Mathematics 3205 Grading Standards
June 2006

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations.

Post-Marking Report

a) Marking Standard and Consistency

A trial run of 40 randomly selected examinations was scored on a separate back flap with no physical markings on the original examinations. These examinations were re-corrected later in the normal course of marking. The two scores were compared. Any discrepancies were reviewed and discussed with the marker. Discrepancies were rare. In addition, each marker made on-going notes regarding partial marks and scoring schemes for their particular questions. Whenever a non-common error appeared, it was scored by a consensus of the board and made note of for consistency.

b) Summary

Students did well with most questions on the examination. Many students however, performed poorly on questions requiring simplification of radicals or knowledge of the properties of logarithms.

c) Commentary on Responses:

Part II – Constructed Response – Total Value: 50%

4% 51. Algebraically determine the **EXACT** roots in simplest form for $\frac{\sqrt{6}-x}{\sqrt{6x}}=\frac{\sqrt{6x}}{\sqrt{6}+x}$.

Answer:

$$\left(\sqrt{6}+x\right)\left(\sqrt{6}-x\right)=\left(\sqrt{6x}\right)\left(\sqrt{6x}\right)$$ **0.5marks**

$$6-x^2=6x$$ **0.5marks**

$$0=x^2+6x-6$$ **0.5marks**

$$x=\frac{-6\pm\sqrt{6^2-4(1)(-6)}}{2}$$ **0.5marks**

$$x=\frac{-6\pm\sqrt{36+24}}{2}=\frac{-6\pm\sqrt{60}}{2}$$ **0.5marks**

$$x=\frac{-6\pm2\sqrt{15}}{2}$$ **1mark**

$$x=-3\pm\sqrt{15}$$ **0.5marks**

The marking board agreed that students would not lose marks for rejecting $-3-\sqrt{15}$, though the question did not warrant this rejection.

Common Errors

Students:

- used $\sqrt{6x}\cdot\sqrt{6x}=6x^2$.
- did not know the quadratic formula.
- made computational errors in using the correct quadratic formula (e.g., $x=\frac{-6\pm\sqrt{36-4(1)(-6)}}{2}=\frac{-6\pm\sqrt{36-24}}{2}$).

4% 52. Chris purchased a motorboat and used it to travel 6 km upstream and then 6 km back downstream in a total of 4 hours. If the river's current is constant at 2 km/h, algebraically determine an equation which models this situation and use it to find the speed of the boat in still water.

Answer:

	D	v	t
Down	6	$x+2$	$\frac{6}{x+2}$
Up	6	$x-2$	$\frac{6}{x-2}$

$$\frac{6}{x+2}+\frac{6}{x-2}=4$$ **1mark**

$$\cancel{(x+2)}(x-2)\frac{6}{\cancel{x+2}}+(x+2)\cancel{(x-2)}\frac{6}{\cancel{x-2}}=4(x+2)(x-2)$$ **0.5marks**

$$6x-12+6x+12=4x^2-16$$

$$0=4x^2-12x-16$$ **0.5marks**

$$0=x^2-3x-4$$

$$0=(x-4)(x+1)$$ **1mark**

$$\boxed{x=4} \text{ or } \cancel{x=-1}$$ **1mark**

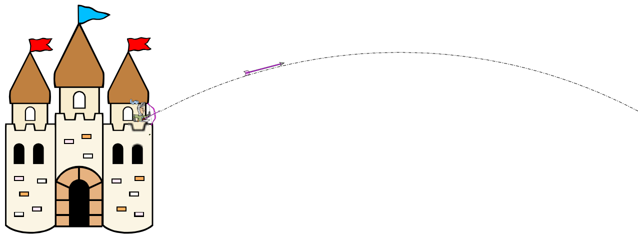
Speed of boat in still water is 4 km/h.

Common Errors

Students:

- did not account for the speed upstream $\left(\text{i.e., } \frac{6}{x+2}+\frac{6}{x}=4\right)$.
- did not account for speed downstream $\left(\text{i.e., } \frac{6}{x-2}+\frac{6}{x}=4\right)$.
- used $t=\frac{v}{d}$ instead of $t=\frac{d}{v}$ $\left(\text{i.e., } \frac{x+2}{6}+\frac{x-2}{6}=4\right)$.

