Grading Standards
Mathematics 3205
June 2009

Pre-Marking Appraisal

The examination was similar in length and had parallel structure to previous public examinations. There was no change in value for the common items or the Math 3204 appropriate items for the Math 3204 sub-score. The examination was considered fair and had coverage of each unit of study and each level of cognitive learning as per the table of specifications.

Post-Marking Report

a) Marking Standard and Consistency

Marker reliability was checked by obtaining a random sample of 50 examinations. These examinations were scored on separate back flaps with no physical markings on the original examinations and were held by the Chief Marker for recirculation throughout the marking period. These papers were corrected by the marking board again, and the initial and subsequent marks were compared. Any discrepancies in marking were reviewed and discussed with individual markers. Each marker also made on-going notes regarding partial marks and scoring for their particular question. Whenever a non-common error occurred, it was scored by consensus of the board and made note of, for scoring consistency.

b) Summary

While the overall performance in the Math 3205 examination was slightly down from June 2008 to June 2009, it was still the second highest provincial score in the past five years.

c) Commentary on Responses

Part I – Selected Response – Total Value: 50%

Item #35:

Both A and B were accepted for item #35.

The domain of $2\log_2 x$ is $\{x \mid x > 0, x \in R\}$. Here, $x \neq -4$.

The domain of $\log_2 x^2$ is $\{x \mid x \neq 0, x \in R\}$. Here, $x = \pm 4$.

Although students should have referred to the original equation, $2\log_2 x + \log_2 4 = \log_2 64$, to determine their final answer, it was decided to accept $x = \pm 4$ in order to negate any confusion caused by the introduction of the equations $\log_2 x^2 = \log_2 64 - \log_2 4$ and $\log_2 x^2 = 4$. This decision was supported by a review of the statistical results.

Solution 1:

$$2\log_2 x + \log_2 4 = \log_2 64$$
$$\log_2 x^2 = \log_2 64 - \log_2 4$$
$$\log_2 x^2 = \log_2 \tfrac{64}{4}$$
$$\log_2 x^2 = \log_2 16$$
$$\therefore x^2 = 16$$
$$x = \pm 4$$
$$\therefore x = 4$$

Reject -4 since $x \neq -4$ in the original equation:

$$2\log_2 x + \log_2 4 = \log_2 64$$

Solution 2:

$$2\log_2 x + \log_2 4 = \log_2 64$$
$$\log_2 4 = 2,\ \log_2 64 = 6$$
$$2\log_2 x + 2 = 6$$
$$2\log_2 x = 6 - 2$$

$$2\log_2 x = 4$$
$$\tfrac{2}{2}\log_2 x = \tfrac{4}{2}$$
$$\log_2 x = 2$$
$$\therefore 2^2 = x$$
$$x = 4$$

or

$$2\log_2 x = 4$$
$$\log_2 x^2 = 4$$
$$\therefore 2^4 = x^2$$
$$x^2 = 16$$
$$x = \pm 4$$
$$\therefore x = 4$$

(Reject -4)

Items #30, 31, 35 & 36:

Students did not perform well on these items. Students had difficulty with items involving the properties of logarithmic functions.

Item #38:

Students had difficulty finding the slope of the altitude perpendicular to $\overline{\text{AC}}$.

Items #49 & 50:

Students had difficulty with the coordinate geometry of a circle.

Part II – Constructed Response – Total Value: 50%

Value

4 51. Algebraically determine the **exact** roots in simplest form for $\frac{x}{x+2} - \frac{2}{x-4} = -\frac{1}{2}$.

Answer:

$\text{LCD}: 2(x+2)(x-4)$ Multiply each term by LCD and cancel.

$$\frac{2(x+2)(x-4)x}{(x+2)} - \frac{2(x+2)(x-4)2}{(x-4)} = -\frac{1(2)(x+2)(x-4)}{2}$$ **0.5 marks**

$$2x(x-4) - 4(x+2) = -(x+2)(x-4)$$ **0.5 marks**

$$2x^2 - 8x - 4x - 8 = -(x^2 - 2x - 8)$$

$$2x^2 - 12x - 8 = -x^2 + 2x + 8$$

$$3x^2 - 14x - 16 = 0$$ **1 mark**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(3)(-16)}}{2(3)}$$ **0.5 marks**

$$x = \frac{14 \pm \sqrt{196 + 192}}{6}$$ **0.5 marks**

$$x = \frac{14 \pm \sqrt{388}}{6} = \frac{14 \pm 2\sqrt{97}}{6}$$ **0.5 marks**

$$x = \frac{7 \pm \sqrt{97}}{3}$$ **0.5 marks**

Common Errors

Students:

