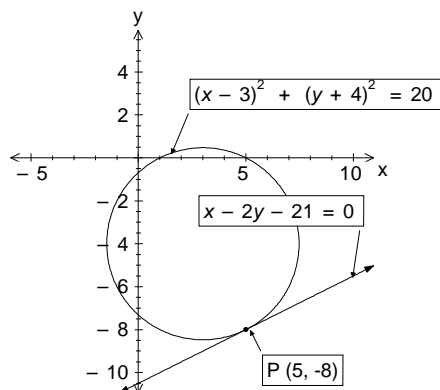
This question was relatively well done.

Common Errors

- Students thought the chord was 10cm and not the radius.
- Students used the central angle as double or half of the arc.
- It appeared that some students used their calculator in radian mode.
- Students didn't know how to find area of Δ .
- Students used the incorrect formula for area of sector and Δ .
- Students used the wrong trig. function to find height of Δ .
- Students found the area of half of the Δ .

Value

- 4 63. Prove that the line with equation $x - 2y - 21 = 0$ is tangent to the circle with equation $(x - 3)^2 + (y + 4)^2 = 20$ at the point $(5, -8)$.



Alternative Solution:

Show that the point $(5, -8)$ is on the line & circle:

$$\begin{array}{ll} (5-3)^2 + (-8+4)^2 = 20 & 5 - 2(-8) - 21 = 0 \\ 20 = 20 & 0 = 0 \end{array}$$

Slope of Line

$$\begin{aligned} y &= \frac{1}{2}x + \frac{21}{2} \\ m &= \frac{1}{2} \end{aligned}$$

Slope of radius

$$\begin{aligned} m &= \frac{-8-(-4)}{5-3} \\ m &= -2 \end{aligned}$$

Slope of line is negative reciprocal of slope of radius.
 \therefore The line is tangent to the circle at $(5, -8)$.

Intersection of line and circle:

$$\begin{aligned} x - 2y - 21 &= 0 \\ \therefore x &= 2y + 21 \end{aligned}$$

$$(2y + 21 - 3)^2 + (y + 4)^2 = 20$$

$$(2y + 18)^2 + (y + 4)^2 = 20$$

$$4y^2 + 72y + 324 + y^2 + 8y + 16 = 20$$

$$5y^2 + 80y + 320 = 0$$

$$y^2 + 16y + 64 = 0$$

$$(y + 8)^2 = 0$$

$$y = -8, \quad x = 5$$

Line intersects circle at one point only!... $(5, -8)$

$\therefore x - 2y - 21 = 0$ is tangent to $(x - 3)^2 + (y + 4)^2 = 20$.

Commentary on Response

This question was not well done.

Common Errors

- Students didn't show that the point $(5, -8)$ was on the circle or the line.
- Students didn't show that the radius and tangent had negative reciprocal slopes.

**MATHEMATICS 3205
PART 1
SELECTED - RESPONSE ITEM ANALYSIS**

Item	Responses				
	Multiple Answers or No Response	A	B	C	D
	%	%	%	%	%
1. Correct answer is B	.1	2.8	92.0	.9	4.1
2. Correct answer is A	.1	73.1	23.0	1.6	2.2
3. Correct answer is D	.3	6.6	4.3	21.3	67.6
4. Correct answer is D	.1	12.2	18.5	2.0	67.1
5. Correct answer is A	.1	62.6	11.5	15.7	10.0
6. Correct answer is D	.6	12.8	5.8	12.7	68.0
7. Correct answer is D	.1	6.5	7.3	4.9	81.2
8. Correct answer is D	.9	17.6	6.3	7.4	67.8
9. Correct answer is C	.1	7.1	21.6	68.7	2.4
10. Correct answer is D	.2	14.6	1.2	3.6	80.5
11. Correct answer is B	.1	3.3	90.0	4.1	2.5
12. Correct answer is A	.7	71.3	10.7	11.9	5.5
13. Correct answer is A	.1	63.1	20.9	5.6	10.3
14. Correct answer is D	.2	.7	2.5	4.1	92.5
15. Correct answer is B	.6	9.8	62.5	11.4	15.6
16. Correct answer is C	.5	5.5	10.3	77.1	6.7
17. Correct answer is B	.1	.4	90.4	4.9	4.1
18. Correct answer is C	.1	3.4	1.2	93.1	2.3
19. Correct answer is A	.4	42.1	19.9	19.4	18.2
20. Correct answer is B	.1	5.5	86.5	1.5	6.5
21. Correct answer is B	.1	11.9	66.7	11.7	9.7
22. Correct answer is C	.4	7.6	9.8	72.1	10.0
23. Correct answer is C	.1	5.7	13.0	77.4	3.9
24. Correct answer is B	.1	5.6	90.1	3.2	1.0
25. Correct answer is B	.2	20.5	66.5	7.8	5.1

**MATHEMATICS 3205
PART 1
SELECTED - RESPONSE ITEM ANALYSIS**

Item	Responses				
	Multiple Answers or No Response	A	B	C	D
	%	%	%	%	%
26. Correct answer is D	.1	5.5	6.2	4.9	83.2
27. Correct answer is B	.3	23.1	45.4	7.9	23.2
28. Correct answer is B	.3	3.3	89.5	2.8	4.1
29. Correct answer is C	.2	6.0	3.2	87.4	3.2
30. Correct answer is D	.1	4.5	4.1	1.2	90.1
31. Correct answer is A	.3	79.0	9.4	4.1	7.2
32. Correct answer is B	.1	19.6	66.1	6.8	7.4
33. Correct answer is C	1.0	12.9	27.7	47.8	10.6
34. Correct answer is A	.1	88.9	6.0	1.7	3.3
35. Correct answer is C	.4	28.5	11.9	31.1	28.1
36. Correct answer is B	.2	31.3	62.9	2.5	3.1
37. Correct answer is C	.1	5.1	7.7	79.5	7.6
38. Correct answer is C	.1	.8	2.4	94.6	2.1
39. Correct answer is A	.2	77.9	9.9	7.4	4.7
40. Correct answer is A	.1	83.1	8.0	4.1	4.7
41. Correct answer is A	.3	86.0	8.9	4.0	.9
42. Correct answer is C	.3	29.8	3.4	62.9	3.6
43. Correct answer is D	.2	6.8	3.6	26.2	63.1
44. Correct answer is D	.4	6.0	21.8	23.4	48.3
45. Correct answer is C	.3	18.8	26.6	47.9	6.5
46. Correct answer is C	.2	26.3	16.4	46.6	10.5
47. Correct answer is B	1.1	17.4	42.2	20.2	19.1
48. Correct answer is A	.1	30.1	1.7	66.7	1.4
49. Correct answer is A	.9	41.9	13.8	33.7	9.6
50. Correct answer is A	.8	58.6	14.8	10.0	15.7

**MATHEMATICS 3205
PART II
CONSTRUCTED - RESPONSE ITEM ANALYSIS**

Item	Students Completing Item	Value	Average	Average % Per Item
PART II		50		
51	1355	4	2.92	73.0
52	1355	4	2.51	62.8
53	1355	4	3.09	77.3
54(a)	1355	2	1.72	86.0
54(b)	1355	2	.93	46.5
55	1355	3	1.84	61.3
56	1355	4	3.20	80.0
57	1355	4	3.20	80.0
58	1355	4	2.423	60.6
59	1355	3	2.11	70.3
60	1355	4	3.29	82.3
61	1355	4	3.01	75.3
62	1355	4	2.67	66.8
63	1355	4	1.58	39.5

**Math 3205
June 2004**

