

Math 3205
Grading Standards
June 2005

Pre-Marking Appraisal

The board considered the exam fair and of the appropriate difficulty and length. Item #32 had no answer and was therefore dropped from the exam (i.e., exam marked out of 99). For item #62, the use of the word ‘a’ instead of ‘any’, allowed students to receive full marks if they proved a right triangle (numerical or variables).

Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 40 papers that went through the marker panel and marks were assigned to each question on a separate sheet of paper. The 40 exams were put back into the original stack of exams and corrected again when they appeared. The two values were compared and if there were discrepancies, the chief marker would review the scoring with the individual marker.

Throughout the marking process there was statistical analysis run on item data to enhance reliability and consistency of marking.

Commentary on Responses

Students did very well with most questions on the exam, however questions #52 and #60 were not well done by many students.

PART 11
Total Value: 50%

Instructions: Answer ALL items in the space provided. Show ALL workings.

Value

- 4 51. Algebraically determine the **EXACT** roots in simplest form for $-\frac{3}{x} = 3 - \frac{x}{x+1}$.

$$\begin{aligned} \frac{x}{x+1} &= 3 + \frac{3}{x} & x &= \frac{-6 \pm \sqrt{36 - 24}}{2(2)} \\ x(x+1) \left[\frac{x}{x+1} = 3 + \frac{3}{x} \right] & & x &= \frac{-6 \pm \sqrt{12}}{4} \\ x^2 - 3x &= 3x^2 + 3x + 3 & x &= \frac{-6 \pm 2\sqrt{3}}{4} = \boxed{\frac{-3 \pm \sqrt{3}}{2}} \\ 0 &= 2x^2 + 6x + 3 & & \end{aligned}$$

Commentary on Response

This question was well done.

Common Errors

- Students made the following errors;
 $(x)(x+1) \cdot \frac{-3}{x} = 3(x)(x+1) - (x)(x+1) \cdot \frac{x}{x+1}$
 $-3x + 1, \quad = 3x^2 + 3, \quad "+" \quad x^2$
Distributive law error leave out x sign error
- Students did not simplify radical (e.g., $\frac{-6 \pm \sqrt{12}}{4}$).
- Students did not reduce $\frac{-6 \pm 2\sqrt{3}}{4}$.
- Students did not use common denominator.

52. A theatre seats 400 people per show and is currently sold out with a ticket price of \$10. A survey shows that for every \$1 per ticket price increase, 25 fewer tickets will be sold. Write a function to model this situation and use this function to determine the ticket price that will result in the greatest revenue per show.

Let $x = \#$ of \$1 price increases

$R(x) = \text{Revenue}$

$$\begin{aligned} R(x) &= (10 + x)(400 - 25x) \\ &= 4000 - 250x + 400x - 25x^2 \\ &= -25x^2 + 150x + 4000 \end{aligned}$$

Max. occurs where...

$$x = \frac{-b}{2a} = \frac{-150}{-50} = 3$$

Maximum revenue occurs after
3 one dollar price increases for a price of \$13.

Commentary on Response

Question was done very well or very poorly by students.

Common Errors

- Students used $R(x) = (400 - 25x)(10 - x)$ or $(4000 - 25x)(10x)$.
- Students found the vertex (3,4225) but stated the ticket price is \$3 instead of \$13.
- Students took $R(x) = -25x^2 + 150x + 4000$ and divided 'through' by -25 producing $R(x) = x^2 - 6x - 160$.

Value

4 53.