- multiplied by an incorrect LCD.
- cross-multiplied when they should not have.
- reduced the quadratic formula incorrectly.
- rearranged the quadratic equation incorrectly.
- made errors with the distributive property. (+ and –)

Value

4 52. A local restaurant averages 200 customers per day who spend \$30 per meal. The manager estimates a loss of 10 customers per day for each \$1 increase in meal price. If the average cost to prepare each meal is \$12, write a quadratic function to model the daily profit and use it to determine the meal price that will maximize the profit.

Answer:

\# of customers = 200 – 10x
price per meal = 30 + x (meal price)
cost per meal = 12

profit per meal = price per meal – cost per meal
= (30 + x) – 12
= 30 – 12 + x
= 18 + x **0.5 marks**

Let P represent the total profit.

$$\text{P} = (\#\text{ of customers})(\text{profit per meal})$$

$$\text{P} = (200 - 10x)(18 + x)$$ **1 mark**

$$\text{P} = -10x^2 + 20x + 3600$$ **1 mark**

Maximum profit occurs at the vertex (x , P) where:

$$x = -\frac{b}{2a} = \frac{-20}{-20} = 1$$ **1 mark**

If x = 1, the meal price is:
price per meal = 30 + x
= 30 + 1
= \$31 **0.5 marks**

Commentary on Response
This question can also be answered by setting up the quadratic using the difference between revenue and costs (expenses).

$$\text{P} = \text{Revenue} - \text{Expenses}$$

$$\text{P} = (\#\text{ of customers})(\text{price per meal}) - (\#\text{ of customers})(\text{cost per meal})$$

$$\text{P} = (200 - 10x)(30 + x) - (200 - 10x)(12)$$

$$\text{P} = -10x^2 + 20x + 3600$$

In general, students had difficulty and displayed confusion with this question.

Common Errors
Students:

- did not subtract the cost from revenue to determine profit.
- did not develop the quadratic equation correctly.
- solved for the roots of the quadratic.
- incorrectly determined the meal price after x = 1 was obtained.

Value

4 53. A person standing on the surface of Mars throws a ball from a height of 1 m and the data below is collected on the ball's height over time. Algebraically determine the function, *h(t)*, that defines the height of the ball above the ground, in metres, *t* seconds after leaving the person's hand.

t(s)	1	2	3	4
h(m)	12	19	22	21

Answer:

Find a, b and c for the function: $h(t) = at^2 + bt + c$

For a, find D_2:

h(m)	12	19	22	21
D_1	7	3	–1	
D_2	–4	–4		

$D_2 = 2a$

$-4 = 2a$ **0.5 marks**

$\therefore a = -2$ **1 mark**

$h(t) = at^2 + bt + c$

$h(t) = -2t^2 + bt + c$

Set up equations to find b and c:

Using point (1,12):

$12 = -2(1)^2 + b(1) + c$

$12 = -2 + b + c$

$b + c = 14$ **0.5 marks**

Using point (2,19):

$19 = -2(2)^2 + b(2) + c$

$19 = -8 + 2b + c$

$2b + c = 27$ **0.5 marks**

Solve the System of Equations:

$c = 14 - b$

$2b + (14 - b) = 27$

$b = 13$ **0.5 marks**

$c = 1$ **0.5 marks**

$h(t) = -2t^2 + 13t + 1$ **0.5 marks**

Common Errors

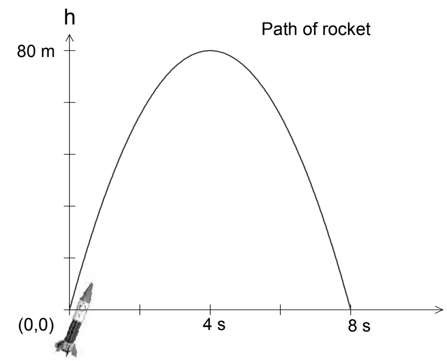
Students:

- substituted coordinates incorrectly. (i.e., used (12,1) instead of (1,12).)
- did not determine the correct value for D_1 and D_2.
- performed an incorrect operation to obtain the value of c.
- assumed c = 12 or c = 23.

