Fractions

Suggested Time: 3 Weeks


Unit Overview

Focus and Context

In Grade 6, students will deepen their understanding of number by continuing to work on fractions. They will develop an understanding of fractions that are greater than one and how these improper fractions can be expressed as mixed numbers. Students will have many opportunities to develop these concepts as they model, draw, name and write about mixed numbers and improper fractions while solving meaningful problems. These ‘hands on’ experiences will enable students to visualize the connections between improper fractions and mixed numbers.

The study of improper fractions and mixed numbers should build on students’ prior knowledge of whole number and proportional concepts and skills and their encounters with decimals, proper fractions, ratios and percents in previous work and in their everyday life. Students will be engaged in problem solving situations where they will have to compare improper fractions and mixed numbers. As they work through various situations, students will continue to develop their problem solving strategies as they learn how to effectively use models, pictures and logical reasoning to solve problems.

Math Connects

Fractions are a part of everyday life; cooking, measuring, building and understanding quantities are just a few examples of where people encounter fractions. Learning about fractions that have a value greater than one becomes merely an extension of what students have already learned previously. This not only helps in the development of a strong sense of number, but also gives students a better understanding of how these numbers are used in everyday life. Thus, developing this number sense, understanding what a number means, what it represents and how it can be used becomes a reason to understand and become familiar with the concepts of improper fractions and mixed numbers.

As students learn about these concepts through solving problems in context, they can consider the advantages and disadvantages of various representations of these quantities. For example, they could consider the reasonableness of showing how much flour is needed to make pizza dough as a mixed number rather than an improper fraction. Their use of logical reasoning to help solve problems such as this becomes stronger as they add more strategies to helping them solve everyday, real world problems.
**Process Standards Key**

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**Curriculum Outcomes**

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<td>Number</td>
<td>6N4 Relate improper fractions to mixed numbers.</td>
<td>[CN, ME, R, V]</td>
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Outcomes

Students will be expected to

6N4. Relate improper fractions to mixed numbers.

[CN, ME, R, V]

Achievement Indicator:

6N4.1 Demonstrate, using models, that a given improper fraction represents a number greater than 1.

Elaborations—Strategies for Learning and Teaching

Improper fractions and mixed numbers are new to Grade 6 students. This work provides students with opportunities to build on their prior understanding of equivalent fractions and their ability to compare fractions with like and unlike denominators where the numerator is less than the denominator. In Grade 6, students now work on fractions that are greater than 1 and relate this to mixed numbers. This is completed through the use of models and pictures such as pattern blocks, fraction pieces and number lines.

Students will have an understanding of fractional parts, or equal shares which they refer to as thirds, fourths, fifths, tenths, etc. They will recognize that these fractional parts can be counted in the same way as any other set of objects. Fractions greater than one whole can be understood this way. For example, six fourths is just six parts called fourths. By counting fractional parts we can help students develop a completely generalized system for naming fractions before they learn about fraction symbolism. Counting fractional parts lays the groundwork for several important ideas. The idea that fifths are smaller than halves, for example, can be a difficult concept for students to understand since 5 is usually 'more' than 2. Counting the different size parts and seeing how many parts it takes to make one whole is an opportunity to begin thinking on this idea.

To illustrate this idea, show students manipulatives representing five or six fourths. Ask students to tell how many fourths. Ask if the collection is more or less than one whole, two wholes? While doing this, prompt students to make informal comparisons among different counts. For example, ask them to explore the reasons they would get almost two wholes out of eight fifths when they would not get one whole out of five eighths. Ask them questions such as “What is another way we could say seven fourths? To introduce this topic, use fractional parts that students are most comfortable with, such as thirds, fourths and halves. Challenge students to come up with ways to represent what the fractional pieces show. For example, if they have $\frac{5}{4}$, ask students to explore other ways of showing how they are thinking about this fraction using models, pictures and numbers.

