Grading Standards
Mathematics 3205
June 2007

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations.

Post-Marking Report

Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examinations and were held by the Chief Marker for recirculation throughout the marking period. These papers were corrected by the marking board again, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Each marker also made on-going notes regarding partial marks and scoring for their particular question. Whenever a non-common error occurred, it was scored by consensus of the board and made note of, for scoring consistency.

Throughout the marking process there were statistical analysis ran on item data to enhance reliability and consistency of marking.

Summary

Overall performance in the Math 3205 examination improved from June 2006 to June 2007. Many students however, performed poorly on questions requiring simplification of radicals or knowledge of the properties of logarithms.

Part II – Constructed Response – Total Value: 50%

Value

4% 51. Algebraically determine the **exact** roots in simplest form for $\frac{3}{x-5} = 4 - \frac{x}{x+3}$.

Answer:

$3(x+3) = 4(x-5)(x+3) - x(x-5)$ **0.5 mark**

$0 = 3x^2 - 6x - 69$ **1 mark**

$0 = x^2 - 2x - 23$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-23)}}{2(1)}$ **0.5 mark**

$x = \frac{2 \pm \sqrt{96}}{2}$ **1 mark**

$x = \frac{2 \pm 4\sqrt{6}}{2}$ **0.5 mark**

$x = 1 \pm 2\sqrt{6}$ **0.5 mark**

Common Errors

Students:

- did not multiply 4 by the LCD
- reduced $3x + 9 = 3x^2 - 3x - 60$ to $3x^2 - 6x - 51 = 0$
- multiplied $-x(x-5)$ to get $-x^2 - 5x$

Value

4% 52. An orange grower has 400 crates of oranges ready for market and will have 20 more crates each day that shipment is delayed. The present price is $60 per crate; however, for each day shipment is delayed, the price per crate decreases by $2. Write a quadratic function to model the grower's revenue and use it to determine how many days the grower should delay shipment in order to maximize revenue. What is the maximum revenue?

Answer:

$R = (400 + 20x)(60 - 2x)$ **1.5 marks**

$R = -40x^2 + 400x + 24000$ **0.5 mark**

$$x = \frac{-b}{2a}$$

$$x = \frac{-400}{2(-40)}$$

$x = 5$ **1 mark**

$R = -40(5)^2 + 400(5) + 24000$

$R = 25000$ **1 mark**

The maximum revenue is $25 000

Common Errors

Students:

- divided the right side only of $R = -40x^2 + 400x + 24000$ by – 40
- copied 24000 as 2400

Value

4% 53. A soccer player kicks a ball from the ground towards a wall that is 28 m away. The ball reaches a maximum height of 9 m when it is a horizontal distance of 15 m away from the player. At what height from the floor will the ball hit the wall?

Answer:

$$\frac{1}{a}(y-9)=(x-15)^2$$ **0.5 mark**

$$\frac{1}{a}(0-9)=(0-15)^2$$ **0.5 mark**

$$\frac{1}{a}=-25$$ **1 mark**

$$-25(y-9)=(x-15)^2$$

$$y=\frac{-1}{25}(28-15)^2+9$$ **1 mark**

$$y=2.24\text{m}$$ **1 mark**

Commentary on Response
Many students did not attempt this question.

Common Errors
Students assumed that the ball hit the ground at (28, 0).

Value

2% 54. (a) The height of a pebble fired by a sling shot is given by $h(t) = 25t - 4.9t^2$, where $h(t)$ is height in metres and t is time in seconds after the pebble leaves the sling shot. Determine an approximation for the instantaneous rate of change in the height of the pebble at 3 seconds and describe how the height of the pebble is changing at that instant.

Answer:

$$IRoC = \frac{h(3.1) - h(2.9)}{3.1 - 2.9}$$

$$= \frac{30.411 - 31.291}{0.2} \qquad \textbf{1 mark}$$

$$= \frac{-0.88}{0.2}$$

$$= -4.4 \qquad \textbf{0.5 mark}$$

The height is decreasing at a rate of 4.4m/s. **0.5 mark**

Commentary on Response

It is best to maintain at least four places after the decimal when solving rate of change problems. Only the answer should be rounded. Some students did not answer both parts of the question.

