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# Introduction

## *Purpose*

This document describes the standards and conventions used in Mathematics 3204/3205 public examinations.

## *Guiding Principles & Documents*

The Mathematics 3204/3205 Curriculum Guide describes the courses in terms of breadth and depth of treatment and nature of applications. The authorized textbook, Mathematical Modeling Book 3 (2002), and the Teachers' Resource Binder, both published by Nelson Thomson Learning, support the delivery of the course. The public examination is consistent with the curriculum guide.

Provincially authorized curriculum guides list course outcomes and suggested treatment. The guides are supplied by the Learning Resources Distribution Centre (LRDC) and are available online at:

http://www.gov.nl.ca/edu/sp/mathlist.htm.

Other support materials available at this site include:

- ✓ Math Companion 3204/3205 Supporting Document (the Supplementary Resource Material Binder)
- ✓ Solutions for Exercises in the above binder
- ✓ Mathematics 3204/3205 Study Guide: Quadratics Unit
- ✓ Calculators in Mathematics: Instruction and Assessment
- ✓ Professional Development Math Resources (software tutorials)

Public examinations, regulations for candidates (which include information on calculator use), and other resources are available online at:

http://www.gov.nl.ca/edu/pub/public.htm.

Other resources include grading standards which contain information relevant to the scoring of each June examination, and comments about individual questions.

*Important Notes:*

(1) Where clarifications are not associated particularly with Mathematics 3205, they can be assumed to apply to both courses.

(2) Unless otherwise specified, positions taken apply only to the public examinations in Mathematics.

## Public Examination Conventions

Mathematical information should be communicated according to accepted conventions. The Department of Education of Newfoundland and Labrador follows the conventions below in developing provincial Mathematics examinations.

### Screenshots

Graphing calculator screen shots simulating aspects of an investigation may occur on the examination.

### Roots, Zeros, *x*-intercepts, Solutions

The terms roots, zeros, *x*-intercepts, and solutions may all appear on public examinations. These are related, but not synonymous terms (see also *Terminology*).

### Imaginary Numbers

The use of imaginary numbers is restricted to describing the square root of a negative as an '$i$' number (i.e., $i^2$ will not be tested).

### Number Format and Units

The format for numerals is the International System of Units (SI). Variables and quantity symbols are in italic type while unit symbols are not, with unit symbols placed one space to the right of the quantifier. Unit symbols are unaltered in the plural.

For example,

| Correct Format | | Incorrect Format | Reason |
|---|---|---|---|
| 7 cm | not | 7cms | (cm is a unit) |
| 20 °C | not | 20°C | (°C is a unit) |
| *e* | not | e | (*e* is the quantity that is the base for natural logarithm) |
| 7*xy* | not | 7xy | (*x* and *y* are variables) |

Where a number contains more than four digits to the left or right of a decimal, a thin fixed space will be used as a separator per group of three digits as shown:

For example,

- 2000
- 20 000 not 20000 or 20,000
- 0.0001
- 0.000 01 not 0.00001

# Public Examination Standards

Public Examination Standards will set expectations for student responses on public examinations. These expectations are outlined for each unit followed by other broad standards.

## *Public Examination Standards: Quadratics*

### Creating a Quadratic Sequence [Mathematics 3205 only]

In creating or determining a quadratic function from data, $x$-values will only change by increments of 1.

### Sequences [Mathematics 3205 only]

If $m$ is the degree of a power sequence, then $D_m = m!a$.

### Imaginary Numbers

Students will not be expected to create a quadratic function from given imaginary roots.

### Rational Expressions in Equations

For Mathematics 3204, questions where set up involves a rational equation, or where a rational equation is to be solved, will be limited to simple 'cross-multiplication' involving binomials and will not include radicals.

### Radicals

For radicals, simplest form requires that all radicands be completely reduced.

For Mathematics 3204, when radical coefficients are involved in an equation which may be solved by the quadratic formula, the only radical coefficient will be b. This eliminates rationalizing the denominator.

For Mathematics 3205, rationalizing the denominator is a necessary simplification for a solution to warrant full marks.

## *Public Examination Standards: <u>Rate of Change</u>*

**Rate of Change [functions]**

Mathematics 3204 students will have the function provided.

Mathematics 3205 students are expected to generate the function from a context.

**Approximating Rate of Change [difference quotient]**

In determining the average/approximate instantaneous rate of change of a function, a student's response must show the difference quotient prior to the average/approximate instantaneous rate of change value being shown. That is, a $\frac{dy}{dx}$ value on its own, from a graphing calculator or other means, will not warrant full marks unless supported by a correct related difference quotient.

<u>Example</u>

Determine the approximate instantaneous rate of change of $f(x) = x^2$ at $(3, 9)$.

$$\text{Inst. RoC} \doteq \frac{f(3.1) - f(2.9)}{0.2} \doteq 6 \quad \text{or} \quad \text{Inst. RoC} \doteq \frac{f(3) - f(2.9)}{0.1} \doteq 5.9$$

However, the derivative *calculated from correct f ' algebra* is also acceptable as an approximation of the instantaneous rate of change of a function.

$$\begin{aligned} f'(x) &= 2x \\ \therefore \text{Inst. RoC} &= f'(3) \\ &= 6 \end{aligned}$$

**Approximating Rate of Change [maximum *x*-interval]**

The maximum *x*-interval which will warrant full marks in the approximation of an instantaneous rate of change is 0.2 *x*-units (i.e., $\pm 0.1$ *x*-units at the point in question).