Models can help students clarify ideas that are often confused in a purely symbolic mode. Sometimes it is useful to do the same activity with two different models. From the view point of the student, the activity may be quite different. Using two different models, for example pattern blocks and fraction tiles, students visually see how the same fraction can be modelled in two different ways. This may help students understand the concept of improper fractions holding a value that is greater than 1.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Tell students Billy went to the store. While he was there he noticed a sign saying bars were on sale. “4 for $1.00. Billy thought this was a great deal and bought 4 bars. Billy wanted to share the bars among his two friends. Show how Billy can share this equally among him and his two friends. 

Student-Teacher Dialogue

- Ask students to explain why \( \frac{5}{3} \) must be greater than one whole.

- Ask students to model \( \frac{7}{4} \) to show that it is greater than one whole.

- Tell students you modelled the improper fraction \( \frac{9}{5} \) using pattern blocks. Ask them how they know you used 9 blocks. Ask them to determine which pattern block was used to model \( \frac{9}{5} \) and explain how they know.

- Tell students that the improper fraction \( \frac{10}{3} \) can be modelled using pattern blocks. How many different ways can you model this improper fraction using these pattern blocks?

- Ask students to represent the following improper fractions using rectangles:
  
  (i) \( \frac{5}{4} \)
  
  (ii) \( \frac{3}{2} \)

Ask students to explain their thinking.

Resources/Notes

Math Focus 6

Lesson 1: Modelling Fractions

6N4

TG pp. 13 – 17
Strand: Number

Outcomes

*Students will be expected to*

6N4 Continued

**Achievement Indicator:**

6N4.1 Continued

Elaborations—Strategies for Learning and Teaching

Making meaningful connections to the world can also help students understand the concepts they are learning. While talking about improper fractions, and even mixed numbers, money can be used to help students understand these concepts. A dime, for example can be seen as $\frac{1}{10}$ of a dollar where 10 dimes make one whole dollar. Use this context to ask students to explore ways to make more than one dollar using these coins. For example, if they use 5 quarters, which can be represented as $\frac{5}{4}$, they will have $1.25$ or $1\frac{1}{4}$ of a dollar. Since students are really familiar with money, it may be a good place to start when talking about these concepts. This can also help students see the connections between improper fractions and mixed numbers.

It is important that students have a strong conceptual understanding of improper fractions. Students need to be able to understand and explain that an improper fraction represents more than one whole and that its numerator is greater than its denominator. To create this conceptual understanding it is suggested that students engage in many hands on activities that require them to solving problems using improper fractions. Using this approach and encouraging the use of models and pictures, students will begin to get a good idea about what improper fractions and mixed numbers really mean.

Two central concepts students need to develop about improper fractions is that of the purpose of the denominator and the numerator and the fact that the number of parts in a whole may change depending on the situation. Remind students that the denominator names the number of parts in a whole and the numerator names how many parts in all. Similarly, students also need to recognize that the number of parts in a whole can change. Doing activities such as using pattern blocks with the yellow hexagon as being one whole and then changing the whole to be two yellow hexagons, can help students understand this concept.

- When using one hexagon as a whole, the red trapezoid is $\frac{1}{2}$, the blue rhombus is $\frac{1}{3}$, and the green triangle is $\frac{1}{6}$.
- When two yellow hexagons becomes one whole, the red trapezoid, then becomes $\frac{1}{4}$, the blue rhombus becomes $\frac{1}{6}$, and the green triangle becomes $\frac{1}{12}$.

It may be a good idea to do several examples of this using these blocks to help students recognize this difference.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Journal**

- Write the following on the board “All improper fractions must be greater than one whole”. Ask students to explore this statement using models and pictures to agree or disagree with the statement. Ask them to justify or prove their thinking about the statement.

  \[(6N4.1, 6N4.2)\]

**Performance**

- Tell students there are 12 eggs in a carton. Ask them if they had 17 eggs, would \(\frac{17}{12}\) cartons be enough to hold the eggs? \((6N4.1, 6N4.2)\)

- Ask students to use any type of manipulative available to model an improper fraction. Ask students to pass this model to their shoulder partner where their partner will determine if the model represents an improper fraction. Ask each partner to explain how they know it is or is not an improper fraction and use a picture and the symbolic form to explain.

  \[(6N4.1, 6N4.2)\]

- Using snap cubes, show students a model of a whole. (E.g., 5 same coloured snap cubes would equal one whole) Ask students to explore different ways to create an improper fraction using this whole that would come between 1 and 2.

  \[(6N4.1)\]

- Using the numbers 2, 5, 7, 8, ask students to create as many improper fractions as they can. Ask them to choose one improper fraction and represent it using a model, picture and in symbolic form.

  \[(6N4.1, 6N4.2)\]

Resources/Notes

*Math Focus 6*

**Lesson 2: Fractions Greater Than 1**

6N4

TG pp. 18 - 22
Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.2 Translate a given improper fraction between concrete, pictorial and symbolic forms.