4% 53. An archer atop a turret 30 m above the ground shoots an arrow as shown. The height of the arrow, *h*, in metres above the ground, 1, 2, and 3 seconds after shooting is 45.1 m, 50.4 m, and 45.9 m respectively. Algebraically determine the function that defines the height *h* of the arrow above the ground *t* seconds after shooting.



t	0	1	2	3
h	30	45.1	50.4	45.9

0.5marks $d_2 = -9.8 = 2a$

0.5marks $a = -4.9$

1mark $c = 30$

Answer:

$h = -4.9t^2 + bt + 30$

$45.1 = -4.9(1)^2 + b(1) + 30$

$15.1 + 4.9 = b$

$20 = b$ **1mark**

$\boxed{h = -4.9t^2 + 20t + 30}$ **1mark**

Common Errors

Students:

- assumed the vertex was $(2, 50.4)$.
- substituted incorrect points (i.e., (2, 45.1)).
- made sign errors in subtracting to determine D_2 i.e., $D_2 = 9.8$.

2% 54(a). The volume of water at a given time in a 2000 L tank is represented by the formula $V = 2000\left(1-\frac{t}{45}\right)^2$, where t is time in minutes. Determine the average rate of change in the volume of water in the tank from minute 0 to minute 10, and use it to describe how the volume of water in the tank is changing during that time.

Answer:

$$\text{Average RoC} = \frac{V(10)-V(0)}{10-0}$$

$$= \frac{2000(1-\frac{10}{45})^2 - 2000}{10} \qquad \textbf{0.5marks}$$

$$= \frac{1209.9-2000}{10} \qquad \textbf{0.5marks}$$

$$= \boxed{-79.01 \text{ L/minute.}} \qquad \textbf{0.5marks}$$

On average, the tank is losing 79.01 L of water each minute.

0.5marks

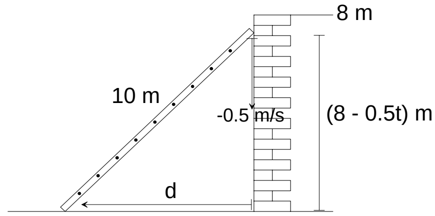
Common Errors

Students:

- did not describe how the volume of water in the tank was changing during the 10-minute interval.
- calculated average rate of change as $\frac{V(10)-V(0)}{10-1}$ versus $\frac{V(10)-V(0)}{10-0}$.
- treated the question as an instantaneous rate of change problem using $\frac{V(10.1)-V(9.9)}{10.1-9.9}$.
- used the reciprocal of the rate of change calculation $\left(\text{i.e., } \frac{10-0}{V(10)-V(0)}\right)$.

2% (b). A 10 m long ladder is leaning against the top of an 8 m high wall. The ladder is slipping down the wall at a rate of 0.5 m/s. Algebraically determine the function that represents the distance, *d*, between the base of the ladder and the wall at any given instant.

Answer:



$a^2 + b^2 = c^2$ **1mark**

$d^2 + (8 - 0.5t)^2 = 10^2$ **1mark**

$d^2 + 64 - 8t + 0.25t^2 = 100$

$d^2 = -0.25t^2 + 8t + 36$

$d = \sqrt{-0.25t^2 + 8t + 36}$ or $d = \sqrt{100 - (8 - 0.5t)^2}$

Common Errors

Students:

- used $6 \pm 0.5t$ for the value of *d*.
- found $d^2 = 100 - (8 - 0.5t)^2$ correctly but failed to take the square root to determine *d*.
- determined that $d^2 = 10^2 - \left[8(0.5t)\right]^2$.
- simplified $d = \sqrt{100 - (8 - 0.5t)^2}$ incorrectly with many simplifying it as $d = \sqrt{0.25t^2 + 8t + 36}$.
- treated the question as an instantaneous rate of change problem using $\dfrac{10(0.5) - 10(0.49)}{0.5 - 0.49}$

4% 55. Algebraically solve: $\log_4(x+7)-\log_4(x^2+3x)=\frac{1}{2}$.