**Trigonometric Ratios & Rate of Change**

There will be no questions from the Rate of Change unit involving trigonometric ratios.

## *Public Examination Standards: <u>Exponential and Logarithmic Functions</u>*

**Logarithmic Equations**

Mathematics 3204 students will be expected to solve logarithmic equations which produce more than one potential solution, and to identify and reject extraneous values among potential solutions.

**Logarithmic Scales**

Problems involving logarithmic scales will not be tested.

## *Public Examination Standards: <u>Circle Geometry</u>*

**Euclidean Proof**

Euclidean proof will not be tested.

**Properties of Circles**

Properties of circles will be tested in non-Euclidean proof contexts only. Students are expected to know the properties of circles (tangent, chord, and angle properties) in missing measure or other questions, and in the context of coordinate proof.

**Transformational Proof**

Transformational proof will not be tested.

**Coordinate Proof**

For Mathematics 3204, coordinate proofs will contain numerical coordinates only.

For Mathematics 3205, coordinate proofs will contain conditions such that some or all coordinates will be variable.

**Exploring Circles on a Graphing Calculator**

Exploring circles on the graphing calculator using parametric graphing will not be tested.

## *Public Examination Standards: <u>Other</u>*

**Domain and Range**

Domain and range will be tested. When students are writing domain or range, set notation or interval notation is acceptable.

**Exact Value, Simplest Form**

Where exact value or simplest form appears in a question, the following conditions must be met for a response to warrant full marks.

- Negative exponents should not appear in a final answer.
- Fractions should be simplified (i.e., positive denominator and lowest terms)
- Trigonometric ratios other than $\pm\left\{0, \frac{1}{2}, 1\right\}$ should not be in decimal form unless a question specifies a decimal answer.

**Graph**

A response to the prompt, "*graph*", must include a minimum of…

- a scale on each axis
- an appropriate label on each axis using the variables applied in the question

Further specifics of the graph may be required in the language of a question (intercepts, asymptotes, maximum, minimum, vertex, etc.).

In cases where "*graph*" is not requested in the question, but a grid is helpful, the comment: "*This grid is provided for your assistance only*" will accompany a supplied grid. This indicates use of the grid is optional but, if used, will be considered in scoring the question.

**Algebraically Determine**

"*Algebraically determine…*" in a constructed response question requires that:

- variable assignment be explicit and valid
- the function/equation be set up correctly
- methods of solution involve manipulation of variables, for example
  - finding the maximum or minimum of a quadratic function using
    - completing the square
    - $x = \dfrac{-b}{2a}$

- solving a quadratic equation using
    - completing the square
    - quadratic formula
    - factoring
- applying laws of logarithms and exponents
- solving a system of equations, using
    - substitution
    - elimination
    - matrices - set up must be shown
- approximating instantaneous rate of change
    - must include the algebra of the difference quotient or the derivative calculated using $f'$

- the final answer is sufficiently justified and clearly stated (e.g., The width of the rectangle is 3 m.)

"*Algebraically determine…*" partnered with "…*and use this function to*…" requires that algebraic methods of solution involving manipulation of variables be used to complete the question.

**Set up the function…**

"*Set up the function*…" partnered with "…and *use this function to*…" requires that the algebra of the function model and algebraic methods of solution involving manipulation of variables be used to complete the question.

**Graphing Calculator Use**

The Mathematics 3204 and 3205 Public Examinations are graphing calculator neutral.

Where a graphing calculator utility is used in a problem-solving situation, its use must be accompanied by appropriate set up or appropriate explanation.

✓ For example, if in solving a problem it is necessary to determine an intersection, a student might choose to calculate $[A]^{-1}B$ from the coefficients of the given equations. If so, then as a minimum, $[A]^{-1}B$ (with the actual related values) must be given prior to stating the intersection value(s) produced.

**Inadmissible Roots and Extraneous Values**

Inadmissible roots and extraneous values must be rejected from any solution set for a response to warrant full marks. This most commonly occurs in solving rational equations and logarithmic equations, but may also occur due to the context of a problem or due to the particular value the student is asked to determine from the question. Students should be advised to determine whether any values arrived at by calculations are within the domain of the original statement of the function or problem.

**Accuracy, Precision, and Rounding**

Students should use a minimum of four decimal place accuracy for computations involving rate of change, logarithms and trigonometric ratios. It is not necessary for students to write four decimal place accuracy in intermediate steps as calculators can carry such accuracy easily.

Precision of final answers will be determined by the context of the question.

Students are expected to use a calculator $\pi$ value since it reduces rounding error and provides greater precision.

## Terminology

**Inadmissible Roots** are roots that are inadmissible in the context of the problem. For example, if a question asks for the second occurrence of a given height, and solving produces two reasonable domain values, the first is inadmissible in the context of the problem and should be rejected and the second is the solution.

**Extraneous Values** are not potential roots since they are not in the domain of the problem/function to begin with. Examples include values which produce division by zero, negative arguments in a logarithm, or dimension conflicts in a problem (e.g., a negative length).

**Roots, Zeros, and *x*-intercepts** are related but not synonymous terms. Roots are the potential solution value(s) for an equation or problem. Such values can be verified as solutions by testing them against the conditions of the problem to ensure admissibility, or to determine they are not outside the domain of the problem (extraneous). Zeros and *x*-intercepts generally relate to the algebraic or graphical representation of a function rule respectively.