Elaborations—Strategies for Learning and Teaching

To help students understand that an improper fraction represents a number greater than one, use a hundreds grid. Presenting the idea that 100 blocks in the grid equals one whole (grid), students can explore ways they can represent more than one whole. In the example below, students are asked to shade in 245 blocks and use their picture to name the improper fraction. Using this type of activity will also help students strengthen their understanding of the purpose of the denominator and numerator.

Once students have had ample opportunity building, creating, modeling, drawing, and naming improper fractions, they will be ready to use symbolic forms to represent the fractions they are working with. Students should be able to easily translate a given improper fraction between various representations such as models, pictures and then in numbers. Ask students to model an improper fraction and then draw this representation. Ask students to then use numbers to name this fraction.

To help students see the relevance of translating their models of improper fractions to pictures and then to a symbolic form ask students to prove that \( \frac{8}{6} \) is less than \( \frac{11}{2} \). To do this ask students to use pattern blocks to build \( \frac{8}{6} \) and then draw this on paper as a part of their written response to the question. Students could then go on to show how their picture of the pattern blocks show that \( \frac{8}{6} \) is less than \( \frac{11}{2} \).
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Ask students to explore the different possibilities of creating an improper fraction with two hexagons being one whole. Challenge them by limiting the number of blocks they can use. For example, if two hexagons equals one whole, ask students to find various ways to represent an improper fraction with 7 blocks. Ask them to record their answers in a chart. Make this a class challenge where students can find as many representations as possible, receiving one point for each representation found and 5 points for each representation that is unique among the students. The winner would be the student with the most points. (6N4.2)

- Provide index cards and ask students to create cards that name an improper fraction and that represent the improper fractions. E.g., students can draw five rhombuses to represent when one hexagon is a whole and then create the corresponding card with the symbolic form of . Students can then combine their completed cards and play a matching game whereby they have to match the picture with the number. (6N4.2)

**Resources/Notes**

*Math Focus 6*
*Lesson 2 (Cont'd): Fractions Greater Than 1*
*6N4*
*TG pp. 18 - 22*
Strand: Number

Outcomes

Students will be expected to

6N4 Continued
Achievement Indicator:

6N4.3 Express improper fractions as mixed numbers.

Elaborations—Strategies for Learning and Teaching

After working with improper fractions, students are formally introduced to mixed numbers. Students need to see the connection between improper fractions and mixed numbers as both represent numbers greater than one whole. They also need to recognize that every improper fraction can be converted to a mixed number and every mixed number can be converted to an improper fraction.

Pattern blocks can be used to help students see these connections. Ask students to model \( \frac{7}{2} \) when one yellow hexagon equals one whole. As they build this model they will see that they have created 3 whole hexagons with one piece or one half of another hexagon left over. Ask students to then think about how they could report what they see. Ask them to think of another way to represent this improper fraction.

Using effective math language while modeling these concepts may become important with regards to student understanding. E.g., if a student was asked to model seven over two, or \( \frac{7}{2} \), he/she may not have a good understanding of what this really means. Compare this to asking a student to model seven halves. As long as a student has a good sense of what a half is, it may be easier to think about seven halves rather than seven over two.

Through working with these numbers, students may discover the relationship in multiplying the denominator with the whole number and adding the numerator to get the improper fraction, but it is not the recommended way to introduce or teach the topic.

“There is absolutely no reason ever to provide a rule about multiplying the whole number by the bottom number and adding the top number. Nor should students need a rule about dividing the bottom number into the top to convert fractions to mixed numbers.” Van de Walle (2006) p. 141.