Value

4 54. A toy rocket is launched into the air and reaches a maximum height of 80 m after a time of 4 seconds. If the rocket lands after 8 seconds, determine the quadratic function that describes the flight path of the rocket. Use the function to determine the height of the rocket at 6.5 seconds.

Answer:



vertex (4,80) = (h, k) **0.5 marks**

Along with the vertex, use one other point on the parabola to determine 'a'.
The point (0,0) can be used.

$$\frac{1}{a}(y-k)=(x-h)^2$$

$$\frac{1}{a}(y-80)=(x-4)^2$$ **0.5 marks**

$$\frac{1}{a}(0-80)=(0-4)^2$$

$$\frac{-80}{a}=16$$

$$a=-5$$ **1 mark**

$$\therefore -\frac{1}{5}(y-80)=(x-4)^2$$ **1 mark**

The height of the rocket at x = 6.5 sec:

$$y=-5(6.5-4)^2+80$$ **0.5 marks**

$$y=-5(2.5)^2+80$$

$$y=48.75\,\text{m}$$ **0.5 marks**

Common Errors

Students:

- did not square $(x-4)$ in the quadratic equation.
- squared $(y-80)$.
- switched the x-coordinate and the y-coordinate when substituting a point into the quadratic equation.

Value

4 55. A ball is thrown vertically upward with an initial speed of 30 m/s. Its height, in metres, t seconds after release is given by $h(t) = 1 + 30t - 5t^2$. Calculate the instantaneous rate of change at 2.5 seconds and describe how the height of the ball is changing at that instant.

Answer:

$h(2.51) = 44.8$ **1 mark**

$h(2.5) = 44.75$ **1 mark**

$$IRoC = \frac{h(2.51) - h(2.5)}{2.51 - 2.5} = \frac{44.8 - 44.75}{0.01} = \frac{0.05}{0.01} = 5$$ **1 mark**

The height is increasing at 5 m/s. **1 mark**

Commentary on Response

It is best to maintain as many places after the decimal as possible when solving rate of change problems. Only the answer should be rounded. A range of time between a minimum of 2.4 seconds and a maximum of 2.6 seconds was accepted. The use of derivatives was also an acceptable approach to solving this problem.

Common Errors

Students:

- did not give a concluding statement.
- let the $\text{rate} = \frac{x_2 - x_1}{y_2 - y_1}$.

Value

4 56. Algebraically solve for x: $\frac{1}{3}\log_2 125 + \log_2(x+2) = 4$.

Answer:

$\log_2 125^{\frac{1}{3}} + \log_2(x+2) = 4$	**0.5 marks**
$\log_2\left[(5)(x+2)\right] = 4$	**1 mark**
$\log_2(5x+10) = 4$	**0.5 marks**
$\therefore 2^4 = 5x+10$	**1 mark**
$16 = 5x+10$	
$5x = 6$	
$x = \frac{6}{5}$	**1 mark**

Common Errors

Students:

- used the properties of logarithms incorrectly.
  - placed the $\frac{1}{3}$ in front of all logs.
  - did not define $2^4 = 5x+10$.
- incorrectly solved the linear equation.

Value

4 57. Algebraically solve for x: $3^{2x} - 5 \cdot 3^{x} = -4$.

Answer:

$3^{2x} - 5(3^{x}) + 4 = 0$

$(3^{x})^{2} - 5(3^{x}) + 4 = 0$ **1 mark**

Let $3^{x} = m$

$m^{2} - 5m + 4 = 0$

$(m-1)(m-4) = 0$ **1 mark**

$m = 1$ $m = 4$

$\therefore 3^{x} = 1$ $3^{x} = 4$ **1 mark**

$3^{x} = 3^{0}$ $x = \dfrac{\log 4}{\log 3}$

$x = 0$ $x = 1.26$ **1 mark**

Common Errors

Students:

- did not treat this as a quadratic. They introduced logs in an attempt to solve.
- did not form the correct quadratic when substituting a variable for 3^{x}. They used $3m^{2} - 5m + 4 = 0$.