# Examination Design and Scoring

## *Introduction*

This section will elaborate on the different question types used on the examination and how questions are placed and scored.

## *Question Types*

There are two question types used in the assessment: selected-response and constructed-response valued at 50% each.

The selected response type used is multiple-choice. Multiple-choice questions on the examination use 4 alternatives A, B, C, and D. These items are equally weighted at 1 point each.

The constructed response questions generally range in value from 3 to 5 points and require students to write, graph or draw, and explain to form their responses. Sufficient justification and appropriate explanation are required to warrant full marks.

## *Scoring Criteria*

<u>Multiple-Choice</u>
Multiple-choice items are machine-scored dichotomously as correct or incorrect.

<u>Constructed Response</u>
A solution key, including the distribution of point values for each question, is developed during the item writing process. These solutions are then verified by the marking board. A student who reaches an incorrect final result but has elements of the process correct, will be awarded credit for the correct portions of the process.

The marking board notes all variations in student responses and submits these to the Chief Marker. A synthesis of this collected information, including a validated answer key, is produced as the *Grading Standards* for that examination for that year.

### *Sub-score Generation*

A portion of the questions on the Mathematics 3205 examination is used to generate a Mathematics 3204 sub-score which is used to calculate a Mathematics 3204 non-shared grade. A student doing Mathematics 3205 will receive two grades on his/her transcript: a grade for Mathematics 3205 that is shared equally from the school and the public examination, and a Mathematics 3204 grade that is based only on the sub-score obtained from the Mathematics 3205 public examination.

Questions used in the sub-score form approximately 60% of the Mathematics 3205 examination. Outcomes where the breadth and depth of treatment are identical in Mathematics 3204 and Mathematics 3205 have identical items on both examinations and are filed as common items. Outcomes where the treatment in Mathematics 3204 and Mathematics 3205 is very similar, have corresponding but slightly different items written for both examinations and are filed as 'Mathematics 3204-appropriate' questions. Together, the common and 'Mathematics 3204-appropriate' questions provide a sub-score and produce a Mathematics 3204 non-shared grade.

This model encourages students to get the most out of the Mathematics 3205 course and put forward their best effort on the examination. At the same time, the Mathematics 3205 examination remains at an appropriate level of difficulty.

# Appendices

## *Mathematics Public Examination Framework*

### Introduction

This section organizes the outcomes from the curriculum guide into a framework for examination development. The following are included:

- Cognitive Taxonomy - in this case, a version of Bloom's Taxonomy containing descriptors for the three levels associated with public examination questions in Mathematics.

- Outcome List by Unit.

- Table of Specifications - a formal evaluation framework for the Mathematics examination, mapping content and cognitive level.

## Cognitive Taxonomy

| Taxonomy of Cognitive Processes in Mathematics | | |
|---|---|---|
| Level 1 | Level 2 | Level 3 |
| Demonstrates **understanding of terms, principles, relationships, processes, and theorems** by being able to…<br><br>- use definitions and terms<br>- identify when and why a formula, procedure, or theorem is appropriate<br>- read mathematical words, tables, and graphs with understanding<br>- apply simple computational algorithms and formulae in simple contexts<br><br>Demonstrates mathematical *literacy* by being able to…<br><br>- represent numeracy through multiple representations<br>- comprehend and appropriately use mathematical terminology<br>- translate from one representation to another<br><br><u>Selected Verbs</u>: what, why, when, where, which, choose, find, how, define, label, show, list, name, tell, recall, select, compare, contrast, sequence, classify, explain, identify, summarize, outline, interpret, infer, extend, generalize, conclude | Demonstrates **conceptual understanding in various contexts and via multiple representations in routine applications** by being able to…<br><br>- make interconnections among mathematical operations and concepts, other subject areas, and daily life<br>- choose and apply appropriate formulae, procedures, theorems, and computational algorithms in routine applications<br>- use words, tables, and graphs to communicate mathematical understanding<br>- use words, tables, and graphs to model mathematical situations and ideas<br><br>- manipulate algebraic symbols and formulae<br>- analyze a given procedure and give reasons for a procedure<br><br><u>Selected Verbs</u>: apply, construct, model, use, practise, organize, simulate, translate, experiment, analyze, diagram, compare, contrast, sequence, classify, simplify, describe, explain, justify, support, differentiate, test for, inspect | Combines conceptual understanding (in various contexts and via multiple representations in routine applications) with the ability to **choose, apply, and adapt strategies to solve problems, and monitor and reflect on ideas and the reasonableness of answers** while solving problems by being able to…<br><br>- use appropriate formulae, procedures, theorems, and computational algorithms in novel situations<br>- justify and explain thinking using mathematical terms and arguments<br>- interconnect among mathematical operations and concepts, other subject areas, and daily life, as illustrated through the use of various mathematical representations (e.g., patterns, pictures, symbols, models, tables, charts, graphs, lists, diagrams, numbers)<br>- write about and/or represent mathematics; explain answers and describe strategies; use appropriate mathematical language with precision and clarity<br><br><u>Selected Verbs</u>: compose, design, develop, propose, adapt, elaborate, formulate, originate, solve, invent, improve, predict, plan, compose, criticise, appraise, estimate, value, deduct, determine, decide, describe, judge, rank, rate, evaluate, assess, defend, explain, justify, support, prove |

**Outcome List by Unit**

Note: *Develop and apply* in an outcome is *apply* for public examination purposes.