Providing students with ample opportunities to explore these concepts through the use of hands on activities, using models and pictures will help students develop an understanding but in their own words and in their own way.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students: Susan wrote the improper fraction $\frac{7}{4}$. Ask students to think about what this improper fraction could represent.  
  (6N4.1, 6N4.3, 6N4.2)

- Ask students to think of a mixed number that is a little less than $\frac{9}{4}$. Ask them to show how they know.  
  (6N4.1, 6N4.3, 6N4.6)

- Ask students to make a model to represent $\frac{15}{6}$. Ask them to explain how this number can be expressed as a mixed number using models, picture and numbers.  
  (6N4.3, 6N4.4)

- Ask students the following: If Daniel ate 9 half donuts, how do you know he ate between 4 and 5 full donuts? Use models, pictures and numbers to show your thinking.  
  (6N4.3, 6N4.2 6N4.4)

**Resources/Notes**

*Math Focus 6*

**Lesson 3**: Representing Improper Fractions as Mixed Numbers

6N4

TG pp. 23 - 27

**Additional Reading** (provided):

Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

6N4.3 Continued

Give students a mixed number such as $\frac{2}{5}$. Their task is to find a single fraction that names the same amount. They may use any familiar materials or make drawings, but they must be able to give an explanation for their result. Similarly, ask students to start with a fraction greater than 1, such as $\frac{17}{4}$, and ask them to determine the mixed number and provide a justification for their result. This activity can be repeated using other numbers. Similarly, students may find it easier if they are able to choose the numbers they work with for this task. (Van de Walle, 2006).

Give students 5 green triangles and using the yellow hexagon as a whole. Ask students to write the mixed number that represents the triangles. (The purpose of this activity is to help students realize that in order to write a mixed number, the numerator must be greater than the denominator, or there has to be at least one full whole in the number.)

Provide students with opportunities to use concrete, pictorial and symbolic forms to represent numbers. This helps students become exposed to the number in more than one way, where they are physically working with the number using materials. Drawing a picture to represent the number they are working with helps students solidify the concrete image of the number. The next step in this progression of learning is naming the number using symbols.

Ask students to model a given mixed number, for example 3 and $\frac{2}{5}$ using manipulatives. You may wish to give them a choice of several different mixed numbers depending upon their understanding of mixed numbers. Allow students time to discuss their choice of manipulatives and how their model represents their chosen mixed number. Ask students to then draw a picture to represent this number (it could be a picture of the model they already used, or the number in a different context). Again, ask them to explain how their drawings represent the given mixed number. Ask them to then represent this mixed number in symbolic form (using numbers).

6N4.4 Translate a given mixed number between concrete, pictorial and symbolic forms.

Elaborations—Strategies for Learning and Teaching

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students that a cookie recipe calls for \( \frac{4}{3} \) cups of flour. Mr. Bob is not sure what this means. Ask students to help Mr. Bob by explaining to him what \( \frac{4}{3} \) means and tell him how much flour he needs using a mixed number. (6N4.3)

- Ask students to model and then draw a picture to show that \( \frac{5}{2} = \frac{21}{12} \). (6N4.3, 6N4.5, 6N4.2, 6N4.4)

- Tell students to explain a situation when it would be a good idea to express an improper fraction as a mixed number. (6N4.3)

**Resources/Notes**

*Math Focus 6*

**Lesson 3 (Cont’d): Representing Improper Fractions as Mixed Numbers**

6N4

TG pp. 23 - 27
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

6N4.3 Continued

6N4.4 Continued

Elaborations—Strategies for Learning and Teaching

As students continue to work with improper fractions and mixed numbers, they may begin to use and refine personal strategies they have created to make sense of this concept. Encourage the use of these strategies and provide students with a lot of opportunities to talk about the strategies they are using and ask them to demonstrate these to other students.

Many types of manipulatives can be used to explore improper fractions and mixed numbers. For example, coloured snap cubes can be used to create different lengths where students can compare the lengths of the joined cubes. Students could create a length of 4 blue cubes and another with 8 yellow cubes. Students could then talk about how the lengths of these two compare. So, blue is $\frac{1}{2}$ the length of yellow. If they then create a length of 6 red cubes, they would say that red is $1 \frac{2}{4}$ the length of blue.