A flare is fired as a distress signal and its height, h in metres above the ground, t seconds after firing, is provided in the table below. Algebraically determine the quadratic function that defines the height of the flare above the ground t seconds after firing, and use it to determine the flare's height at 2.4 seconds.

t	0	1	2	3	4
h	2	97	182	257	322

$$h(t) = at^2 + bt + c$$

$$97 = -5(1)^2 + b(1) + 2$$

$$97 = -5 + b + 2$$

$$2 = a(0)^2 + b(0) + c$$

$$100 = b$$

$$c = 2$$

$$\therefore h(t) = at^2 + bt + 2$$

$$\boxed{h(t) = -5t^2 + 100t + 2}$$

$$d_1 = \{95, 85, 75, 65\}$$

$$d_2 = \{-10, -10, -10, -10\}$$

$$h(2.4) = -5(2.4)^2 + 100(2.4) + 2$$

$$\therefore 2a = -10, \quad a = -5$$

$$\boxed{h(2.4) = 213.2 \text{ m}}$$

$$h(t) = -5t^2 + bt + 2$$

Commentary on Response

This question was well done.

Common Errors

- Students used (97, 1) & (182, 2), (i.e., reversed ordered pair).
- Students used $D_2 = 10$ instead of -10.

Value

- 2 54(a) The power P , in watts, supplied to a circuit by a battery is given by the formula $P = 6I - 0.5I^2$, where I is the current in amperes. What is the approximate instantaneous rate of change of power when the current is 4 amperes?

Inst. RoC:

$$\frac{P(4.1) - P(3.9)}{4.1 - 3.9} = \frac{16.2 - 15.8}{0.2} = \frac{0.4}{0.2} = 2$$

About 2 watts per ampere.

Commentary on Response

This question was well done.

Common Errors

- Students used $\frac{P(4.1)+P(3.9)}{2}$.

- 2 54(b) A flashlight projects a circular image with a radius of 10 cm. As the flashlight is moved away from the wall, the radius of the circular image increases at a rate of 2.5 cm/sec. Write the function used to determine the area of the circular image at a given instant.

$$A(r) = \pi r^2$$

$$r = 10 + 2.5t, \text{ t in seconds}$$

$$A(t) = \pi(10 + 2.5t)^2, \text{ t in seconds, A in cm}^2$$

Commentary on Response

This question was well done.

Common Errors

- Students did not know the correct area formula, (e.g., $A = 2\pi r$, $A = 2\pi r^2$ or $A = \frac{4}{3}\pi r^3$).
- Students used $r = (10 - 2.5t)$.
- Students used $r = (10t + 2.5)$.

Value

3 55. Solve: $\log(x-2)^2 - \log(x+3.6) = 1$

$$\frac{(x-2)^2}{(x+3.6)} = 10$$

$$x^2 - 4x + 4 = 10x + 36$$

$$x^2 - 14x - 32 = 0$$

$$(x-16)(x+2) = 0$$

$$x = 16 \text{ or } \cancel{x = -2}$$

Therefore, $x = 16$

Commentary on Response

This question was poorly done. Students struggled with the laws of logarithms.

Common Errors

- Students didn't reject the $x = -2$ because they substituted it back in and squared before taking the log.
- Students divided the logs instead of the arguments, (e.g., $\frac{\log(x-2)^2}{\log(x+3.6)} = 1$), and then ignored the logs, further compounding the errors.
- Students ignored the logs right away (i.e., $\cancel{\log}(x-2)^2 - \cancel{\log}(x+3.6) = 1$).
- Students made the following algebraic errors;
$$\frac{(x^2-4x+4)}{x+3.6} = x^2 - 5x + 0.4$$

or $x^2 - 4x + 4 = 10x + 36$ to $x^2 + 6x - 32 = 0$

- Students incorrectly raised both sides as a power of 10
 $\cancel{\log}(x-2)^2 = \cancel{\log}(x+3.6) + 1$ then cancelling off the logs.
- Students miscopied, (e.g.,
 $(x-2)$ as $(x+2)$ and $(x+3.6)$ as $(x-3.6)$)

Value

4 56. Solve: $9^{x+y} = 3$

$$4^{x-y} = \left(\frac{1}{2}\right)^{-3}$$

$$\begin{array}{l} 3^{2x+2y} = 3^1 \\ 2^{2x-2y} = 2^3 \end{array} \quad \begin{array}{l} \left\{ \begin{array}{l} 2x + 2y = 1 \\ 2x - 2y = 3 \end{array} \right. \\ \hline 4x = 4 \\ \boxed{x = 1} \\ \boxed{y = -\frac{1}{2}} \end{array}$$

Commentary on Response

This question was answered fairly well, however many students didn't take advantage of adding the equations to simplify the system. They used substitution method. Many students just wrote the system of equations but did not try to solve it.