| **Unit 1:** | **Quadratics** |
|---|---|
| C4 | Demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions |
| C29 | Analyze tables and graphs to distinguish between linear, quadratic, and exponential relationships |
| C3 | Sketch graphs from descriptions, tables, and collected data |
| C1 | Model real-world phenomena using quadratic functions |
| C8 | Describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships |
| C31 | Analyze and describe the characteristics of quadratic functions |
| A7 | Describe and interpret domains and ranges using set notation |
| F1 | Create and analyze scatter plots and determine the equations for curves of best fit, using appropriate technology |
| C23 | Solve problems involving quadratic equations |
| C32 | Demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions |
| C10**ADV** | Determine the equation of a quadratic function using finite differences |
| C9 | Translate between different forms of quadratic equations |
| B1 | Demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations |
| C22 | Solve quadratic equations |
| B10 | Derive and apply the quadratic formula |
| B11**ADV** | Analyze the quadratic formula to connect its components to the graphs of quadratic functions |
| A4 | Demonstrate an understanding of the nature of the roots of quadratic equations |
| C15 | Relate the nature of the roots of quadratic equations and the $x$-intercepts of the graphs of the corresponding functions |
| A9 | Represent non-real roots of quadratic equations as complex numbers |
| A3 | Demonstrate an understanding of the role of irrational numbers in applications |

| Unit 2: | Rate of Change |
|---|---|
| C17 | Demonstrate an understanding of the concept of rate of change in a variety of situations |
| B4 | Calculate average rates of change |
| C16 | Demonstrate an understanding that slope depicts rate of change |
| C30 | Describe and apply rates of change by analyzing graphs, equations and descriptions of linear and quadratic functions |
| C28 | Solve problems involving instantaneous rate of change |
| C18 | Demonstrate an understanding that the slope of a line tangent to a curve (at a point) is the instantaneous rate of change of the curve at the point of tangency |
| C27 | Approximate and interpret slopes of tangents to curves at various points on the curves, with and without technology |

| Unit 3: | Exponential Growth |
|---|---|
| C2 | Model real-world phenomena using exponential functions |
| B2 | Demonstrate an intuitive understanding of the recursive nature of exponential growth |
| C25 | Solve problems involving exponential and logarithmic equations |
| C11 | Describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships |
| C4 | Demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions |
| C33 | Analyze and describe the characteristics of exponential and logarithmic functions |
| C29 | Analyze tables and graphs to distinguish between linear, quadratic, and exponential relationships |
| A7 | Describe and interpret domains and ranges using set notation |
| C34 | Demonstrate an understanding of how the parameter changes affect the graphs of exponential functions |
| A5 | Demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations |
| C3 | Sketch graphs from descriptions, tables, and collected data |
| F1 | Create and analyze scatter plots and determine the equations for curves of best fit, using appropriate technology |
| C35**ADV** | Write exponential functions in transformational form, and as mapping rules to visualize and sketch graphs |
| B1 | Demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations |
| B12 | Apply real number exponents in expressions and equations |
| C24 | Solve exponential and logarithmic equations |
| C19 | Demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function |
| B13 | Demonstrate an understanding of the properties of logarithms and apply them |

| **Unit 4:** | **Circle Geometry** |
|---|---|
| E4 | Apply properties of circles |
| E5 | Apply inductive reasoning to make conjectures in geometric situations |
| E11 | Write proofs using various axiomatic systems and assess the validity of deductive arguments |
| E7 | Investigate and make and prove conjectures associated with chord properties of circles |
| E8 | Investigate and make and prove conjectures associated with angle relationships in circles |
| E9 | Investigate and make and prove conjectures associated with tangent properties of circles |
| E12 | Demonstrate an understanding of the concept of converse |
| D1 | Develop and apply formulas for distance and midpoint |
| A3 | Demonstrate an understanding of the role of irrational numbers in applications |
| E15 | Solve problems involving the equations and characteristics of circles and ellipses |
| E13 | Analyze and translate between symbolic, graphical, and written representations of circles and ellipses |
| E3 | Write the equations of circles and ellipses in transformational form, and as mapping rules to visualize and sketch graphs |
| E16 | Demonstrate the transformational relationship between the circle and the ellipse |
| E14 | Translate between different forms of the equations of circles and ellipses |
| C20**ADV** | Represent circles using parametric equations |
| C36 | Demonstrate an understanding of the relationship between angle rotation and the coordinates of a rotating point |
| C37**ADV** | Describe and apply parameter changes within parametric equations of circles |

**Table of Specifications**

| | Level 1 | Level 2 | Level 3 | Total |
|---|---|---|---|---|
| **Quadratics** | 8 | 16 | 9 | **33** |
| **Rate of Change** | 1 | 1 | 4 | **6** |
| **Exponential Growth** | 8 | 17 | 8 | **33** |
| **Circle Geometry** | 3 | 16 | 9 | **28** |
| **Total** | **20** | **50** | **30** | **100** |

## *Exemplars*

This section contains items that are typically asked by teachers or items that students often have difficulty with. Common student errors or misconceptions are highlighted and some teaching suggestions are provided.

**Unit 1: Quadratics**

**ITEM 1.1**

Algebraically determine the EXACT roots in simplest form for $x(x-2)=-3$.

