Grading Standards
Mathematics 3205
June 2008

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations. A change in value for quadratics and circles was implemented. A change in value for the common items and the Math 3204 appropriate items for the Math 3204 sub-score was implemented.

Post-Marking Report

a) Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examinations and were held by the Chief Marker for recirculation throughout the marking period. These papers were corrected by the marking board again, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Each marker also made on-going notes regarding partial marks and scoring for their particular question. Whenever a non-common error occurred, it was scored by consensus of the board and made note of, for scoring consistency.

b) Summary

Overall performance in the Math 3205 examination improved from June 2007 to June 2008.

c) Commentary on Responses

Part I – Selected Response – Total Value: 50%

Part II – Constructed Response – Total Value: 50%

Value

4 51. Algebraically determine the **exact** roots in simplest form for $x+\frac{4}{3}=\frac{-2}{x}$.

Answer:

$x(3x)+4(x)=-2(3)$

$3x^2+4x+6=0$ **1 mark**

$$x=\frac{-4\pm\sqrt{(4)^2-4(3)(6)}}{2(3)}$$ **1 mark**

$$x=\frac{-4\pm\sqrt{-56}}{6}$$ **1 mark**

$$x=\frac{-4\pm 2i\sqrt{14}}{6}$$ **0.5 marks**

$$x=\frac{-2\pm i\sqrt{14}}{3}$$ **0.5 marks**

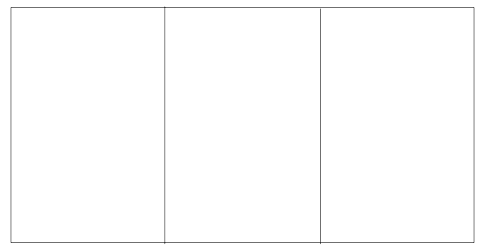
Common Errors

Students:

- did not multiply by the LCD of 3*x*.
- reduced the quadratic formula incorrectly.
- rearranged the quadratic equation incorrectly.

Value

4 52. A farmer is constructing a rectangular pen for his animals, consisting of three sections as shown. If he has 100 m of fencing and wants to use it all, create a quadratic function that models the area of the pen, and use it to determine the maximum area of the pen.



Answer:

Let x = width

Let y = length

$4x + 2y = 100$

$y = 50 - 2x$ **1 mark**

$A = (x)(50 - 2x)$

$A = -2x^2 + 50x$ **1 mark**

$x = \frac{-b}{2a}$

$x = \frac{-50}{2(2)}$

$x = 12.5m$ **1 mark**

$A = -2(12.5)^2 + 50(12.5)$

$A = 312.5m^2$ **1 mark**

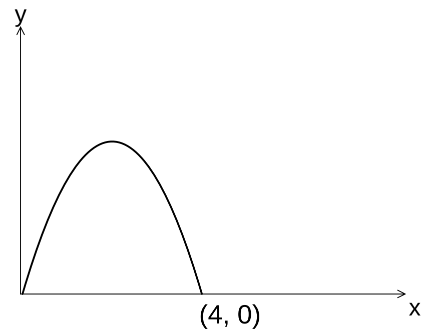
Common Errors

Students:

- did not divide the length by 2.
- did not develop the quadratic equation.
- solved for the roots of the quadratic.

Value

4 53. A golf ball is hit from the ground and lands on the green after 4 seconds. If the golf ball reaches a maximum height of 20 m, algebraically determine the quadratic function representing its path, and use it to determine the approximate height of the ball at 3 seconds.



Answer:

Vertex (2,20) **1 mark**

$$\frac{1}{a}(y-20)=(x-2)^2$$

$$\frac{1}{a}(0-20)=(0-2)^2$$

$$\frac{1}{a}=\frac{-1}{5}$$ **1 mark**

$$\frac{-1}{5}(y-20)=(x-2)^2$$ **1 mark**

$$y=-5(3-2)^2+20$$

$$y=15\,\text{m}$$ **1 mark**

Common Errors

Students were unable to find the vertex.

Value

4 54. A cannonball is shot into the air and its height *h*, in metres, after *t* seconds is recorded in the table below. Algebraically determine the quadratic function that models the height of the ball above the ground *t* seconds after it is shot into the air.

t	0	1	2	3	4
h	3	18.1	23.4	18.9	4.6

Answer:

t	0	1	2	3	4
h	3	18.1	23.4	18.9	4.6

D_1 15.1, 5.3, -4.5, -14.3

D_2 -9.8, -9.8, -9.8 **1 mark**

$D_2 = 2a$

$a = -4.9$ **0.5 marks**

$h = -4.9t^2 + bt + 3$ **1 mark**

$18.1 = -4.9(1)^2 + b(1) + 3$

$b = 20$ **1 mark**

$h = -4.9t^2 + 20t + 3$ **0.5 marks**

Common Errors

Students:

- found D_2 to be positive 9.8.
- switched the *x*-coordinate and the *y*-coordinate when substituting a point into the quadratic equation.