Common Errors

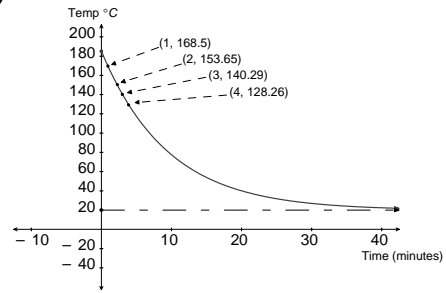
- Students didn't use distributive property in the powers and just used
 $2x + y = 1$
 $2x - y = 3$
- Students wrote 9 as 3^3 and got $3x + 3y = 1$.
- Students wrote 1st equation as $2x + 2y = 0$, taking 3 as 3^0 .
- Students took the 1st equation and solved it for x and y $\left[\begin{array}{l} x = \frac{1}{2} - y \\ y = \frac{1}{2} - x \end{array} \right]$ [doing the same for the second equation].

Value

4 57.

A roast turkey is removed from an oven and left on a countertop to cool. The graph of its temperature is shown below. What is the equation describing the relationship between temperature and time?

time	temp	adj. temp
1	168.5	148.5
2	153.65	133.65
3	140.29	120.29
4	128.26	108.26



$$b = \frac{133.65}{148.5} = 0.9$$

$$a = \frac{148.5}{0.9} = 165$$

$a=165$, so equation of column 1 vs. column 3:

$C = 165b^t$, using (1, 148.5) gives

$$148.5 = 165b^1$$

$$b = 0.9$$

$$\therefore C = 165(0.9)^t + 20$$

Commentary on Response

Many students didn't get full credit because they couldn't determine "a" (frequent guessing)

Common Errors

- Students did not adjust the table before finding "b".
- Students added 20 to the table rather than subtracting 20.
- Students estimated y - int. as 180, 182 or 185 then - 20 to find "a".
- Students used common differences and wrote as a quadratic.
- Students couldn't find 'a' by dividing 148.5 by 0.9.
- Students found ratio $\frac{t_1}{t_2}$ instead of adjusted values.

Value

3

58. Joan purchases a car for \$20 000 and it depreciates at 18% per year. Dan puts \$5000 in a savings account that increases in value at a rate of 4% compounded annually. How long will it take for the car and the savings account to be equal in value?

$$\text{Car} = 20000(0.82)^x \quad \text{Savings} = 5000(1.04)^x$$

$$20000(0.82)^x = 5000(1.04)^x$$

$$4(0.82)^x = (1.04)^x$$

$$\log 4(0.82)^x = \log(1.04)^x$$

$$\log 4 + x \log(0.82) = x \log(1.04)$$

$$\log 4 = x(\log 1.04 - \log 0.82)$$

$$0.602 = 0.1032x$$

$$\boxed{5.8} = x \quad \text{They will be the same value in about 5.8 years.}$$

Commentary on Response

- This question was relatively well done.

Common Errors

- Students used (0.18) instead of (0.82)
- Students made the following division error $\left(\frac{20000}{5000}\right) \Rightarrow 4000$ or 40.
- Students used $\frac{\log 1.04}{\log 0.82}$ instead of $\log\left(\frac{1.04}{0.82}\right)$.
- Students multiplied $4(0.82)$.
- Students used $\log 4(0.82)^x = \log 4 \cdot \log 0.82^x$ instead of $\log 4 + \log 0.82^x$.