**Solution:**

$$x^2-2x+3=0$$

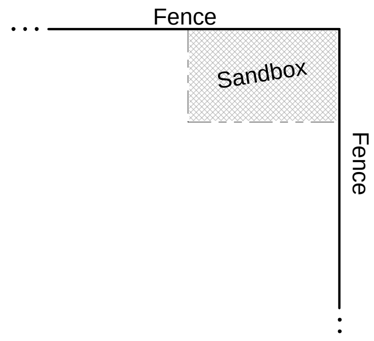
$$\begin{aligned} x &= \frac{2 \pm \sqrt{(-2)^2-4(1)(3)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4-12}}{2} \\ &= \frac{2 \pm \sqrt{-8}}{2} \\ &= \frac{2 \pm 2i\sqrt{2}}{2} \\ &= 1 \pm i\sqrt{2} \end{aligned}$$

**Common Errors:**

1. expanding and transposing incorrectly in $x(x-2)=-3$
2. trying to factor $x^2-2x+3$, resulting in incorrect roots
3. using an “incorrect” quadratic formula
4. not simplifying $\frac{2 \pm 2i\sqrt{2}}{2}$ or simplifying incorrectly to $\pm i\sqrt{2}$
5. not recognizing that the roots are imaginary and using a calculator to find decimal approximations

**ITEM 1.2**

A day care centre bought 20 m of board to form two sides of a rectangular sandbox against a corner in a fenced yard as shown. If all 20 m of board is used, write the quadratic function that models the area of the sandbox and use it to determine the maximum area the sandbox can have.




**Solution:**

Length + Width = 20

Length = $x$
Width = $20-x$

$A = x(20-x)$

$A = -x^2 + 20x$

$x = \frac{-b}{2a} = \frac{-20}{2(-1)} = 10$

Length = 10 m and Width = 10 m
$\therefore$ maximum area is 100 m$^2$

**Alternate Solution:**

Length + Width = 20

Length = $x$
Width = $20-x$

$A = x(20-x)$

$A = -x^2 + 20x$

$A = -(x^2 - 20x)$

$A - 100 = -(x^2 - 20x + 100)$

$A - 100 = -(x-10)^2$

Vertex $(10,100)$

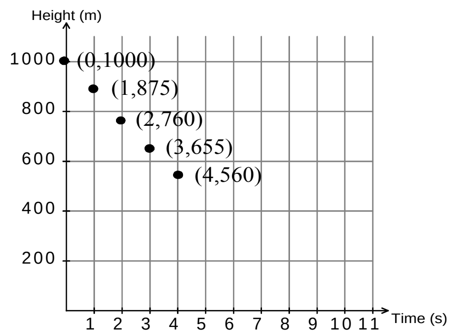
$\therefore$ maximum area is 100 m$^2$

**Common Errors:**

1. using $A = x(20-2x)$ instead of $A = x(20-x)$
2. getting -10 instead of 10 using $x = -\frac{b}{2a}$
3. getting the correct set up and width, but failing to determine the maximum area
4. treating this as a three-sided instead of a two-sided diagram
5. incorrectly factoring out the negative in $A = -x^2 + 20x$
6. incorrectly balancing the equation when completing the square

**ITEM 1.3 (3205 only)**

A small plane begins a parabolic dive from 1000 m. The graph shows the plane’s height, in metres, above the ground over time, in seconds, during a parabolic dive. Algebraically determine the quadratic function that defines the path of the plane over time and use it to determine the plane’s height at 9.5 seconds.




**Solution:**

Note: $c = 1000$

$$\begin{array}{lcccc} & 875 & 760 & 655 & 560 \\ & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \\ D_1 & -115 & -105 & -95 & \\ & \searrow\swarrow & \searrow\swarrow & & \\ D_2 & 10 & 10 & & \end{array}$$

$D_2 = 2a$
$10 = 2a$
$5 = a$

$h(t) = 5t^2 + bt + 1000$
$875 = 5(1)^2 + b(1) + 1000$
$b = -130$

$h(t) = 5t^2 - 130t + 1000$
$h(9.5) = 216.25$ m
At 9.5 s the plane’s height is 216.25 m.

**Alternate Solution:**

$$875 = a(1)^2 + b(1) + c$$
$$760 = a(2)^2 + b(2) + c$$
$$655 = a(3)^2 + b(3) + c$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 875 \\ 760 \\ 655 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 875 \\ 760 \\ 655 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ -130 \\ 1000 \end{bmatrix}$$

$h(t) = 5t^2 - 130t + 1000$
$h(9.5) = 216.25$ m

**Common Errors:**

1. misinterpreting the points (e.g., (1,875) and (2,760) as the numbers 1,875 and 2,760)
2. using $D_2 = -10$
3. forgetting to determine the plane’s height

**Unit 2: Rate of Change**

**ITEM 2.1**

The volume of water at a given time in a 2000 L tank is represented by the formula $V = 2000\left(1-\frac{t}{45}\right)^2$, where $t$ is time in minutes. Determine the average rate of change in the volume of water in the tank from minute 0 to minute 10, and use it to describe how the volume of water in the tank is changing during that time.

**Solution:**

$$V(0) = 2000 \qquad V(10) = 1209.9$$

$$\begin{aligned}\text{Average RoC} &= \frac{V(10)-V(0)}{10-0}\\ &= \frac{2000(1-\frac{10}{45})^2-2000}{10}\\ &= \frac{1209.8765-2000}{10}\\ &= -79.01\ \text{L}/\text{min}\end{aligned}$$

On average, the tank is losing 79.01 L of water each minute.