Another suggested activity in using the snap cubes would be to provide students with 20 same colored snap cubes. Ask students to represent an improper fraction such as $\frac{17}{5}$. They should understand from previous work that $\frac{17}{5}$ means there are 5 in a whole with 17 parts in all. They should then go on to create towers of 5 snap cubes where they will see that they can create 3 complete towers with 2 cubes left over. This can be used then to help them see that $\frac{17}{5}$ is the same as 3 and $\frac{2}{5}$. 

![Diagram of improper fractions using snap cubes]
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

• Invite students to make a plan to teach their parents about improper fractions and mixed numbers. Ask students to use models, pictures, numbers and words to show their parents how to express a mixed number as an improper fraction.  

(6N4.3, 6N4.5)

• Improper Fraction War - Using a deck of cards containing numerals 1-9, ask students to work in small groups. Shuffle the cards and deal 4 cards to each player. Students can use any two of the 4 cards in their hand to create the greatest possible improper fraction. In turn, each player will reveal their improper fraction determining who has the greatest number. Students may have to convert these improper fractions to mixed numbers to help them compare the numbers. The player with the greatest improper fraction will score one point. The first player with 5 points wins.  

(6N4.3)

• Ask students to create their own riddles about improper fractions and mixed numbers. Ask their classmates to solve the riddles. E.g.,

  I am a unit of time.

  My denominator is 2.

  My numerator is the number of days in one week.

  How many days did Mary work?  

  (7/2 = 3 1/2 days)  

(6N4.3, 6N4.5)

Resources/Notes

Math Focus 6
Lesson 4: Exploring Improper Fractions and Mixed Numbers
6N4
TG pp. 28 – 31

The lesson activity calls for the use of colored rods. If these are not available, colored snap cubes can be used instead.

Children’s Literature (provided):
Merrill, Jean The Toothpaste Millionaire

Please note: The book is written using Imperial units. Please modify to reflect the metric system.
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.5 Express mixed numbers as improper fractions.

Elaborations—Strategies for Learning and Teaching

Ask students to use any manipulative to model or represent $\frac{5}{3}$. Discuss with students their choice of models and lead the discussion to help them see or link the $\frac{5}{3}$ to $1 \frac{2}{3}$. Invite students to explain some of their personal strategies that help them to understand that $\frac{5}{3}$ is the same as $1 \frac{2}{3}$.

Using pattern blocks, have the yellow hexagon represent 1 whole. Give students 7 blue rhombuses and ask them how many complete hexagons they could cover. Ask them to explore and decide what fraction of the hexagon they could cover with the left over blue rhombus. Then ask students to think about how to write the mixed number and the improper fraction that the 7 blue rhombuses represent.

Students have been modeling, creating, describing, drawing and naming improper fractions and mixed numbers. They have been exploring with naming improper fractions as mixed numbers and mixed numbers as improper fractions. Now, students will strengthen their understanding that every improper fraction can be written as a mixed number and every mixed number can be written as an improper fraction. They will learn that when an improper fraction can be expressed as a mixed number these numbers then are equivalent. Equivalent fractions were taught in Grade 5.

The figure below illustrates this idea by showing that the improper fraction $\frac{7}{4}$ and the mixed number $1 \frac{3}{4}$ refer to the same fraction of a region, or area, therefore they are equivalent.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Tell students that it takes 4 – 250 mL cups to fill a 1 L bottle. If they had 10 – 250 mL cups, how many 1 L bottles would they fill? Represent this answer as a mixed number and an improper fraction. (6N4.3, 6N4.5)

- Ask students to choose one mixed number and one improper fraction. Ask them to write these numbers on separate pieces of paper. Next ask students to create a model for each number and draw a picture to represent each. Next, students to switch papers and models with a partner and asks their partner to express the improper fraction as a mixed number and express the mixed number as an improper fraction. Finally, partners model these new numbers using a different type of manipulative and draw their representation of the numbers and show the number in symbolic form. (6N4.2, 6N4.4)