Answer:

$$\log_4(x+7)-\log_4(x^2+3x)=\tfrac{1}{2}$$

$$\log_4\left(\frac{x+7}{x^2+3x}\right)=\frac{1}{2}$$ **1mark**

$$\frac{(x+7)}{(x^2+3x)}=4^{\frac{1}{2}}=2$$ **1mark**

$$x+7=2(x^2+3x)$$

$$x+7=2x^2+6x$$ **1mark**

$$0=2x^2+5x-7$$

$$0=(2x+7)(x-1)$$

$$\boxed{x=-\tfrac{7}{2}} \text{ or } \boxed{x=1}$$ **1mark**

Common Errors

Students:

- rejected the value $x=-\frac{7}{2}$. This value was acceptable in the problem.
- failed to apply laws of logarithms as indicated by the following first approaches:
  $$\frac{\log_4(x+7)}{\log_4(x^2+3x)}=\frac{1}{2} \Rightarrow \frac{(x+7)}{(x^2+3x)}=\frac{1}{2}$$
  or $\frac{(x+7)}{(x^2+3x)}=\frac{1}{2}$
- made algebraic errors in determining the resulting quadratic.
- incorrect factoring or quadratic formula errors in solving $2x^2+5x-7=0$.
- immediately 'dropping' logs incorrectly as follows:
  $(x+7)-(x^2+3x)=2$ where a 2 was determined as $4^{\frac{1}{2}}$ or as $4\left(\frac{1}{2}\right)$.

3% 56. Algebraically solve: $2^{4x}-6(2)^{2x}-16=0$.

Answer:

$(2^{2x})^2-6(2)^{2x}-16=0$

Let $y=2^{2x}$

$y^2-6y-16=0$ **1mark**

$(y-8)(y+2)=0$

$y=8$ or $y=-2$ **1mark**

$2^{2x}=8$ or ~~$2^{2x}=-2$~~ **0.5marks**

$2^{2x}=2^3$

$2x=3$

$\boxed{x=\frac{3}{2}}$ **0.5marks**

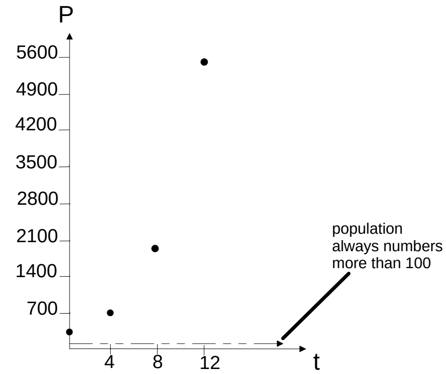
Common Errors

Students:

- did not reject $2^{2x}=-2$ and gave responses such as $2x=-1$ $\left(\text{i.e., } x=-\frac{1}{2}\right)$, or $2x=1$ $\left(\text{i.e., } x=\frac{1}{2}\right)$, as the second value for *x*.
- misapplied logarithms to the original equation. For example, $\log 2^{4x}-6\log 2^{2x}-\log 2^4=0$ and going on again incorrectly to obtain $4x-12x-4=0$.
- changed 16 to 2^4 to produce $2^{4x}-6(2)^{2x}-2^4=0$ and obtained $4x-12x-4=0$.
- solved the quadratic equation $2y^2-6y-16=0$.
- rearranged the equation as $2^{4x}-6(2)^{2x}=16$ and again incorrectly obtained $2^{4x}-(2)^{2x}=\frac{16}{6}$ or $\frac{2^3}{3}$ and equated exponents as $4x-2x=3$.
- solved $2x=3$ as $x=\frac{2}{3}$.
- incorrectly factored $y^2-6y-16$.

4% 57. A bacteria population is well cared for so it always numbers more than 100. An experiment was begun to study how a particular type of light affected the number of bacteria. Upon starting the experiment, the population, P, at time, t, in hours, was recorded in the table and shown on the graph below. Algebraically determine the function that models this population and use the function to determine the time it will take for the bacteria to number 48 700.

t	0	4	8	12
P	300	700	1900	5500



Answer:

period		4 /\		4 /\		4 /\	
t	0		4		8		12
P	300		700		1900		5500
$Adj.P$	200		600		1800		5400
ratio		\/ 3		\/ 3		\/ 3	

0.5marks

$$P = a(b)^{\frac{x}{c}} + d$$

$$a = 200,\ b = 3,\ c = 4,\ d = 100$$

$$\boxed{P = 200(3)^{\frac{x}{4}} + 100}$$

0.5 marks for 200
0.5 marks for 3
0.5 marks for exponent
0.5 marks for 100

$$48700 = 200(3)^{\frac{x}{4}} + 100$$ **0.5marks**

$$48600 = 200(3)^{\frac{x}{4}}$$

$$243 = (3)^{\frac{x}{4}}$$

$$3^5 = (3)^{\frac{x}{4}}$$ **0.5marks**

$$5 = \frac{x}{4}$$

$$20 = x$$ **0.5marks**

The bacteria will number 48 700 in 20 hours.