**Common Errors:**

1. misusing the slope formula (i.e. $\frac{y_2-y_1}{x_1-x_2}$ or $\frac{x_2-x_1}{y_2-y_1}$ )
2. having difficulty calculating $2000(1-\frac{10}{45})^2$
3. calculating $\frac{V(0)+V(10)}{2}$
4. incorrectly using 200 instead of 2000
5. not describing how the volume of water in the tank was changing
6. treating the question as an instantaneous rate of change question using $\frac{V(10.1)-V(9.9)}{10.1-9.9}$

**ITEM 2.2**

A diver jumps off a spring board. Her height $h$, in metres, above the water, $t$ seconds after she jumps, is given by $h(t) = -4.9t^2 + 8t + 5$. Algebraically determine the approximate instantaneous rate of change in her height at 2 seconds.

**Solution:**

| $t$ | $h$ |
|---|---|
| 1.99 | 1.5155 |
| 2.01 | 1.2835 |

$$= \frac{h(2.01) - h(1.99)}{2.01 - 1.99}$$
$$= \frac{1.2835 - 1.5155}{2.01 - 1.99}$$
$$= \frac{-0.232}{0.02}$$
$$= -11.6 \text{ m/s}$$

At 2 seconds, the instantaneous rate of change is – 11.6 m/s.

**Alternate Solution:**

Using the derivative:

$$h'(t) = -9.8t + 8$$
$$h'(2) = -9.8(2) + 8$$
$$h'(2) = -11.6 \text{ m/s}$$

**Common Errors**:

1. not following the correct order of operations
2. using too large a time interval for instantaneous rate of change
3. using an inadequate number of decimal places
4. evaluating $h(2)$ and getting 1.4 and leaving it as the answer
5. dropping the negative sign

**Unit 3: Exponential and Logarithmic Functions**

**ITEM 3.1**

Solve for $x$:

$$\log_4(x+7) - \log_4(x^2+3x) = \frac{1}{2}$$

**Solution:**

$$\log_4\left(\frac{x+7}{x^2+3x}\right) = \frac{1}{2}$$

$$\frac{x+7}{x^2+3x} = 4^{\frac{1}{2}}$$

$$x+7 = 2(x^2+3x)$$

$$x+7 = 2x^2+6x$$

$$2x^2+5x-7=0$$

$$(2x+7)(x-1)=0$$

$$x = -\tfrac{7}{2} \text{ or } x = 1$$

**Common Errors:**

1. rejecting the value $x = -\tfrac{7}{2}$ (this value is acceptable in the problem)
2. failing to apply laws of logarithms as indicated by the following first approaches:
   $\frac{\log_4(x+7)}{\log_4(x^2+3x)} = \frac{1}{2} \quad \Rightarrow \quad \frac{(x+7)}{(x^2+3x)} = \frac{1}{2}$ or $\frac{(x+7)}{(x^2+3x)} = \frac{1}{2}$
3. making algebraic errors in determining the resulting quadratic
4. incorrect factoring or making quadratic formula errors in solving $2x^2+5x-7=0$
5. immediately 'dropping' logs incorrectly as follows:
   $(x+7)-(x^2+3x) = 2$ where a 2 was determined as $4^{\frac{1}{2}}$ or as $4\left(\tfrac{1}{2}\right)$
6. subtracting the logarithmic arguments instead of dividing

**ITEM 3.2**

Solve for *x*:

$$3^{5x-1} = \sqrt[3]{9}$$

**Solution:**

$3^{5x-1} = \sqrt[3]{9}$
$3^{5x-1} = 3^{\frac{2}{3}}$
$5x - 1 = \frac{2}{3}$
$15x - 3 = 2$
$15x = 5$
$x = \frac{1}{3}$

**Common Errors:**

1. rewriting the radical incorrectly (i.e. $\sqrt[3]{9} = \left(3^2\right)^3$ or $\sqrt[3]{9} = \left(9^{\frac{1}{2}}\right)^{\frac{1}{3}}$)
2. incorrectly changing $9$ to $3^3$
3. cross-multiplying incorrectly resulting in $15x - 1 = 2$
4. solving incorrectly (i.e. $15x = 5 \rightarrow x = 3$)

**ITEM 3.3 (3205 only)**

Algebraically solve: $2^{4x} - 6(2)^{2x} - 16 = 0$ .

**Solution:**

$\left(2^{2x}\right)^2 - 6(2)^{2x} - 16 = 0$

Let $y = 2^{2x}$

$y^2 - 6y - 16 = 0$

$(y-8)(y+2) = 0$

$y = 8$ or $y = -2$

$2^{2x} = 8$ or ~~$2^{2x} = -2$~~

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

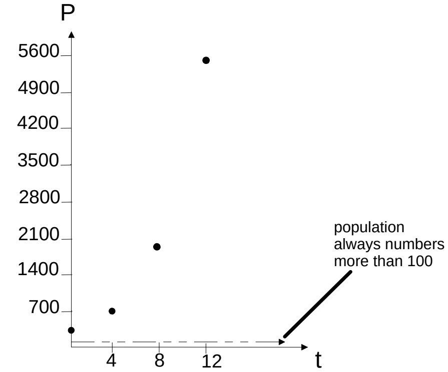
**Common Errors:**

1. not rejecting $2^{2x} = -2$ and giving responses such as $2x = -1$ $\left(\text{i.e. } x = -\frac{1}{2}\right)$ or $2x = 1$ $\left(\text{i.e. } x = \frac{1}{2}\right)$ as the second value for $x$
2. applying logarithms to the original equation (i.e. $\log 2^{4x} - 6\log 2^{2x} - \log 2^4 = 0$ ) and going on to obtain $4x - 12x - 4 = 0$
3. changing 16 to $2^4$ to produce $2^{4x} - 6(2)^{2x} - 2^4 = 0$ and obtaining $4x - 12x - 4 = 0$
4. solving the quadratic equation $2y^2 - 6y - 16 = 0$
5. rearranging the equation as $2^{4x} - 6(2)^{2x} = 16$, obtaining $2^{4x} - (2)^{2x} = \frac{16}{6}$ or $\frac{2^3}{3}$, and equating exponents (i.e. $4x - 2x = 3$ )
6. solving $2x = 3$ and obtaining $x = \frac{2}{3}$
7. incorrectly factoring $y^2 - 6y - 16$
8. solving for "$y$" correctly but not substituting back to find the value of $x$