Common Errors

Students:

- did not give any concluding statement
- used an interval of 3 to 2 or 4 to 2, which is an average rate of change
- let $rate = \frac{x_2 - x_1}{y_2 - y_1}$

Value
2% 54. (b) A spherical snowball with a diameter of 8 cm is cut in half, and one of the pieces is discarded. Write an expression for the volume of the half snowball remaining if the radius decreases at a uniform rate of 1 mm/min. $\left(V_{\text{sphere}} = \frac{4}{3}\pi r^3\right)$

Answer:

$$V = \frac{1}{2}\left(\frac{4}{3}\pi\left(40-t\right)^3\right)$$

radius 4 cm or 40 mm **0.5 mark**

half circle **0.5 mark**

$$= \frac{2}{3}\pi\left(40-t\right)^3 \ \ or\ \ V = \frac{2}{3}\pi(4-0.1t)^3$$

1 mark

Common Errors

Students:

- did not use half a sphere
- used 0.01t instead of 0.1t
- did not use a variable
- wrote $V = \frac{2}{3}\pi\left(4-0.1t\right)^2$ or $V = \frac{2}{3}\pi\left(4-0.1t\right)$
- used 8 for the radius

Value

3% 55. Algebraically solve the system of equations:

$$(2)^{3x-2y} = 64$$
$$\left(\sqrt{6}\right)^{2x+2y} = 36$$

Answer:

$$2^{3x-2y} = 2^6$$
$$6^{\frac{1}{2}(2x+2y)} = 6^2$$

$3x-2y=6$ **0.5 mark**

$x+y=2$ **1 mark**

$$3x-2y=6$$
$$2x+2y=4$$

$$5x=10$$

$x=2$ **1 mark**

$$x+y=2$$
$$2+y=2$$

$y=0$ **0.5 mark**

Common Errors

Students:

- wrote $\sqrt{6}$ as 6^2
- did not solve for the variable y
- believed that $\sqrt{36}$ had a common base of 6 resulting in the equation $2x+2y=1$
- wrote $(6^2)^{2x+2y}=6^2$

Value

4% 56. Algebraically solve: $\log_2(x+2)+\frac{1}{3}\log_2(8x^3)=16^{\frac{1}{2}}$.

Answer:

$\log_2(x+2)+\log_2(8x^3)^{\frac{1}{3}}=4$	**0.5 mark**
$\log_2(x+2)+\log_2(2x)=4$	
$\log_2 2x(x+2)=4$	**0.5 mark**
$2x(x+2)=16$	**1 mark**
$2x^2+4x-16=0$	**0.5 mark**
$x^2+2x-8=0$	
$(x+4)(x-2)=0$	**0.5 mark**
$x=-4 \quad x=2$	
reject $-4,\ x=2$	**1 mark**

Commentary on Response
Many students did not understand the logarithmic properties required for this question.

Common Errors

Students:

- wrote $(8x^3)^{\frac{1}{3}}=8x$
- did not reject $x=-4$
- did not know how to change from logarithmic form to exponential form
- wrote $\frac{1}{3}\log_2(8x^3)$ as $\frac{8x}{3}$
- multiplied by a common denominator of 3

Value

4% 57. A truck is purchased for $51 000 and depreciates by 23% annually. At the same time a minivan is purchased for $38 000 and depreciates by 17% annually. Write an equation to model this situation and use it to determine when the two vehicles will be of equal value.

Answer:

$$y = 51000(1-0.23)^x \qquad y = 38000(1-0.17)^x$$

$$y = 51000(0.77)^x \qquad y = 38000(0.83)^x$$

$$51000(0.77)^x = 38000(0.83)^x \qquad \textbf{2 marks}$$

$$1.3421 = \left(\frac{0.83}{0.77}\right)^x$$

$$\log 1.3421 = x \log 1.0779$$

$$x = 3.9 \qquad \textbf{2 marks}$$

The two vehicles will have equal value in 3.9 years.

Commentary on Response

Many students did not understand the logarithmic properties and did not using four places after the decimal when working with logarithmic equations.

Common Errors

Students:

- used $51000\left(0.23\right)^x = 38000(0.17)^x$
- wrote $1.34(0.77)^x$ as $x\log 1.34(0.77)$
- wrote $1.34(0.77)^x$ as $(\log 1.34)(x\log 0.77)$
- wrote $1.34(0.77)^x$ as $1.34x\log 0.77$

Value

4% 58. A teacher walks into a classroom of temperature 23 °C with a cup of coffee. The temperature of the coffee is recorded in the table shown. Determine the equation of the exponential function that models the cooling behaviour of the coffee, and use it to determine the temperature of the coffee at the end of a 56 minute class period.