Common Errors
Students:
- ran the original data through an exponential regression analysis on a calculator without regard for the vertical shift.
- did not adjust for the horizontal asymptote in the table of values before determining the common ratio.
- used 300 as the initial value in the equation **(**after the table had been adjusted).

- did not adjust for the period (i.e. used x as the exponent on 3 instead of $\frac{x}{4}$).
- did not apply the horizontal asymptote value back on once the initial value, base and period were determined.
- determined the model correctly but did not calculate the time for the bacteria to number 48 700.

4% 58. A rich relative gave Pat a gift of $100 000 as a college fund. Pat can withdraw half of the money that remains in the fund once every 5 months. Set up an equation that models the money remaining in the fund and use it to determine, to the nearest 5 months, when the amount remaining first drops below $500.

Answer:

$y = ab^{\frac{x}{c}}$ $\quad a = 100000$ $\quad 500 = 100000\left(\frac{1}{2}\right)^{\frac{x}{5}}$ **1.5marks**

$b = \frac{1}{2}$ $\quad 0.005 = \left(\frac{1}{2}\right)^{\frac{x}{5}}$

$c = 5$ $\quad \log 0.005 = \frac{x}{5}\log 0.5$ **1.5marks**

$y = 500$ $\quad 5 \cdot \frac{\log 0.005}{\log 0.5} = x \doteq 38.2$ **0.5marks**

The fund will have less than $500 left 40 months after Pat starts withdrawing the money.

0.5marks

Alternate solution:

$500 = 100000\left(\frac{1}{2}\right)^{x}$ **1.5 marks**

$0.005 = \left(\frac{1}{2}\right)^{x}$

$\log 0.005 = x \log 0.5$ **1.5 marks**

$x \doteq 7.64$ **0.5 marks**

so first drops below $500 after 8 five month periods. **0.5 marks**

Common Errors

Students:

- multiplied 100 000 by ½ to obtain $500 = (50000)^{\frac{x}{5}}$.
- did not round answer to the nearest 5 months.

4% 59. Algebraically show that the centre of the ellipse defined by $9x^2+4y^2-90x-16y+205=0$ lies on the parabola defined by $y=2x^2-16x+32$.

Answer:

Find the centre of the ellipse.

$9x^2-90x+4y^2-16y=-205$

$9(x^2-10x+\underline{25})+4(y^2-4y+\underline{4})=-205+\underline{225}+\underline{16}$ **1mark**

$9(x-5)^2+4(y-2)^2=36$ **1mark**

$C(5,2)$ **1mark**

Determine whether the centre satisfies the equation of the parabola.

$2\overset{?}{=}2(5)^2-16(5)+32$

$2\overset{?}{=}50-80+32$

$2\overset{\checkmark}{=}2$ **1mark**

Since the centre of the ellipse satisfies the equation for the parabola, the centre of the ellipse does lie on the parabola.

Common Errors

Students:

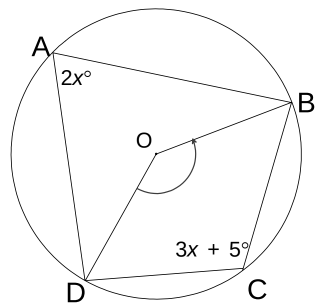
- ignored the effect of the value outside brackets when balancing the perfect squares.
- subtracted 225 and 16 on the RHS instead of adding as on the LHS.
- did not verify that the centre point lies on the parabola.
- when attempting to verify the centre point lies on the parabola, rearranged the equation thus **assuming** that LHS=RHS which is what they were required to demonstrate. For example,

$$2\overset{?}{=}50-80+32$$

$$2-32\overset{?}{=}50-80$$

$$-30\overset{?}{=}-30$$

3% 60. The points A, B, C, and D are on the circumference of the circle with centre O shown. Determine the measure of $\angle DOB$ in degrees.