**ITEM 3.4 (3205 only)**

A bacteria population is well cared for so it always numbers more than 100. An experiment was begun to study how a particular type of light affected the number of bacteria. Upon starting the experiment, the population, *P*, at time, *t*, in hours, was recorded in the table and shown on the graph below. Algebraically determine the function that models this population and use the function to determine the number of bacteria at 20 hours.

| *t* | 0 | 4 | 8 | 12 |
|---|---|---|---|---|
| *P* | 300 | 700 | 1900 | 5500 |




**Solution:**

| *t* | 0 | 4 | 8 | 12 |
|---|---|---|---|---|
| *P* | 300 | 700 | 1900 | 5500 |
| *P* - 100 | 200 | 600 | 1800 | 5400 |

Ratio $= \dfrac{600}{200} = 3$

$P(t) = a\left(b\right)^{\frac{x}{c}} + d$

$P(t) = 200(3)^{\frac{x}{4}} + 100$

$P(20) = 200(3)^{\frac{20}{4}} + 100$

$P(20) = 48\,700$ At 20 h, there are $48\,700$ bacteria.

**Common Errors:**

1. not adjusting for the horizontal asymptote in the table of values before determining the common ratio
2. using 300 as the initial value in the equation (after adjusting the table)
3. not adjusting for the "*c*" value (i.e. using *x* as the exponent instead of $\frac{x}{4}$)
4. not including the "*d*" value of 100 in the final equation
5. determining the equation correctly but not calculating *P*(20)
6. writing equation as $P(t) = 200(3)^{\frac{x}{4}} - 100$

**ITEM 3.5**

Technetium-99, a radioactive isotope used in nuclear medicine, has a half-life of 6 hours. Set up an equation and use it to determine how long it would take for 500 micrograms of Technetium-99 to reduce to 100 micrograms.

**Solution:**

$$y = 500\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$100 = 500\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\tfrac{1}{5} = \left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\log\left(\tfrac{1}{5}\right) = \log\left(\tfrac{1}{2}\right)^{\frac{x}{6}}$$

$$\log\left(\tfrac{1}{5}\right) = \tfrac{x}{6}\log\left(\tfrac{1}{2}\right)$$

$$x = \frac{6\log\left(\tfrac{1}{5}\right)}{\log\left(\tfrac{1}{2}\right)}$$

$$= 13.93 \text{ or } 14 \text{ hours}$$

It takes 14 hours for 500 micrograms of Technetium-99 to reduce to 100 micrograms.

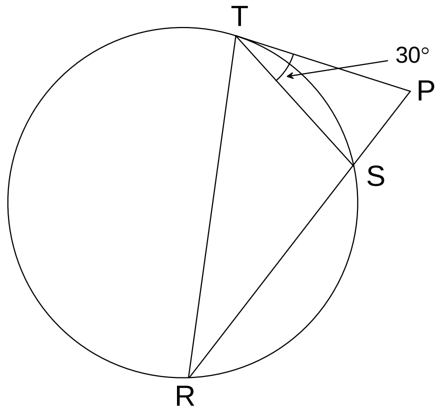
**Common Errors:**

1. multiplying 500 by $\frac{1}{2}$
2. dividing $\log\frac{1}{2}$ by $\log\frac{1}{5}$
3. dividing $\dfrac{\log\left(\frac{1}{5}\right)}{\log\left(\frac{1}{2}\right)}$ by 6 instead of multiplying
4. substituting values into $y = a \cdot b^{\frac{x}{c}}$ incorrectly
5. substituting 100 in for *x* and solving for *y*

## Unit 4: Circle Geometry

### ITEM 4.1

If $\overline{TP}$ is tangent to the circle, $\overset{\frown}{RS} = 100°$, and $\angle PTS = 30°$, determine the measure of $\angle TRS$, $\angle RTS$, and $\angle TPS$.




**Solution:**

$\angle TRS = \angle PTS$

$\angle TRS = 30°$

$\angle RTS = 50°$

$\angle TPS = 180° - 80° - 30° = 70°$

**Common Errors:**

1. assuming $\overline{TR}$ is a diameter and thus concluding that $\angle RTP = 90°$
2. assuming $\overline{TS} \perp \overline{PR}$
3. incorrectly naming or misinterpreting names of angles

**ITEM 4.2**

Write $9x^2 + 4y^2 - 90x - 16y + 205 = 0$ in transformational form.