Time (minutes)	0	15	30	45
Temperature (°C)	93	65	48	38

Answer:

Time (minutes)	0	15	30	45	
Temperature (°C)	93	65	48	38	
Temp - VT	70	42	25	15	**1 mark**

Ratio 0.60

$$y = a(b)^{\frac{x}{c}} + d$$

$$y = 70(0.6)^{\frac{x}{15}} + 23$$ **2 marks**

$$y = 70(0.6)^{\frac{56}{15}} + 23$$

$$y = 33.4^0 C$$ **1 mark**

Commentary on Response

Many students had a good understanding of horizontal asymptotes.

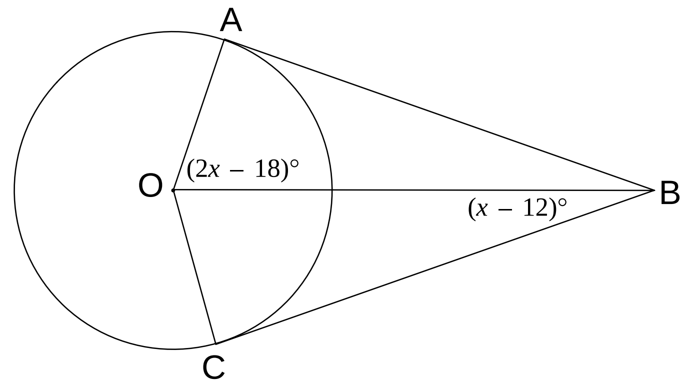
Common Errors

Students:

- did not translate the values in the table
- used the initial value of 93 as the value for a
- divided 65 by 93 to get a common ratio of 0.7

Value

3% 59. $\overline{BA}$ and $\overline{BC}$ are tangents to the circle with centre O shown. If $\angle AOB = (2x - 18)°$ and $\angle CBO = (x - 12)°$, what is the numerical measure of angle $\angle ABO$?



Answer:

$(2x-18)+(x-12)=90$ **1 mark**

$3x-30=90$

$3x=120$

$x=40$ **1 mark**

$m\angle ABO = 40-12$

$= 28^0$ **1 mark**

Common Errors

Students:

- did not find the measure of $\angle ABO$
- found the measure of $\angle AOB$
- wrote $3x-30=90$ as $3x=60$
- used $2x-18+90+x=180$
- wrote $-(2x-18)$ as $-2x-18$

Value

4% 60. Determine the equation of the line joining the centres of the ellipses

$x^2+25y^2-6x-100y+84=0$ and $9(x+1)^2+(y-3)^2=9$.

Answer:

$$\left(x^2-6x+9\right)+25\left(y^2-4y+4\right)=-84+9+100$$

$$\left(x-3\right)^2+25\left(y-2\right)^2=25$$ **1 mark**

$center(3,2)$ **0.5 mark**

center of second ellipse (-1, 3**)** **0.5 mark**

$$m=\frac{3-2}{-1-3}$$

$$=\frac{1}{-4}$$ **0.5 mark**

$$y=\frac{-1}{4}x+b \quad (3,2)$$

$$2=\frac{-1}{4}(3)+b$$

$$b=\frac{11}{4}$$ **1 mark**

$$y=\frac{-1}{4}x+\frac{11}{4}$$ **0.5 mark**

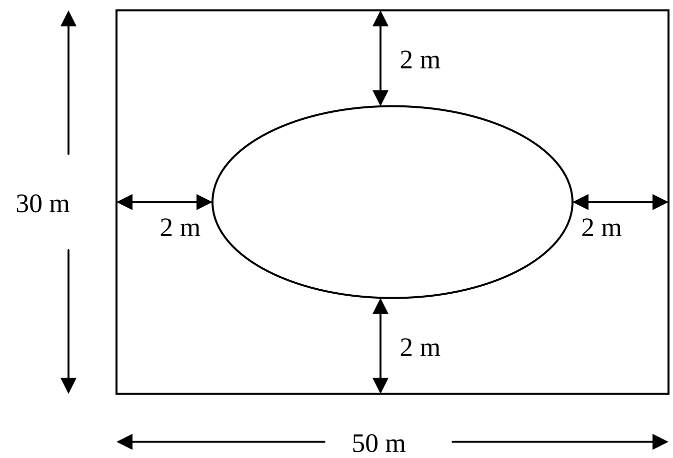
Common Errors

Students:

- did not multiply 25 by 4
- wrote centers as (-3,-2) and (1,-3)
- copied equation incorrectly
- wrote slope as $m=\dfrac{x_2-x_1}{y_2-y_1}$
- used the negative reciprocal of the slope to write the equation
- did not change the sign on 84 when moving the value to the opposite side of the equation
- substituted the values incorrectly into the slope formula
- completed the square before factoring out 25

Value

4% 61. A university plans to construct an elliptical indoor running track inside a gym with dimensions 30 m by 50 m. The track requires a minimum of 2 m clearance from each wall. If the equation for the track is $x^2 + 4y^2 - 50x - 120y = -1125$, calculate whether or not the track will fit inside the gym.