Value

4 58. The amount of a certain antibiotic drug remaining in a person's body decreases by 15% each hour. If the initial dose was 250 mg, algebraically determine an exponential equation to model this situation and use it to determine how long it will take for the amount of drug in the person's body to reduce to 55 mg.

Answer:

$r = 0.15$

$b = (1 - 0.15) = 0.85$ **1 mark**

$A = 250(0.85)^t$ **0.5 marks**

$55 = 250(0.85)^t$ **0.5 marks**

$\frac{55}{250} = (0.85)^t$

$0.22 = (0.85)^t$ **0.5 marks**

$\log(0.22) = t\log(0.85)$ **1 mark**

$t = \frac{\log(0.22)}{\log(0.85)}$

$t = 9.3$ hrs **0.5 marks**

Commentary on Response

Many students had a good understanding of this question. It was very well done.

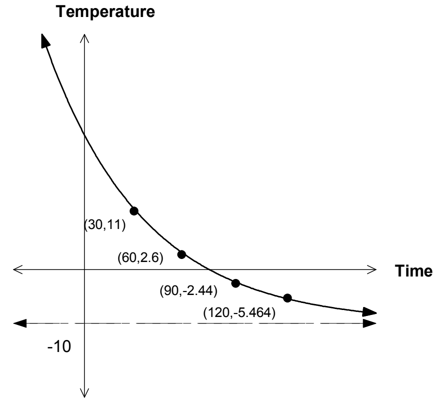
Common Errors

Students:

- used 0.15 instead of 0.85 for the *b*–value.
- made errors in calculating *b*.
- calculated *t* incorrectly. Example: $t = \frac{\log 0.85}{\log 0.22}$.

Value

4 59. During a power failure caused by a winter storm, the temperature inside a house, with respect to time, in minutes, is shown in the graph below. Based on an outside temperature of $-10°\text{C}$, create a function to model this situation.



Answer:

Find *a*, *b*, *c* and *k* for the function:

$T = a(b)^{\frac{t}{c}} + k$

Add 10 to each temperature value.
Use this to find the common ratio, *b*.

t	30	60	90	120
$T+10$	21	12.6	7.56	4.536

1 mark

$b = 0.6$ (common ratio) **1 mark**

Using the common ratio, find $T + 10$, a, when $t = 0$.

t	**0**	30	60	90	120
$T+10$	***a***	21	12.6	7.56	4.536

$a = \frac{21}{0.6} = 35$ **1 mark**

$c = 30$ (Temperature recorded every 30 minutes.)

$k = -10$ (Horizontal asymptote occurs at $T = -10$.)

$T = 35(0.6)^{\frac{t}{30}} - 10$ **1 mark**

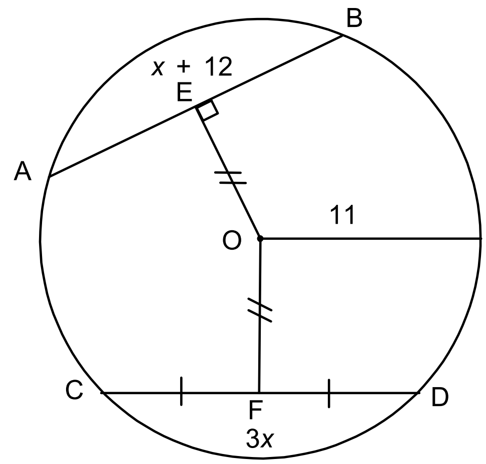
Commentary on Response
Some students found *a* by substituting a point from the group (t, T) into the equation: $T = a(0.6)^{\frac{t}{30}} - 10$.

Common Errors
Students:

- used 21 as the value of *a*.
- did not have the horizontal asymptote in the final equation.
- claimed 11 + 10 = 22 .
- substituted the wrong coordinates when calculating *a*.

Value

3 60. In the circle with centre O shown, the radius is 11 cm and $\overline{OE} \cong \overline{OF}$. If $\overline{AB} = x + 12$ and $\overline{CD} = 3x$, determine the length of $\overline{OF}$.