**Solution:**

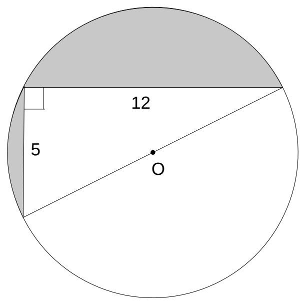
$$9x^2 - 90x + 4y^2 - 16y = -205$$
$$9(x^2 - 10x + \underline{25}) + 4(y^2 - 4y + \underline{4}) = -205 + \underline{225} + \underline{16}$$
$$9(x-5)^2 + 4(y-2)^2 = 36$$
$$\tfrac{1}{4}(x-5)^2 + \tfrac{1}{9}(y-2)^2 = 1$$
$$\left[\tfrac{1}{2}(x-5)\right]^2 + \left[\tfrac{1}{3}(y-2)\right]^2 = 1$$

**Common Errors:**

1. incorrectly completing the square (i.e. $x^2 - 10x \rightarrow x^2 - 10x + 10$ )
2. ignoring the number outside brackets when balancing the equation
3. subtracting 225 and 16 on the right hand side instead of adding
4. leaving the answer as $\tfrac{1}{4}(x-5)^2 + \tfrac{1}{9}(y-2)^2 = 1$

**ITEM 4.3**

In this circle with center O, determine the total area of the shaded region.




**Solution:**

$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$c = 13$$
$$\therefore \text{Radius} = 0.5(13) = 6.5$$

$$A_{\text{triangle}} = \tfrac{1}{2}(5)(12) = 30$$

$$A_{\text{semicircle}} = \tfrac{1}{2}\pi(6.5)^2$$

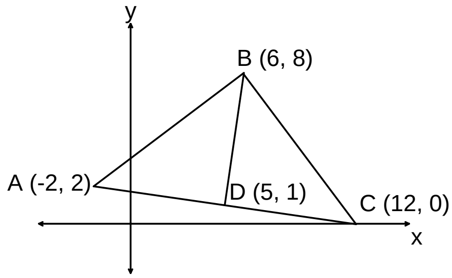
$$A_{\text{shaded region}} = 66.4 - 30 = 36.4 \text{ square units}$$

**Common Errors:**

1. using 90° instead of 180° for the portion of the circle involved in the shading
2. using 13 instead of 6.5 for the radius
3. attempting to get the area of the triangle by using $2\pi r$ or $2\pi r^2$ or $b \times h$
4. adding rather than subtracting $A_{\text{semicircle}}$ and $A_{\text{triangle}}$

**ITEM 4.4**

Given $\Delta ABC$ as shown, use coordinate geometry to prove $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.




**Solution:**

slope $\overline{BD} = \frac{8-1}{6-5} = 7$

slope $\overline{AC} = \frac{2-0}{-2-12} = -\frac{1}{7}$

$\therefore\ \overline{BD} \perp \overline{AC}$

Midpoint of $\overline{AC}$:

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+12}{2}, \frac{2+0}{2}\right) = (5, 1)$ i.e. point D

$\therefore\ \overline{BD}$ bisects $\overline{AC}$

Since $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ is the midpoint of $\overline{AC}$, then $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

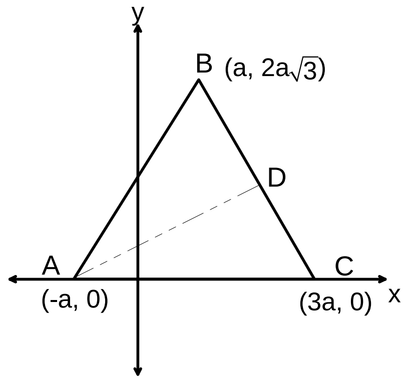
**Note:** Students could also find the lengths of sides in the smaller triangles and show via the Pythagorean theorem they were right triangles and $\overline{AD} = \overline{DC}$.

**Common Errors:**

1. using an "incorrect" slope formula $\left(\text{i.e. } m = \frac{x_2 - x_1}{y_2 - y_1}\right)$
2. finding the slopes of $\overline{AB}$ and $\overline{BC}$
3. using the distance formula to determine the lengths of $\overline{BD}$ and $\overline{AC}$ with no logical conclusion
4. stating $-\frac{2}{14} = -7$
5. using the distance formula to determine all sides and stating that this proved a perpendicular bisector
6. proving perpendicularity but not the bisector
7. proving the bisector but not perpendicularity

**ITEM 4.5 (3205 only)**

Using coordinate geometry, prove the median from A to $\overline{BC}$ is perpendicular to $\overline{BC}$.




**Solution:**

$D: \left(2a, a\sqrt{3}\right)$

$m_{\overline{AD}} = \dfrac{a\sqrt{3}}{3a} = \dfrac{\sqrt{3}}{3}$

$m_{\overline{BC}} = \dfrac{2a\sqrt{3}}{-2a} = -\sqrt{3}$

$\left(m_{\overline{AD}}\right)\left(m_{\overline{BC}}\right) = \dfrac{\sqrt{3}}{3} \cdot -\sqrt{3} = -1$ OR $\overline{AD}$ and $\overline{BC}$ have opposite reciprocal slopes.

Thus, $\overline{AD} \perp \overline{BC}$.

**Common Errors:**

1. using an "incorrect" slope formula $\left(\text{i.e., } m = \dfrac{x_2 - x_1}{y_2 - y_1}\right)$
2. not recognizing or having difficulty determining that the required slopes were opposite reciprocals $\left(\text{i.e., } m_{\overline{AD}} = \dfrac{\sqrt{3}}{3} \text{ and } m_{\overline{BC}} = \sqrt{3}\right)$
3. not making a concluding statement indicating why/how lines were perpendicular