Answer:

$$(x^2 - 50x + 625) + 4(y^2 - 30y + 225) = -1125 + 625 + 900 \quad \textbf{1 mark}$$

$$(x-25)^2 + 4(y-15)^2 = 400$$

$$\frac{1}{400}(x-25)^2 + \frac{4}{400}(y-15)^2 = 1 \quad \textbf{0.5 mark}$$

$$\left[\frac{1}{20}(x-25)\right]^2 + \left[\frac{1}{10}(y-15)\right]^2 = 1 \quad \textbf{0.5 mark}$$

Major horizontal axis length is 40 and minor vertical axis length of 20 **1 mark**

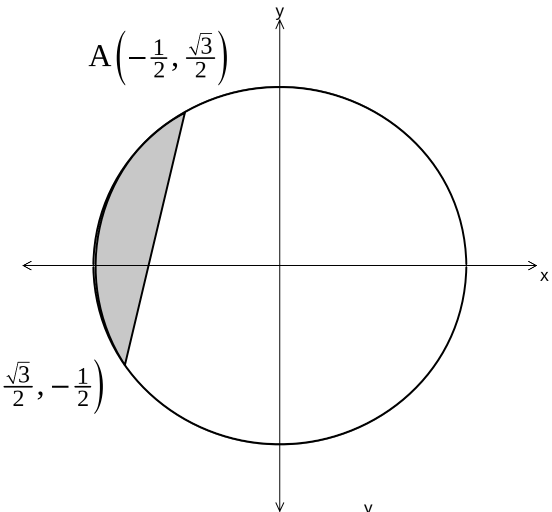
Yes, it will fit. **1 mark**

Common Errors

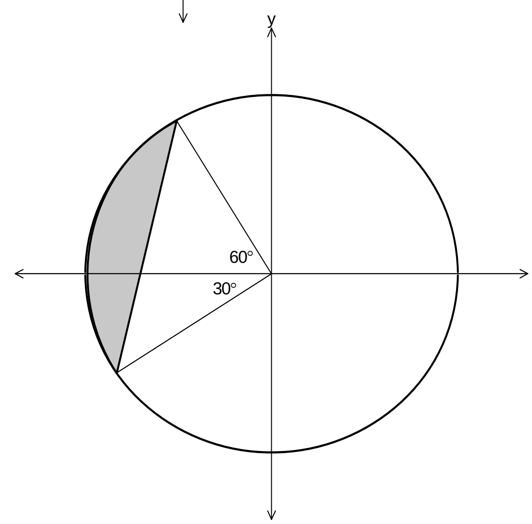
Students:

- did not multiply 225 by 4
- found $\sqrt{400}$ and called it a circle
- forgot the square brackets $\frac{1}{20}(x-25)^2 + \frac{1}{10}(y-15)^2 = 1$

Value
4% 62. Determine the area of the shaded region in the unit circle shown.



Answer:



Radius = 1
Central angle = 60 + 30
90 **1 mark**

$$Area = \frac{\theta}{360}\pi r^2 - \frac{1}{2}bh$$

$$= \frac{90}{360}\pi(1)^2 - \frac{1}{2}(1)(1)$$ **2 marks**

$$= 0.285 \text{ or } \frac{\pi}{4} - \frac{1}{2}$$ **1 mark**

Commentary on Response

Many students did not attempt this question. Students should realize that an alternative method for finding the area of a triangle is $A = \frac{1}{2}ab\sin C$.

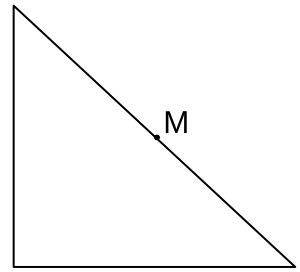
Common Errors

Students:

- used a central angle of 60° or 120°
- did not know that the unit circle has a radius of one

Value

4% 63. Using coordinate geometry, prove that the midpoint of $\overline{AC}$ is equidistant from all three vertices.