Value

4 55. The motion of a ball thrown upward from the ground is described by $h(t) = -4.9t^2 + 11t$, where h is the height of the ball in metres and t is the time in seconds. Algebraically determine the approximate instantaneous rate of change in the height of the ball at 2 seconds, and describe how the height of the ball is changing at that instant.

Answer:

$$\begin{aligned} IRoC &= \frac{h(2.1) - h(1.9)}{2.1 - 1.9} \\ &= \frac{1.491 - 3.211}{0.2} \qquad \textbf{2 marks} \\ &= \frac{-1.72}{0.2} \\ &= -8.6 \text{ m/s} \qquad \textbf{1 mark} \end{aligned}$$

The ball is falling at a rate of 8.6 m/s. 1 mark

Commentary on Response

It is best to maintain at least four places after the decimal when solving rate of change problems. Only the answer should be rounded.

Common Errors

Students:

- did not give any concluding statement.
- let $rate = \frac{x_2 - x_1}{y_2 - y_1}$.

Value

4 56. Algebraically solve for x: $5\left(5^{2x}\right) + 7 = 36\left(5^{x}\right)$.

Answer:

let $y = 5^x$

$5(y)^2 - 36y + 7 = 0$ **1 mark**

$(5y-1)(y-7) = 0$

$y = \frac{1}{5}$ $y = 7$ **1 mark**

$5^x = \frac{1}{5}$ $5^x = 7$

$x = -1$ $x = \frac{\log 7}{\log 5}$ **2 marks**

Common Errors

Students:

- did not factor correctly: $(y-1)(y-35)$.
- used the properties of exponents and logarithms incorrectly.
- multiplied the base by the coefficient.

Value

4 57. Algebraically solve for *x*: $\log_7 4 + \log_7(x+3) = 2\log_7 x$.

Answer:

$\log_7 4(x+3) = \log_7 x^2$	**(1 mark)**
$4x + 12 = x^2$	**(1 mark)**
$x^2 - 4x - 12 = 0$	**(0.5 marks)**
$(x-6)(x+2) = 0$	**(0.5 marks)**
$x = 6$, $x = -2$ ***(reject)***	**(1 mark)**

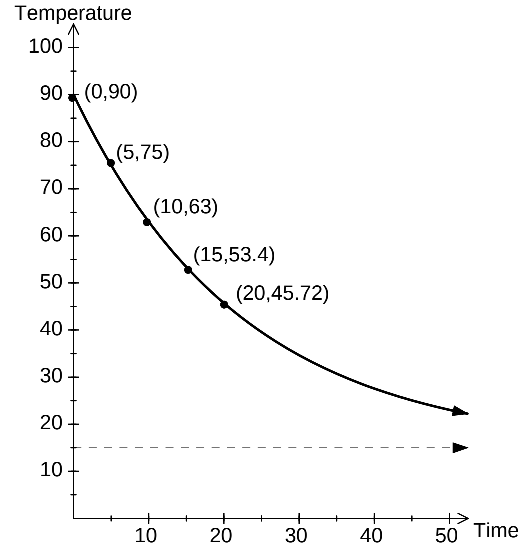
Common Errors

Students:

- wrote $\log_7 4 + \log_7(x+3) = \log_7(4x+3)$.
- did not reject $x = -2$.
- wrote $\log_7 4 + \log_7(x+3) = 2\log_7 x$ as $4 + (x+3) = 2x$.
- wrote $\log_7(4x+12) = 2\log_7 x$ as $\log_7(2x+6) = \log_7 x$.

Value

4 58. A mug of hot chocolate is laid on a table to cool and its temperature, in degrees Celsius, over time, in minutes, is shown in the graph below. Algebraically determine the equation that models the temperature of the hot chocolate over time, and use it to determine the temperature of the hot chocolate after 30 minutes.



Answer:

t	0	5	10	15	20
h	90	75	63	53.4	45.72

Temp - VT 75 60 48 38.4 30.72 1 mark

Ratio 0.80

$y = a(b)^{\frac{x}{c}} + d$

$y = 75(0.8)^{\frac{x}{5}} + 15$ **2 marks**

$y = 75(0.8)^{\frac{30}{5}} + 15$ **0.5 marks**

$y = 34.7^0 C$ **0.5 marks**

Commentary on Response

Many students had a good understanding of horizontal asymptotes.

Common Errors

Students:

- did not translate the values in the table.
- used the initial value of 90 as the value for a.
- divided 75 by 90 to get a common ratio of 0.83.
- used an exponent of *x*.
- used -15 for the value of d .