Answer:

$2x + 3x + 5 = 180$ **1mark**

$5x + 5 = 180$

$5x = 175$

$x = 35°$ **0.5marks**

$2x = 70°$ **0.5marks**

$\boxed{\angle DOB = 140°}$ **1mark**

Common Errors

Students:

- determined the value for x but did not substitute to determine the measure of $\angle DOB$.
- made algebraic errors. For example,

$180 - 3x + 5 = 2x$ instead of $180 - (3x + 5)$

$185 = 5x$

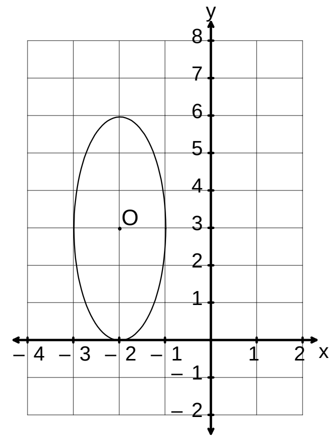
$x = 37$

$2x + 3x + 5 = 180$

$5x = 185$ instead of 175

$x = 37$

4% 61. Determine the equation of the ellipse, in general form, having centre O as shown.



Answer:

Centre $(-2, 3)$

$(x+2)^2 + \left[\tfrac{1}{3}(y-3)\right]^2 = 1$

$(x+2)^2 + \tfrac{1}{9}(y-3)^2 = 1$ **0.5marks**

$9(x^2+4x+4)+(y^2-6y+9)=9$ **0.5marks**

$9x^2+36x+36+y^2-6y+9-9=0$ **0.5marks**

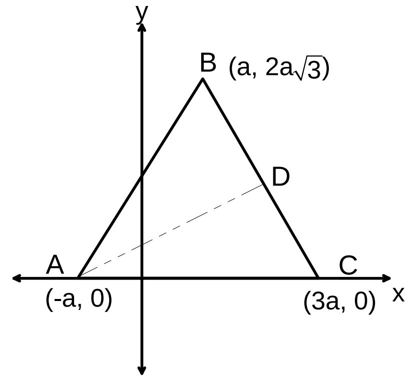
$\boxed{9x^2+y^2+36x-6y+36=0}$ **0.5marks**

$\left[(x+2)^2\right]+\left[\frac{1}{3}(y-3)^2\right]=1$ **2marks**

Common Errors
Students:

- determined transformational form only $\left(\text{i.e.,} (x+2)^2 + \left[\tfrac{1}{3}(y-3)\right]^2 = 1\right)$.
- did not include the vertical stretch factor within the squares $\left(\text{i.e.,} (x+2)^2 + \tfrac{1}{3}(y-3)^2 = 1\right)$.
- multiplied through by 3 and not by 9 when converting to general form.
- multiplied incorrectly through by 9 when converting to general form $\left(\text{e.g.,} (x+2)^2 + \tfrac{1}{9}(y-3)^2 = 1 \text{ leading to } (x+2)^2 + (y-3)^2 = 9\right)$.
- used major and minor axis lengths (instead of semi-major and semi-minor) as the stretch factors.

4% 62. Using coordinate geometry, prove the median from A to $\overline{BC}$ is perpendicular to $\overline{BC}$.



Answer:

$D: \left(2a, a\sqrt{3}\right)$ **1mark**

$m_{AD} = \frac{a\sqrt{3}}{3a} = \frac{\sqrt{3}}{3}$ **1mark**

$m_{BC} = \frac{2a\sqrt{3}}{-2a} = -\sqrt{3}$ **1mark**

$(m_{AD})(m_{BC}) = \frac{\sqrt{3}}{3} \cdot -\sqrt{3} = -1$

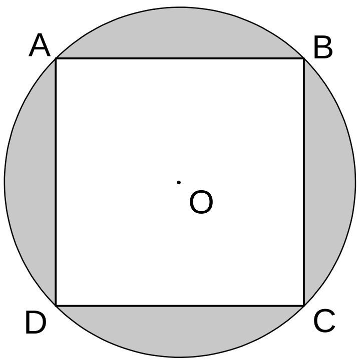
$\boxed{\therefore \overline{AD} \perp \overline{BC}}$ **1mark**

Common Errors

Students:

- used incorrect slope formula $\left(\text{i.e., } m = \frac{x_2 - x_1}{y_2 - y_1}\right)$.
- did not recognize or had difficulty determining that the required slopes were opposite reciprocals $\left(\text{i.e., } m_{\overline{AD}} = \frac{\sqrt{3}}{3} \text{ and } m_{\overline{BC}} = \sqrt{3}\right)$.
- did not make a concluding statement indicating why/how lines were perpendicular.