Value

4

59. An elliptical pool can be described by the equation $16x^2 + 9y^2 - 96x + 90y + 225 = 0$. A rope is secured at each end of the longer axis. What is the length of the rope?

$$16(x^2 - 6x + 9) + 9(y^2 + 10y + 25) = -225 + 144 + 225$$

$$16(x - 3)^2 + 9(y + 5)^2 = 144$$

$$\frac{(x-3)^2}{9} + \frac{(y+5)^2}{16} = 1$$

$$\left[\frac{1}{3}(x-3)\right]^2 + \left[\frac{1}{4}(y+5)\right]^2 = 1$$

The rope is 8 units long.

Commentary on Response

This question was well done.

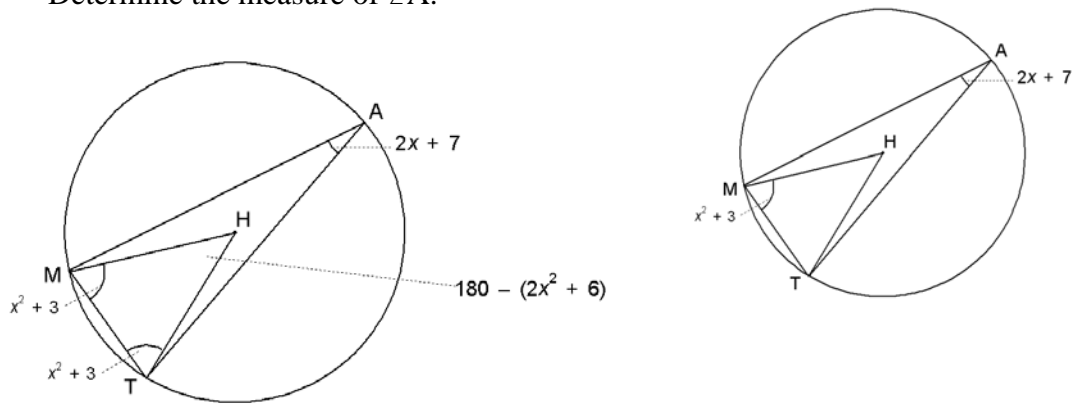
Common Errors

- Students did not have a negative with 225 when transposing.
- Students added 9 and 25 to right side instead of 144 and 225.
- Students did not take $\sqrt{16}$ thus getting 32 for rope length.
- Students made the following arithmetic error, $9y^2 + 90y \rightarrow 9(y^2 + 9y)$.
- Students forgot to answer the “length of rope” question.

Value

3

60. The points M, A, and T are on the circumference of a circle with centre H. Determine the measure of $\angle A$.



$$\begin{aligned}\angle MHT &= 180 - (x^2 + 3) - (x^2 + 3) \\ &= 174 - 2x^2\end{aligned}$$

$$2\angle A = \angle MHT$$

$$2(2x + 7) = 174 - 2x^2$$

$$2x^2 + 4x - 160 = 0$$

$$2(x + 10)(x - 8) = 0$$

$$x = -10 \text{ or } \boxed{x = 8} \quad \text{So, } \angle A = 2(8) + 7 = 23^\circ$$

Commentary on Response

This question was poorly done and often left out.

Common Errors

- Students used the sum of \angle 's of $\Delta = 360^\circ$ instead of 180° .
- Students indicated $m\angle H = 2(x^2 + 3)$.
- students assumed $\angle AMT$ and $\angle ATM$ are 90° \angle 's .
- Students used $m\angle H = \frac{1}{2}m\angle A$ instead of $m\angle H = 2m\angle A$.
- students had sign errors in algebra.

Value

4

61. Determine the equation of the perpendicular bisector of the line segment joining the centres of: $(x+3)^2 + (y-5)^2 = 1$ and $\left[\frac{1}{2}(x-1)\right]^2 + \left[\frac{1}{3}(y-1)\right]^2 = 1$.

Centres $(-3,5)$ and $(1,1)$

Slope of segment through centres is $m = \frac{1-5}{1-(-3)} = \frac{-4}{4} = -1$

Slope of \perp to this segment is 1.