Value

4 59. The amount of road salt in a storage shed decreases by 25% every 3 weeks. Write an equation that models this situation and use it to determine when $\frac{3}{5}$ of the original pile remains.

Answer:

$$y = 1(1-0.25)^{\frac{x}{3}}$$

$$y = 1(0.75)^{\frac{x}{3}}$$

$$\frac{3}{5} = (0.75)^{\frac{x}{3}}$$ **2 marks**

$$\log\left(\frac{3}{5}\right) = \frac{x}{3}\log 0.75$$ **1 mark**

$$1.7757 = \frac{x}{3}$$ **0.5 marks**

$x = 5.3$ **weeks** **0.5 marks**

Commentary on Response

Students should use four places after the decimal when working with logarithmic equations and only round the answer.

Common Errors

Students:

- used a base of 0.25.
- wrote an exponent of *x*.
- did not know how to find the initial amount.

3 60. Write $4x^2 + y^2 - 8x - 12 = 0$ in transformational form, and state the coordinates of the centre and the length of the major axis.

$$4(x^2 - 2x + 1) + (y^2) = 12 + 4 \qquad \textbf{1 mark}$$

$$4(x-1)^2 + (y)^2 = 16 \qquad \textbf{0.5 marks}$$

$$\frac{4}{16}(x-1)^2 + \frac{1}{16}(y)^2 = 1$$

$$\left[\frac{1}{2}(x-1)\right]^2 + \left[\frac{1}{4}y\right]^2 = 1 \qquad \textbf{0.5 marks}$$

center (1,0) **0.5 marks**

Major axis 8 **0.5 marks**

Common Errors

Students:

- did not know how to complete the square.
- completed the square before factoring the 4.

Value

3 61. Determine the equation of the line segment that joins the centre $(-3,-5)$ of one circle, with the centre of another circle given by $x^2+y^2-10x+2y-7=0$.

Answer:

$$\left(x^2-10x+25\right)+\left(y^2+2y+1\right)=7+25+1$$

$$(x-5)^2+(y+1)^2=33$$ **1 mark**

$$m=\frac{-5-(-1)}{-3-5}$$

$$=\frac{1}{2}$$ **0.5 marks**

$$y=\frac{1}{2}x+b$$ **(-3,-5)** **0.5 marks**

$$-5=\frac{1}{2}(-3)+b$$

$$b=\frac{-7}{2}$$ **0.5 marks**

$$y=\frac{1}{2}x-\frac{7}{2}$$ **0.5 marks**

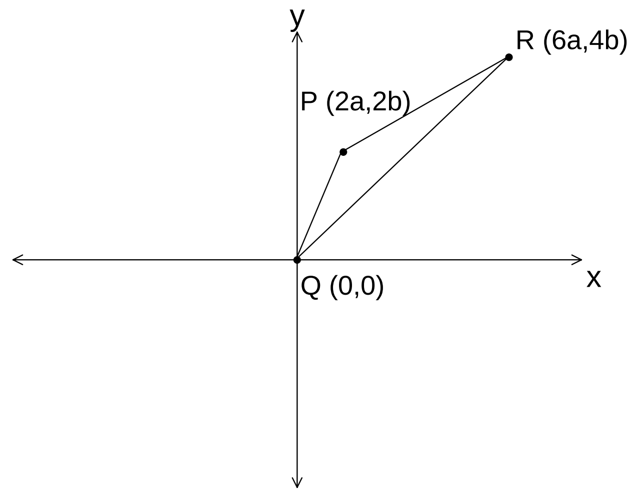
Common Errors

Students:

- wrote slope as $m=\frac{x_2-x_1}{y_2-y_1}$.
- had problems when subtracting a negative.
- substituted the values incorrectly into the slope formula.
- switched the *x*-coordinate and the *y*-coordinate when substituting a point into the equation of a line.

Value

4 62. Using coordinate geometry, prove that the segment joining the midpoints of sides $\overline{PQ}$ and $\overline{PR}$, is parallel to $\overline{QR}$.



Answer:

$$\textbf{midpoint}_{\textbf{PQ}} = \left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$$
$$= (a, b)$$ **0.5 marks**

Point X(a,b)

$$\textbf{midpoint}_{\textbf{PR}} = \left(\frac{2a+6a}{2}, \frac{2b+4b}{2}\right)$$
$$= (4a, 3b)$$ **0.5 marks**

Point Y(4a,3b)

$$slope_{RQ} = \frac{4b-0}{6a-0} = \frac{2b}{3a}$$ **1 mark**

$$slope_{XY} = \frac{3b-b}{4a-a} = \frac{2b}{3a}$$ **1 mark**

Equal slopes, therefore the lines are parallel. **1 mark**

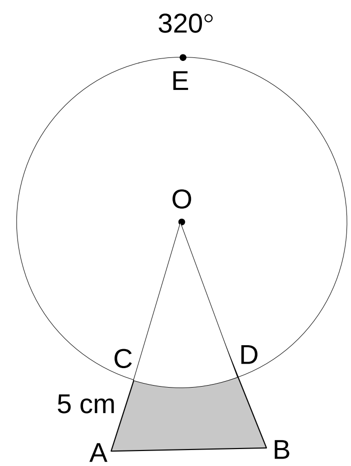
Common Errors

Students:

- did not know the midpoint formula.
- did not provide a clearly written concluding statement.