- Ask students to think of possible values for when is an improper fraction that is between 2 and 3. Ask students to determine if there is more than one answer and explain how they know. (6N4.3, 6N4.5, 6N4.6)

- Tell students it takes of an hour to bake one batch of cookies. If they had 5 batches of cookies to bake, how long would it take. Ask them to represent their answer using a mixed number and an improper fraction. (6N4.3, 6N4.5)

**Resources/Notes**

*Math Focus 6*

**Lesson 5: Representing Mixed Numbers as Improper Fractions**

6N4

TG pp. 36 - 40
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.5 Continued

The figure below illustrates another way to show that an improper fraction can be expressed as a mixed number, where both name the same quantity. In this example, it can be seen that there is one full egg carton, or \( \frac{12}{12} \) eggs and a part of another carton, namely \( \frac{5}{12} \). The resulting mixed number then, would be \( 1 \frac{5}{12} \) or the equivalent improper fraction that would name the amount of the set would be \( \frac{17}{12} \).

Using the book *Funny and Fabulous Fraction Stories* ask students to read the section on Improper Fractions and Mixed Numbers found on page 22. Set up this activity as a reader’s theatre. Students take on the roles of the various characters but extend the scenes, whereby, the callers of the radio station need further clarification about the nature of converting mixed numbers to improper fractions. Students can rewrite the script to do this. (It may not be necessary to do the second scene with Joe and Al as this section deals with simplifying the fractions which is not a requirement for Grade 6). Once Joe and the caller is established, give the remaining students the role of the Math Experts in which Joe calls upon to help out the callers “Betty” and “Pauline”. In the scenes Betty and Pauline call Joe Trella, the Fraction Fella, to get advice on a problem they have involving converting mixed number and improper fractions. The Math Experts have to come up with a plan on how to show Betty and Pauline how to convert these numbers. Encourage students to use different ways of teaching Betty and Pauline as they may need to see how it can be done in different ways. These scenes should be done separately with Joe and the callers being different people allowing all students to be involved in the lesson. You may also decide that Joe could have a role where he is showing Betty a way to convert and then have the others demonstrate their thinking to coincide with Joe’s lesson. Alternatively or as a follow up, other types of situations can be created where students role play the scenarios demonstrating their understanding of a concept.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

**Performance**

- Mixed Number War - Using a deck of cards containing numerals 1-9, ask students to work in small groups. Shuffle the cards and deal 3 cards to each player. Students use the cards to create the greatest possible mixed number. In turn, each player will reveal their mixed number. The player with the greatest mixed number will score one point. The first player with 5 points wins.  
  
  (6N4.5)

- Place a mixed number/improper fraction on students’ back. Instruct the class to question the student about the number to determine what it is. Limit the number of questions the student can ask. This could also be done as a class activity where all students have improper fractions/mixed numbers on their back where they have to go around the class asking questions to get clues about what their number is.  
  
  (6N4.3, 6N4.5)

- Tell students that you have 18 quarters in a coin collection. How many dollars would this be? Can you model the amount as an improper fraction? Mixed number? Which one would be easier to understand how much money you have altogether?  
  
  (6N4.3, 6N4.5)

- Mario and Sydney are great hockey players. To prepare for the season, Mario practiced \(3 \frac{3}{4}\) weeks. Sydney practiced \(25 \frac{5}{7}\) weeks. Who practiced more? Explain your thinking using pictures, numbers and words.  
  
  (6N4.3, 6N4.5)

**Resources/Notes**

*Math Focus 6*

Lesson 5 (Cont’d): Representing Mixed Numbers as Improper Fractions

6N4

TG pp. 36 - 40

*Children’s Literature (provided):*

Greenberg, Dan. *Funny and Fabulous Fraction Stories*
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicator:

6N4.5 Continued

Elaborations—Strategies for Learning and Teaching

Give students a mixed number such as \( 2 \frac{1}{3} \). Using pattern blocks, ask students to use the blue rhombi to create a design on triangular dot paper that represents this mixed number. Ask them to use their design to explain how this mixed number also represents the equivalent improper fraction, where they would write the improper fraction in symbolic form.