4% 63. Square ABCD is inscribed in the circle with centre O as shown. If the diameter of the circle is 20 cm, determine the area of the shaded region.



Answer:

$s^2 = 10^2 + 10^2$

$s^2 = 200$

$s = \sqrt{200} = 10\sqrt{2}$ **1mark**

$A_{circle} = \pi(10)^2 \doteq 314.16 \text{ cm}^2$ **1mark**

$A_{square} = \left(10\sqrt{2}\right)^2 = 200 \text{ cm}^2$ **1mark**

$A_{shaded} = 314.16 - 200 \doteq 114.16 \text{ cm}^2$ **1mark**

Common Errors

Students:

- used $r = 20$ as the radius when calculating the area of the circle (the diameter was 20, radius was 10).
- used $r = 20$ as the side length of the square.
- used $\frac{d}{2}$ for the side length of the square (i.e., 10).

TABLE 1
MATHEMATICS 3205 ITEM ANALYSIS
SELECTED RESPONSE (PART I)[1]

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	B	0.7	98.3	0.8	0.2
2	A	73.4	10.8	10.7	5.0
3	C	2.0	1.4	89.5	7.1
4	C	0.2	0.3	94.6	5.0
5	B	1.4	60.1	0.7	37.8
6	B	2.4	78.4	1.8	17.1
7	A	57.7	17.7	14.4	9.8
8	C	5.5	16.0	65.3	12.8
9	D	3.1	9.7	12.0	75.2
10	D	0.9	28.8	1.2	69.1
11	B	1.2	89.9	5.3	3.6
12	B	11.8	69.6	12.6	5.7
13	C	7.0	3.1	82.0	7.5
14	B	3.5	73.9	19.9	2.6
15	C	8.1	8.7	70.4	12.5
16	D	1.8	0.8	6.3	91.1
17	D	0.6	1.0	5.6	92.5
18	D	11.4	2.0	9.3	77.3
19	D	1.5	25.0	0.6	72.8
20	B	1.3	77.6	3.1	18.0
21	A	81.2	17.7	0.5	0.5
22	A	52.0	2.9	40.7	4.2
23	A	47.7	21.5	11.6	19.1
24	B	14.2	66.8	15.0	3.9
25	C	1.0	1.0	57.2	40.7
26	A	85.4	7.5	4.2	2.8
27	A	74.1	12.5	9.8	3.3
28	B	8.5	75.6	10.4	5.3
29	B	0.4	98.0	1.4	0.2
30	C	1.4	8.4	80.1	10.1
31	B	2.0	56.5	25.3	15.9
32	D	2.7	4.2	60.9	31.9
33	D	1.8	7.1	5.9	84.9
34	A	83.9	2.0	7.1	7.0
35	A	79.4	6.5	9.3	4.6

[1] Note: Percentages may not add to 100% due to multiple responses, missing values, or rounding.

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
36	A	62.8	25.0	8.6	3.6
37	A	94.1	2.7	1.5	1.5
38	B	0.7	89.6	2.1	7.6
39	C	2.2	10.1	85.7	1.7
40	C	2.6	5.3	91.5	0.5
41	D	32.1	2.9	10.1	54.8
42	B	2.8	96.3	0.5	0.4
43	B	13.7	63.4	8.8	13.4
44	C	2.2	4.5	87.0	5.9
45	D	3.4	14.2	13.5	68.8
46	A	52.0	11.8	8.5	27.5
47	B	2.3	70.9	20.2	6.4
48	B	2.1	94.3	2.8	0.7
49	C	3.5	4.5	85.1	6.5
50	B	19.7	54.6	21.7	3.5

TABLE 2
MATHEMATICS 3205 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	1151	4	3.15
52	1151	4	1.42
53	1151	4	3.54
54(a)	1151	2	1.69
54(b)	1151	2	0.81
55	1151	4	2.72
56	1151	3	2.19
57	1151	4	2.36
58	1151	4	3.24
59	1151	4	3.27
60	1151	3	1.92
61	1151	4	2.39
62	1151	4	2.67
63	1151	4	3.06