Midpoint of segment through centres is $\left(\frac{-3+1}{2}, \frac{5+1}{2}\right) = (-1,3)$

So \perp bisector has slope 1 and contains the midpoint $(-1,3)$.

$y = 1x + b$ using $(-1,3)$

$$3 = 1(-1) + b$$

$$3 = -1 + b$$

$$4 = b \quad \boxed{\text{Equation is } y = x + 4}$$

Commentary on Response

This question was relatively well done.

Common Errors

- Students found the equation of the line segment joining the centres of the circles, rather than the perpendicular bisector.
- Students did not take the negative reciprocal slope for the perpendicular bisector.
- Students had incorrect centres (i.e., opposite signs).

Value

4

62. Using coordinate geometry, prove that the midpoint of the hypotenuse of a right triangle is equidistant from all vertices.

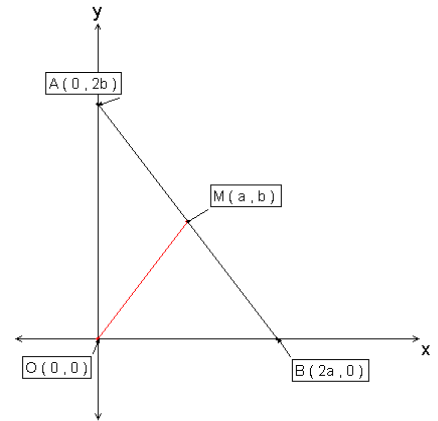
M, Midpoint of hypotenuse is $(\frac{0+2a}{2}, \frac{2b+0}{2}) = (a, b)$

$$d_{AM} = \sqrt{(0-a)^2 + (2b-b)^2} = \sqrt{a^2 + b^2}$$

$$d_{BM} = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

$$d_{CM} = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

Since $AM = BM = CM$, the midpoint of the hypotenuse is equidistant from all three vertices.



Commentary on Response

Because the wording of the question was, “the hypotenuse of a right triangle” instead of “any” right triangle, many students used numbers instead of variables.

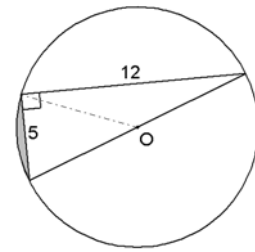
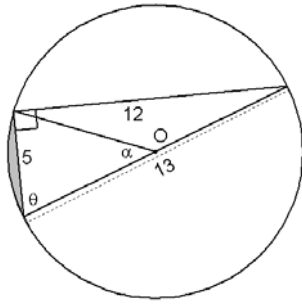
Common Errors

- Students drew the diagram incorrectly.
- Students simplified the radical incorrectly
(e.g., $\sqrt{a^2 + b^2} = a + b$ or $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{a}{2} + \frac{b}{2}$).

Value

4

63. In this circle with centre O, determine the area of the shaded region.



$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$c = 13$$

$$\sin \theta = \frac{12}{13}$$

$$\theta \doteq 67.4^\circ$$

$$\therefore \text{central } \angle \theta \doteq 45.2^\circ$$

$$A_{\text{sector}} = \frac{45.2}{360} \pi (6.5)^2 \doteq 16.7 \text{ units}^2$$

$$A_{\text{triangle}} = \frac{1}{2} (6.5)(6.5) \sin(45.2^\circ) \doteq 15.0 \text{ units}^2$$

$$A_{\text{shaded region}} \doteq 16.7 \text{ units}^2 - 15.0 \text{ units}^2 \doteq 1.7 \text{ units}^2$$

Commentary on Response

This question was fairly well done , however finding the angle caused the greatest difficulty.

Common Errors

- Students calculated the radius to be 7.5 (half of 13 is 6.5).
- Students used an incorrect ratio to find θ .
- Students used sine to find α even though it is not a right triangle.