Answer:

$$\overline{AB} = \overline{CD}$$

$x + 12 = 3x$ **1 mark**

$12 = 2x$

$x = 6$ **0.5 marks**

$\overline{CD} = 18$

$\overline{FD} = 9$ **0.5 marks**

$$\overline{OF} = \sqrt{11^2 - 9^2}$$
$$= \sqrt{121 - 81}$$
$$= \sqrt{40}$$

$= 2\sqrt{10}$ **1 mark**

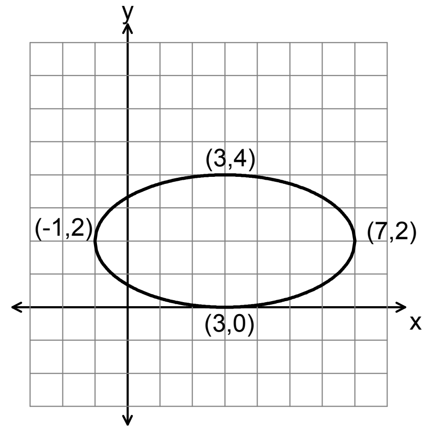
Commentary on Response
This question was very poorly done.

Common Errors
Students:

- failed to realize the chords were congruent (equidistant chords).
- incorrectly calculated $\overline{FD}$.
- made errors in the Pythagorean Theorem calculation.
- assumed $\overline{OE}$ and $\overline{OF}$ were *x*.

Value

3 61. Write, in transformational form, the equation for the ellipse shown.



Answer:

Centre (3,2) **1 mark**

Horizontal stretch = 4 **0.5 marks**

Vertical stretch = 2 **0.5 marks**

$\left[\frac{1}{4}(x-3)\right]^2+\left[\frac{1}{2}(y-2)\right]^2=1$ **1 mark**

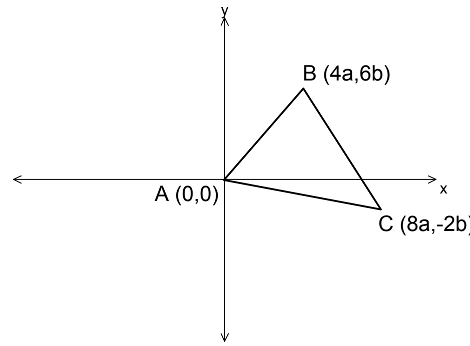
Common Errors

Students:

- incorrectly identified the horizontal and vertical stretch.
- switched the horizontal stretch with the vertical stretch.
- left out square brackets.
- made mistakes writing the final formula.

Value

4 62. In ΔABC, the coordinates of the vertices are $A(0,0)$, $B(4a, 6b)$, and $C(8a, -2b)$. Prove that the segment joining the midpoints of $\overline{AB}$ and $\overline{CB}$ is one half the length of $\overline{AC}$.



Answer:

Let M be the midpoint of AB.
Let P be the midpoint of BC.

$M(2a, 3b)$ **0.5 marks**

$P(6a, 2b)$ **0.5 marks**

$\overline{MP} = \sqrt{(6a-2a)^2 + (2b-3b)^2}$ **0.5 marks**

$= \sqrt{(4a)^2 + (-b)^2}$

$= \sqrt{16a^2 + b^2}$ **0.5 marks**

$\overline{AC} = \sqrt{(8a)^2 + (-2b)^2}$ **0.5 marks**

$= \sqrt{64a^2 + 4b^2}$ **0.5 marks**

$\overline{AC} = \sqrt{64a^2 + 4b^2}$

$= \sqrt{4(16a^2 + b^2)}$

$= 2\sqrt{16a^2 + b^2}$

$\therefore \overline{AC} = 2\overline{MP}$ **1 mark**

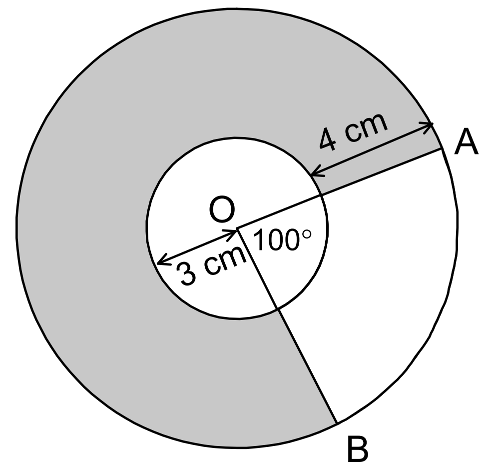
Commentary on Response
This question was poorly done.