This is just one example of how the blue rhombi can be used to model \( 2 \frac{1}{3} \). Encourage students to be creative in their designs.

The picture below shows another example of how to use pattern blocks, namely the green triangles, to represent an improper fraction on triangular dot paper. Ask students to represent \( \frac{13}{6} \) and then explain how this also shows \( 2 \frac{1}{6} \).
General Outcome: Develop Number Sense

Suggested Assessment Strategies

<table>
<thead>
<tr>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Ask students to choose a mixed number and tell them to keep it a secret from their classmates. Set up centers around the room where students could model their number in one center, draw it in another and then represent it using an improper fraction in yet another center. After everyone has had an opportunity to represent their number in the three centers, bring the class together. Ask them to match all the models to the corresponding pictures and numbers.</td>
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<tr>
<td>(6N4.2)</td>
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<table>
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<td>Lesson 5 (Cont’d): Representing Mixed Numbers as Improper Fractions</td>
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<tr>
<td>6N4</td>
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<tr>
<td>TG pp. 36 - 40</td>
</tr>
</tbody>
</table>
Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

- **6N4.6 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.**

Elaborations—Strategies for Learning and Teaching

Students have been working with mixed numbers and improper fractions by representing, modelling, naming and expressing them in different forms. As students continue working with these numbers they can begin thinking about strategies that would enable them to compare improper fractions and mixed numbers. In Grade 5, students worked with comparing proper fractions with like and unlike denominators which will now help them compare mixed numbers and allow them to extend their personal strategies to compare improper fractions.

When comparing improper fractions, encourage students to recognize that it may be easier to express the improper fraction as a mixed number where they would compare the whole number first and then look at the proper fraction if needed. For example, when comparing \( \frac{6}{4} \) and \( \frac{9}{5} \), students could express both as a mixed number namely, \( 1 \frac{2}{4} \) and \( 1 \frac{4}{5} \). Here they could easily see that they are both 1 whole with the first having an extra \( \frac{2}{4} \) or \( \frac{1}{2} \) and the second having an extra \( \frac{4}{5} \). Students should be able to see that \( \frac{2}{4} \) is greater than \( \frac{1}{2} \) because \( \frac{4}{5} \) is greater than \( \frac{1}{2} \).

Using a number line when solving problems is another strategy that students can use to help show their understanding. As students encounter problems that require them to compare mixed numbers and improper fractions, a logical representation of the understanding would be to place the given numbers on a number line.

Fractional Clothesline - Stretch string across the classroom with various points marked for 0, 1, 2, 3, and 4. You may want to ask students to do this. The clothesline will be used to show students that all proper fractions are between 0 and 1 and all mixed numbers and improper fractions are greater than 1. Students will clip index cards with various proper fractions, improper fractions and mixed numbers on the clothesline. You may wish to ask students to place various benchmarks and ask them to choose the numbers they want to place. For example, you may wish to ask them to write any mixed number or improper fraction that would come between 1 and 2, or 3 and 5 that could go on the number line. After each student has had a chance to place some numbers on the line, have a class discussion to decide if all the placed numbers are in relative positions. You may even want to give the class a chance to change some already placed numbers before the discussion takes place.

(continued)
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students: Bill said he ate $1 \frac{1}{3}$ bags of popcorn. Ed said he ate $\frac{4}{3}$ bags of popcorn. Assuming each bag of popcorn is the same size, is it possible for Bill to have eaten more popcorn than Ed? Explain your thinking with pictures, numbers and words.
  
  (6N4.3, 6N4.5, 6N4.6)

- Ask students to determine which number is greater – $\frac{26}{5}$ or $4 \frac{3}{4}$. Ask students to explain using pictures, numbers and words how they know.
  
  (6N4.6)

- Tell students: 4 friends were at a party. Joe stated he ate $\frac{5}{3}$ of pizza while Amy stated she ate $\frac{5}{4}$ of pizza. Larry said that Amy ate more pizza than Joe. Is Larry correct? Explain your thinking.
  