Answer:

$$\text{midpoint} = \left(\frac{-a+5a}{2}, \frac{4a\sqrt{2}+0}{2}\right)$$
$$= \left(2a, 2a\sqrt{2}\right) \quad \textbf{1 mark}$$

$$d_{AM} = \sqrt{(2a+a)^2 + \left(2a\sqrt{2} - 4a\sqrt{2}\right)^2}$$
$$= \sqrt{(3a)^2 + \left(-2a\sqrt{2}\right)^2}$$
$$= \sqrt{9a^2 + 8a^2}$$
$$= \sqrt{17a^2}$$
$$= a\sqrt{17}$$

$$d_{CM} = \sqrt{(5a-2a)^2 + \left(0 - 2a\sqrt{2}\right)^2}$$
$$= \sqrt{(3a)^2 + \left(-2a\sqrt{2}\right)^2}$$
$$= \sqrt{9a^2 + 8a^2}$$
$$= \sqrt{17a^2}$$
$$= a\sqrt{17}$$

$$d_{BM} = \sqrt{(2a+a)^2 + \left(2a\sqrt{2} - 0\right)^2}$$
$$= \sqrt{(3a)^2 + \left(2a\sqrt{2}\right)^2}$$
$$= \sqrt{9a^2 + 8a^2}$$
$$= \sqrt{17a^2}$$
$$= a\sqrt{17} \quad \textbf{2 marks}$$

The midpoint is equidistant from all vertices. **1 mark**

Common Errors

Students:

- did not know the distance and/or midpoint formulae
- were unable to simplify the radical expression correctly
- did not provide a concluding statement

TABLE 1
MATHEMATICS 3205 ITEM ANALYSIS
SELECTED RESPONSE (PART I)

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	B	2.4	76.6	1.9	19.1
2	A	86.7	2.6	4.9	5.8
3	C	7.7	4.8	81.6	5.6
4	D	1.0	4.3	5.0	89.7
5	B	22.4	67.4	5.4	4.8
6	D	2.9	8.8	24.9	63.3
7	C	21.3	23.5	48.8	6.0
8	C	1.5	0.3	95.2	3.0
9	A	83.1	8.3	6.0	2.2
10	B	1.5	94.5	3.7	0.2
11	B	8.0	75.4	9.1	7.3
12	D	4.9	7.2	13.5	74.0
13	C	3.6	13.7	77.6	5.0
14	D	7.6	22.0	6.1	63.5
15	D	1.6	5.4	7.8	85.1
16	D	0.8	0.5	1.0	97.7
17	A	82.3	10.1	1.3	6.3
18	D	2.1	14.2	0.6	83.1
19	A	94.2	0.5	5.3	0.1
20	A	66.1	17.2	10.1	6.3
21	D	2.6	36.0	3.8	57.6
22	C	1.9	4.2	83.9	9.9
23	B	2.9	92.7	3.0	1.1
24	B	6.8	79.4	8.2	5.6
25	A	60.7	37.0	1.5	0.6
26	B	22.7	48.6	12.3	16.1
27	C	0.7	6.4	91.3	1.6
28	D	5.3	15.7	13.3	65.6

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
29	C	10.4	14.4	70.0	4.7
30	D	13.6	1.2	11.1	74.1
31	A	79.2	8.6	7.3	4.9
32	B	3.9	79.4	12.8	3.6
33	A	51.1	10.4	9.0	29.3
34	B	5.1	76.8	9.7	8.4
35	D	3.5	4.1	27.3	64.9
36	A	55.3	3.0	38.1	3.6
37	D	3.1	14.6	10.3	71.7
38	A	96.0	1.7	1.8	0.5
39	B	4.7	89.7	1.9	3.6
40	D	1.2	15.2	1.4	82.2
41	A	42.5	24.7	22.5	9.9
42	B	0.4	92.6	3.8	3.1
43	C	10.7	19.6	64.3	4.6
44	B	2.7	78.6	1.5	17.2
45	C	14.7	9.8	55.1	19.6
46	B	1.7	84.8	5.2	8.0
47	B	2.5	89.1	2.4	5.6
48	B	20.3	68.8	7.0	3.2
49	B	15.6	57.9	18.1	7.7
50	B	4.6	81.9	4.0	8.7

NOTE: Percentages may not add to 100% due to multiple responses or missing values.

TABLE 2
MATHEMATICS 3205 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	1293	4	3.2
52	1293	4	3.0
53	1293	4	2.1
54(a)	1293	2	1.8
54(b)	1293	2	1.3
55	1293	3	2.6
56	1293	4	2.5
57	1293	4	3.1
58	1293	4	3.1
59	1293	3	1.9
60	1293	4	3.1
61	1293	4	2.8
62	1293	4	2.4
63	1293	4	3.1