MATHEMATICS 3205
PART 1
SELECTED - RESPONSE ITEM ANALYSIS

Item				
	A	B	C	D
	%	%	%	%
1. Correct answer is B	1	98.1	0.6	0.3
2. Correct answer is C	8.1	3.6	71.7	16.6
3. Correct answer is C	8.5	2.7	81.9	6.8
4. Correct answer is D	21.2	4.3	10.4	64
5. Correct answer is C	2	0.7	86.7	10.6
6. Correct answer is C	10.4	9.3	68	12
7. Correct answer is B	11	70.8	12.9	5.3
8. Correct answer is A	66.6	24.2	3.8	5.1
9. Correct answer is C	9.2	5.1	73.1	12.1
10. Correct answer is A	67.2	3.1	4.8	24.9
11. Correct answer is C	6.9	5.7	82.3	5.1
12. Correct answer is B	19.8	51.3	15.2	13.1
13. Correct answer is D	5.6	1.3	1	92.2
14. Correct answer is D	10	15.2	26.6	48.1
15. Correct answer is A	88	3.7	5.4	2.7
16. Correct answer is A	78.2	2.5	2.9	16.5
17. Correct answer is A	87.3	5.6	2.2	4.8
18. Correct answer is B	6.1	80.9	11.2	1.8
19. Correct answer is D	3	9.1	3.9	84
20. Correct answer is C	4.3	8.6	59.7	27.4
21. Correct answer is A	96.3	1.4	1.8	0.5
22. Correct answer is B	10.1	63.5	14.6	11.7
23. Correct answer is A	77.2	18.6	1.7	2.5
24. Correct answer is C	1.6	1	66.8	30.6
25. Correct answer is B	22.6	64	7.1	6.2

MATHEMATICS 3205
PART 1
SELECTED - RESPONSE ITEM ANALYSIS

Item				
	A	B	C	D
	%	%	%	%
26. Correct answer is C	17.1	19.7	57.3	5.7
27. Correct answer is C	16.7	5.5	58.4	19.1
28. Correct answer is A	68.5	18.3	7.8	4.9
29. Correct answer is D	0.8	1.2	0.6	97.4
30. Correct answer is D	2.9	6.9	6.9	83.2
31. Correct answer is D	2.3	17.8	3.6	76.3
32. Correct answer is	Had No Solution			
33. Correct answer is B	6.3	73.8	15.6	3.9
34. Correct answer is A	94.5	2.9	1.8	0.8
35. Correct answer is C	3.4	2.9	41.3	52.4
36. Correct answer is A	27.5	2	66.8	3.5
37. Correct answer is A	76	6.9	2.1	15.1
38. Correct answer is B	2.1	85.7	9.7	2.6
39. Correct answer is D	0.4	1.2	4.6	93.8
40. Correct answer is C	30.9	4.6	61.2	3.3
41. Correct answer is C	29.7	8.5	53.4	8.3
42. Correct answer is B	6	86.8	4.5	2.6
43. Correct answer is A	52.4	15.1	13.9	17.9
44. Correct answer is A	76.6	8.5	8.9	5.8
45. Correct answer is B	14.6	52.2	17.5	15.2
46. Correct answer is B	23.5	66.8	4.1	5.2
47. Correct answer is D	6	4	18	71.9
48. Correct answer is C	5.9	2.2	90.4	1.4
49. Correct answer is C	55.1	16.5	24.2	3.7
50. Correct answer is B	8.4	75.8	9.3	5.9

**MATHEMATICS 3205
PART II
CONSTRUCTED - RESPONSE ITEM ANALYSIS**

Item	Students Completing Item	Value	Average	Average % Per Item
PART II		50		
51	3499	4	3.3	82.5
52	3499	4	2.2	55
53	3499	4	3.6	90
54 (a)	3499	2	1.8	90
54 (b)	3499	2	1.4	70
55	3499	3	1.1	36.7
56	3499	4	3.3	82.5
57	3499	4	1.8	45
58	3499	4	3.2	80
59	3499	4	3.3	82.5
60	3499	3	1.7	56.7
61	3499	4	2.5	62.5
62	3499	4	3.3	82.5
63	3499	4	2.7	67.5