  (6N4.1, 6N4.6)

- Ask students why someone would think that $\frac{5}{4}$ is greater than $\frac{3}{2}$? Use pictures, numbers and words to explain.
  
  (6N4.1, 6N4.3, 6N4.5, 6N4.6)

- On the number line below, place the following: $\frac{3}{2}$, $3 \frac{1}{2}$, $\frac{5}{4}$, $\frac{15}{4}$, $1 \frac{2}{5}$.
  
  Choose one to explain its placement.
  
  (6N4.6)

- Ask students how they would immediately know that $2 \frac{2}{3}$ is greater than $1 \frac{7}{8}$? Explain.
  
  (6N4.6)

- Ask students to choose two improper fractions or two mixed numbers, order them and compare them. Ask them to explain to a friend how they know they have ordered their numbers correctly.
  
  (6N4.1, 6N4.3, 6N4.5, 6N4.6)

- Tell students two athletes were competing in a biathlon. Athlete one finished the race in $\frac{12}{5}$ hours and the other athlete finished in $\frac{12}{10}$ hours. Ask students to determine who won.
  
  (6N4.6)

Resources/Notes

Math Focus 6
Lesson 6: Comparing Fractions and Mixed Numbers
6N4
TG pp. 41 – 45

Math Game: Spinner Fractions
6N4
TG pp. 46 - 47

Additional Reading (provided):
Strand: Number

Outcomes

Students will be expected to

6N4 Continued

Achievement Indicators:

6N4.3 Continued

6N4.5 Continued

6N4.6 Continued

Elaborations—Strategies for Learning and Teaching

As students continue working with improper fractions and mixed numbers, they can use this understanding to help them solve meaningful problems using logical thinking. Using logical reasoning is yet one strategy for solving problems students have developed. Engaging students in problems that allow them to synthesize and analyze the information presented, having them plan out their solutions and then represent their thinking using pictures, numbers and words will help strengthen their problem solving process.

Another strategy that may help students solve problems is that of solving a simpler problem that is related to a given problem. For example, if a problem deals with both improper fractions and mixed numbers, ask students to solve the problem in which only improper fractions are used. Similarly, students could also change the numbers in the problem (if that is what is causing difficulty) to numbers they are more comfortable with and note how they would go about solving the problem using simpler numbers and how this could be transferred to solving the problem with the given numbers.

Encourage students to pick out the most important parts of the problem, draw or use a model to help them visualize the given information and identify exactly what the problem is asking. Students should be given many opportunities to work together while solving problems and talk out their thinking with their partner, as it is through this collaboration that students learn best.

Students may find it interesting to explore how the fraction name changes if the whole changes. If the blue rhombus is the whole, using pattern blocks, for example, the green triangle would be $\frac{1}{2}$ and the red trapezoid would be $1 \frac{1}{2}$ or $\frac{3}{2}$. Playing with these different scenarios will help students better understand the relationship between the improper fractions and mixed numbers.
General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

• Tell students there are 24 cans of soft drink in one case. 60 cans of drink are needed for the volleyball tournament. How could knowing about mixed numbers and improper fractions help you determine the number of cases of drinks that are needed for the tournament? (Remind students that a part of a case of drink can be bought) (6N4.1, 6N4.3)

• Ask students to write two improper fractions and two mixed numbers that are between 4 and 5. Ask them to explain their thinking. (6N4.6)

• Pose the following to students: You just baked one dozen brownies. Seven of your friends just arrived and you want to share the brownies equally among all of you. Show how you would do this and write the amount you all get using a mixed number and an improper fraction. (6N4.3, 6N4.5)

• Tell students that the answer to a problem is $2 \frac{1}{3}$. Ask them what the question might be? (6N4.6)

Student-Teacher Dialogue

• Ask students: Which of the following improper fractions and mixed numbers are between 2 and 3? Explain. $\frac{7}{5}, 2 \frac{1}{5}, 3 \frac{1}{2}, \frac{7}{4}$ (6N4.6)

Resources/Notes

Math Focus 6
Lesson 7: Solving Problems Using Logical Reasoning
6N4
TG pp. 48 – 50

Curious Math: Growing Shapes
6N4
TG pp. 51 – 52