Value

4 63. In the circle with centre O shown, the diameter is 20 cm, ΔAOB is isosceles, and $\overline{AC}$ = 5 cm. If $\overset{\frown}{CED}$ = 320° determine the area of the shaded region.



Answer:

radius is 10 cm **0.5 marks**

$\angle$COD is 40^0 **0.5 marks**

$$Area_{shaded} = \frac{1}{2}ab\sin\theta - \frac{\theta}{360}\pi r^2$$

$$= \frac{1}{2}(15)(15)\sin 40 - \frac{40}{360}\pi\left(10\right)^2$$ **2 marks**

$$= 72.31 - 34.91$$

$$= 37.4\,\text{cm}^2$$ **1 mark**

Commentary on Response

Students should realize that an alternative method for finding the area of a triangle is $A = \frac{1}{2}ab\sin C$.

Common Errors

Students:

- found an incorrect central angle.
- used the radian mode when making calculations.
- did not know the area formula.
- called the sector a segment.

TABLE 1
MATHEMATICS 3205 ITEM ANALYSIS
SELECTED RESPONSE (PART I)

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	C	5.7	2.3	91.4	0.6
2	B	1.8	91.9	4.8	1.3
3	D	4.5	7.0	9.0	79.3
4	D	5.2	14.5	3.9	76.5
5	B	8.7	83.2	4.1	3.8
6	D	0.3	2.4	1.6	95.6
7	D	1.8	0.5	1.2	96.4
8	C	9.2	7.7	83.0	0.0
9	D	8.0	8.9	24.2	58.8
10	B	11.3	82.9	1.6	4.1
11	C	20.8	17.1	49.6	11.9
12	B	2.0	96.7	0.8	0.6
13	A	83.9	2.5	2.7	10.7
14	A	92.8	5.4	0.3	1.5
15	A	78.4	4.4	6.2	10.6
16	C	19.1	9.3	67.5	4.1
17	C	8.4	7.7	79.5	3.9
18	B	1.0	97.5	0.5	1.0
19	A	87.2	6.4	5.5	0.8
20	B	7.4	69.1	3.7	19.8
21	C	2.3	2.2	68.6	26.9
22	B	4.6	83.0	5.2	7.0
23	A	63.3	16.5	13.0	7.2
24	D	8.6	8.5	11.7	71.1
25	C	22.3	22.5	49.7	5.2
26	B	10.6	73.6	10.2	5.5
27	D	6.1	15.5	19.3	58.9
28	C	7.3	1.1	88.3	3.3
29	C	7.7	10.0	74.8	7.1
30	A	66.8	16.7	8.9	7.6
31	D	4.1	30.7	3.9	61.2
32	B	6.3	62.7	24.4	6.4
33	C	2.8	3.7	91.2	2.3
34	D	2.9	6.6	5.1	85.2
35	D	3.4	6.4	8.9	81.1
36	C	3.9	8.7	42.7	44.6
37	A	96.8	0.3	0.1	2.8

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
38	A	87.2	4.5	7.3	0.9
39	A	88.7	4.7	3.9	2.6
40	A	85.2	6.0	3.5	5.3
41	B	2.2	85.0	1.0	11.7
42	B	10.5	53.4	10.8	25.2
43	B	15.2	58.6	17.0	8.8
44	B	13.1	83.6	2.0	1.3
45	C	10.4	21.4	57.7	10.2
46	D	1.1	21.4	2.0	75.5
47	A	46.5	29.5	4.2	19.6
48	B	17.2	73.0	5.0	4.5
49	C	9.2	10.3	56.7	23.1
50	D	8.3	2.3	18.2	70.9

NOTE: Percentages may not add to 100% due to multiple responses or missing values.

TABLE 2
MATHEMATICS 3205 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	1318	4	3.5
52	1318	4	3.1
53	1318	4	3.0
54	1318	4	3.5
55	1318	4	3.5
56	1318	4	2.9
57	1318	4	3.1
58	1318	4	3.2
59	1318	4	3.3
60	1318	3	2.4
61	1318	3	2.4
62	1318	4	3.4
63	1318	4	3.4