Common Errors
Students:

- did not use the midpoint or distance formula correctly.
- did not correctly simplify the radical expression.
- did not attempt to show $\sqrt{64a^2 + 4b^2} = 2\sqrt{16a^2 + b^2}$.
- failed to provide a clearly written concluding statement.

Value

4 63. In the circle with centre O shown, calculate the area of the shaded region if $\angle AOB = 100°$.



Answer:

$$A = \frac{\theta}{360^o}\pi r^2$$

Big radius − small radius

$$A = \frac{260}{360}\pi(7)^2 - \frac{260}{360}\pi(3)^2$$ **2 marks**

$$A = \frac{637\pi}{18} - \frac{117\pi}{18}$$ **1 mark**

$$A = \frac{260\pi}{9}$$

$$A = 90.8 \text{ cm}^2$$ **1 mark**

Commentary on Response

There were many different approaches to this question that could have lead to the correct answer.

Common Errors

Students:

- used an incorrect formula for area.
- counted the small sector area twice.
- did not use the correct shaded region.
- used the formula for the area of a triangle to calculate the area of a sector.

TABLE 1
MATHEMATICS 3205 ITEM ANALYSIS
SELECTED RESPONSE (PART I)

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
1	B	3.7	77.9	16.9	1.4
2	B	12.4	77.0	5.0	5.1
3	D	0.2	1.1	1.4	97.3
4	D	0.7	19.8	3.9	75.5
5	B	14.5	81.7	1.5	2.3
6	D	13.8	0.9	2.1	83.3
7	B	7.6	76.5	3.9	11.9
8	D	0.7	1.2	21.9	76.1
9	C	17.7	27.2	51.6	3.1
10	A	60.5	13.5	16.0	9.7
11	B	4.9	77.3	3.3	14.5
12	C	1.1	1.1	92.9	5.0
13	C	17.8	12.4	58.5	11.2
14	C	10.9	3.6	75.0	10.5
15	C	5.0	7.2	82.2	5.5
16	A	82.7	9.1	5.3	2.9
17	A	70.0	9.6	1.5	18.7
18	A	85.6	11.3	2.1	1.0
19	B	3.0	92.8	3.5	0.7
20	C	4.8	1.9	84.5	8.8
21	A	70.8	20.1	5.3	3.8
22	A	73.0	21.1	4.2	1.7
23	D	4.9	26.2	0.4	68.5
24	A	55.8	7.3	6.5	30.3
25	D	7.6	32.1	9.9	50.1
26	A	56.9	4.3	8.4	30.2
27	B	18.1	72.8	5.4	3.4
28	D	0.9	4.6	25.2	69.1
29	C	1.3	7.8	85.7	5.3
30	D	34.4	0.3	5.7	59.5
31	B	30.1	51.7	8.4	9.6
32	A	68.5	8.8	16.5	5.9
33	B	5.7	90.0	3.6	0.7
34	C	2.9	3.2	89.4	4.5
35	A or B	38.0	49.1	5.6	7.1
36	D	10.9	42.1	11.2	35.5
37	C	3.2	1.3	87.8	7.7
38	A	44.4	17.3	30.5	7.3
39	B	13.7	63.9	13.7	8.4

Item	Answer	Responses			
		A	B	C	D
		%	%	%	%
40	C	16.9	24.4	52.6	5.9
41	B	26.9	64.5	3.4	5.3
42	A	91.5	2.7	2.2	3.6
43	C	5.3	21.4	64.1	8.9
44	D	1.4	3.2	6.5	88.8
45	A	62.4	8.1	26.0	3.5
46	C	8.6	15.9	51.5	23.6
47	D	21.9	3.8	2.7	71.5
48	D	20.2	10.4	8.0	61.3
49	D	9.9	15.6	24.9	49.1
50	A	48.2	9.9	33.2	8.6

NOTE: Percentages may not add to 100% due to multiple responses or missing values.

TABLE 2
MATHEMATICS 3205 ITEM ANALYSIS
CONSTRUCTED RESPONSE (PART II)

Item	Number of Students Completing Item	Value	Average
51	1391	4	3.3
52	1391	4	2.6
53	1391	4	3.6
54	1391	4	3.5
55	1391	4	3.5
56	1391	4	3.4
57	1391	4	3.1
58	1391	4	3.7
59	1391	4	2.6
60	1391	3	1.8
61	1391	3	2.4
62	1391	4	2.4
63	1391	4	3.3